define: $Q = m_n - m_e = 1.293\ \mathrm{MeV}$

"neutron binding energy"

switch to:

$$\epsilon = \frac{E_e}{m_e},\ q = \frac{Q}{m_e},\ z = \frac{m_e}{T},\ z_\nu = \frac{m_e}{T_\nu}$$

$$\Rightarrow \Gamma_{ep\to n\nu} = \frac{G_F^2}{2\pi^3}(1+3g_A^2)\, m_e^5 \times \int_q^\infty d\epsilon \frac{\epsilon(\epsilon-q)^2\sqrt{\epsilon^2-1}}{\left(1+e^{\epsilon z}\right)\left(1+e^{(q-\epsilon)z_\nu}\right)} \qquad (54)$$

13.5.

the prefactor also appears in free neutron decay: no $f_i(E_i)$ for single n

$$\Rightarrow \Gamma_{n\to pe\bar\nu} = \frac{G_F^2}{2\pi^3}(1+3g_A^2)\, m_e^5 \cdot \underbrace{\int_1^q d\epsilon \cdot \epsilon(\epsilon-q)^2\sqrt{\epsilon^2-1}}_{\lambda_0} \qquad (55)$$

$\lambda_0 \simeq 1.633$ phase space factor

& life time $\tau_n = \Gamma^{-1}_{n\to pe\bar\nu}$ (56)

$$\Rightarrow \frac{G_F^2}{2\pi^3}(1+3g_A^2)\, m_e^5 = (\tau_n \lambda_0)^{-1}$$

$$\Rightarrow \Gamma_{ep\to n\nu} = (\tau_n\lambda_0)^{-1} \cdot \int_q^\infty d\epsilon \frac{\epsilon(\epsilon-q)^2\sqrt{\epsilon^2-1}}{\left(1+e^{\epsilon z}\right)\left(1+e^{(q-\epsilon)z_\nu}\right)} \qquad (57)$$

prefactor can be fixed by neutron life time measurement

$\Downarrow$

(58) $g_A \simeq 1.26$ axial vector coupling of the nucleon

relativistic & non-relativistic limit:

$$\Gamma_{ep\to n\nu} = \begin{cases} \tau_n^{-1}\left(\frac{T}{m_e}\right)^3 \cdot e^{-Q/T}, & T \ll Q, m_e \\ \frac{7}{30}\pi\,(1+3g_A^2)G_F^2 T^5 \simeq G_F^2 T^5 & \text{for } T \gg Q, m_e \end{cases} \quad (59)$$

and: $H = \frac{\pi}{\sqrt{90}} \cdot g_*^{1/2} \cdot \frac{T^2}{M_p}$

and: γ, ν's relativistic

$\Rightarrow g_* \simeq 3.36$

$$\Rightarrow \frac{\Gamma_{ep\to n\nu}}{H} \simeq \left(\frac{T}{0.8\,\text{MeV}}\right)^3 \quad (60)$$

$\Rightarrow$ freeze-out at: $T_F \simeq 0.8$ MeV

self-consistent with use of approximation of:

- $\Gamma_{ep\to n\nu}$ for $T > m_e \simeq 0.5$ MeV
- only γ, ν's safely relativistic d.o.f in g_* for $T <$ MeV

We can calculate

$$\frac{n_n}{n_p} = e^{-\frac{m_n - m_p}{T} + \frac{\overbrace{\mu_n - \mu_p}^{= \mu_e - \mu_\nu}}{T}} = e^{-\frac{Q}{T}} \cdot e^{-\frac{\mu_e - \mu_\nu}{T}} \quad (61)$$

by (29) we have:

$$\frac{n_{e^+} - n_{e^-}}{n_\gamma} \sim \frac{\mu_e T^2}{T^3} = \frac{\mu_e}{T} \qquad (62)$$

and: $n_{e^+} - n_{e^-} = n_p - n_{\bar{p}} \simeq n_B$

$$\Rightarrow \frac{n_{e^+} - n_{e^-}}{n_\gamma} \simeq \frac{n_B}{n_\gamma} = \eta_B \sim \frac{\mu_e}{T}$$

$$\eta_B \sim 10^{-9} \quad \Rightarrow \frac{\mu_e}{T} \sim \eta_B \sim 10^{-9}$$

thus we know n_n/n_e at freeze-out:

$$\left.\frac{n_n}{n_p}\right|_{T_F} = e^{-\frac{Q}{T_F}} \simeq \frac{1}{6} \qquad (63)$$

step ii): at $T \lesssim T_F \simeq 0.8$ MeV

production of D and ^{3}He from n, p slow

$\rightarrow$ small n_D, $n_{^3He}$

$\rightsquigarrow$ blocks build-up of ^{4}He by fusion until about $T_{NUC} \simeq 0.1$ MeV

"deuterium bottleneck"

thus within:

$$0.1\ \text{MeV} < T < T_F \simeq 0.8\ \text{MeV}$$

$\Rightarrow$ radioactive neutron-decay reduces n_n/n_p somewhat

radiation domination:

$$H = \frac{\pi}{\sqrt{90}}\, g_*^{1/2} \cdot \frac{T^2}{M_P}$$

$$\Rightarrow t = \frac{\sqrt{90}}{2\pi}\, g_*^{-1/2}\, \frac{M_P}{T^2}$$

$$\Rightarrow t(0.1\ \mathrm{MeV}) \simeq t_{NUC} \simeq 100\,s$$

and: $\tau_n \simeq 900\,s$

$$\Rightarrow \left.\frac{n_n}{n_p}\right|_{T_{NUC}} = e^{-\frac{Q}{T}} \cdot e^{-\frac{t_{NUC}}{\tau_n}} \simeq \frac{1}{7} \qquad (64)$$

at t_{NUC} n_D and $n_{^3He}$ have grown large, and all n left fuse rapidly into ^4He:

$$\Rightarrow \underbrace{X_{^4He}}_{\text{He mass fraction}} = \frac{N_{^4He}\, m_{^4He}}{\underbrace{N_N}_{\substack{\text{nucleon number}\\ = N_p + N_n}} \cdot m_N} = \frac{4 n_{^4He}}{n_N}$$

$$= \frac{2 \cdot n_n}{n_n + n_p} = \frac{2}{1 + \frac{n_p}{n_n}} \qquad (65)$$

$\Rightarrow$ Synthesized ^4He mass fraction:

$$X_{^4He}\Big|_{t_{NUC}} = \frac{2}{1+\frac{n_p}{n_n}}\Bigg|_{T_{NUC}} = \frac{2}{1+7} = \frac{1}{4} = 25\,\% . \tag{66}$$

2) freeze-out of thermally produced weakly interacting dark matter

Why dark matter (DM)?

- galaxy rotation curves:

$$\frac{mv^2}{r} \sim G\frac{Mm}{r^2} \Rightarrow v^2(r) \sim \frac{1}{r}$$

$\rightsquigarrow$ observation:

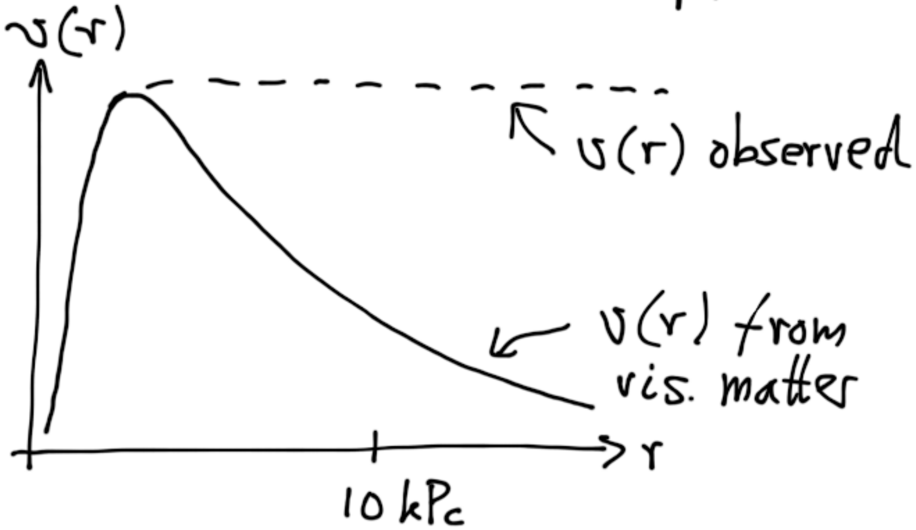
$$v(r) \simeq \text{const.}$$

$\rightsquigarrow$ need $M(r) \sim r$ to compensate:

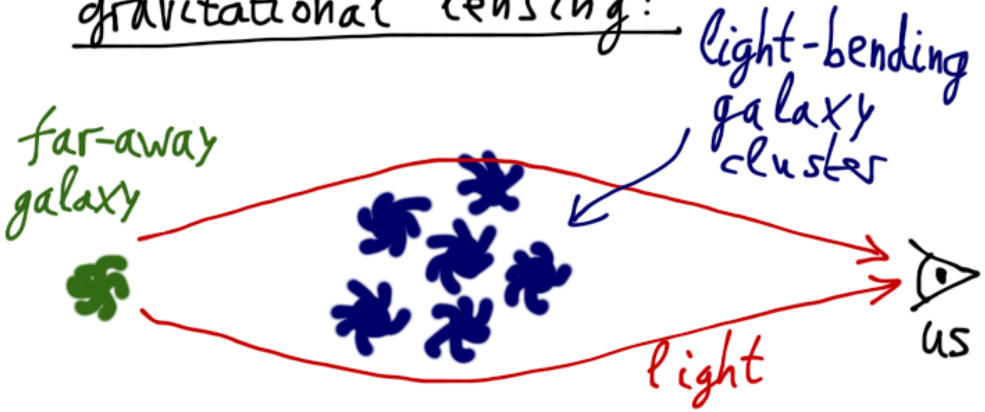
invisible/dark ...

galaxy rotation curves:

visible matter: $M(r) \sim \frac{1}{r^{\#}}, \# > 0$



gravitational lensing:



- X-ray observations of gas bound in galaxies:

  requires some extra dark gravitating stuff to keep gas inside the galaxy

- WMAP – CMB + BBN:

  BBN fixes $\eta_B \Rightarrow \Omega_B \simeq 0.04$

  WMAP: $\Omega_\Lambda \simeq 0.7$

  $\Omega_0 \simeq 1$

  $\Rightarrow \Omega_{DM} \simeq 0.3$

- gravitational lensing

  $\rightsquigarrow \Omega_M \simeq 0.3$

- structure formation requires some 'extra' gravitational wells : $\Omega_m \simeq 0.3$

$\Rightarrow$ There are many possibilities for DM.

Will focus on WIMP:

new "Weakly Interacting Massive Particle" beyond the SM

$\longrightarrow$ has weak-scale interactions

$\longrightarrow$ heavy

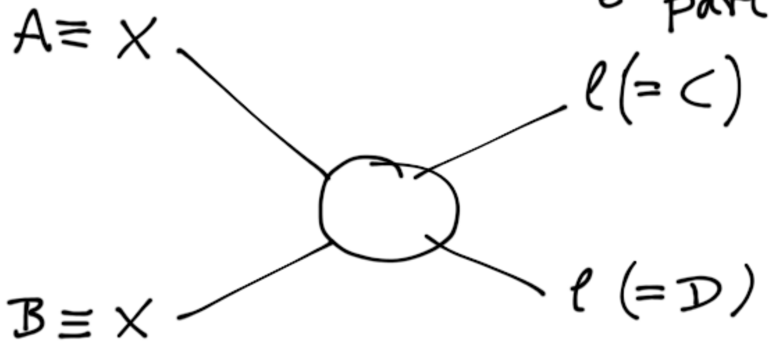
$\rightsquigarrow$ well-motivated e.g. from SUSY.

single constituent DM:

analysis similar to BBN

process:

light visible particle



(68)

X : heavy DM WIMP

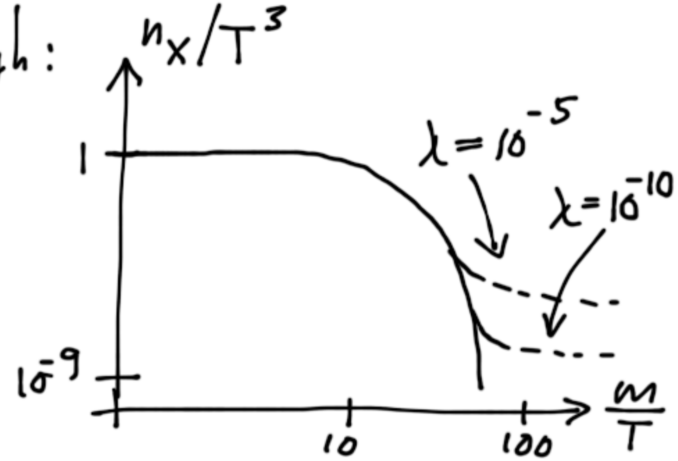
the Boltzmann eq. is again (48).

big picture:

DM particles X start at high T in equilibrium. If they stayed so always, then:

$$n_X \sim e^{-\frac{m_X}{T}} \quad \text{for } T < m_X$$

but, if $\Gamma_X < H$, DM density freezes out & survives, if it lives long enough:



here: $\lambda = \dfrac{m^2 \langle \sigma v \rangle}{H(m)}$ (69)

$\rightsquigarrow$ will turn out that WIMPS with weak scale masses give $\Omega_{DM} \simeq 0.3$

note: at high T the reaction is in kinetic equilibrium, but not always in chemical equilibrium...

This implies:

$$(70) \quad \begin{cases} f(E) \sim e^{-\frac{E-\mu}{T}} \\ T \ll E - \mu \end{cases}$$

⇝ put differently, μ need not be at its equilibrium value.

⇒ Can rewrite the bracket $\{\dots\}$ of eq. (48) as:

$$\{\dots\} = e^{-\frac{E_A+E_B}{T}} \cdot \left(e^{\frac{\mu_C+\mu_D}{T}} - e^{\frac{\mu_A+\mu_B}{T}}\right) \qquad (71)$$

$$(E_A + E_B = E_C + E_D)$$

use the n_i for $T \ll m_i$:

$$\Rightarrow \{\dots\} = e^{-\frac{E_A+E_B}{T}} \cdot \left[\frac{n_C n_D}{n_C^{(0)} n_D^{(0)}} - \frac{n_A n_B}{n_A^{(0)} n_B^{(0)}}\right] \qquad (72)$$

with: $\dfrac{n_i}{n_i^{(0)}} = e^{\mu_i/T}$ equil. n_i at $\mu_i = 0$.

now define the thermally averaged cross section:

$$\langle \sigma v \rangle = \frac{1}{n_A^{(0)} n_B^{(0)}} \cdot \int \frac{d^3 p_i}{(2\pi)^3 E_i} e^{-\frac{E_A+E_B}{T}} \cdot |\mathcal{M}|^2 \cdot (2\pi)^4 \times \delta^{(4)}(p) \qquad (73)$$

Simplifies the Boltzmann eq. (48):

$$\frac{1}{a^3} \frac{d(n_A a^3)}{dt} = n_A^{(0)} n_B^{(0)} \langle \sigma v \rangle \cdot \left[\frac{n_C n_D}{n_C^{(0)} n_D^{(0)}} - \frac{n_A n_B}{n_A^{(0)} n_B^{(0)}}\right]$$

$$\Downarrow$$

$$\frac{1}{a^3} \frac{d(n_X a^3)}{dt} = \left(n_X^{(0)}\right)^2 \langle \sigma v \rangle \cdot \left[1 - \left(\frac{n_X}{n_X^{(0)}}\right)^2\right] \qquad (74)$$