Anti-de Sitter space (AdS_d)

like de Sitter space hyperboloid, defined by embedding into $(d+1)$-dim flat space:

$$-X_0^2 + \sum_{i=1}^{d-1} X_i^2 - X_d^2 = \underbrace{-\ell^2}_{\text{negative curvature}}$$

$$ds^2 = -dX_0^2 - dX_d^2 + \sum_{i=1}^{d-1} dX_i^2$$

by construction: isometry $SO(2, d-1)$, maximal symmetry, homogeneous + isotropic

$$\left(G_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & 1 & \\ & & & \ddots \; 1 \end{pmatrix} \right)$$

coordinates:

$$X_0 = \ell \cdot \cosh\left(\frac{\rho}{\ell}\right) \cdot \cos\left(\frac{\tau}{\ell}\right)$$

$$X_i = \ell \cdot \sinh\left(\frac{\rho}{\ell}\right) \cdot \omega_i \;, \quad \sum_{i=1}^{d-1} \omega_i^2 = 1$$

$$X_d = \ell \cdot \cosh\left(\frac{\rho}{\ell}\right) \cdot \sin\left(\frac{\tau}{\ell}\right)$$

$$\rho \geq 0 \;, \quad 0 \leq \frac{\tau}{\ell} < 2\pi$$

metric on AdS_d:

$$ds^2 = -\cosh^2\left(\frac{\rho}{\ell}\right) d\tau^2 + d\rho^2 + \ell^2 \sinh^2\left(\frac{\rho}{\ell}\right) d\Omega_{d-2}^2$$

coordinates periodic in τ; closed time-like curves?

$\rightarrow$ causal space-time by changing

$S^1 \rightarrow \mathbb{R}$ (covering space)

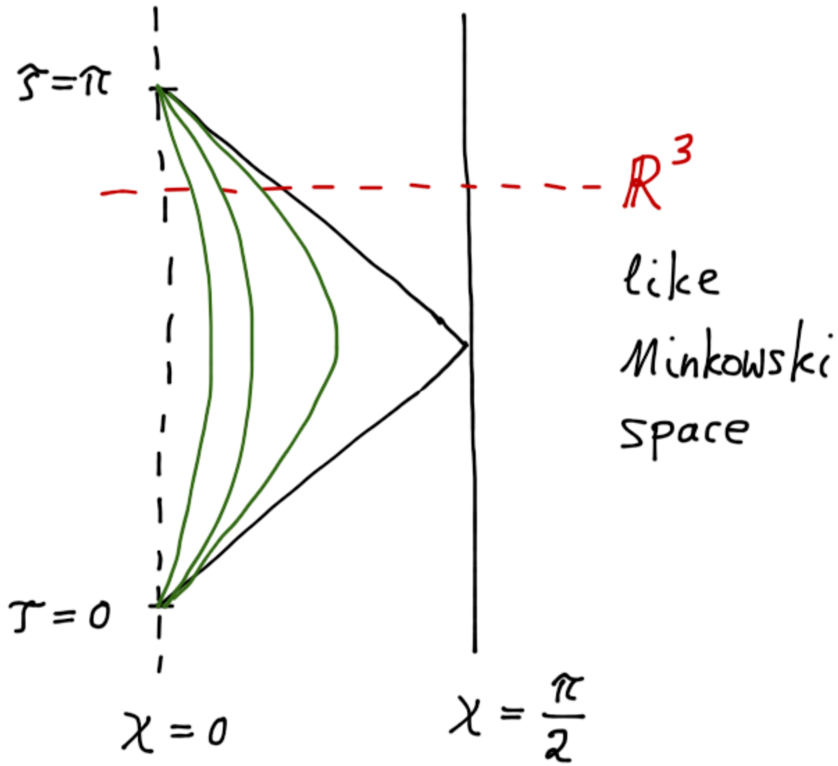
$\rightsquigarrow$ same metric!

conformal coordinates (cf. de Sitter):

$$\cosh\left(\frac{\rho}{\ell}\right) = \frac{1}{\cos\chi} \quad , \quad 0 \leq \chi \leq \frac{\pi}{2}$$

$$\rightsquigarrow ds^2 = \frac{1}{\cos^2\chi}\left[-d\tau^2 + \ell^2 \cdot \left(d\chi^2 + \sin^2\chi \cdot d\Omega^2_{d-2}\right)\right]$$

conformal diagram:



II.4 Horizons & Holography

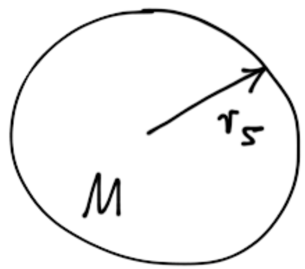
some literature:

- Susskind & Lindesay '05:
  "[...] The holographic universe"
- Susskind & Witten '98:
  "The holographic bound in AdS space"
  hep-th/9805114
- Kaloper & Linde '99:
  "Cosmology vs. Holography"
  hep-th/9904120

black hole entropy

for outside observer BH of mass M has temperature:

$$T = \frac{1}{4\pi \cdot r_S} , \quad r_S = 2GM$$



1st law of thermodynamics implies entropy:

$$dE = T \cdot dS$$ (energy of infalling matter, <u>no</u> work done on BH)

$$\curvearrowright \quad dE = dM = \frac{1}{8\pi G M}\, dS\,, \quad S\big|_{M=0} = 0$$

$$\Leftrightarrow \quad S_{BH} = 4\pi G M^2 = \frac{4\pi \cdot r_s^2}{4G} = \frac{A_{horizon}}{4G}$$

horizon area

Bekenstein-Hawking entropy of BH

Bekenstein-Hawking entropy

$= \frac{1}{4}$ area of event horizon in Planck units

Planck length : $\ell_P = \sqrt{\frac{G\hbar}{c^3}}\,, \quad G = \frac{\ell_P^2 c^3}{\hbar}$

$$\curvearrowright \quad \boxed{S_{BH} = \frac{\hbar}{4c^3} \cdot \frac{A_{horizon}}{\ell_P^2}}$$

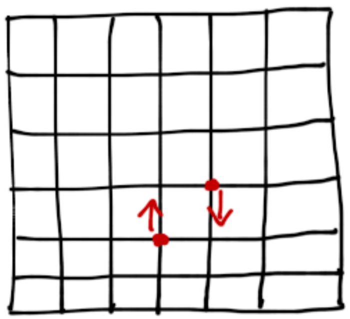
($k = 1$ Boltzmann const.)

$\ell_P \simeq 10^{-33}$ cm

quantum effect $(O(\hbar))$!!

Maximum entropy

system of spins in volume V;



$\}\,a$

V

a: lattice spacing

number of states:

$$N_{tot} = 2^{\frac{V}{a^3}}$$

entropy of macroscopic state with statistical weight ΔP

$$\left(\text{e.g. } \frac{\Delta p \, \Delta q}{2\pi\hbar}\right)$$

(# of corresponding quantum states):

$$S = \ln \Delta P$$

no knowledge about system:

$$\Delta P = N_{tot}$$

maximum entropy of spin system:

$$S = \frac{V}{a^3} \cdot \ln 2$$

proportional to volume, extensive quantity.

effect of gravity