

11.5.

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Freeze out of heavy particles and decoupling

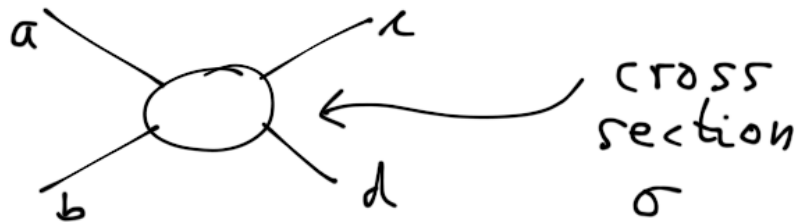
→ 'freeze-out' = decoupling from the thermal bath

↪ need to know reaction

rate Γ :

$$(43) \Gamma_a = n_a \cdot \sigma \cdot v$$

particle speed $\approx c$
production of a-particles



2 cases - rule of thumb:

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interaction via massless gauge boson (like γ):

$$QFT \Rightarrow \sigma \sim \frac{\alpha^2}{T^2} \quad (44)$$

α : gauge coupling

or via massive gauge boson X with mass m_X :

QFT \Rightarrow for $T < m_X$

$$\sigma \sim g_X^2 T^2 \quad (45)$$

$g_X \sim \frac{\alpha}{m_X^2}$ gauge coupling

for relativistic species. 77

$$n \sim g T^3$$

\Rightarrow

$$\Rightarrow \Gamma \sim \begin{cases} g \cdot \alpha^2 T & \text{massless} \\ & \text{gauge boson} \\ g \cdot \frac{\alpha^2}{m_x^4} T^5 & \text{massive} \\ & \text{gauge} \\ & \text{boson} \end{cases} \quad (46)$$

how many particles are produced?

$$N = \int_0^t \Gamma(t') dt', \quad \Gamma \sim T^n$$

$$(34): t \sim T^{-2} \Rightarrow dt \sim \frac{dT}{T^3}$$

$$\Rightarrow N \sim \frac{1}{n-2} T^{n-2} \sim \frac{1}{n-2} \frac{\Gamma}{H}$$

\Rightarrow production stops once 78

$$\boxed{\Gamma < H} \quad (47)$$

'freeze-out condition'

example: decoupling of neutrinos

\sim interact electro-weakly:

$$\Gamma \sim G_F^2 T^5$$

\nwarrow Fermi constant of weak interaction

$$\text{and } H \sim \frac{T^2}{M_P}$$

freeze-out:

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$$1 = \frac{\Gamma}{H} \sim M_P G_F^2 T^3$$

$$M_P \sim 2 \cdot 10^{18} \text{ GeV}$$

$$G_F \sim (200 \text{ GeV})^{-2}$$

$$\sim \frac{10^9}{\text{GeV}^3} \cdot T^3 = \left(\frac{T}{\text{MeV}} \right)^3$$

\Rightarrow ν 's decouple at
 $T \simeq 1 \text{ MeV}$.

can use this to calculate
neutrino temperature today ...

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after ν 's freeze out, at
 $T \lesssim m_e = 0.5 \text{ MeV} < 1 \text{ MeV}$

the e^-, e^+ annihilate

thus, before $t_{\text{annih.}}$

$$g_{*S} = 1 \cdot 2 + \frac{7}{8} \cdot 2 \cdot 2 + \nu\text{'s}$$

$\frac{11}{2}$

e^+, e^-

and after $t_{\text{annih.}}$:

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$$g_{*S} = 2 + \nu'_S$$

γ \nearrow

but entropy conserved :

$$S_{t < t_{\text{annih}}} = g_{*S}^{\text{before}} \cdot (T_V a)^3 + S_V(T_V)$$

$\searrow T_V = T_\gamma$ still

$$= \frac{11}{2} \cdot (T_V a)^3 + S_V(T_V)$$
$$= S_{t > t_{\text{annih.}}} = g_{*S}^{\text{after}} (T_\gamma a)^3 + S_V(T_V)$$

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$$= 2 (T_\gamma a)^3 + S_V(T_V)$$

$$\Rightarrow T_V = \left(\frac{4}{11}\right)^{1/3} \cdot T_\gamma \approx 1.91 \text{ K}$$

today

another example: photon decoupling

the process $\gamma + H \rightarrow p + e^-$
prevents H from forming if
too rapid - Thomson scattering

$$\Gamma_\gamma = n_e \sigma_T, \quad \sigma_T \approx 6.7 \cdot 10^{-29} \text{ m}^2$$

photon decoupling

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\leftrightarrow mean free path \sim Hubble horizon $\sim \frac{1}{H}$

$$\Rightarrow t_{\gamma} = \frac{1}{\Gamma_{\gamma}} = \frac{1}{H} = t_H \quad (*)$$

$$\Gamma_{\gamma} = n_e \sigma_T = \sigma_T X_e n_B = X_e n_B \sigma_T$$

ionization fraction $\left\{ \begin{array}{l} n_B = n_B^0 \cdot (1+z)^3 \\ n_B^0 = 0.23/\text{m}^3 \end{array} \right.$

and matter domination after decoupling:

$$\Rightarrow \frac{t}{t_0} = \left(\frac{a}{a_0} \right)^{3/2} = (1+z)^{-3/2}$$

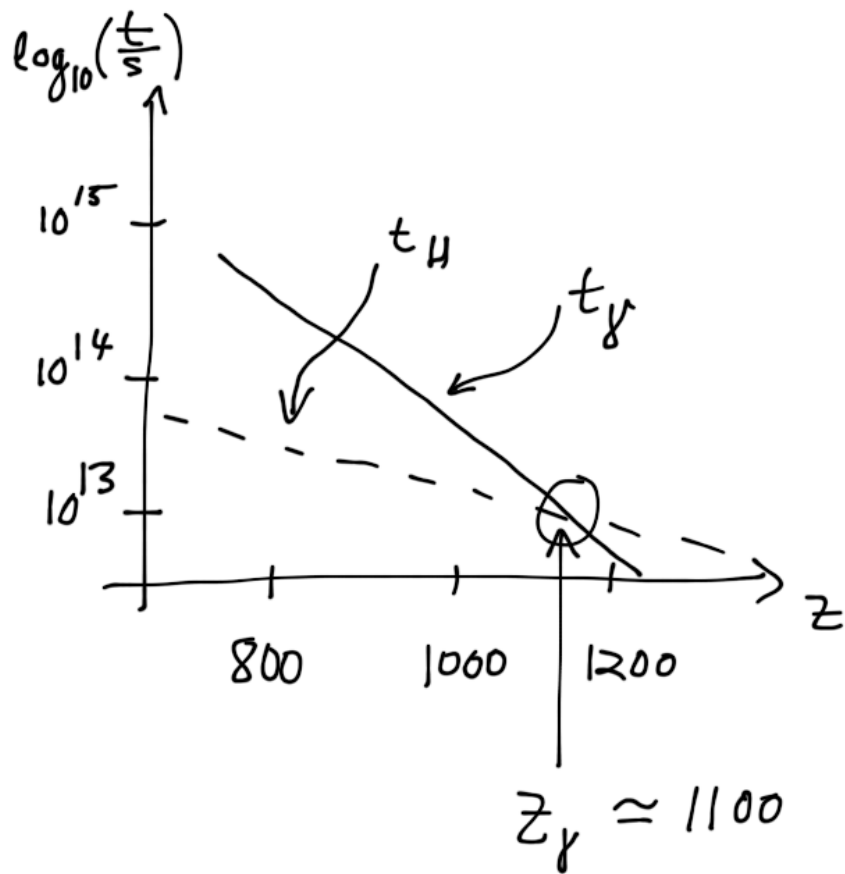
$$t_0 = \frac{2}{3H_0}$$

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$$\Rightarrow t_H = \frac{2}{3H_0} (1+z_H)^{-3/2}$$
$$\approx 3 \cdot 10^{17} \text{ s} \cdot (1+z_H)^{-3/2}$$

plug into (*), and let's say that decoupling is complete once $X_e \approx 0.02$ (% level reionization) ...

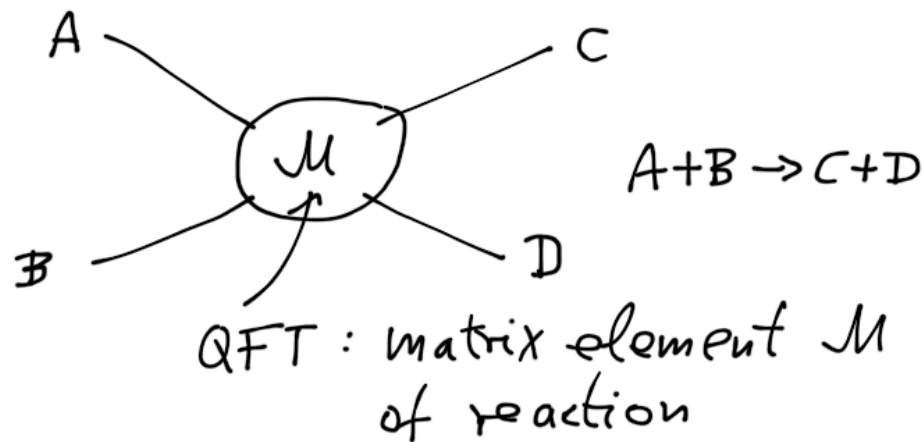
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description of freeze-out

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reaction



Semi-classically:

$$\frac{d\#}{dt} = \Gamma_{\text{production}} - \Gamma_{\text{annihil./decay}}$$

for e.g. particle type A in more detail:

$$\#(A) = N_A = n_A V \sim n_A a^3$$

and QFT

$$\Rightarrow \frac{1}{a^3} \frac{d}{dt} (n_A a^3) =$$

$$= \int \frac{d^3 p_A}{(2\pi)^3 E_A} \int \frac{d^3 p_B}{(2\pi)^3 E_B} \int \frac{d^3 p_C}{(2\pi)^3 E_C} \int \frac{d^3 p_D}{(2\pi)^3 E_D} \quad (48)$$

$$\times (2\pi)^4 \cdot \delta^{(3)}(\vec{p}_A + \vec{p}_B - \vec{p}_C - \vec{p}_D) \cdot \delta(E_A + E_B - E_C - E_D)$$

$$\times |\mathcal{M}|^2 \cdot \left\{ \underbrace{f_C f_D (1 \pm f_A) (1 \pm f_B)}_{\text{production of A}} - \underbrace{f_A f_B (1 \pm f_C) (1 \pm f_D)}_{\text{annihilation/decay of A}} \right\}$$

Matrix element

+ : boson
- : fermion

$$f_i(E), \quad i = A, B, C, D$$

partition function of particle species i

(48) is the Boltzmann equation.

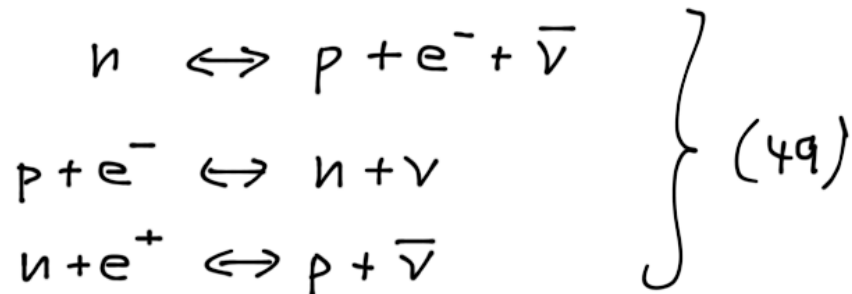
↓
look at two examples...

1) nucleosynthesis: Helium synthesis

step i): neutron freeze-out

Weak interactions

→ reactions:



at high T chemical equilibrium:

$$\mu_e + \mu_p = \mu_n + \mu_\nu \quad (50)$$

and energy conservation:

$$E_e + E_p = E_n + E_\nu \quad (51) \quad 89$$

rate for this process: 90

$$\Gamma_{ep \rightarrow n\nu} = \int \frac{d^3 p_e}{(2\pi)^3 E_e} \int \frac{d^3 p_\nu}{(2\pi)^3 E_\nu} \int \frac{d^3 p_n}{(2\pi)^3 E_n} \quad (52)$$

$$\times \frac{(2\pi)^4}{8} \delta^{(4)}(p_e + p_p - p_n - p_\nu)$$

$$\times f_e(E_e) \cdot [1 - f_\nu(E_\nu)] \cdot |\mathcal{M}|^2$$

$$|\mathcal{M}|^2 \simeq G_F^2 (1 + 3g_A^2) \quad (53)$$

$$= \frac{G_F^2}{2\pi^3} (1 + 3g_A^2) \quad \left\{ \begin{array}{l} \text{Fermi} \\ \text{constant} \end{array} \right.$$

$$\times \int_{m_e}^{\infty} dE_e \frac{E_e \sqrt{E_e^2 - m_e^2} (E_e + E_p - E_n)^2}{(1 + e^{E_e/T}) \cdot (1 + e^{-\underbrace{(E_e + E_p - E_n)}_{E_\nu}/T})}$$

define: $Q = m_n - m_e = 1.293 \text{ MeV}$ 91
 "neutron binding energy"

switch to:

$$\epsilon = \frac{E_e}{m_e}, \quad q = \frac{Q}{m_e}, \quad z = \frac{m_e}{T}, \quad z_V = \frac{m_e}{T_V}$$

$$\Rightarrow \Gamma_{ep \rightarrow n\nu} = \frac{G_F^2}{2\pi^3} (1 + 3g_A^2) m_e^5 \quad (54)$$

13.5. $\times \int_q^\infty d\epsilon \frac{\epsilon (\epsilon - q)^2 \sqrt{\epsilon^2 - 1}}{(1 + e^{\epsilon z})(1 + e^{(q - \epsilon)z_V})}$

the prefactor also appears in free neutron decay: no $f_i(E_i)$ for single n

$$(55) \quad \Rightarrow \Gamma_{n \rightarrow pe\bar{\nu}} = \frac{G_F^2}{2\pi^3} (1 + 3g_A^2) m_e^5 \underbrace{\int_1^q d\epsilon \cdot \epsilon (\epsilon - q)^2 \sqrt{\epsilon^2 - 1}}_{\lambda_0}$$