Problems for Cosmology

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1. Using the scale factor – red shift relation one can derive an expression for cosmological time as a function of redshift for arbitrary present-day composition of the energy density of the universe. Compute the age of the present universe from this relation as a function of the cosmological density parameters of the different matter and energy sources in cosmological energy budget.

   The result simplifies if one imposes a flat universe and thus dominant composition $\Omega_\Lambda + \Omega_{matter} = 1$. Calculate the age of the universe for $\Omega_\Lambda = 0.7$, and for $\Omega_\Lambda = 0$, respectively.

   Try to sketch the age of the universe as a function of Ω_Λ for this flat case, and compare with the age of oldest known stars $t_{star} \simeq 1 \times 10^{10}\,yr$. Which problem did the advent of non-zero Ω_Λ solve?

2. The non-dark (baryonic) matter consists almost completely of protons and neutrons with an average number density today of $n_{B,0} \simeq 0.23\,m^{-3}$. Calculate the energy density parameter of the CMB photons Ω_γ and their number density n_γ using the CMB temperature $T_{CMB} \simeq 2.73\,K$. Which value has the 'baryon asymmetry' $\eta_B \equiv n_B/n_\gamma$?

3. At photon decoupling (CMB generation) all involved particles e^- , p , H are non-relativistic. The ionization fraction is defined as $X_e \equiv n_p/n_B$ with $n_B = n_p + n_H$ the total baryon number (H atoms and protons). Charge neutrality $n_e = n_p$ holds always, and the ionization reaction $H + \gamma \to e + p$ implies $\mu_H = \mu_e + \mu_p$ for the chemical potentials in chemical equilibrium. Using the expressions for the number densities n_e, n_p, n_H, and the CMB photon number density n_γ as a function of temperature, and rewriting n_B via the baryon asymmetry as $n_B = \eta_B n_\gamma$, derive an equation for the ionization fraction X_e as a function of temperature. This Saha equation reads

   $$\frac{1 - X_e}{X_e^2} = \frac{8\zeta(3)}{\sqrt{2\pi}} \eta_B \left(\frac{T}{m_e}\right)^{3/2} e^{\frac{B_H}{T}}$$

   where $B_H = m_e + m_p - m_H = 13.6\,eV$ is the ionization energy of hydrogen, and $g_e = g_p = 2$, $g_H = 4$ denote the species degrees of freedom of electrons, protons and hydrogen atoms, respectively. Here the expression for n_γ as a function of temperature has been used.

Finally, using that the temperature T at redshift z is given by $T = T_0(1+z)$ with $T_0 = 2.73\,K$, and that the baryon asymmetry

$$\eta_B = \Omega_B \left(\frac{H_0}{100\,km\,s^{-1}\,MPc^{-1}} \right)^2 2.7 \times 10^{-8}$$

compute X_e as a function of redshift z for $\Omega_B = 0.04$ and $\Omega_B = 0.4$.