# IV Inflation



## recall of the horizon problem:

light propagates as $ds^2 = 0$

$\Rightarrow \; dr = \frac{dt}{a(t)}$ can see $d = a_0 \int_0^{t_0} \frac{dt'}{a(t')} \sim t_0 \sim \frac{1}{H_0}$

"Hubble radius"

today:

$\sim 4 \cdot 10^{10}$ ly, CMB

matter or radiation

CMB : photon decoupling at

$3000\,K \simeq 0.3\,eV \rightarrow z = 1100$

$\rightarrow 3K$ today

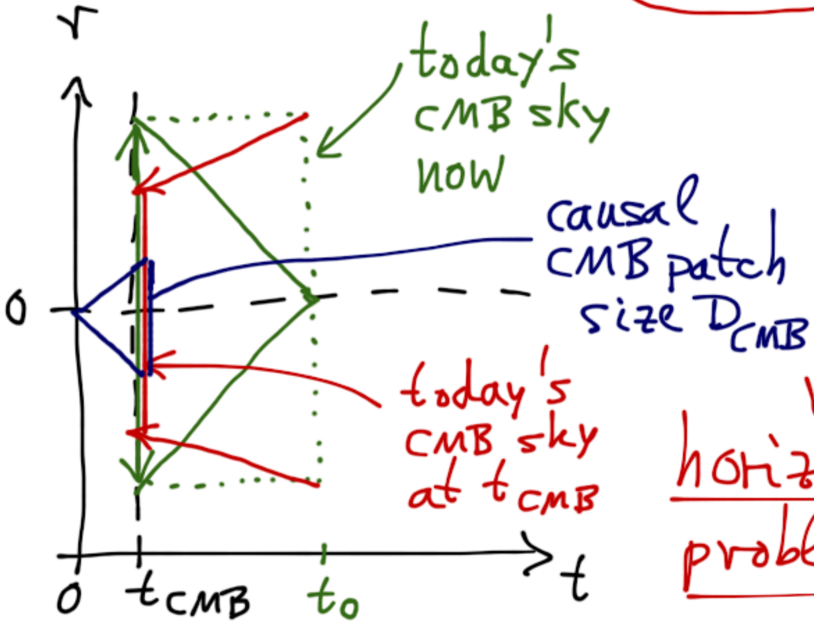
$\boxed{t_{CMB} \simeq 400.000\ yr}$

to-day's CMB sky was smaller at $t_{CMB}$ by:

$$\frac{a(t_{CMB})}{a_0} = \left(\frac{t_{CMB}}{t_0}\right)^{2/3} = \frac{1}{1+z} \simeq 10^{-3}$$

matterdomination since recombination

so $4 \cdot 10^7\, ly$ then. But causal patch at $t_{CMB}$ only $\boxed{D_{CMB} \simeq 4 \cdot 10^5\, ly}$.




**horizon problem!**



<u>another problem:</u> spatial flatness

<u>recall:</u> $\Omega_k \sim \frac{k}{a^2}$, $\Omega_m \sim \frac{1}{a^3}$, $\Omega_{rad.} \sim \frac{1}{a^4}$

$\Rightarrow \Omega_k / \Omega_{rad.} \sim a^2$ <u>grows!</u>

<u>today:</u> $\langle E_{CMB} \rangle \simeq T_{CMB} \simeq 3K \simeq 10^{-3} eV$

<u>at nucleosynthesis:</u> $T_{NUC} \sim 10^6 eV$

<u>after quantum gravity epoch:</u> $T_P \sim M_P \sim 10^{27} eV$

<u>we have:</u> $T \sim \frac{1}{a}$

<u>and:</u> $|\Omega_k(\text{today})| \lesssim 10^{-2}$

$$\Rightarrow \begin{cases} |\Omega_k(T_{NUC})| \lesssim 10^{-18} \\ |\Omega_k(T_P)| \lesssim 10^{-60} \end{cases}$$

↯ **HOW?**

an idea: try getting $\ddot{a} > 0$

$\Rightarrow$ $a(t)$ grows faster than $t$

$\Rightarrow$ today's CMB sky was smaller than causal patch at some early time

solves horizon problem

$\Downarrow$

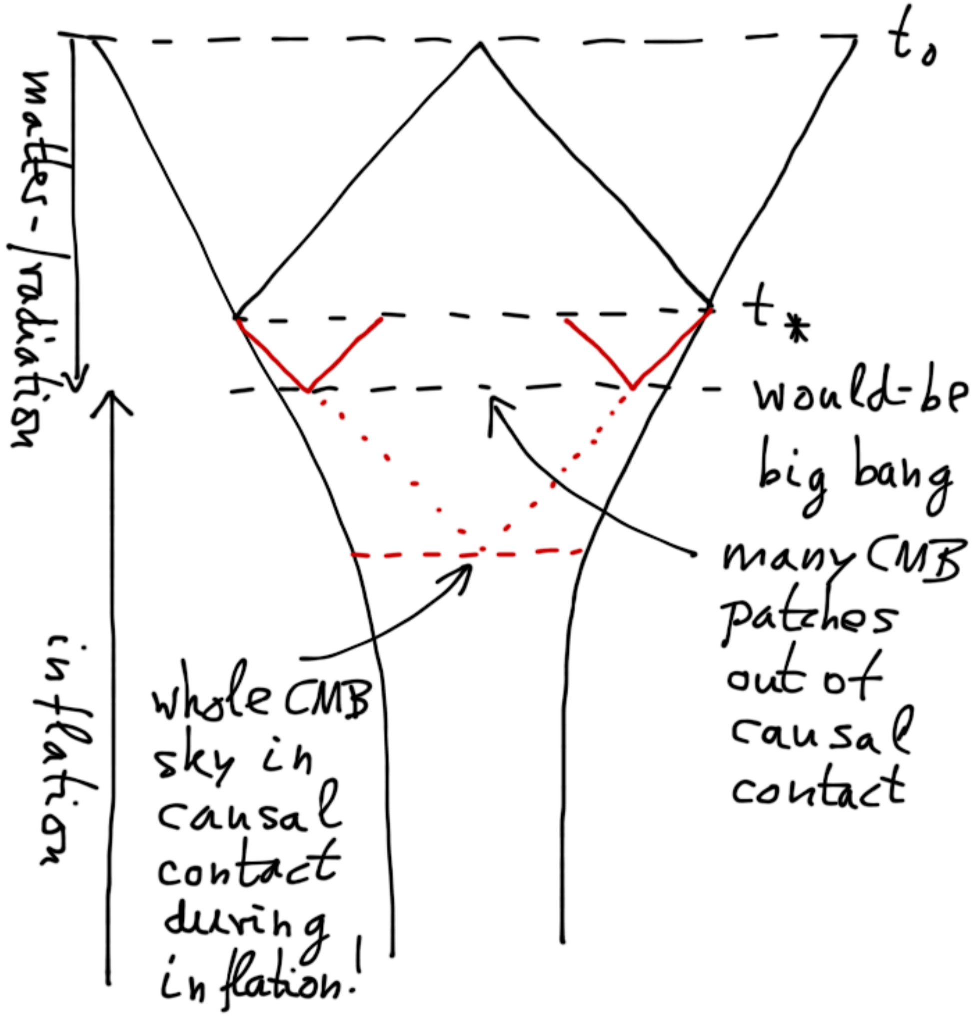
try a finite epoch of a quasi-cosmological constant - a large one:

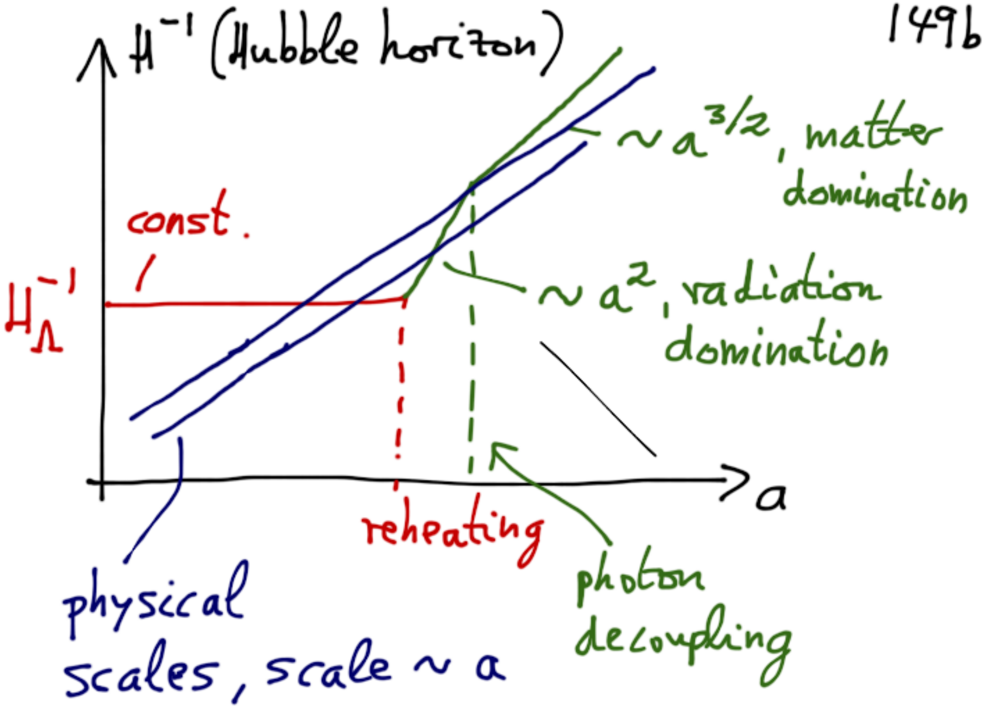
$\Rightarrow \rho_\Lambda = \text{const.} = -p_\Lambda$

$\Rightarrow H_\Lambda^2 = \frac{\rho_\Lambda}{3M_P^2} = \text{const.} = \frac{\dot{a}^2}{a^2}$

$\Rightarrow \boxed{a \sim e^{H_\Lambda \cdot t}}$ inflation








↘ modes left the horizon during inflation & re-entered afterwards → were in causal contact early on

→ solves horizon problem & some others (flatness, ...)



## horizon during inflation:

↝ calcute distance, that light can reach at time $t_e > 0$ after emission at $t_i < t_e$:

$$d_{\text{inf.}} = a(t_i) \cdot \int_{t_i}^{t_e} \frac{dt}{a(t)} = e^{H_\Lambda t_i} \, \frac{e^{-H_\Lambda t_e} - e^{-H_\Lambda t_i}}{-H_\Lambda} = \frac{1}{H_\Lambda}\left(1 - e^{-H_\Lambda (t_e - t_i)}\right)$$

$$\longrightarrow \frac{1}{H_\Lambda} \quad \text{for } t_e - t_i \to \infty$$

while during matter- or radiation domination:

$$d_{hor.} \sim t_e - t_i \to \infty$$

for $t_e - t_i \to \infty$

$\rightsquigarrow$ totally different behaviour !

$\longrightarrow$ during inflation there is a <u>true event horizon</u> at $H_\Lambda^{-1}$ distance around us, similar to a black hole event horizon, but turned 'inside-out'...



solves flatness problem:

$$\Omega_\Lambda \simeq \text{const.}$$

$$|\Omega_k| \sim \left|\frac{k}{a^2}\right| \lesssim 10^{-60}$$

$$\text{if} \quad \underbrace{H_\Lambda \cdot \Delta t_{inf.}}_{\equiv N_e} \gtrsim 60 \qquad \boxed{\text{"e-folds"}}$$

also:

$$\frac{a(t+\Delta t_{inf.})}{a(t)} = e^{H_\Lambda \cdot \Delta t_{inf.}} \equiv e^{N_e} \gtrsim e^{60}$$

would have all visible CMB scales today originating from a tiny causal region early on...

## how to do this?

(i) historically 1st: Guth '80

→ supercool Universe into a meta-stable state of positive vacuum energy → problems with exit & reheating

↓

use:

1 scalar field $\phi$ ...

action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - V(\phi) \right]$$

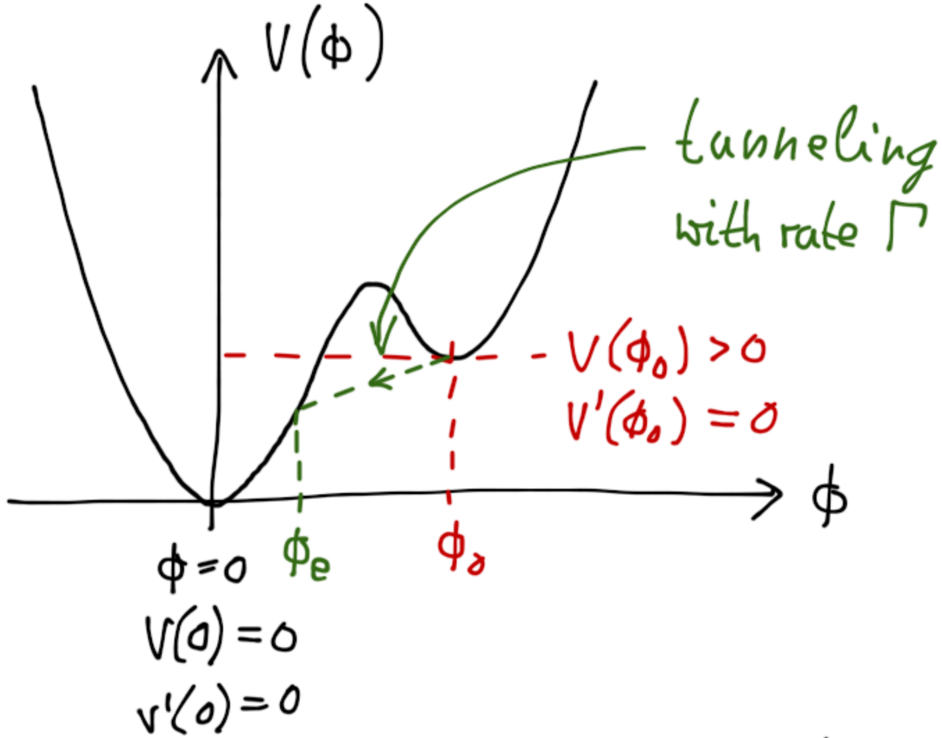
scalar potential

$\downarrow \delta S / \delta g^{\mu\nu}$

$T_{\mu\nu}$ : $\rho = \frac{1}{2}(\partial\phi)^2 + V$, $p = \frac{1}{2}(\partial\phi)^2 - V$



metastable vacuum of potential $V(\phi)$ emulates $\Lambda > 0$:




at $\phi = \phi_0$ : local minimum, $\dot{\phi} = 0$

$\Rightarrow \rho = V(\phi_0) = \text{const.} > 0$, $p = -V(\phi_0) \underline{\underline{= -\rho}}$

$\Rightarrow$ while $\phi$ at $\phi_0$: inflation

$\leftrightarrow$ $a \sim e^{\sqrt{V(\phi_0)} \cdot t}$

$\Rightarrow$ once $\phi$ tunnels:

$\rightsquigarrow$ $\phi$ exits at $\phi_e < \phi_0$,
then rolls quickly to
$\phi = 0$ & oscillates
around $\phi = 0$:
non-relativistic matter
($\phi$ has mass $m$)

$\leftrightarrow$ $a \sim t^{2/3}$ $\Leftrightarrow$ $\ddot{a} < 0$
inflation has ended



$\Rightarrow$ if $\phi$ coupled to other fields (SM fields):

$\rightsquigarrow$ decays to SM fields
while oscillating
around $\phi = 0$, once
inflation ended via
tunneling to $\phi_e$ ...
decay converts part
of $V(\phi_0)$ into new
radiation & heat —
"reheating"

<u>problem:</u>

- if tunneling slow, life-time $\tau = \frac{1}{\Gamma}$ of $\phi$ at $\phi_0$ long

  $\Rightarrow$ many efolds $N_e = H(\phi_0) \cdot \tau$

  but too few regions with exit by tunneling – one such region not big enough for our universe: need many that merge ...

- need fast tunneling, short lifetime $\rightarrow$ <u>not</u> enough efolds $\lightning$