conformal coordinates (T, θ_i):

$$\cosh\left(\frac{\tau}{\ell}\right) = \frac{1}{\cos\left(\frac{T}{\ell}\right)}\,, \quad -\frac{\pi}{2} \leq T \leq \frac{\pi}{2}$$

finite interval!

$$\left(\tau : -\infty \ldots 0 \ldots +\infty\,,\ \cosh\left(\frac{\tau}{\ell}\right) : \infty \ldots 1 \ldots \infty\right)$$

$$\Downarrow$$

$$\frac{1}{\ell}\sinh\left(\frac{\tau}{\ell}\right) d\tau = \frac{1}{\ell}\,\frac{1}{\cos^2\left(\frac{T}{\ell}\right)}\sin\left(\frac{T}{\ell}\right) dT$$

$$\Downarrow$$

$$\sinh^2\left(\frac{\tau}{\ell}\right) d\tau^2 = \left[\cosh^2\left(\frac{\tau}{\ell}\right) - 1\right] d\tau^2$$

$$= \frac{1-\cos^2\left(\frac{T}{\ell}\right)}{\cos^4\left(\frac{T}{\ell}\right)}\, dT^2$$

$$= \frac{1}{\cos^4\left(\frac{T}{\ell}\right)} \cdot \frac{\cosh^2\left(\frac{\tau}{\ell}\right) - 1}{\cosh^2\left(\frac{\tau}{\ell}\right)}\, dT^2$$

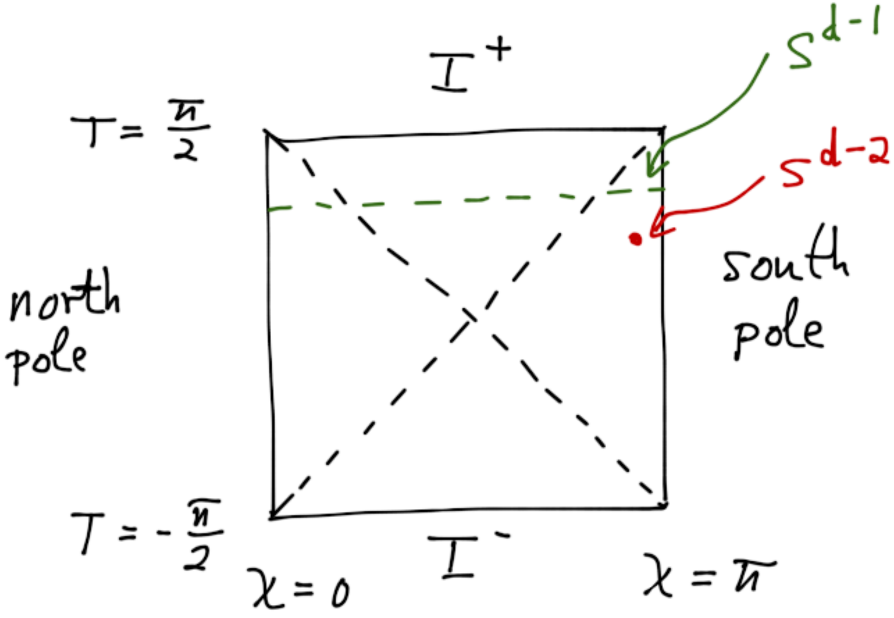
$$\Longleftrightarrow \quad d\tau^2 = \frac{1}{\cos^2\left(\frac{T}{\ell}\right)}\, dT^2$$

$$\Rightarrow \quad ds^2 = \frac{1}{\cos^2\left(\frac{T}{\ell}\right)} \cdot \underbrace{\left(-dT^2 + \ell^2 \cdot d\Omega^2_{d-1}\right)}_{\text{flat Minkowski metric } (T, \omega_i)}$$

light rays: $ds^2 = 0$, light cone $90°$!

($\mathcal{J}^+$, "scri-plus")

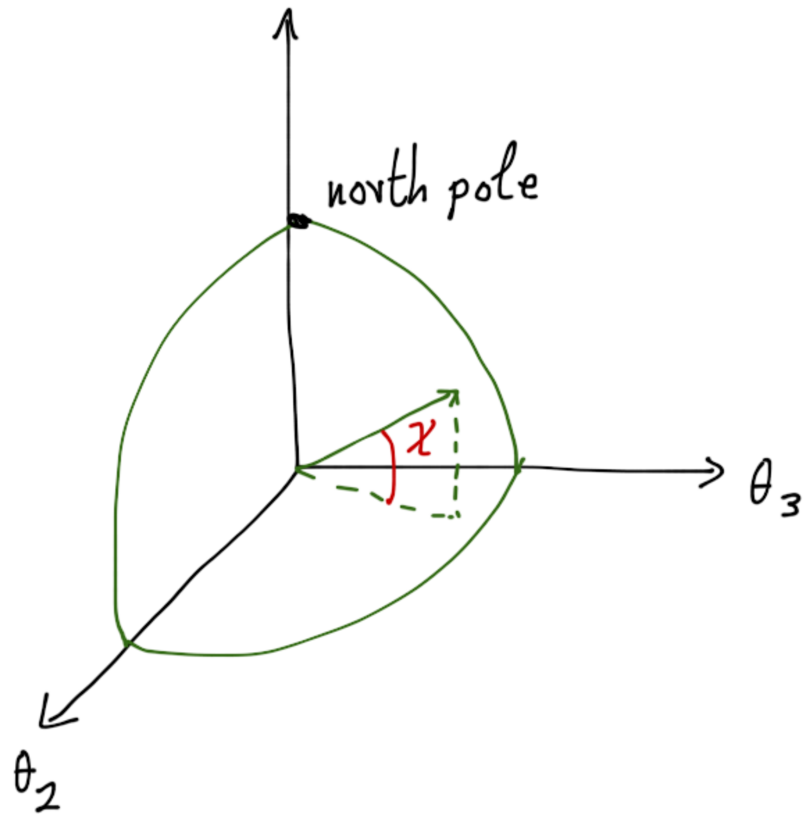
Penrose (conformal) diagram



$\theta_1 = \chi \quad , \ell = 1$

$$d\Omega_{d-1}^2 = d\chi^2 + \sin^2\chi \cdot d\Omega_{d-2}^2$$

$\theta_2, \theta_3, \ldots = \text{const.}$



sphere of dS_2, χ angle covers whole manifold

$\mathscr{I}^+$: future null infinity (all light rays end here)

$\mathscr{I}^-$: past null infinity (all light rays originate here)

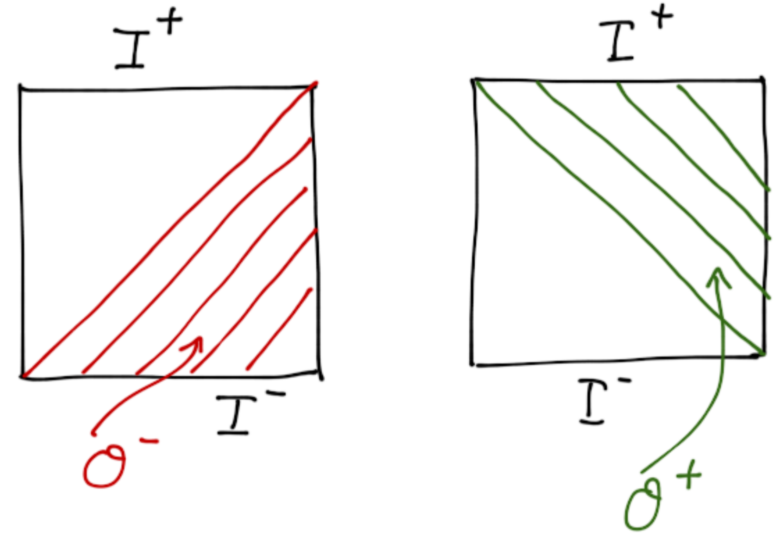
light rays = null geodesics

<u>causal structure</u>

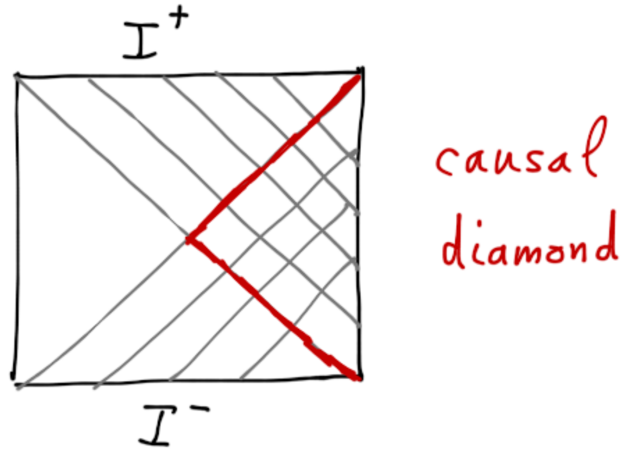
consider observer at south pole;

causal past: $\mathcal{O}^-$

accessible future: $\mathcal{O}^+$



region with possible interaction: $\mathcal{O}^- \cap \mathcal{O}^+$ (question + answer)

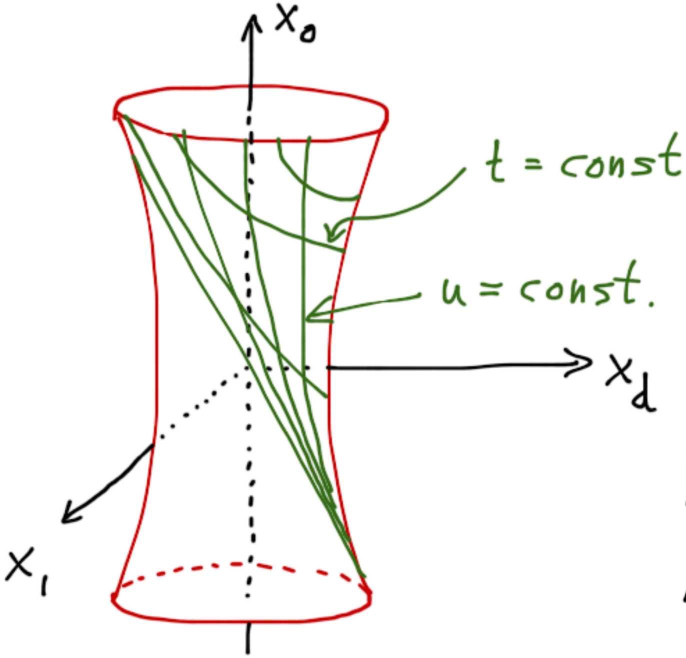


Planar coordinates:

$$x_0 = \ell \cdot \sinh\left(\frac{t}{\ell}\right) + \frac{1}{2}\ell^{-1} \cdot e^{t/\ell} \cdot \vec{u}^{\,2}$$

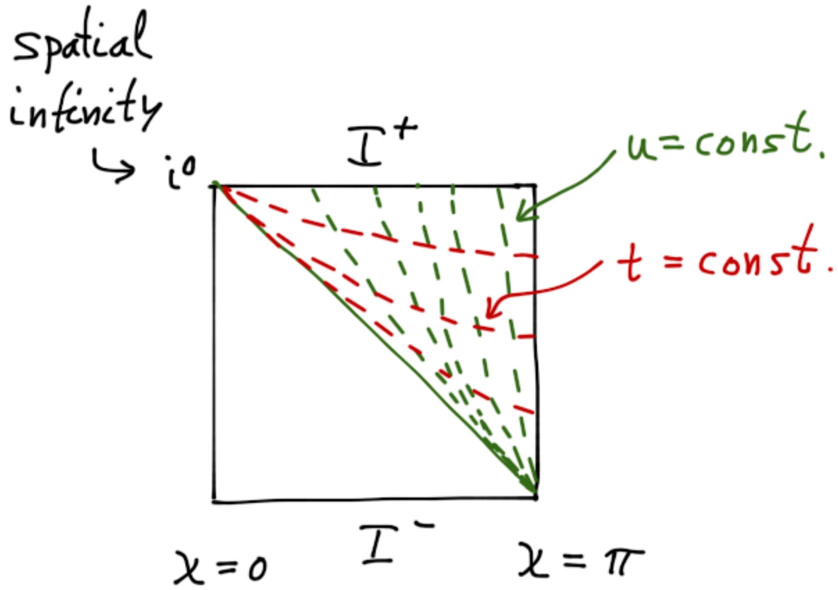
$$x_i = e^{t/\ell} \cdot u_i \quad , \quad i = 1 \dots d-1$$

$$x_d = \ell \cosh\left(\frac{t}{\ell}\right) - \frac{1}{2}\ell^{-1} \cdot e^{t/\ell} \cdot \vec{u}^{\,2}$$



$x_0 + x_d > 0$

half of hyperboloid



domain which can be influenced from the south pole

$$ds^2 = -dt^2 + e^{2t/\ell} \cdot d\vec{u}^{\,2}$$

flat Friedmann universe with Hubble parameter $H = \frac{1}{\ell}$

$t = \text{const.}$: infinite-volume $d-1$ planes with flat Euclidean metric

<u>static coordinates</u>: (t, r, θ_i) $i = 1 \ldots d-1$

$$x_0 = \sqrt{\ell^2 - r^2} \cdot \sinh\left(\frac{t}{\ell}\right)$$

$$x_i = u_i \quad , \quad i = 1 \ldots d-1 \quad , \quad \vec{u}^{\,2} = r^2$$

$$x_d = \sqrt{\ell^2 - r^2} \cdot \cosh\left(\frac{t}{\ell}\right)$$

$x_0 + x_d > 0$, i.e., same part of hyperboloid as for planar coordinates

metric (originally proposed by de Sitter):

$$ds^2 = -\left(1 - \frac{r^2}{\ell^2}\right) dt^2 + \left(1 - \frac{r^2}{\ell^2}\right)^{-1} \cdot dr^2 + r^2 d\Omega^2_{d-2}$$

with: $r < \ell$

embedding time-dependent, but metric time-independent!

compare to <u>Schwarzschild metric</u>:

$$ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \left(1-\frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2_{d-2}$$

with: $r > r_s$

region outside black hole,

r_s: Schwarzschild radius, <u>horizon</u>

static dS_d-metric: 'inside black hole'

<u>event horizon</u> (events which can <u>be</u> <u>influenced</u> by observer)

$$d_e(t) = a(t)\int_t^\infty \frac{dt'}{a(t')} = \ell, \quad a(t) = e^{t/\ell}$$

[horizon: $d_H(t) = a(t)\cdot r_H$, $dr = \frac{dt}{a(t)}$]

<u>but:</u> particle horizon (events which can <u>influence</u> observer)

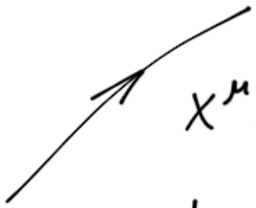
$$d_p(t) = a(t)\cdot\int_{t_i}^{t} \frac{dt'}{a(t')} = \ell\cdot\left(e^{\frac{t-t_i}{\ell}} - 1\right) \xrightarrow[t_i \to -\infty]{} +\infty$$

[$ds^2 = 0$, $dt = a(t)\,dr$]

Does the static patch on dS_d describe a static universe?

$\rightsquigarrow$ No!

Consider freely falling observer (geodesic):



$x^\mu(\lambda)$, $\lambda = f(\tau)$

$$d\tau = \left(-g_{\mu\nu}\, dx^\mu dx^\nu\right)^{1/2}$$

proper time

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu{}_{\nu\lambda} \frac{dx^\nu}{d\tau} \cdot \frac{dx^\lambda}{d\tau} = 0$$

'static universe': for time-independent metric, coordinates are comoving, i.e. const.: $\dfrac{dx^i}{d\tau} = 0$

check:

$$\Gamma^\mu{}_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma} \left(\partial_\nu g_{\sigma\lambda} + \partial_\lambda g_{\sigma\nu} - \partial_\sigma g_{\nu\lambda}\right)$$

$$\Gamma^r{}_{00} = \frac{1}{2} g^{rr} \cdot \left(-\partial_r g_{00}\right) = -\frac{1}{2}\left(1 - \frac{r^2}{\ell^2}\right) \cdot \frac{2r}{\ell^2} \neq 0$$

i.e. static coord.s are <u>not</u> comoving coordinates!

(unknown to de Sitter ($\to$ Weinberg))