Excercise Sheet 10 to General Relativity

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Discussion on 25.01.2013 in the exercise classes

1. **The Harmonic Gauge**

   Show that the harmonic gauge condition on the metric perturbation,

$$\partial_\nu h^{\nu\mu} = \frac{1}{2}\partial^\mu h^\nu{}_\nu \tag{1}$$

   is equivalent to the condition

$$\Box x^\mu = \nabla^\nu \nabla_\nu x^\mu = 0\,, \tag{2}$$

   where x^μ is considered as a scalar function on the manifold.

2. **Gravitational Waves from a Head-On Collision**

   Two identical point particles of mass M undergo a head-on collision at the origin of the coordinate system, $x^\mu = (t, x, y, z) = (0, 0, 0, 0)$. In the distant past, $t \to -\infty$, the two masses started at $x^\mu \to (-\infty, \pm\infty, 0, 0)$ and at rest.

   (a) Show that $x(t) = \pm(9G_{\rm N}Mt^2/8)^{1/3}$ in Newtonian approximation.

   (b) Determine the range of validity of the Newtonian approximation.

   (c) Calculate the quadrupole moment $Q_{ij}(t)$.

   (d) Using the result in (b) compute $h_{ij}(t)$ at the position $(x, y, z) = (0, R, 0)$.

   please turn over

3. **de Sitter Space**

Consider the metric

$$ds^2 = dt^2 - e^{2Ht}\left(dx^2 + dy^2 + dz^2\right)\,. \tag{3}$$

Solve the geodesic equation for observers moving with respect to the spatial coordinates, i.e. x is not constant (so called non-comoving observers) and determine the affine parameter as a function of t. From this show that the geodesics reach $t = -\infty$ within a finite interval of the affine parameter. This demonstrates that the coordinates in Eq. (3) are incomplete and only describe part of what is known as *de Sitter space* in cosmology.