choose E_γ such that:

$$E_{matter} + E_\gamma = M_{BH}(A)$$

i.e. $A = 4\pi \cdot r_s^2$, $r_s = 2G \cdot M$

$$S_{matter} \leq S_{matter} + S_\gamma = S_{BH} = \frac{A}{4G}$$

<u>Conclusion:</u> entropy of system contained in Γ <u>always</u> <u>smaller</u> than entropy of black hole with $A_{BH} = A(\partial\Gamma)$.

Question: Is all information about the system in the bulk Γ stored on the surface $\partial\Gamma$?

(cf. holography: 3d image from interference pattern on 2d screen)

$\rightarrow$ $\boxed{\text{Holographic Hypothesis}}$

Holographic Hypothesis
(or principle: t'Hooft, Susskind, Witten):
macroscopic region of space Γ with everything inside it can be represented by boundary theory on $\partial\Gamma$; boundary theory should not contain more than 1 d.o.f. per Planck area.

$$\Rightarrow \boxed{N_S \leq e^{\frac{A}{4G}}}$$

\# of states of system

<u>example:</u> type IIB string theory on $S^5 \times AdS_5 \triangleq (3+1)$-dim $\mathcal{N}=4$ super-YM theory on boundary of AdS_5

(AdS/CFT-correspondence, Maldacena '98)

$\rightsquigarrow$ more general validity?

estimate entropy directly from thermal bath of scalar field quanta in dS:

$$T_{dS} = \frac{H}{2\pi}$$

comoving entropy density s_ϕ is constant:

$$s_\phi \sim T_{dS}^3$$

volume integral over event horizon volume:

$$\int dV \sim \int_0^{H^{-1}} R^2\, dR$$

$\rightsquigarrow$ $S = \int s_\phi \, dV \sim V_{EH}$

or $\sim A_{EH}$?

put stationary observer at distance δR from your dS event horizon distance H^{-1} – detector is kept at fixed distance $R = H^{-1} - \delta R$ <u>against</u> free fall accelerated expansion, thus sees large acceleration, and thermal bath at temperature :

$$T(R) = T_{dS} \cdot z(R)$$

<u>enhanced</u> by redshift $z(R)$ relative to you.

$\longleftrightarrow$ radiation with temperature T at $R = H^{-1} - \delta R$ reaches you, sitting just δR from the event horizon of the observer there, redshifted to temperature:

$$\frac{T(R)}{z(R)} = T_{dS}$$

$\rightsquigarrow$ need to calculate:

$$S = 4\pi^2 \int_0^{H^{-1}} s_\phi(T(R)) \cdot R^2 dR$$

~ determine physical distance traveled by radiation emitted at $R = H^{-1} - \delta R$ from you, and compare with redshift $z(R)$ there:

$$R = a(\delta) \int_{\delta}^{t} \frac{dt'}{a(t')} = L_{EH}(t,\delta)$$

$$= e^{H\delta}\left(-\frac{1}{H}\right)\cdot\left(e^{-Ht} - e^{-H\delta}\right)$$

$$= \frac{1}{H}\cdot\left(1 - e^{H\cdot(\delta - t)}\right)$$

$$t \gg \delta \to -\infty$$

$$\Rightarrow \delta R(\delta) = L_{EH}(t, \delta \to -\infty) - R$$

$$= \frac{1}{H} - \frac{1}{H}\left(1 - e^{H\cdot\delta}\right), \delta \ll t$$

$$= \frac{1}{H}\cdot e^{H\cdot\delta}$$

redshift $z(R)$ at $R = H^{-1} - \delta R$:

$$z + 1 = \frac{a(t)}{a(\delta)} = e^{H(t-\delta)} = e^{-H\cdot\delta}, \delta \ll t$$

$$= \frac{1}{H\cdot\delta R}$$

$$\Rightarrow T(\delta R) = T_{dS} \cdot z(\delta R)$$

$$= \frac{T_{dS}}{H \cdot \delta R} = \frac{H/2\pi}{H \cdot \delta R} = \frac{1}{2\pi \cdot \delta R}$$

$$\Rightarrow S \sim \int s \, d\mathcal{V}$$

$$\sim \int_0^{H^{-1}} T^3(R) \, 4\pi R^2 dR$$

$$= \frac{1}{2\pi^2} \cdot \int_0^{H^{-1}} \frac{R^2 dR}{(\underbrace{H^{-1} - R}_{= \delta R})^3}$$

$$\simeq \frac{1}{2\pi^2} \cdot \int_0^{H^{-1} - \epsilon} \frac{R^2 dR}{(H^{-1} - R)^3}$$

$$\simeq \frac{1}{2\pi^2} \cdot \left. \frac{(H^{-1})^2 \cdot \delta R}{\delta R^3} \right|_{\delta R = \epsilon}$$

$$= \frac{1}{2\pi^2} \cdot \frac{H^{-2}}{\epsilon^2} = \frac{1}{(2\pi)^3} \cdot \frac{4\pi H^{-2}}{\epsilon^2}$$

$$= \frac{1}{2\pi^3} \cdot \frac{A_{dS-EH}}{\epsilon^2}$$

$\rightsquigarrow$ <u>Holography rules!</u>

Cosmology & Holography

naive estimates: entropy of our universe

$$S_{\mathcal{V}} \sim s_\gamma \mathcal{V} \sim n_\gamma \mathcal{V} \sim \frac{400}{cm^3} \cdot H^{-3}$$

$$\sim 400 \cdot \left(10^{28}\right)^3$$

$$\sim 10^{87}$$

$$S_{\text{horizon}} \sim \left(\frac{1}{H \cdot \ell_p}\right)^2 \sim 10^{2(28+33)}$$

$$\sim 10^{122}$$

$\Rightarrow$ $S_{\text{horizon}} > S_{\mathcal{V}}$ OK

but: which horizon? constraints on the future?

(i) flat FLRW universe

$$ds^2 = -dt^2 + a^2(t) \cdot \left(dr^2 + r^2 d\Omega_2^2\right)$$

$$T_{\mu\nu} = \text{diag}\,(\rho, p, p, p) \quad , \quad p = \gamma \cdot \rho$$

energy density

pressure

Friedmann eq.: $H^2 = \frac{1}{3}\rho \quad (M_p = 1)$

$$D^\mu T_{\mu\nu} = 0 \; : \; \dot{\rho} + 3H(\rho + p) = 0$$