$$H = \frac{\dot a}{a}, \quad \rho \geq 0, \quad |\gamma| \leq 1$$

$$\rightsquigarrow \frac{\dot\rho}{\rho} = -3(\gamma+1)\cdot\frac{\dot a}{a}$$

$$\Rightarrow \ln\frac{\rho}{\rho_0} = -3(\gamma+1)\cdot \ln a$$

demand : $a_0 = a(1) = 1$

$$\Rightarrow \boxed{\rho = \rho_0 \cdot a^{-3(\gamma+1)}}$$

$$\Rightarrow \dot a^2 = \frac{\rho_0}{3}\cdot a^{-3(\gamma+1)+2} = \frac{\rho_0}{3}\cdot a^{-3\gamma-1}$$

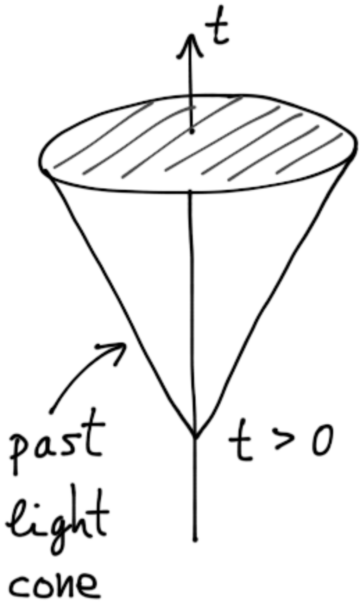
$$\Leftrightarrow a^{\frac{1}{2}(3\gamma+1)} da = \sqrt{\frac{\rho_0}{3}}\, dt$$

$$\Leftrightarrow \frac{2}{3(\gamma+1)}\left[a(t)^{\frac{3(\gamma+1)}{2}} - a(1)^{\frac{3(\gamma+1)}{2}}\right] = \sqrt{\frac{\rho_0}{3}}(t-1)$$

choose : $\rho_0 = \dfrac{4}{3(\gamma+1)^2} = \mathcal{O}(1)$

$$\Rightarrow \boxed{a(t) = t^{\frac{2}{3(\gamma+1)}}}$$

particle horizon



$$L_H(t) = a(t) \cdot \int_0^t \frac{dt'}{a(t')} = a(t) \cdot r_H(t)$$

$$H = \frac{\dot{a}}{a} = \frac{2}{3(\gamma+1)} \cdot \frac{1}{t}$$

$$L_H = \frac{2}{3\gamma+1} \cdot H^{-1} = \frac{3(\gamma+1)}{3\gamma+1} t$$

$$L_H(t_p) = L_H(1) = \mathcal{O}(1)$$

suppose entropy bound fulfilled at Planck time: $\left.\frac{S}{A}\right|_{t_p} \equiv \sigma \lesssim 1$ (ignore $\mathcal{O}(1)$ factors)

later times:

$\sim$ comoving entropy density σ constant:

$$\Rightarrow \quad S \sim \sigma \cdot r_H^3(t) = \sigma \cdot \frac{L_H^3}{a^3}$$

$$\Rightarrow \quad \frac{S}{A_{\text{particle horizon}}} \simeq \sigma \cdot \frac{L_H^3}{a^3} \cdot \frac{1}{L_H^2} \sim \sigma \cdot \underbrace{\frac{3(\gamma+1)}{3\gamma+1}}_{=\mathcal{O}(1)} \cdot t^{1-\frac{2}{\gamma+1}} \sim \sigma \cdot t^{\frac{\gamma-1}{\gamma+1}}$$

$1 \geq \gamma \geq -1$: S/A does not increase with time (HH = Holographic Hypothesis OK)

<u>(ii) closed FLRW universe</u>

$$ds^2 = -dt^2 + a^2(t) \cdot \left(d\chi^2 + \sin^2\chi \cdot d\Omega_2^2\right)$$

$$\chi \in [0, \pi]$$

comoving horizon:

$$\chi_H(t) = \int_0^t \frac{dt'}{a(t')}$$

causal sphere (boundary area):

$$A(t) = 4\pi \cdot a^2(t) \cdot \sin^2 \chi_H(t)$$

comoving volume of causal sphere:

$$V_H(t) = \int_0^{\chi_H} \frac{d\chi}{\sin\chi}\, d\Omega$$

$$= \pi \cdot \left(2\chi_H - \sin 2\chi_H\right)$$

For constant comoving entropy density σ:

$$\frac{S(t)}{A(t)} = \frac{\sigma}{4a^2(t)} \cdot \frac{2\chi_H - \sin 2\chi_H}{\sin^2 \chi_H}$$

consider for simplicity cold matter:

$$p \ll \rho$$

then:

$$a \simeq a_{max} \cdot \sin^2 \frac{\chi_H}{2}$$

$\rightsquigarrow$ close to $\chi_H = \pi$

$A \simeq 0$, HH obviously violated!

(horizon shrinks to zero!)

(iii) flat universe with $\rho_\Lambda < 0$ (AdS)

$$\rho = \rho_0 \cdot a^{-3(\gamma+1)} - |\rho_\Lambda|$$

$|\rho_\Lambda| \ll 1$ (our universe)

$$\rightsquigarrow \dot{a} = \pm \frac{1}{\sqrt{3}} \left(\rho_0 \cdot a^{-3\gamma-1} - |\rho_\Lambda| \cdot a^2 \right)^{1/2}$$

turning point: $\dot{a} = 0$

$$\Leftrightarrow \rho_0 = |\rho_\Lambda| \cdot a^{3(\gamma+1)}$$

later: $\dot{a} < 0$, universe collapses

at turning point:

$$1 \geq \gamma > -1, \qquad \frac{S}{A} \simeq \sigma \cdot |\rho_\Lambda|^{\frac{1}{2} \cdot \frac{1-\gamma}{1+\gamma}}$$

at later times:

$$\frac{S}{A} \sim \frac{\sigma}{a^3} \cdot |\rho_\Lambda|^{-\frac{3\gamma+1}{6(\gamma+1)}}$$

↑ $\mathcal{O}(1)$ at late times

$$\frac{3\gamma+1}{6(\gamma+1)} > 0 \quad \text{for } \forall \gamma > -\frac{1}{3}$$

but then $\frac{S}{A} \gg 1$

again HH violated!

$\rightsquigarrow$ <u>question:</u> are closed & AdS universes excluded?