Excercise Sheet 1 to General Relativity

Prof. Günter Sigl
II. Institut für Theoretische Physik der Universität Hamburg
Luruper Chaussee 149
D-22761 Hamburg
Germany
email: sigl@mail.desy.de
tel: 040-8998-2224

Discussion on 26.10.2012 in the exercise classes

1. **Properties of Lorentz Transformations in the Theory of Special Relativity**
   Consider a general Lorentz transformation

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu}$$

with the matrix $\Lambda^{\mu}{}_{\nu}$ satisfying

$$\eta_{\mu\nu} \Lambda^{\mu}{}_{\rho} \Lambda^{\nu}{}_{\sigma} = \eta_{\rho\sigma}$$

in terms of the *Lorentz metric tensor*

$$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1) \,.$$

(a) Show explicitly that the four dimensional scalar product

$$a \cdot b \equiv \eta_{\mu\nu} a^{\mu} b^{\nu} = a_{\mu} b^{\mu}$$

is invariant under such Lorentz transformations.
(b) Show that the inverse Lorentz transformation is given by

$$\left(\Lambda^{-1}\right)^{\mu}{}_{\nu} = \eta^{\mu\rho} \eta_{\nu\sigma} \Lambda^{\sigma}{}_{\rho} \,.$$

please turn over

2. **Four Dimensional Acceleration in Special Relativity**

(a) Derive the relation

$$a^\mu = \frac{d^2 x^\mu}{d\tau^2} = \Gamma^2 \left[\Gamma^2 \mathbf{v} \cdot \mathbf{a}, \mathbf{a} + \Gamma^2 (\mathbf{v} \cdot \mathbf{a}) \, \mathbf{v} \right] \tag{1}$$

between the four acceleration a^μ and the ordinary three dimensional acceleration $\mathbf{a} \equiv d^2\mathbf{r}/dt^2$ and velocity $\mathbf{v} \equiv d\mathbf{r}/dt$.

(b) Show that the *proper acceleration* $\mathbf{a}_0^2 \equiv -a_\mu a^\mu$ is given by

$$\mathbf{a}_0^2 \equiv -a_\mu a^\mu = \Gamma^4 \left[\mathbf{a}^2 + \Gamma^2 (\mathbf{v} \cdot \mathbf{a})^2 \right] . \tag{2}$$

3. **Constant Force in Special Relativity**

Consider a relativistic point particle with non-vanishing rest mass m_0 which is subject to a constant force $\mathbf{F} = (d\mathbf{p}/dt) = F\mathbf{e}_z =$const in the $z-$direction. The initial conditions are given by $z(0) = z_0$, $v(0) = 0$. Derive the world line $z(t)$ and sketch it on a Minkowski space-time diagram.