4.5. short excurse on horizons:

horizons $\to$ crucial property of FRW expansion

convenient here: conformal time

$$\rightsquigarrow \quad ds^2 = a^2(\eta)\cdot\left[d\eta^2 - d\rho^2 - f^2(\rho)\cdot d\Omega_2^2\right]$$

look at radially outward light rays:

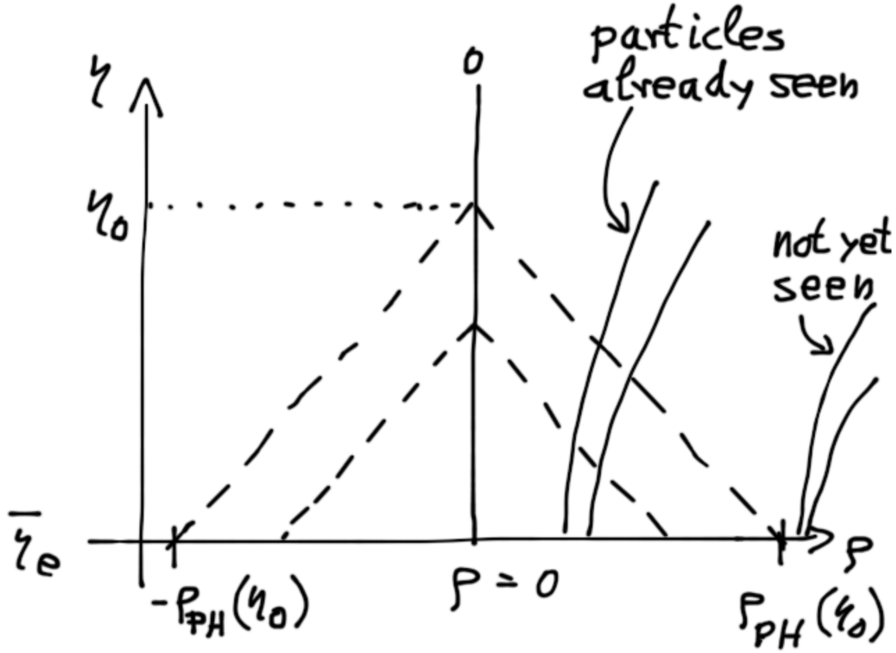
$$\rightsquigarrow \quad ds^2 = 0$$

$$\Rightarrow \quad d\rho = \pm d\eta$$

$$\Rightarrow \quad \rho = \text{const.} \pm \eta$$

two horizons:

i) observer O at $\rho = 0$ receives signal at conformal time η_0, an emitter at $\eta_e < \eta_0$ sends the signal...



event at $\eta < \eta_0$ only in causal contact if comoving distance ρ 48

$$\rho < \rho_e(\eta_e) = \int_{\eta_e}^{\eta_0} d\eta' = \eta_0 - \eta_e$$

$\uparrow$ maximum visible comoving distance at η_e in the past

if universe begins at $\bar{\eta}_e < \eta_0$, then all visible ρ are bounded:

$$\rho < \rho_{PH}(\eta_0) = \int_{\bar{\eta}_e}^{\eta_0} d\eta' = \eta_0 - \bar{\eta}_e$$

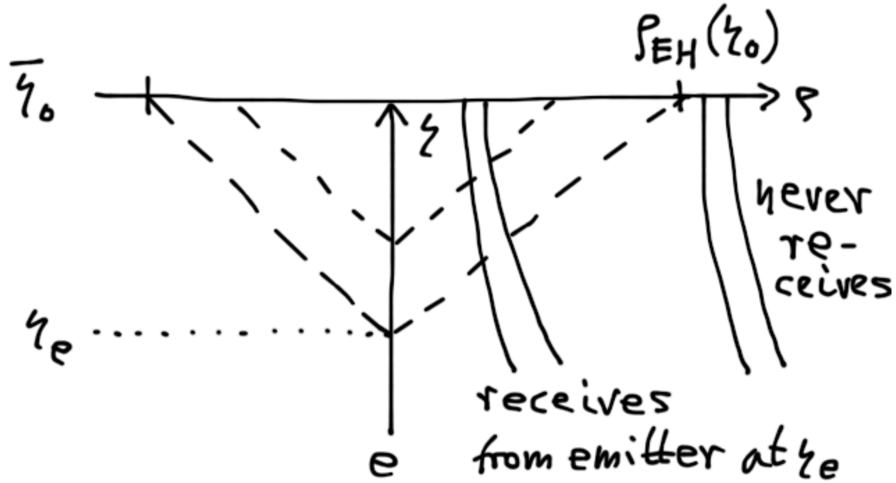
$\uparrow$ "particle horizon" for observer at η_0

$\rightsquigarrow$ the visible universe at η_0! 49

ii) conversely, if the universe ends at $\bar{\eta}_0 > \eta_e$

$$\rightsquigarrow \rho_{EH}(\eta_e) = \int_{\eta_e}^{\bar{\eta}_0} d\eta' = \bar{\eta}_0 - \eta_e$$

"event horizon", maximum distance of causal influence from now & here



conversion into physical horizon distance:

$$\text{e.g.}\quad d_{PH}(t_0) = a(t_0)\cdot \rho_{PH}(\eta_0) = a_0 \cdot \int_{\eta_e}^{\eta_0} d\eta' = a_0 \cdot \int_{t_e}^{t_0} \frac{dt'}{a(t')}$$

matter/radiation: $a(t) \sim t^p$

$$= t_0^p \cdot \frac{1}{-p+1}\left(t_0^{-p+1} - t_e^{-p+1}\right)$$

$t_0 \gg t_e$

$$\simeq \frac{1}{1-p}\cdot t_0 \sim H_0^{-1}$$ "Hubble horizon"

$\rightsquigarrow$ $d_{PH} \sim t_0$ grows faster than comoving length scales

$$\lambda = a(t_0)\,\lambda_{com.} \sim t^p$$

with $p = \frac{1}{2}$ or $\frac{2}{3}$ for radiation or matter

$\rightsquigarrow$ present-day horizon scale H_0^{-1} was smaller by $\frac{1}{1+z_{dec.}} \simeq 1100$ at CMB decoupling:

$$H_0^{-1} \simeq 10^{10}\ ly$$

$$\Rightarrow \frac{1}{1+z_{dec}} H_0^{-1} \simeq 10^{7}\ ly$$

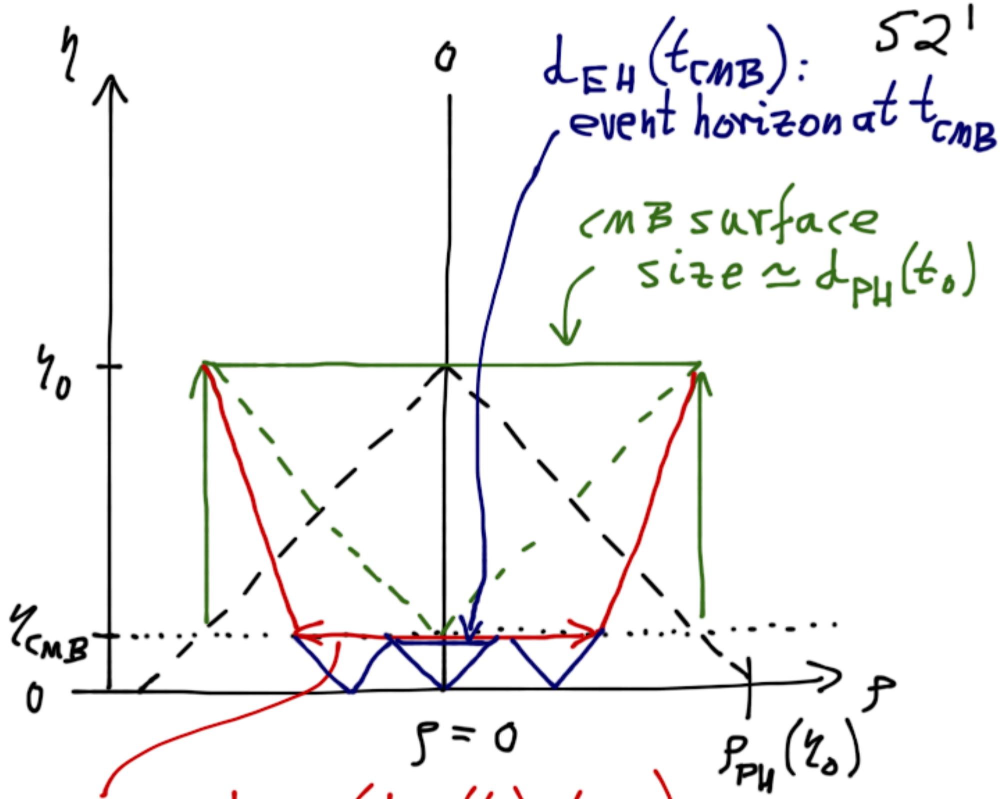
but d_{PH} at decoupling at $t_{dec.} \simeq 400.000$ yr

$\Rightarrow d_{PH}(t_{dec.}) \sim 10^5\, ly$

$\rightsquigarrow$ $O(10^6)$ independent horizon-size patches out of causal contact – so why $\Delta T/T \lesssim 10^{-4}$ everywhere ???

$\downarrow$

"horizon problem"



size $d_{CMB}(d_{PH}(t_0), t_{CMB})$ of the visible CMB region of present-day size $d_{PH}(t_0)$, back at t_{CMB}, shrunken under turned-back matter-dominated expansion $\sim t^{2/3}$

$$— \ll \longleftrightarrow$$

or:

$$\frac{d_{EH}(t_{CMB})}{d_{CMB}(d_{PH}(t_0))} \simeq \frac{t_{CMB}}{\underbrace{t_0}_{d_{PH}(t_0)} \cdot \left(\frac{t_{CMB}}{t_0}\right)^{2/3}}$$

$$= \left(\frac{t_{CMB}}{t_0}\right)^{1/3} \simeq (1+z_{CMB})^{-\frac{1}{2}} \simeq 0.03$$

since:

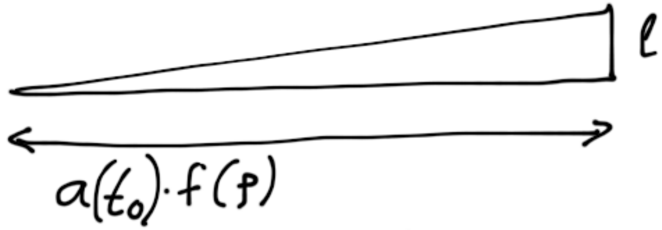
$$1+z_{CMB} = \frac{a_0}{a_{CMB}} = \left(\frac{t_0}{t_{CMB}}\right)^{2/3}$$

$\rightsquigarrow$ $\mathcal{O}(10^3)$ causally disconnected d_{EH}-sized patches at t_{CMB}!

two distance definitions:

i) object of known physical size ℓ

$\rightsquigarrow$ angular distance $\theta_p(\ell)$:



$$\Rightarrow \quad \theta_p(\ell) = \frac{\ell}{a_0 \cdot f(\rho)}$$

$\rightsquigarrow$ $f(\rho)$ depends on k, thus $\theta_p(\ell)$ can test Ω_k

$\longrightarrow$ prime example CMB!

ii) luminosity distance d_L:

source at physical distance d, with luminosity L, produces luminosity F at distance d (sphere of radius d, surface area $4\pi d^2$):

$$F = \frac{L}{4\pi d^2 (1+z)^2} \equiv \frac{L}{4\pi \cdot d_L^2}$$

- redshift of photons : $1+z$
- time dilatation : $1+z$

defines <u>luminosity distance</u> d_L.

measure d_L by magnitudes $m-M$:

$$m - M = 5 \cdot \log_{10}\left(\frac{d_L}{10\,\mathrm{pc}}\right).$$

can compute d_L:

$$d_L = d \cdot (1+z) = a_0 \cdot (1+z) \cdot \int_{\chi(z)}^{\chi_0} d\chi'$$

use:

$$(20)\quad 1+z = \frac{a_0}{a(t_z)} \Rightarrow dz = -\frac{a_0}{a^2}\dot{a}\cdot dt \overset{(21)}{=} -H(1+z)\cdot dt = -a_0 H \cdot d\chi$$

$$\Rightarrow d_L = (1+z)\cdot \int_0^z \frac{dz'}{H(z')}$$

$\rightsquigarrow$ need $H(z)$ – use Friedmann eq.:

$$t_0: \quad H_0^2 + \frac{k}{a_0^2} = \frac{8\pi G}{3} \rho_0 = H_0^2 \cdot \underbrace{\Omega_0}_{= \frac{\rho_0}{\rho_{crit}}}$$

$$t_z: \quad H^2(z) + \frac{k}{a^2(t_z)} = \frac{8\pi G}{3} \cdot \rho(z) = H_0^2 \cdot \Omega_0 \cdot \frac{\rho(z)}{\rho_0}$$

and:

$$\frac{k}{a^2(t_z)} = \frac{k}{a_0^2} \cdot (1+z)^2$$

$$\Rightarrow H(z) = \sqrt{- \frac{k}{a_0^2}(1+z)^2 + H_0^2 \Omega_0 \frac{\rho(z)}{\rho_0}}$$

and $- \frac{k}{a_0^2} = H_0^2 (1 - \Omega_0)$

$$\Rightarrow H(z) = H_0 \sqrt{(1 - \Omega_0)(1+z)^2 + \Omega_0 \cdot \frac{\rho(z)}{\rho_0}} \tag{22}$$

$$1 - \Omega_0 = \Omega_k$$

and: $$\Omega_0 \frac{\rho(z)}{\rho_0} = \Omega_M^0 (1+z)^3 + \Omega_r^0 (1+z)^4 + \Omega_\Lambda$$

$\downarrow$

$m - M$ vs. z – graph

eq. (20) $\Rightarrow$ expansion age t_0 of the universe:

$$dt = -\frac{dz}{H(z)\cdot(1+z)}$$

$$\downarrow$$

$$t_0 = \int_0^{z_{dec.}} \frac{dz'}{(1+z')\cdot H(z')} \quad (22)$$

and use eq. (21) for $H(z)$.