Space Charge Effects in the TESLA Damping Ring

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Damping Ring Parameters
Direct Space Charge
The Cure
Summary

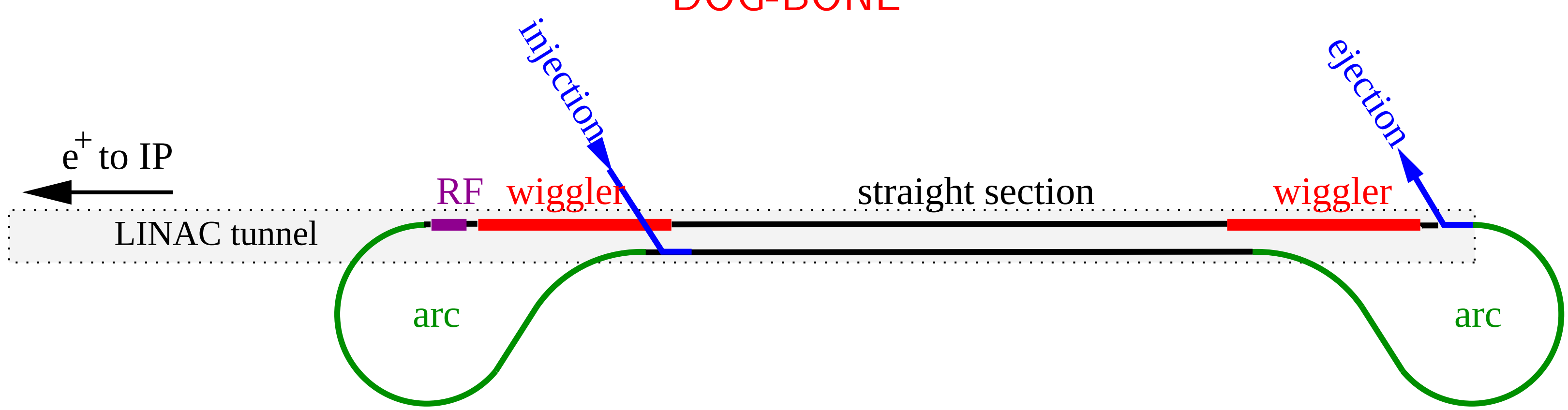
Snowmass 2001

July 2001

Damping Ring - Introduction

- Long TESLA bunch train (2820 bunches, 337 ns bunch-spacing) would require a 280 km circumference damping ring
  $\rightarrow$ compress bunch train with smaller bunch spacing in damping ring
  Circumference is now given by the achievable kicker raise/fall time
- Assume kicker raise/fall time of $20\,\mathrm{ns}$
  $\longrightarrow$ circumference $> 2820 * 20ns * c \approx 17\,\mathrm{km}$
- To avoid excessive additional tunnel cost build most part of the ring in the linac tunnel :

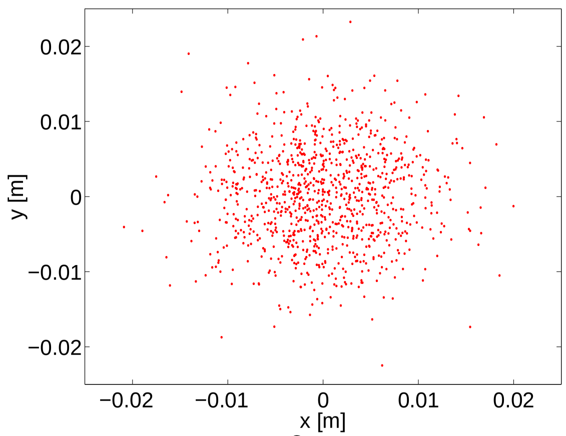
DOG-BONE



- Note: Because of the TESLA positron source scheme the position of an ejected bunch is filled again after ≈ 1.5 turns

Damping Ring - Introduction

Injected positron beam

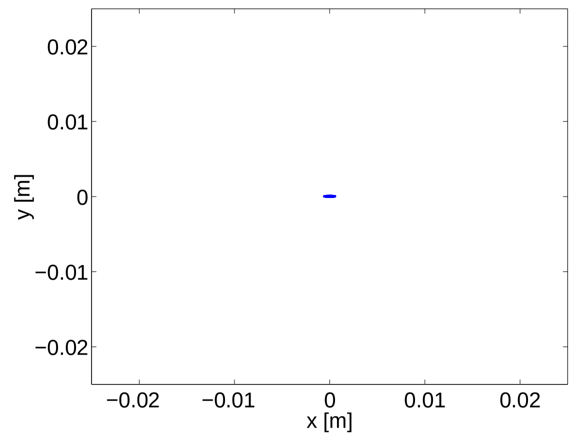


$\varepsilon_{x,inj.} = 1 \times 10^{-2}\,\mathrm{m}$
$\varepsilon_{y,inj.} = 1 \times 10^{-2}\,\mathrm{m}$
$\sigma_{l,inj.} =$
$\sigma_{e,inj.} = 1\,\%$

$\Longrightarrow$

$200\,\mathrm{ms}\ (f_{rep.} = 5\,\mathrm{Hz})$

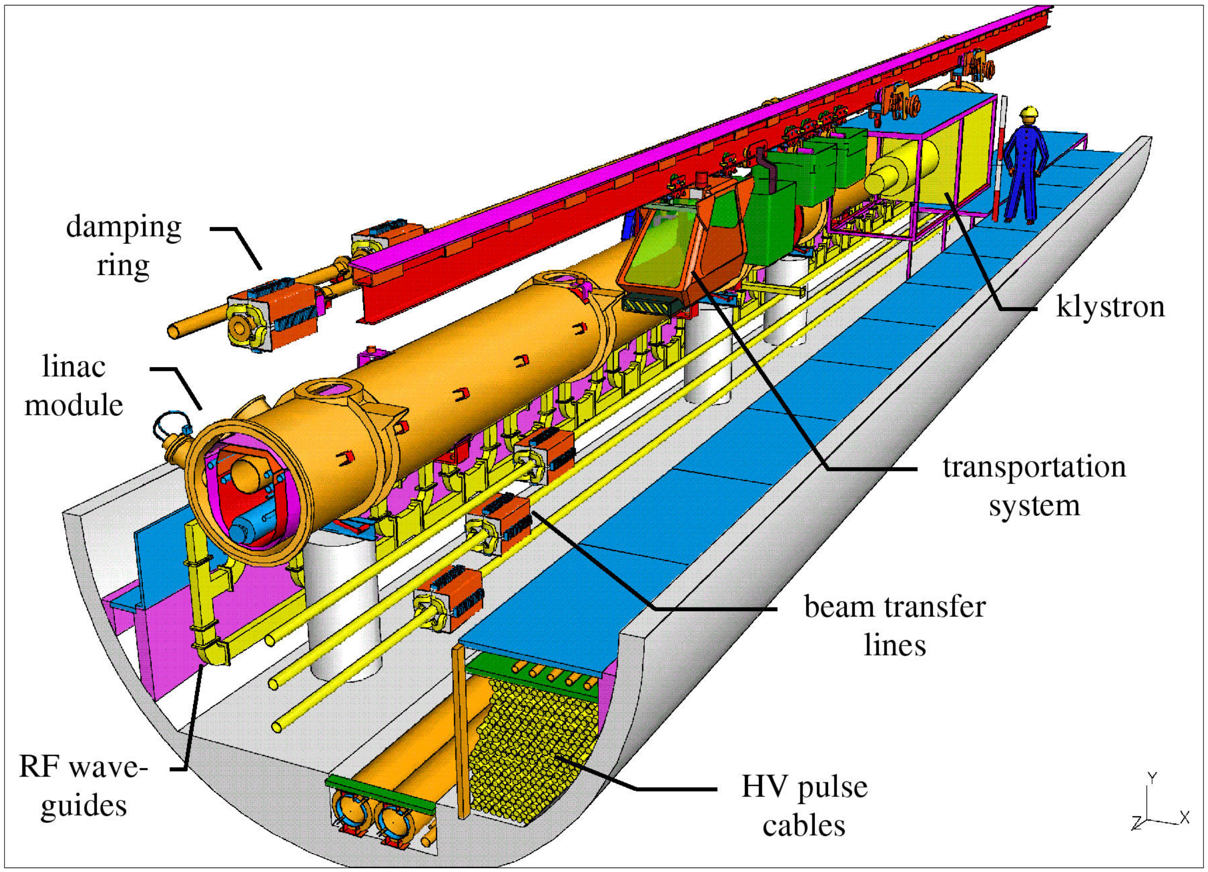
Ejected positron beam



$\varepsilon_{x,ej.} = 8 \times 10^{-6}\,\mathrm{m}$
$\varepsilon_{y,ej.} = 2 \times 10^{-8}\,\mathrm{m}$
$\sigma_{l,ej.} = 6\,\mathrm{mm}$
$\sigma_{e,ej.} = 0.13\,\%$

$\varepsilon_{y,inj.}/\varepsilon_{y,ej.} = 500000$ requires ≈ 7 damping times
$\Longrightarrow$ damping time $\tau_D \approx 28\,\mathrm{ms}$

Tunnel Layout



Incoherent Space Charge Tune Shift

- Space charge force is

$$F_{sc;x|y}(x|y,z) \approx -\frac{2 r_e N_e e^{\frac{-z^2}{2\sigma_z^2}}}{\sqrt{2\pi}\gamma^3 \sigma_{x|y}(\sigma_x+\sigma_y)\sigma_z} x|y$$

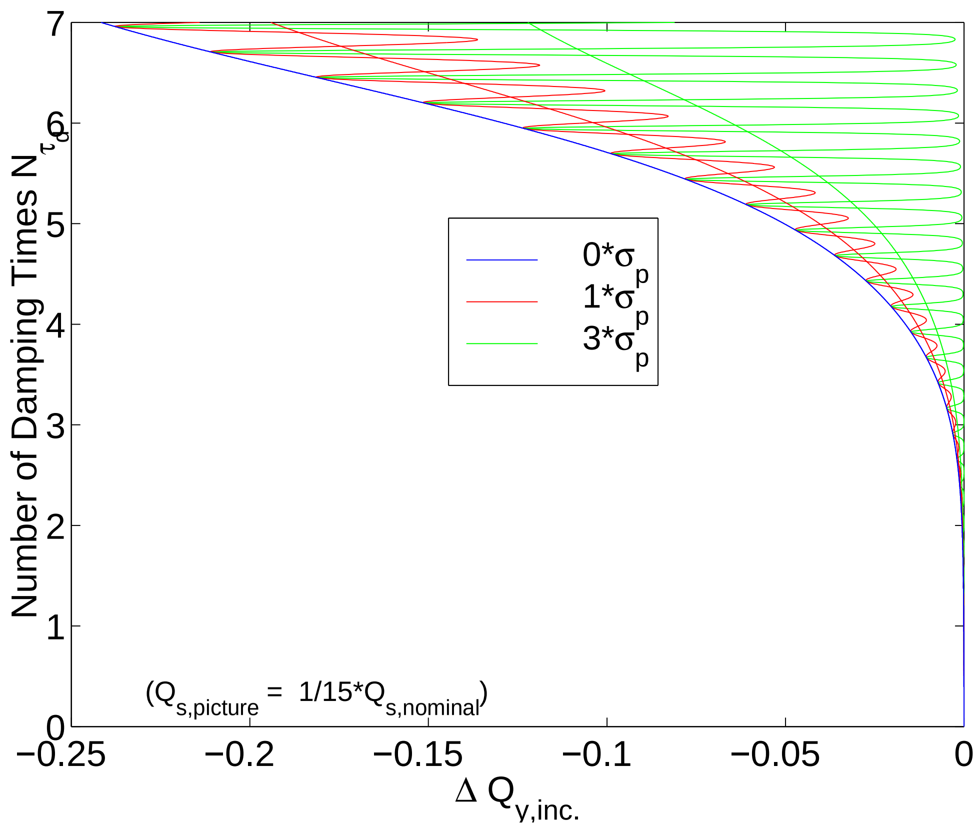
- Tune shift due to space charge force $\Delta Q_{sc} = \frac{1}{4\pi}\oint \beta F_{sc}$
- Large ring length and relative low energy leads to huge incoherent space charge tune shift:

$$\Delta Q_{sc;x|y} \approx \frac{L r_e N_e e^{\frac{-z^2}{2\sigma_z^2}}}{(2\pi)^{\frac{3}{2}}\gamma^2 \sqrt{\varepsilon_{x,n}\varepsilon_{x|y,n}}\sigma_z}$$

- With $z = z_0 \cos(2\pi \#_{turns} \nu_z)$
  $\longrightarrow$ particle tune oscillates with twice synchrotron frequency

Incoherent Space Charge Tune Shift

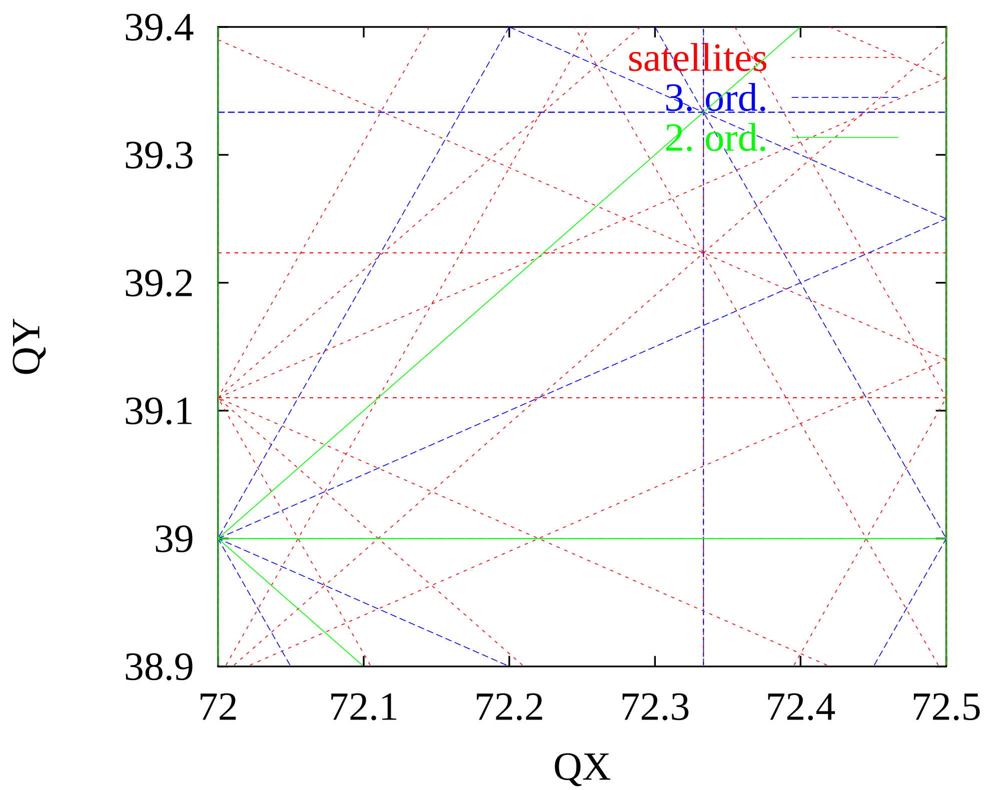
Incoherent tune shift versus damping time and initial longitudinal deviation:



Tune Diagram

Avoid resonances up to third order (always a good idea) **and** their 'satellites' at twice the synchrotron frequency:

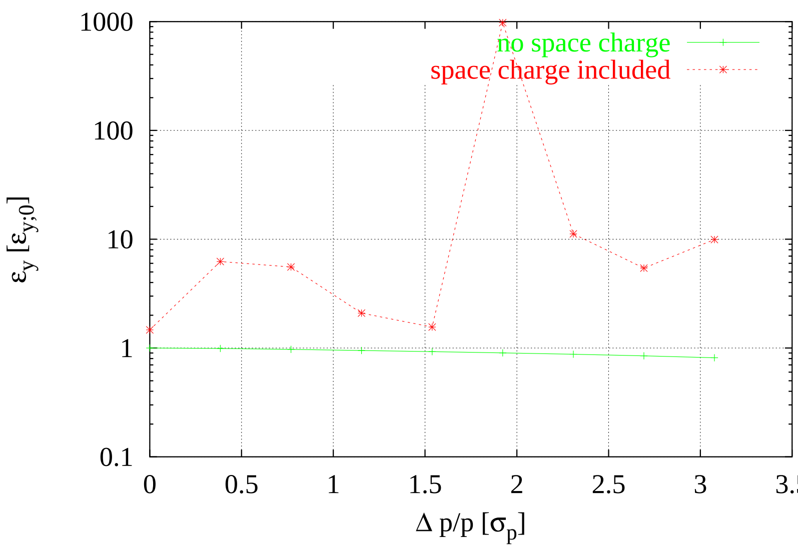
$$nQ_x + m(Q_y \pm 2Q_s) = \text{int.}$$



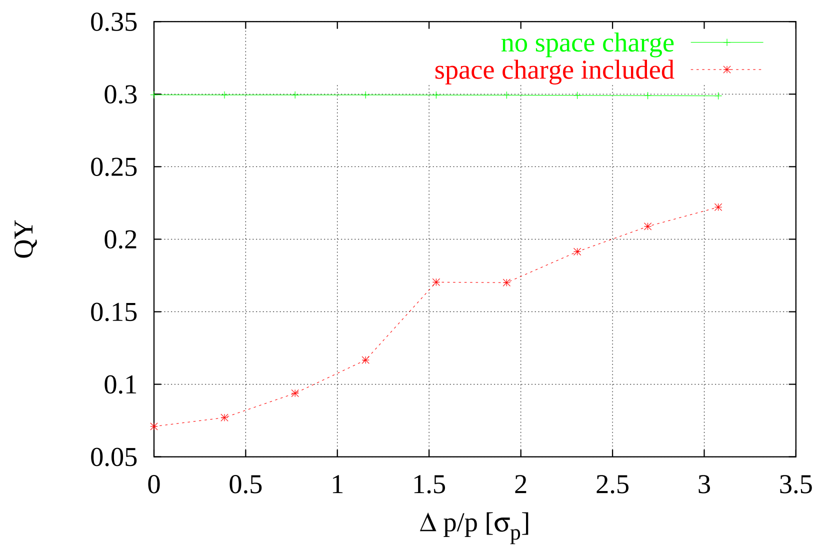
Inc. Space Charge Tune Shift - More Results

- tracking with (non-linear) space charge kick at each element ('weak-strong' model)
- calculate average Courant-Synder invariant as measure of emittance increase
- Include misalignment and orbit distortion (0.2 % coupling)
- Tunes at $Q_x = 72.32, Q_y = 39.30$

Average CS Invariant versus initial $\Delta p/p$



Vertical Tune versus initial $\Delta p/p$



$\longrightarrow$ vertical amplitude growth with space charge

How to Cure the Space Charge Tune Shift

Space charge force is $F_{sc,x/y}(x/y) \approx -\frac{x/y}{\gamma^3 \sigma_{x/y}(\sigma x + \sigma y)\sigma z}$

1 Increase ring energy γ^3
   - needs lattice redesign
   - reason to increase DR energy from $3.2\,\mathrm{GeV}$ to $5.0\,\mathrm{GeV}$
   - Scaling including constant normalized emittance and lattice change shows only weak dependence on γ

2 Increase bunch volume through local vertical dispersion
   - Vertical dispersion has negative impact on IBS emittance growth

3 Increase bunch volume through local coupling in long straights
   - Reduce $\int F_{sc}$ by the ratio

$$\frac{L_{arc} + L_{straight}\sqrt{\frac{\epsilon y}{\epsilon x}}}{L} \approx 5$$

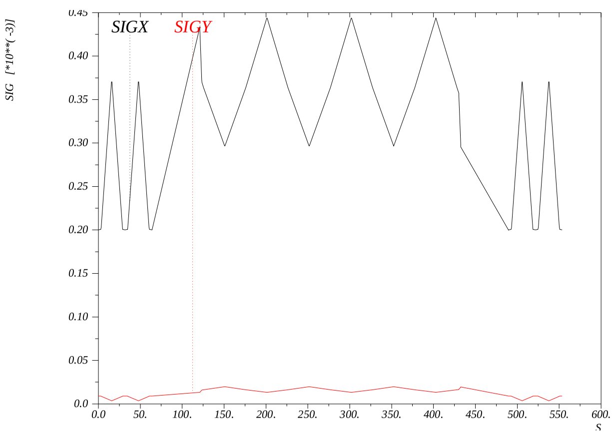
   - Additional coupling in a low-coupling ring

Local Beam Blow Up

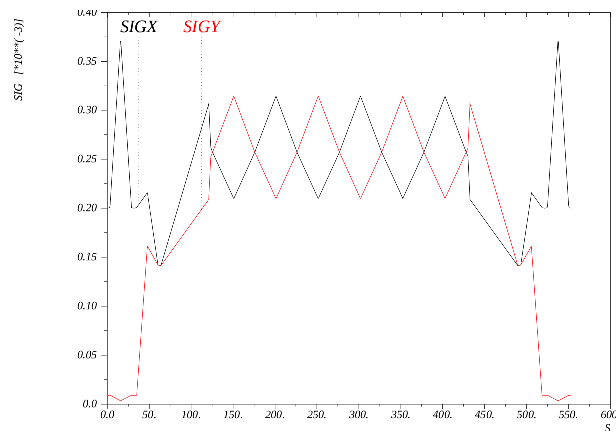
- Use special beam optics transformation to create beam with vortex distribution (Y. Derbenev)
- transformation can be realized with skew quadrupole triplet
- Beam transformed back with inverse transformation
- Drift between the two insertions has to fulfill $\mu_x = \mu_y$
  $\Longrightarrow$ no residual coupling

Local Beam Blow Up

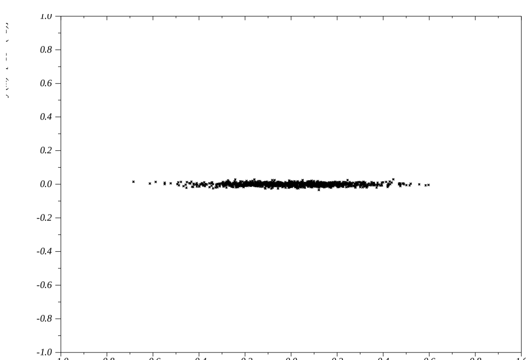
insertion off



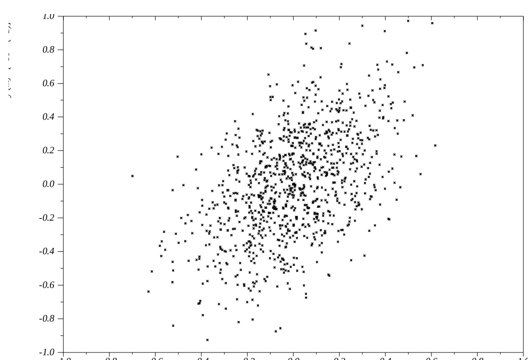
insertion on



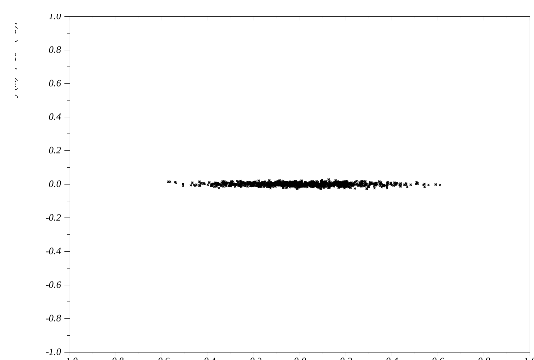
before insertion



between insertions



after insertions



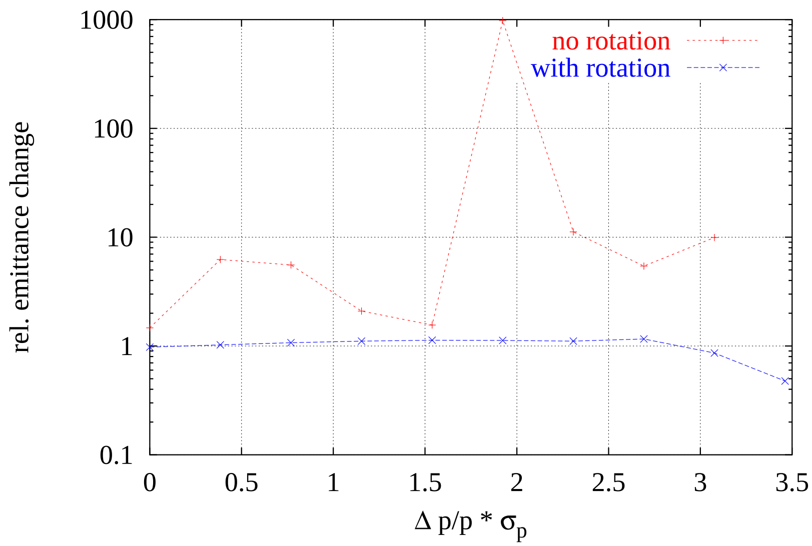
Effects of Emittance Blow Up

- Include misalignment and orbit distortion (0.2 % coupling)
- Tunes at $Q_x = 72.32, Q_y = 39.30$

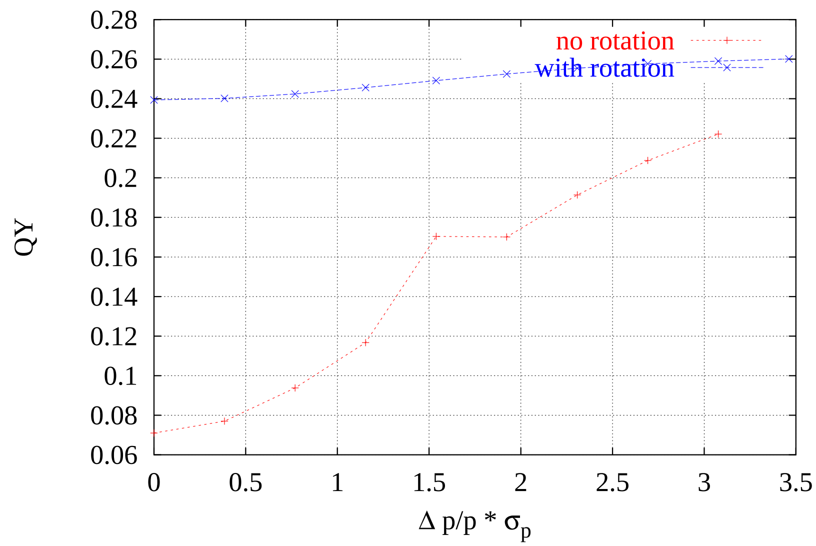
no vertical emittance increase in straight sections

vertical emittance increased in straight sections with local coupling bump

Average CS Invariant versus initial $\Delta p/p$



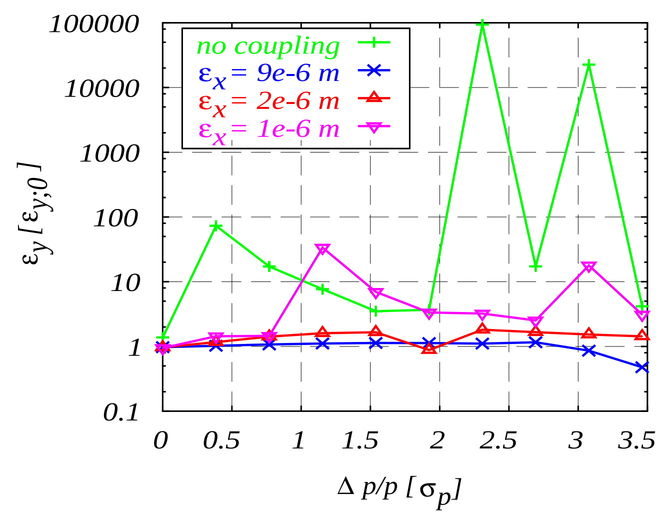
Vertical Tune versus initial $\Delta p/p$



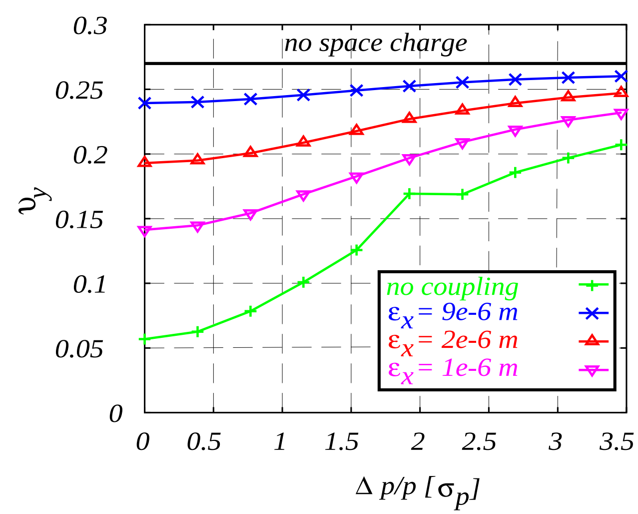
Effects of Smaller Emittance

- Include misalignment and orbit distortion (0.2 % coupling)
- Tunes at $Q_x = 72.32, Q_y = 39.27$
- Decrease horizontal emittance (increase space charge force)

Maximum CS Invariant versus initial $\Delta p/p$



Vertical Tune versus initial $\Delta p/p$



$\longrightarrow Q_{sc}$ up to 0.1 looks tolerable

Intra Beam Scattering

- Intra-beam scattering denotes the effect of many small angle Coulomb scatterings between particles in the bunch leading to diffusion.
- Exact theory difficult, lets try some simplified scalings
- The diffusion rates are:

$$\frac{1}{\tau_{x,y;IBS}} \propto \frac{N_e \mathcal{H}_{x,y}}{\sqrt{\gamma}\sigma_z \varepsilon_{x,y;n}(\varepsilon_{x;n}\varepsilon_{y;n})^{3/4}}$$

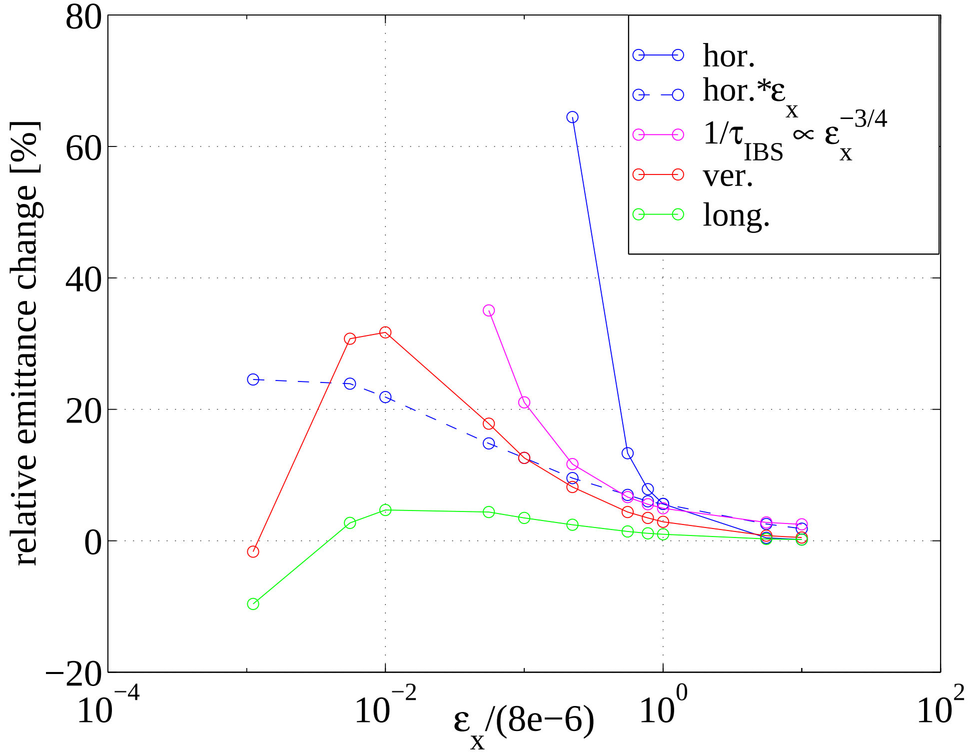
$$\frac{1}{\tau_{z;IBS}} \propto \frac{N_e}{\gamma^{3/2}\sigma_z \sigma_\epsilon^2(\varepsilon_{x;n}\varepsilon_{y;n})^{3/4}}$$

- The horizontal emittance is roughly proportional to $\mathcal{H}_x$ which means that with a decrease of the horizontal emittance the IBS scattering rates scale as $(\varepsilon_{x;n})^{-3/4}$
- The final equilibrium emittance is:

$$\frac{\Delta\varepsilon_{x,IBS}}{\varepsilon_{x,0}} = \frac{1}{1 - \frac{\tau_{x;D}}{\tau_{x;IBS}}}$$

Calculation of Intra Beam Scattering emittance Growth

- Calculation of the emittance growth due to IBS (assuming the damping times to be constant) with Bjoerken-Mitwinga theory using present TESLA DR
- Horizontal IBS scattering rate scaled with emittance (dashed curve) and scaling with $\varepsilon_x^{-3/4}$ (magenta curve) also given



Summary

- Incoherent space charge tune shift at TESLA damping ring can be up to 0.23 even at $5\,\mathrm{GeV}$
- $Q_{inc;SC} < 0.1$ seems to be tolerable
- Reduce space charge with local beam blow-up
- Simulations show that local coupling bump is successful