Emittance Control in the TESLA Damping Ring

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Abstract

This note describes the status of emittance control issues for the TESLA damping ring.

Introduction

The damping ring requires very low transverse emittances. Orbit control is crucial for sufficient performance. We describe a correction model and several error investigations for the TESLA damping ring. The damping ring parameters are:

Energy	5 GeV
Hor. equilibrium emittance	0.8 nm
Ver. Equilibrium emittance	0.0014 nm
Horizontal tune	71.31
Vertical tune	45.18

Model Calibration

Quadrupole misalignment

The simulation is done with 10 different error seeds for each of the different RMS values of quadrupole misalignment. Shown is the average over the different seeds.

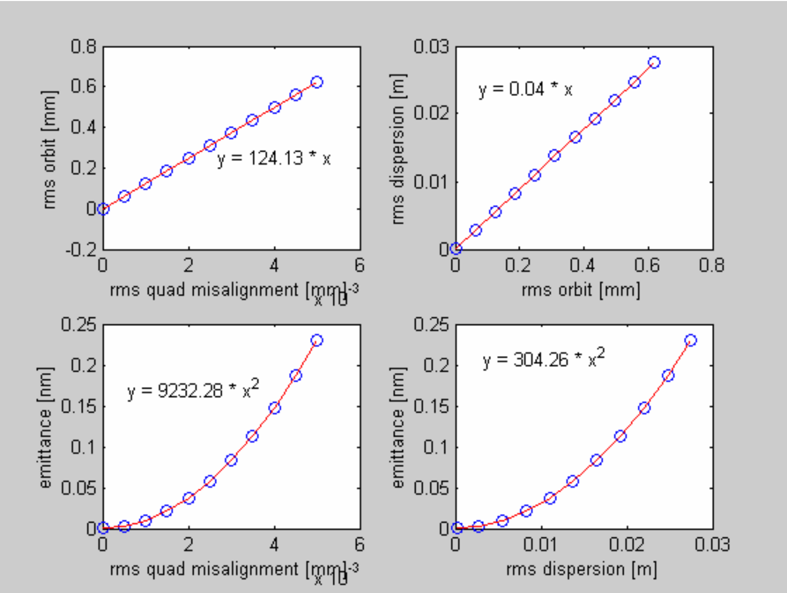

Figure 1 Sensitivity of orbit, dispersion and emittance to RMS quadrupole misalignments.

From this we conclude the following: an orbit RMS of 40 μm will lead to the design emittance. Once it is reached, an additional orbit drift of 13 μm will lead to an emittance increase of 10 %. This orbit change maybe induced by an additional RMS quadrupole jitter of 0.1 μm – a very small number.

Sextupole misalignment

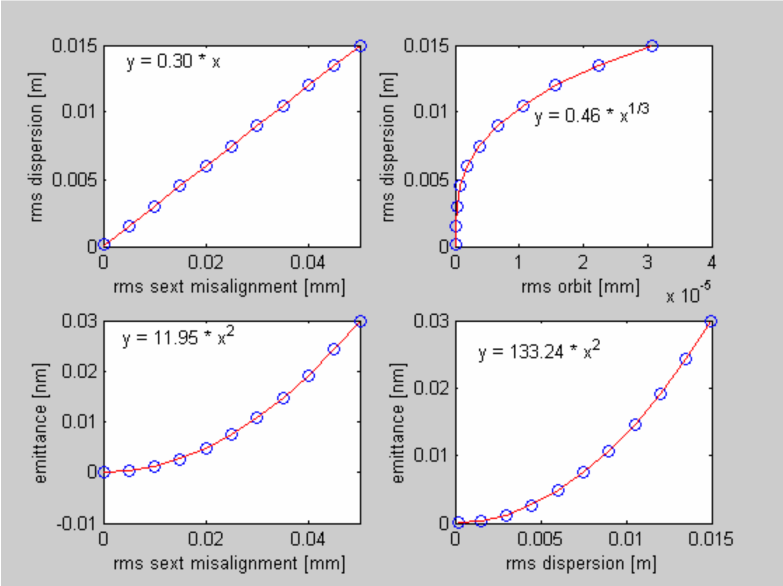

Figure 2 Sensitivity of dispersion and emittance to RMS sextupole misalignments.

From this we get that an RMS sextupole misalignment of 11 μm gives an equilibrium emittance of 0.0014 nm.

Quadrupole Roll

The next figure shows the sensitivity of emittance and dispersion to element roll. An RMS element roll (quadrupoles and sextupoles) of 26 μrad gives the required emittance.

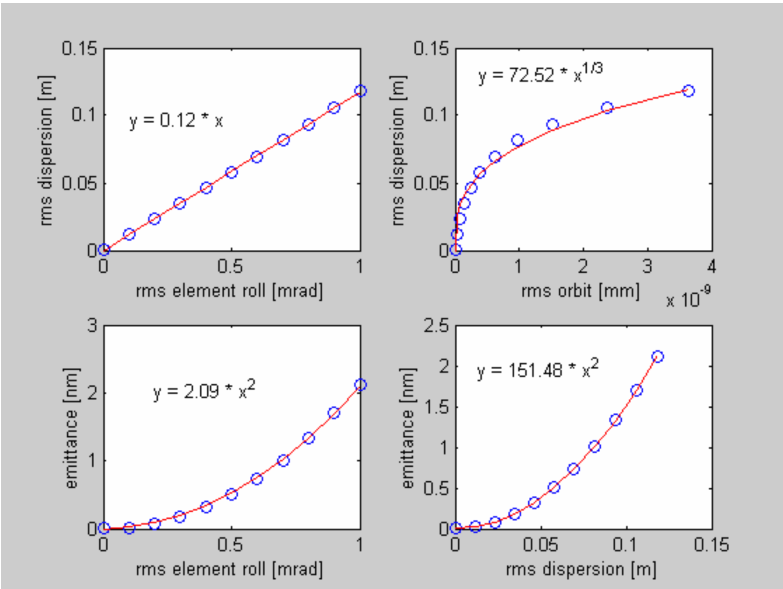

Figure 3 Sensitivity of orbit, dispersion and emittance to RMS quadrupole roll.

Finally we summarize the results from simulations and theoretical estimates based on the following formulas [1]:

Emittance generated only through dispersion in wiggler:

$$\varepsilon_y = 2J_E \frac{\left\langle D_y^2 \right\rangle_{wiggler}}{\left\langle \beta_y \right\rangle_{wiggler}} \sigma_\delta^2$$

Dispersion generated by sextupole offset *Y*:

$$\left\langle D_y^2 \right\rangle \approx \frac{\left\langle \beta_y \right\rangle}{8\sin^2 \pi\nu_y} \left(\sum_{sextupoles} \beta_y \left(k_2 l D_x\right)^2 \right) \left\langle Y_{sextupole}^2 \right\rangle$$

Closed orbit generated by quadrupole offset *Y*:

$$\left\langle y_{co}^2 \right\rangle \approx \frac{\left\langle \beta_y \right\rangle}{8\sin^2 \pi\nu_y} \left(\sum_{quadrupoles} \beta_y \left(k_1 l\right)^2 \right) \left\langle Y_{quadrupole}^2 \right\rangle$$

Dispersion generated by quadrupole roll Θ :

$$\left\langle D_y^2 \right\rangle \approx \frac{\left\langle \beta_y \right\rangle}{2\sin^2 \pi\nu_y} \left(\sum_{quadrupoles} \beta_y \left(k_1 l D_x\right)^2 \right) \left\langle \Theta_{quadrupole}^2 \right\rangle$$

Table 1 Comparison of simulated and analytical calculated misalignment sensitivities.

	Simulation	Analytic
RMS orbit/ RMS quad misalignment	125	118
RMS dispersion/ RMS sextupole misalignment	300	325
RMS dispersion/ RMS quadrupole roll	120	91
Ver. Emittance/ (RMS quad misalignment)^2	9.2 [m^{-1}]	62 [m^{-1}]
Ver. Emittance/ (RMS sextupole misalignment)^2	0.012 [m^{-1}]	0.012 [m^{-1}]
Ver. Emittance/ (RMS quad roll)^2	0.002 [m^{-1}]	0.001 [m^{-1}]

Note that to analytically calculate the sensitivity of emittance against RMS quad alignment we assume that the resulting RMS orbit is equivalent to an uncorrelated RMS sextupole misalignment – an assumption that is not true at all.

Correction Algorithm

The main target function is the vertical dispersion. It is corrected though a global correction algorithm.
A change in a vertical steerer $\Delta\Theta_y$ causes a change in the vertical dispersion due to the dispersion created by the steerer (small) and the new orbit in the sextupoles.
The correction is performed by simultaneously correcting the vertical orbit $\vec{Y}$ and dispersion $\vec{D}_y$ by minimizing it in a least square sense:

$$\begin{pmatrix} \Delta\vec{Y} \\ r\Delta\vec{D}_y \end{pmatrix} = \begin{bmatrix} \mathrm{M_Y} \\ r\,\mathrm{M_D} \end{bmatrix} \times \Delta\vec{\Theta}_y$$

with *r* a weight factor between orbit and dispersion correction.
In addition a skew quadrupole with skew trim coils on all sextupoles:

$$\Delta\vec{D}_y = \mathrm{M_S} \times \Delta\vec{S}$$

M_y is the matrix connecting the change in a corrector to a change of the closed orbit in the BPM, M_D is the matrix connecting the change in a corrector to a change of the dispersion in the BPM, M_S is the matrix connecting the change in a skew quadrupole to a change of the dispersion in the BPM. The weight factor *r* allows distributing the correction between orbit and dispersion.

The present TESLA DR model is equipped with a horizontal and vertical BPM at each quadrupole, a horizontal and vertical corrector at each quadrupole and a skew quadrupole at each sextupole. For the analysis the following initial alignments have been assumed:

Element	horizontal	vertical
Quadrupole	0	0.1 mm
Sextupole	0	0.1 mm
BPM resolution	0	1 μm
BPM (relative to quadrupole)	0	0.1 mm

The correction is performed in the following way:

- Starting from the initial closed orbit usually up to five iterations are needed to correct only the orbit to a value below 0.5 mm RMS.
- After that the orbit and dispersion is corrected by correcting the dispersion to the design vertical dispersion (non-zero in the long straight sections) and the orbit to the previously achieved orbit. The weight factor *r* is set to 1. After approximately 5 iterations the RMS dispersion is of the order of 0.5 mm and the orbit at 0.4 mm RMS.
- In some cases an additional coupling correction (with the theoretical dispersion as the target function) can reduce the resulting emittance even further.

Figure 4 shows the distribution for 100 random seed of initial misalignments.

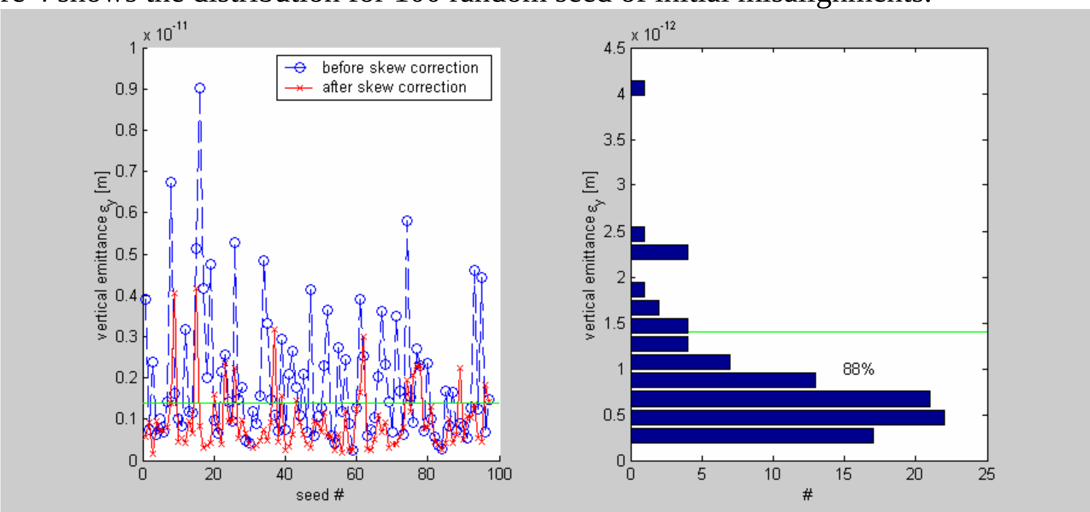

Figure 4 Left: Vertical emittance after orbit and dispersion correction (blue curve) and additional skew correction (red curve) for 100 different seeds of magnet and monitor misalignments. **Right:** Distribution of smallest vertical emittances after correction.

Orbit Jitter

To study the effects of orbit jitter on the emittance we start from a corrected machine and successively add additional quadrupole and sextupole misalignments to the existing misalignments. We simulate for 30 seeds at which additional RMS values an increase of the emittance above 0.0014 nm occurs.

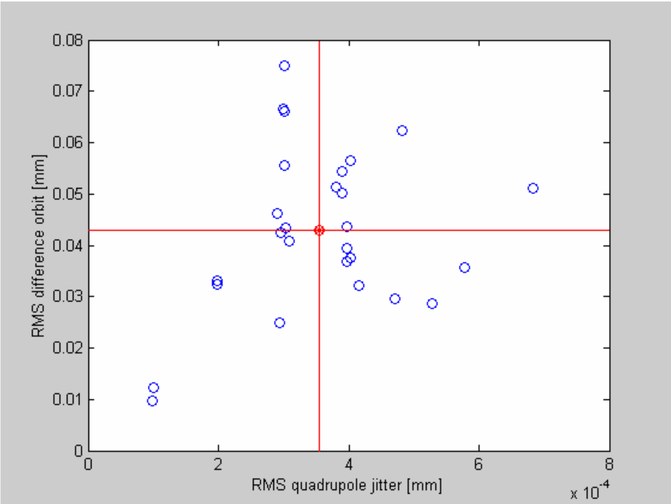

Figure 5 RMS additional quadrupole jitter on top of corrected machine leading to a 10% emittance increase above 0.0014 nm versus RMS difference orbit.

The average starting emittance before applying the additional jitter was 0.00065 nm, the jitter amplitude was increase until 0.00154 nm have been reached, on average a $\Delta\varepsilon$ of 0.0009 nm. On average an RMS quadrupole jitter of 0.35 µm (in agreement for the previously given 0.1 µm for a $\Delta\varepsilon$ of 0.00014 nm) respectively an RMS orbit change with respect to the previously corrected orbit of 43 µm increases the emittance to 0.00154 nm.

In the next step an orbit correction is applied when the RMS of the difference orbit to the initial corrected orbit (the ‘golden orbit’) increases above 35 µm. Because of the ‘ideal’ conditions the orbit correction usually converges to a residual difference orbit is of the order of 1 µm RMS, given by the finite BPM resolution. In all cases (10 seeds) the orbit correction succeeds in restoring the vertical emittance.
The simulation is repeated with a 10 µm BPM resolution. As expected the RMS difference orbit after correction is about 10 µm, but again the vertical emittance is restored.

Finally we repeat the simulations with much larger quadrupole and sextupole offsets, to gauge the ‘sanity’ of the golden orbit. We try additional 10 µm RMS, and the average emittance is 0.0022 nm after correction back to the golden orbit.
With additional 6 µm RMS magnet misalignment the average emittance is 0.0014 nm for the 10 seeds investigated. This means that an uncorrelated additional magnet misalignment of up to 6 µm is correctable with simple orbit correction back to the once established ‘golden’ orbit. This corresponds to an RMS difference orbit of about
0.5 mm.

A rough argument may allow estimating the time scale for corrections. In a ring with FODO lattice the RMS quadrupole misalignment according to the *ATL* law scales with $L = \frac{L_{ring}}{4Q_y}$, i.e. with a quarter of the betatron wavelength. With A=1×10^{-17} [m^2/(m*s)] one gets for the time scale $T = \left\langle Y^2_{quadrupole} \right\rangle * 1.1\times10^{15}\,\mathrm{s/m}^2$. A dispersion correction is thus needed every 11 h or so, while an orbit correction is needed every 2 minutes. This is only a very rough scaling, but agrees with ATL simulations [2].

Quadrupole Roll

Adding quadrupole roll as additional coupling source does show the importance of the final skew correction step. Figure shows the correction procedure as described above with additional RMS element roll (quadrupoles and sextupoles). We derive a tilt tolerance of approx. 0.3 mrad.

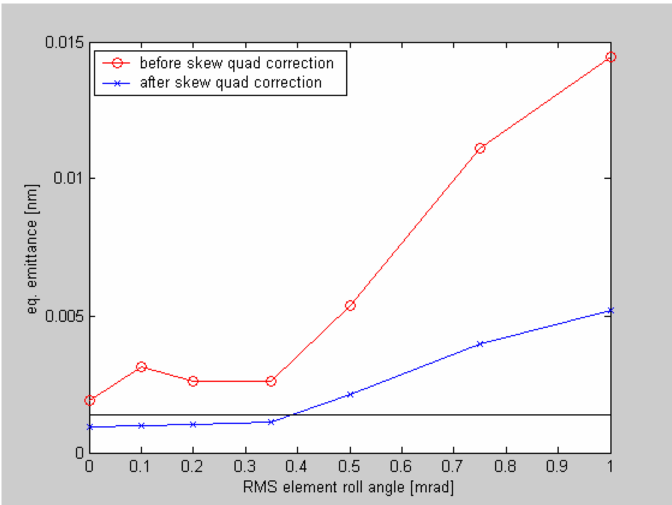

Figure 6 Equilibrium vertical emittance versus element roll angle on top of corrected machine. Each point represents the average of 10 error seeds.

Faulty BPM's and Correctors

The goal is to answer some of the reliability considerations. 10 % of the monitors and correctors are randomly switched off so that they are unavailable for the correction procedure. Because of the large number of elements (946) there is sufficient redundancy within the system to allow for successful orbit correction. The simulation as described in chapter 0 was repeated for 20 different error seeds of magnet misalignments and adding 10 % of disabled monitors and correctors. An emittance of 0.00083 nm is reached on average. The present correction scheme thus is sufficiently robust.

Optic Errors

Convergence of the correction scheme depends strongly on the knowledge of the optical functions. To study the sensitivity against optics errors we randomly changed the quadrupole strength of all quads such that the new β–function differs by RMS 5% from the design β. The tune is re-matched to give the design values. The two quadrupole circuits in the dispersion suppressor are used to minimize the residual horizontal dispersion.
The following simulation includes quadrupole roll of 0.2 mrad as well as 10 % switched off BPM's and correctors. 5 different optic error seeds with 20 different misalignment seeds each have been simulated. The results are shown in Figure 7.

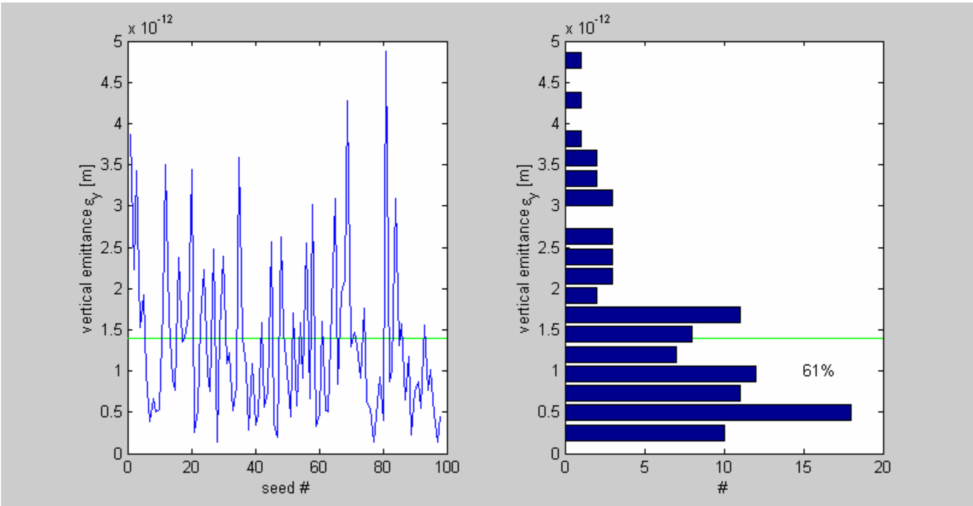

Figure 7 Left: Minimum vertical emittance after orbit, dispersion correction and additional skew correction for 5 different seeds of quadrupole gradient errors and 20 different seeds of magnet and monitor misalignments. **Right:** Distribution of smallest vertical emittances after correction.

As expected the orbit correction converges much slower. About 15-20 correction steps are needed to reach the target values for closed orbit and dispersion. The convergence could be improved by using the measured response matrices M_y, D_s instead of the design ones.
Only 61% of the seeds are below the target value of 0.0014 nm, compared to 88% for the machine without optical errors.

Conclusion

A global correction scheme has proven to be sufficient to reach the vertical design emittance. Further studies should include the fully coupled lattice and horizontal errors as well. In addition empirical emittance tuning with dispersion bumps and skew correctors should be studied.

Acknowledgements

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References

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