(unknown to de Sitter ($\rightarrow$ Weinberg))

Minkowski space

$$ds^2 = -dt^2 + dr^2 + r^2 \cdot d\Omega^2$$

$$-\infty < t < \infty, \quad 0 \leq r < \infty$$

transformation to conformal coordinates:

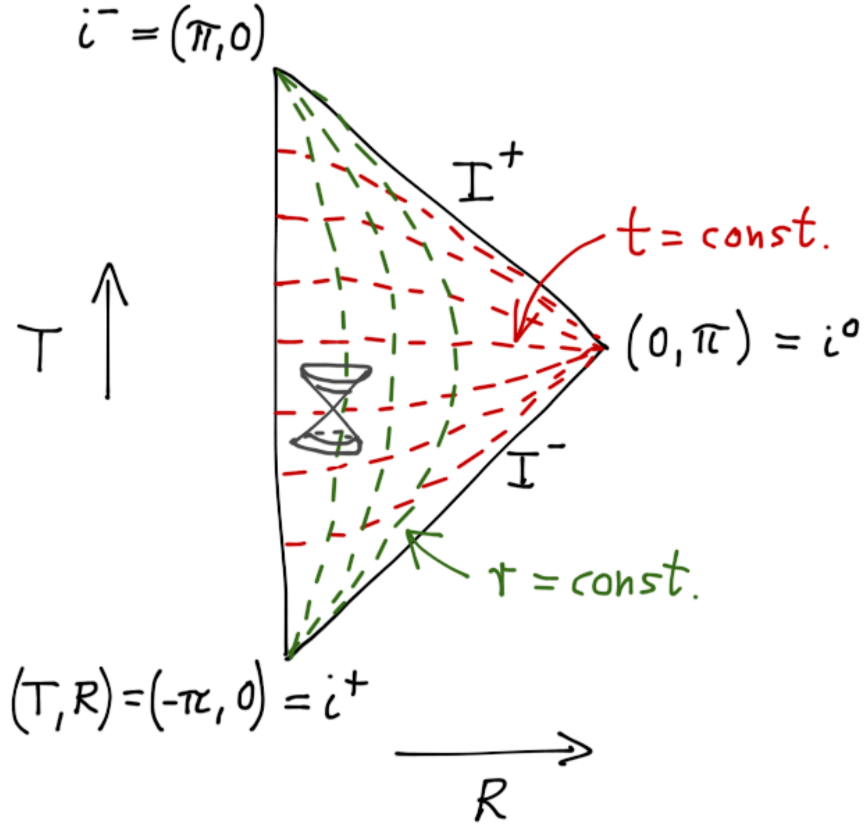
$$t = \frac{1}{2}\left(\tan\frac{T-R}{2} + \tan\frac{T+R}{2}\right)$$

$$r = \frac{1}{2}\left(\tan\frac{T+R}{2} - \tan\frac{T-R}{2}\right)$$

$$\Rightarrow \begin{cases} 0 \leq R \leq \pi \\ -(\pi - R) \leq T \leq \pi - R \end{cases}$$

$$\Downarrow$$

$$\boxed{ds^2 = (\cos T + \cos R)^{-2} \cdot \left(-dT^2 + dR^2 + \sin^2 R \cdot d\Omega^2\right)}$$



boundaries:

i^0 : $T=0, R=\pi$: $t=0,\ r\to\infty$

spacelike infinity

i^+ : $T=-\pi, R=0$: $t\to-\infty,\ r=0$

i^- : $T=\pi, R=0$: $t\to\infty,\ r=0$

timelike infinities

$\mathcal{I}^+$: $T=\pi-R$: $t+r\to\infty,\ t-r<\infty$

future null infinity

$\mathcal{I}^-$: $T=-\pi+R$: $t-r\to-\infty,\ t+r<\infty$

past null infinity

Temperature of de Sitter space

- Consider quantized scalar field $\phi(x)$ in dS, with vacuum state $|0\rangle$;
- 'Unruh' detector A on a worldline of observer can detect ϕ-quanta via interaction:

$$S_I = g\int_{-\infty}^{\infty} d\tau\, A(\tau)\,\phi(x(\tau))$$

- detector states:

$$|E_i\rangle\ ,\quad H|E_i\rangle = E_i|E_i\rangle$$

detection process:

$$|E_i\rangle|0\rangle \longrightarrow |E_j\rangle|\beta\rangle$$

transition amplitude:

$$\mathcal{M}_{\beta j;i} = g\int_{-\infty}^{\infty} d\tau \, \langle E_j|\langle\beta|A(\tau)\,\phi(x(\tau))|0\rangle|E_i\rangle$$

$$A(\tau) = e^{iH\tau}\cdot A(0)\cdot e^{-iH\tau}$$

(Heisenberg picture)

$$= g\int_{-\infty}^{\infty} d\tau \cdot e^{i(E_j-E_i)\tau}\cdot \mathcal{M}_{ji}\,\langle\beta|\phi(x(\tau))|0\rangle$$

$$\mathcal{M}_{ji} = \langle E_j|A(0)|E_i\rangle$$

$$\Downarrow$$

$$\sum_\beta |\mathcal{M}_{\beta j;i}|^2 = g^2|\mathcal{M}_{ji}|^2 \cdot \int_{-\infty}^{\infty} d\tau\, d\tau'\, e^{-i(E_j-E_i)(\tau'-\tau)}$$

$$\times \underbrace{\sum_\beta \langle 0|\phi(x(\tau'))\underbrace{|\beta\rangle\langle\beta|}_{=\mathbb{1}}\phi(x(\tau))|0\rangle}_{\longrightarrow\, G(x(\tau'),\, x(\tau))}$$

can only depend on <u>geodesic</u> <u>distance</u>:

Use embedding $D(x',x) = C_{\mu\nu}(x-x')^\mu (x-x')^\nu$

$$C_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{pmatrix}$$

consider observer at rest in expanding universe, e.g. North pole, $u_i = 0$; then

$$x_0 - x_0' = \ell \cdot \sinh\left(\frac{t}{\ell}\right) - \ell \cdot \sinh\left(\frac{t'}{\ell}\right)$$

$$x_i = x_i' = 0 \,, \quad i = 1 \dots d-1$$

$$x_d - x_d' = \ell \cdot \cosh\left(\frac{t}{\ell}\right) - \ell \cdot \cosh\left(\frac{t'}{\ell}\right)$$

$$ds^2 = -dt^2 = d\tau^2$$

$\Downarrow$

$$D(x,x') = -\left(\sinh\frac{\tau}{\ell} - \sinh\frac{\tau'}{\ell}\right)^2 + \left(\cosh\frac{\tau}{\ell} - \cosh\frac{\tau'}{\ell}\right)^2 = \cosh\left(\frac{\tau-\tau'}{\ell}\right)$$

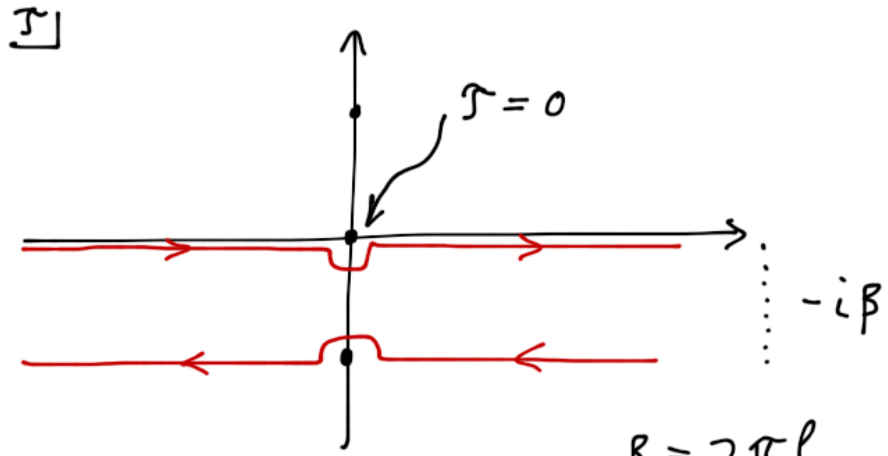
$$\rightsquigarrow \; G(x(\tau'), x(\tau)) = \hat{G}\left(\cosh\left(\frac{\tau-\tau'}{\ell}\right)\right)$$

$\longrightarrow$ transition probability per unit of proper time:

$$P(E_i \to E_j) = g^2 |M_{ji}|^2 \cdot \int_{-\infty}^{\infty} d\tau \, e^{-i(E_j - E_i)\tau} \hat{G}\left(\cosh\frac{\tau}{\ell}\right)$$

$\hat{G}$ invariant under $\tau \to \tau + 2\pi i \cdot \ell$!

$\hat{G}(x,x)$ is singular $\rightsquigarrow$ $\hat{G}\left(\cosh\frac{\tau}{\ell}\right)\Big|_{\tau = 2\pi i \ell}$ is singular



$\beta = 2\pi\ell$

$$\int_{-\infty}^{\infty} d\tau\, e^{-i(E_j - E_i)\tau}\, \hat{G}\left(\cosh\frac{\tau}{\ell}\right) = \int_{+\infty - i\beta}^{-\infty - i\beta} d\tau\, e^{-i(E_j - E_i)\tau}\, \hat{G}\left(\cosh\frac{\tau}{\ell}\right)$$

$$\tau' = -\tau - i\beta$$

$$= e^{-\beta(E_j - E_i)} \int_{-\infty}^{\infty} d\tau'\, e^{-i(E_i - E_j)\tau'}\, \hat{G}\left(\cosh\frac{\tau'}{\ell}\right)$$

$$\Rightarrow P(E_i \to E_j) = e^{-\beta(E_j - E_i)}\, P(E_j \to E_i)$$

If energy levels of detector populated thermally with temperature $T = \beta^{-1}$:

$$N_i = N \cdot e^{-\beta \cdot E_i}$$

then reaction rates must be equal to be in thermal equilibrium:

$$\begin{aligned} R(E_i \to E_j) &= N_i\, P(E_i \to E_j) \\ &= N \cdot e^{-\beta E_i}\, P(E_i \to E_j) \\ &= e^{\beta(E_j - E_i)} N_j\, e^{-\beta(E_j - E_i)}\, P(E_j \to E_i) \\ &= N_j\, P(E_j \to E_i) = R(E_j \to E_i) \end{aligned}$$

principle of detailed balance in thermal ensemble!

$\curvearrowleft$ reaction are equal, and thus system is in equilibrium exactly when levels are thermally populated with:

$$N_i = N \cdot e^{-\beta E_i}$$

$\Rightarrow$ $\beta = T^{-1}$ indeed has meaning of an inverse temperature.

Hence, geodesic observer sees a thermal bath of ϕ-quanta with temperature:

$$T = \frac{1}{\beta} = \frac{1}{2\pi \ell}$$

our universe: $\ell \sim 10^{28}\,\mathrm{cm}$

$$\Rightarrow T \sim 10^{-34}\,\mathrm{eV}$$

for comparison: Hawking radiation of a black hole

M

outside observer sees thermal radiation with temperature:

$$T = \frac{\kappa}{2\pi}, \quad \kappa = \frac{1}{4G\cdot M} = \frac{1}{2r_s}$$

Does de Sitter temperature modify the equation of state of dark energy?

$\rightsquigarrow$ Yes, but negligibly small!

scale factor in dS:

$$a(t) = e^{Ht}$$

$$H^2 = \frac{1}{\ell^2} = \frac{1}{3M_p^2}\rho_\Lambda, \quad M_p^2 = \frac{1}{8\pi G} \simeq (10^{18}\,\mathrm{GeV})^2$$

$$\rho_\Lambda \simeq (10^{-3}\,\mathrm{eV})^4$$

$$p = -\rho_\Lambda = -\Lambda$$

$\rightsquigarrow$ $H \sim 10^{-33}\,\mathrm{eV}$, $\ell \sim 10^{28}\,\mathrm{cm}$

temperature correction:

$$\rho_R = \frac{\pi^2}{30} g_* \cdot T^4 , \quad \rho_\gamma \sim T^4 , \quad P_\gamma = \frac{1}{3} \rho_\gamma$$

$$\rho_{tot} = \rho_\Lambda + \left(\frac{1}{2\pi \ell}\right)^4 \simeq \rho_\Lambda + \frac{1}{9(2\pi)^4} \frac{\rho_\Lambda^2}{M_P^4}$$

$$P_{tot} = - \rho_\Lambda + \frac{1}{3} \frac{1}{16\pi^4} \frac{1}{9 M_P^4} \rho_\Lambda^2$$

$$\Rightarrow \boxed{W = -1 + \frac{1}{108\pi^4} \cdot \frac{\rho_\Lambda}{M_P^4}}$$

$$\Rightarrow \text{today:} \quad W + 1 \simeq \underline{\underline{10^{-124}}} \; !$$