first law:

$$dE = -p\,dV$$ in a closed systems $\leftrightarrow$ total universe

$$\Rightarrow \boxed{dS = 0}$$ Entropy is conserved during expansion!

$$\Rightarrow S = \frac{\rho + p}{T} \cdot a^3 = \text{const.}$$

$$\Rightarrow s = \frac{S}{V} = \frac{S}{a^3} = \frac{\rho + p}{T} \sim \frac{1}{a^3}$$

for relativistic species thus:

$$p_R = \frac{1}{3} \rho_R$$

$$\Rightarrow s = \frac{\rho_R + p_R}{T} = \frac{4}{3} \cdot \frac{\rho_R}{T}$$

$$= \frac{2\pi^2}{45} \cdot g_{*s} \cdot T^3$$

$$g_{*s} = \sum_{i=\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{j=\text{fermions}} g_j \left(\frac{T_j}{T}\right)^3$$

note:

$$s \sim \frac{1}{a^3} \Rightarrow \boxed{N = V \cdot n = \frac{n}{s}}$$ particle number per comoving volume

and:

$$S = s a^3 = \text{const.} \Rightarrow \boxed{T \sim \frac{1}{a}}$$

Freeze out of heavy particles and decoupling

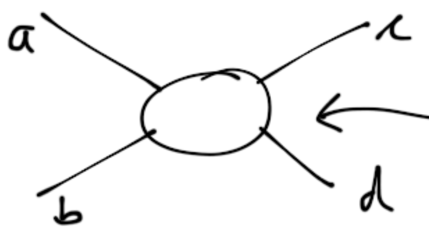
→ 'freeze-out' = decoupling from the thermal bath

↝ need to know reaction rate Γ:

$$\Gamma_a = n_a \cdot \sigma \cdot v$$

particle speed $\simeq c$

production of a-particles



cross section σ

2 cases – rule of thumb:

interaction via massless gauge boson (like γ):

$$\text{QFT} \Rightarrow \quad \sigma \sim \frac{\alpha^2}{T^2}$$

α: gauge coupling

or via massive gauge boson X with mass m_X:

$$\text{QFT} \Rightarrow \text{ for } T < m_X$$

$$\sigma \sim G_X^2 T^2$$

$G_X \sim \frac{\alpha}{m_X^2}$ gauge coupling for relativistic species:

$$n \sim g T^3$$

$$\Rightarrow$$

$$\Rightarrow \Gamma \sim \begin{cases} g \cdot \alpha^2 T & \text{massless gauge boson} \\ g \cdot \dfrac{\alpha^2}{m_X^4} T^5 & \text{massive gauge boson} \end{cases}$$

how many particles are produced?

$$N = \int_t^\infty \Gamma(t')\,dt' \quad , \quad \Gamma \sim T^n$$

$$(t|T): t \sim T^{-2} \Rightarrow dt \sim \frac{dT}{T^3}$$

$$\Rightarrow N \sim \frac{1}{n-2} T^{n-2} \sim \frac{1}{n-2} \frac{\Gamma}{H}$$

$\Rightarrow$ production stops once

$$\boxed{\Gamma < H}$$

'<u>freeze-out condition</u>'

<u>example</u>: decoupling of neutrinos

$\rightsquigarrow$ interact electro-weakly:

$$\Gamma \sim G_F^2 T^5$$

$\leftarrow$ Fermi constant of weak interaction

and $H \sim \dfrac{T^2}{M_P}$

freeze-out:

$$1 = \frac{\Gamma}{H} \sim M_p G_F^2 T^3$$

$M_p \sim 2 \cdot 10^{18}$ GeV

$G_F \sim (200\ \mathrm{GeV})^{-2}$

$$\sim \frac{10^9}{\mathrm{GeV}^3} \cdot T^3 = \left(\frac{T}{\mathrm{MeV}}\right)^3$$

$\Rightarrow$ ν's decouple at
$T \simeq 1$ MeV.

can use this to calculate neutrino temperature today ...

$\downarrow$

after ν's freeze out, at

$T \lesssim m_e = 0.5$ MeV < 1 MeV

the e^-, e^+ annihilate

thus, before $t_{\mathrm{annih.}}$

$$g_{*s} = \underbrace{1 \cdot 2}_{\gamma} + \frac{7}{8} \cdot \underbrace{2 \cdot 2}_{e^+, e^-} + \nu\text{'s}$$

$$= \frac{11}{2}$$

and <u>after</u> $t_{annih.}$:

$$g_{*s} = \underset{\gamma}{2} + \nu's$$

but entropy <u>conserved</u>:

$$S_{t<t_{annih}} = g_{*s}^{before} \cdot (T_\nu a)^3 + S_\nu(T_\nu)$$

$T_\nu = T_\gamma$ still

$$= \frac{11}{2} \cdot (T_\nu a)^3 + S_\nu(T_\nu)$$

$$\stackrel{!}{=} S_{t>t_{annih.}} = g_{*s}^{after} (T_\gamma a)^3 + S_\nu(T_\nu)$$

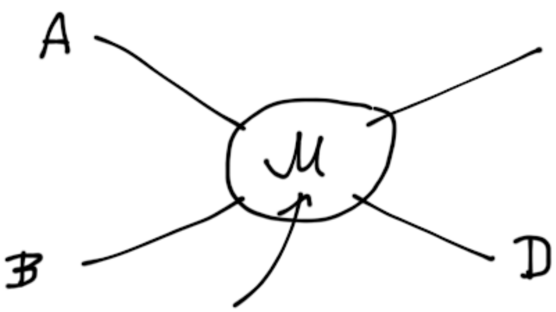
$$= 2\,(T_\gamma a)^3 + S_\nu(T_\nu)$$

$$\Rightarrow T_\nu = \left(\frac{4}{11}\right)^{1/3} \cdot T_\gamma \simeq 1.91 \text{ K}$$

today

description of freeze-out

reaction



QFT : matrix element $\mathcal{M}$ of reaction

$A + B \to C + D$

semi-classically:

$$\frac{d\#}{dt} = \Gamma_{\text{production}} - \Gamma_{\text{annihil./decay}}$$

for e.g. particle type A in more detail:

$$\#(A) = N_A = n_A V \sim n_A a^3$$

and QFT

$$\Rightarrow \frac{1}{a^3}\frac{d}{dt}\left(n_A a^3\right) = \int \frac{d^3 p_A}{(2\pi)^3 E_A} \int \frac{d^3 p_B}{(2\pi)^3 E_B} \int \frac{d^3 p_C}{(2\pi)^3 E_C} \int \frac{d^3 p_D}{(2\pi)^3 E_D} \qquad (B)$$

$$\times (2\pi)^4 \cdot \delta^{(3)}\left(\vec{p}_A + \vec{p}_B - \vec{p}_C - \vec{p}_D\right) \cdot \delta\left(E_A + E_B - E_C - E_D\right)$$

$$\times \underset{\text{Matrix element}}{|\mathcal{M}|^2} \cdot \Big\{ \underbrace{f_C f_D \cdot (1 \pm f_A)(1 \pm f_B)}_{\text{production of } A} - \underbrace{f_A f_B (1 \pm f_C)(1 \pm f_D)}_{\text{annihilation/decay of } A} \Big\}$$

$+$: boson
$-$: fermion

$f_i(E)$, $i = A, B, C, D$
partition function of particle species i

(B) is the Boltzmann equation.

$\downarrow$

look at two examples ...

i) freeze-out of thermally produced weakly interacting dark matter

Why dark matter (DM)?

- galaxy rotation curves:

$$\frac{m v^2}{r} \sim G \frac{Mm}{r^2} \Rightarrow v^2(r) \sim \frac{1}{r}$$

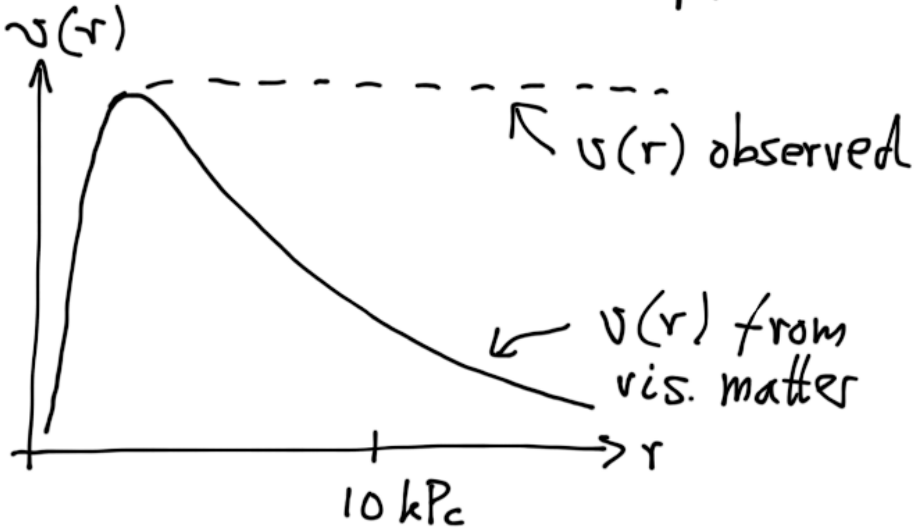
$\rightsquigarrow$ observation:

$$v(r) \simeq \text{const.}$$

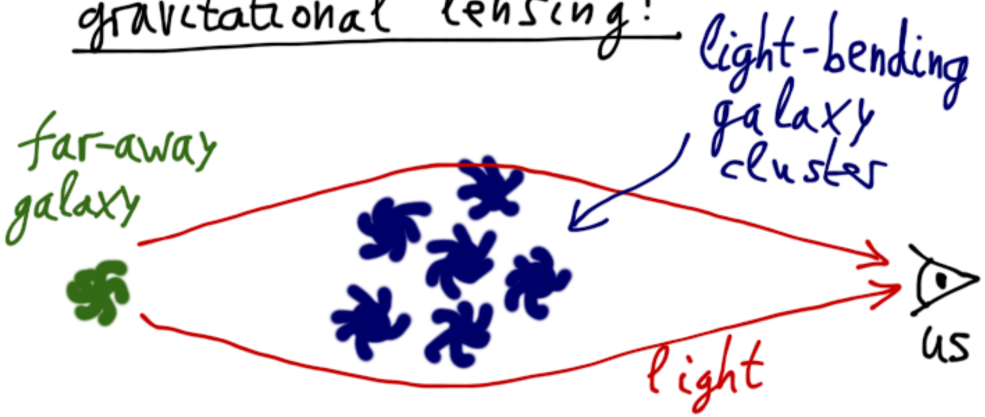
$\rightsquigarrow$ need $M(r) \sim r$ to compensate:
invisible/dark ...

galaxy rotation curves:

visible matter: $M(r) \sim \frac{1}{r^{\#}}, \; \# > 0$



gravitational lensing:



- X-ray observations of gas bound in galaxies:

  requires some extra dark gravitating stuff to keep gas inside the galaxy

- WMAP – CMB + BBN:

  BBN fixes $\eta_B \Rightarrow \Omega_B \simeq 0.04$

  WMAP : $\Omega_\Lambda \simeq 0.7$

  $\Omega_0 \simeq 1$

  $\Rightarrow \Omega_{DM} \simeq 0.3$

- gravitational lensing

  $\rightsquigarrow \Omega_m \simeq 0.3$

- structure formation requires some 'extra' gravitational wells: $\Omega_m \simeq 0.3$

$\Rightarrow$ There are many possibilities for DM.

Will focus on WIMP:

new "Weakly Interacting Massive Particle" beyond the SM

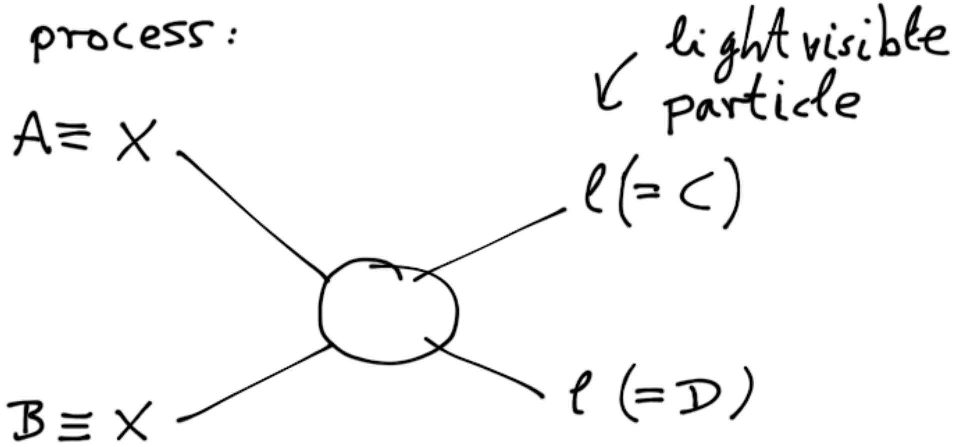
$\rightarrow$ has weak-scale interactions

$\rightarrow$ heavy

$\rightsquigarrow$ well-motivated e.g. from SUSY.

<u>single constituent DM:</u>

process:



X : heavy DM WIMP

the Boltzmann eq. is again (B).

big picture:

DM particles X start at high T in equilibrium. If they stayed so always, then:

$$n_X \sim e^{-\frac{m_X}{T}} \quad \text{for } T < m_X$$

but, if $\Gamma_X < H$, DM density freezes out & survives, if it lives long enough: