Axions

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Werkstattseminar

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Plan:

1) Theta-term in QCD

2) Strong CP violation
   - $d_n(\bar{\theta})$

3) Solutions of strong CP puzzle

4) Axions from UV completions

5) Axion Cold Dark Matter

1) Theta-term in QCD

Most general gauge invariant Lagrangian of QCD up to dim 4 operators:

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu} + \bar{q}\left(i\not{D} - \mathcal{M}_q\right) q + \underbrace{\theta \frac{\alpha_s}{8\pi} G^a_{\mu\nu} \tilde{G}^{a,\mu\nu}}_{\text{Theta-term}}$$

$\left(\tilde{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} G^{\lambda\rho}\right)$

Theta-term

- total derivative:

$$G\tilde{G} = \partial_\mu \underset{\substack{\uparrow \\ \text{Chern-Simons} \\ \text{current}}}{K^\mu}$$

→ does not contribute in perturbation theory

- K^μ not invariant under "large", topol. non-trivial gauge transformations

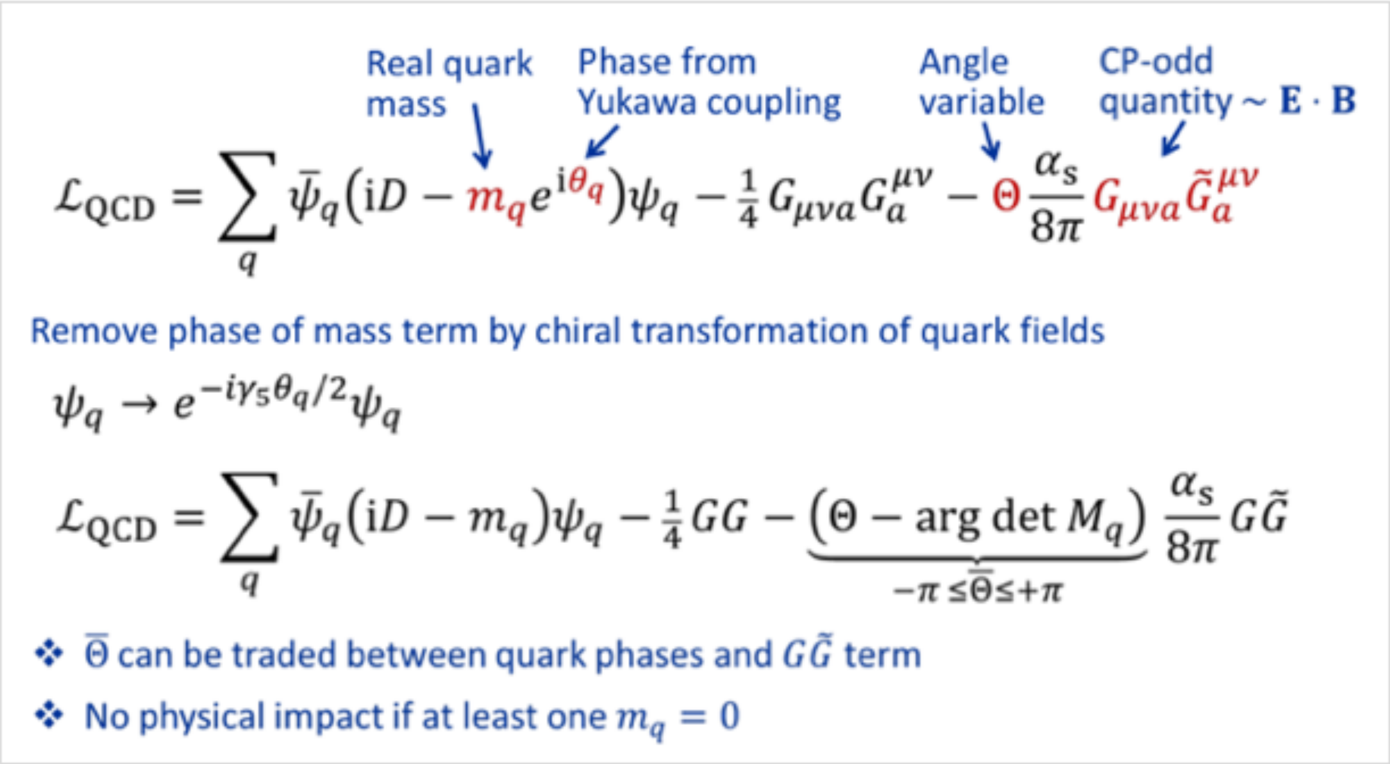
→ ∃ angular par., $-\pi \leq \theta \leq \pi$

→ plays role non-perturbatively

− θ belongs to the fundamental parameters of QCD, on similar footing as α_s and the quark masses m_u, $m_d, \dots$, which have to be determined experimentally.

- In fact, the actual physical parameter is

$$\bar{\Theta} = \Theta + \arg\det M_q$$



Real quark mass

Phase from Yukawa coupling

Angle variable

CP-odd quantity ~ **E · B**

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{\psi}_q(\mathrm{i}D - m_q e^{\mathrm{i}\theta_q})\psi_q - \tfrac{1}{4} G_{\mu\nu a} G_a^{\mu\nu} - \Theta \frac{\alpha_s}{8\pi} G_{\mu\nu a} \tilde{G}_a^{\mu\nu}$$

Remove phase of mass term by chiral transformation of quark fields

$$\psi_q \rightarrow e^{-i\gamma_5\theta_q/2}\psi_q$$

$$\mathcal{L}_{\mathrm{QCD}} = \sum_q \bar{\psi}_q(\mathrm{i}D - m_q)\psi_q - \tfrac{1}{4} GG - \underbrace{(\Theta - \arg\det M_q)}_{-\pi \leq \bar{\Theta} \leq +\pi} \frac{\alpha_s}{8\pi} G\tilde{G}$$

- $\bar{\Theta}$ can be traded between quark phases and $G\tilde{G}$ term
- No physical impact if at least one $m_q = 0$

$\bar{\Theta}$ unphysical if at least one $m_q = 0$

- The chiral anomaly

$$\partial_\mu \left(\bar{q} \gamma_\mu \gamma_5 q \right) = n_f \frac{\alpha_s}{8\pi} \underline{\underline{G \tilde{G}}} + 2i \underline{\underline{\left[\bar{q}_R m_q q_L - h.c. \right]}}$$

allows to shuffle contributions between

$\underline{\underline{G\tilde{G}}} \leftrightarrow \underline{\underline{\text{imaginary quark masses}}}$

2) Detemination of $\bar{\theta}$

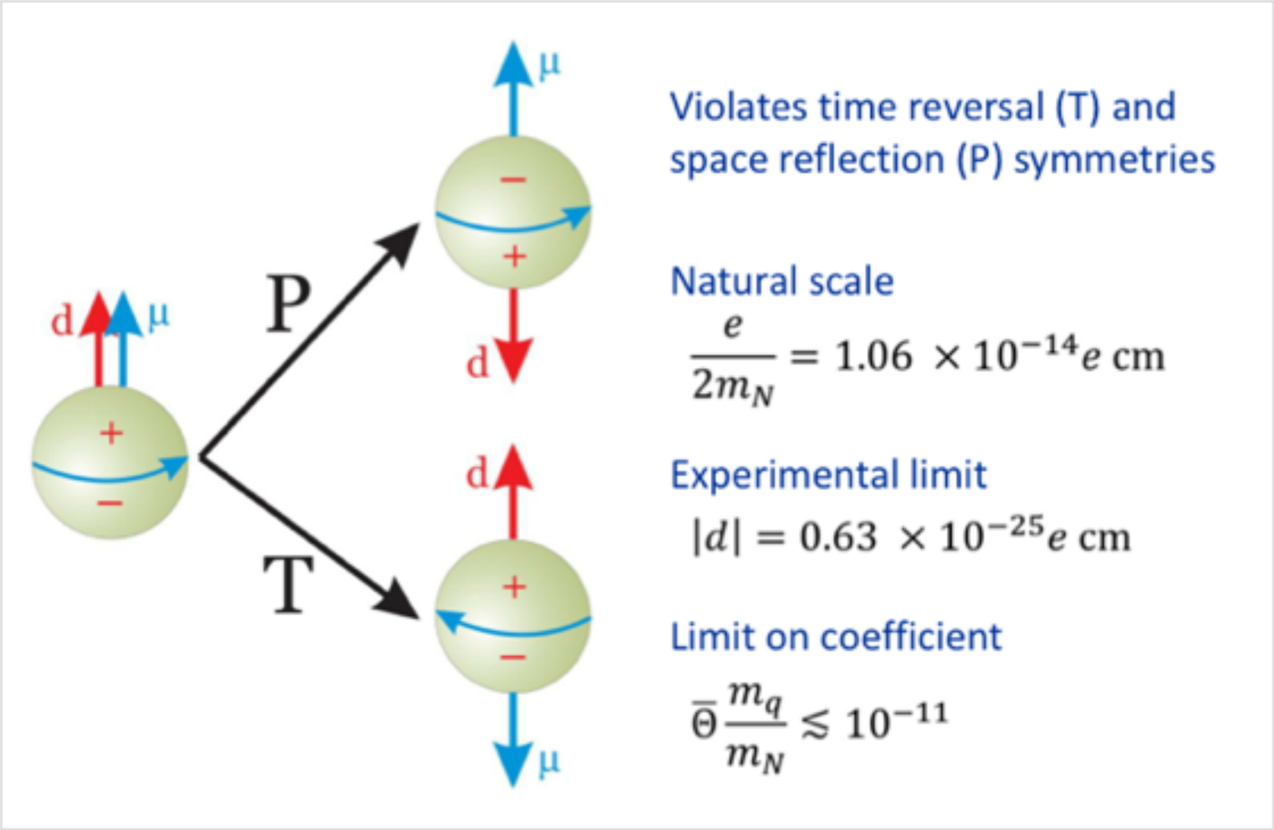
- Theta-term ($G\tilde{G} \sim E \cdot B$) violates P an CP

-> leads to CP violation in flavour conserving interactions in contrast to CKM phase which leads to CP violation in flavour changing interactions

- Electric dipole moment (EDM) of neutron:
  very sensitive probe of CP violation in flavour-conserving interactions

- neutral non-rel. particle placed in E and B field described by

$$H = -\underset{\substack{\uparrow \\ \boxed{\text{MDM}}}}{\mu}\,\vec{B}\cdot\frac{\vec{S}}{S} - \underset{\substack{\uparrow \\ \boxed{\text{EDM}}}}{d}\,\vec{E}\cdot\frac{\vec{S}}{S}$$



Violates time reversal (T) and space reflection (P) symmetries

Natural scale

$$\frac{e}{2m_N} = 1.06 \times 10^{-14} e \text{ cm}$$

Experimental limit

$$|d| = 0.63 \times 10^{-25} e \text{ cm}$$

Limit on coefficient

$$\bar{\Theta}\frac{m_q}{m_N} \lesssim 10^{-11}$$

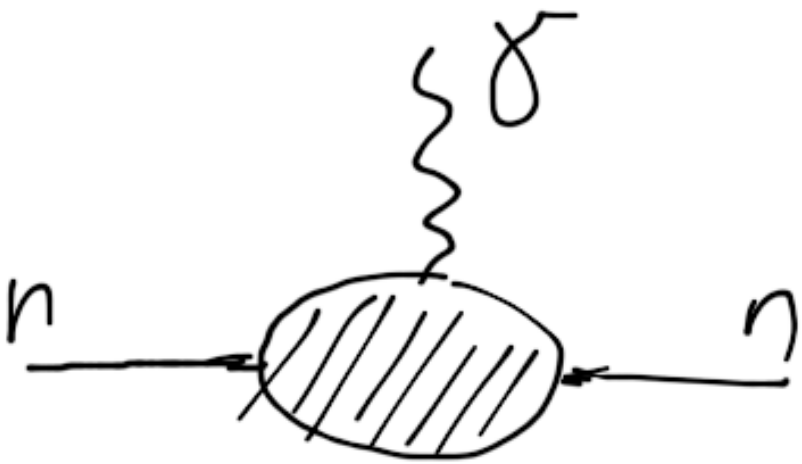
- Nonzero $d_n \Rightarrow$ both $\not{P}$ and $\not{T}$
- Conclusion about $\not{CP}$ relies on validity of CPT

- Operator definition:

$$H_{T,P-odd} = - d_n \vec{E} \cdot \frac{\vec{S}_n}{S_n}$$

$$\Longrightarrow$$

$$\mathcal{L} = - d_n \frac{i}{2} \bar{\psi}_n \sigma^{\mu\nu} \gamma_5 \psi_n F_{\mu\nu}$$



- Calculations of $d_n(\bar{\theta})$:

• educated guess:

$$d_n(\bar{\theta}) \sim e\bar{\theta}\,\frac{m_*}{m_n^2}$$

$$\sim \bar{\theta}\,(6\times10^{-17})\,\text{ecm}$$

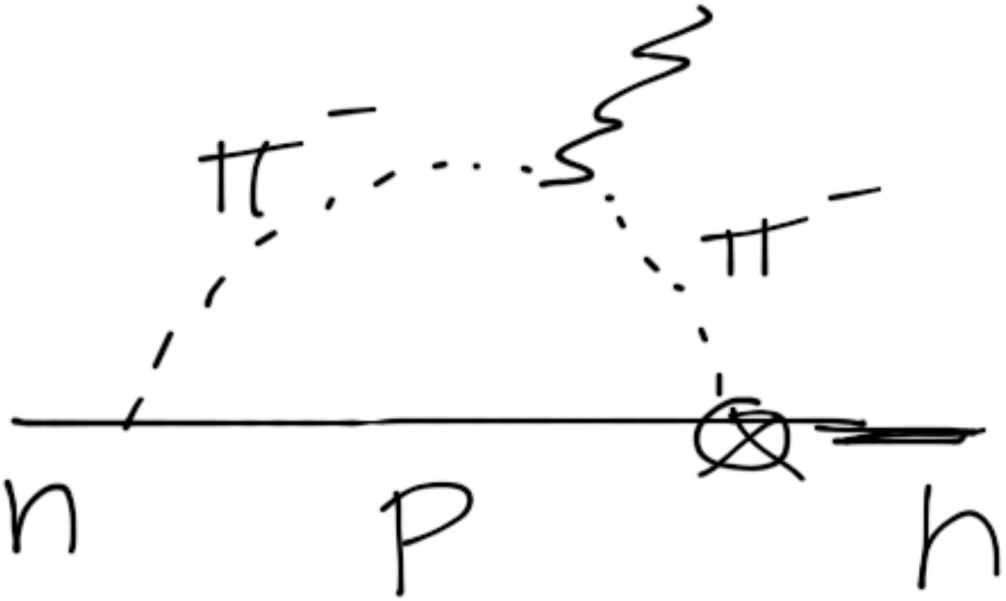
with reduced quark mass

$$m_* = \frac{m_u m_d}{m_u + m_d}$$

- Chiral estimate:

[Crewther, di Vecchia, Veneziano, Witten '79]

$$d_n(\bar{\theta}) = \frac{e\,\bar{\theta}\,m_*}{f_\pi^2} \times \left(\frac{0.9}{4\pi^2} \ln\left(\frac{\Lambda}{m_\pi}\right) + C \right)$$



- Sum rules

$$d_n(\bar{\theta}) = 1.2 \pm 0.5 \times 10^{-16} \, \bar{\theta} \, e \, \mathrm{cm}$$

[Pospelov, Ritz 00]

update:

[Hisano et al. 12]

$$d_n(\bar{\theta}) = 4.2 \times 10^{-17} \, \bar{\theta} \, e \, \mathrm{cm}$$

- CP puzzle:

expectation

$$d_n(\bar{\theta}) \sim 10^{-16}\, \bar{\theta}\ ecm$$

experimental limit:

$$|d_n| < 2.9 \times 10^{-26}\ ecm.$$

i.e.:

$$\boxed{|\bar{\theta}| \lesssim 10^{-10}}$$

3) Solutions of strong CP puzzle

- $m_u = 0$

  inconsistent with quark mass ratios inferred from hadron phenomenology and lattice

- Engineering $\bar{\theta} \simeq 0$:

  assume that at high scales P and CP exact, spontaneously broken at $\Lambda_{P(CP)}$

$$\bar{\theta}_{E>\Lambda_{P(CP)}} \equiv 0$$

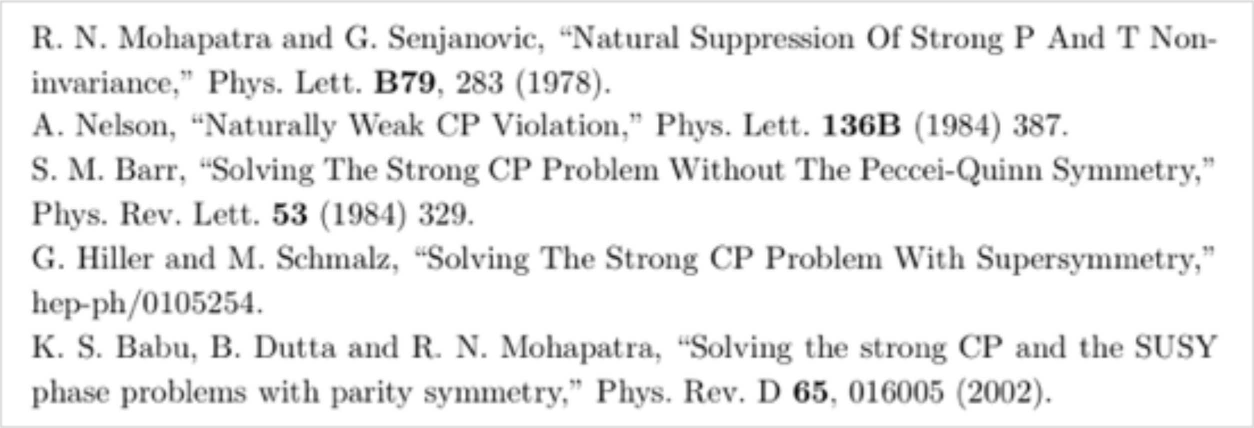
Model engineering problem is then to ensure that corrections below $\Lambda_{P(CP)}$ to $\bar{\theta}$,

$$\bar{\theta}_{E<\Lambda_{P(CP)}} \sim \arg\det(M_u M_d)$$

are small while still allowing for $\mathcal{O}(1)$ CKM phase,

$$\theta_{CKM} \sim \arg\det\left[M_u M_u^\dagger, M_d M_d^\dagger\right]$$

Models of this kind:



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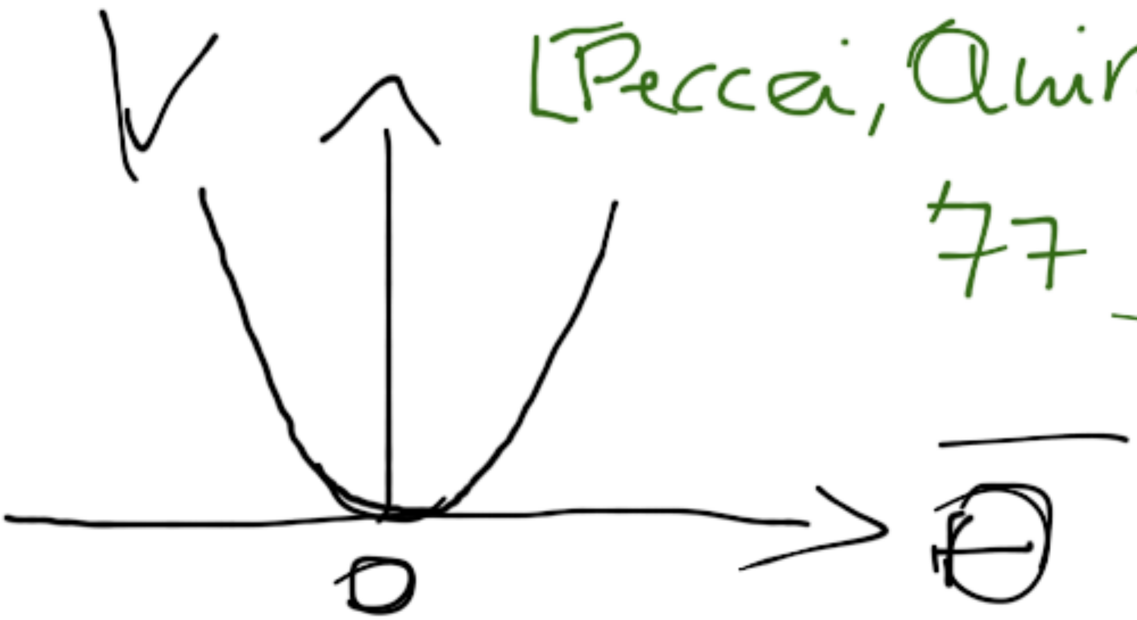
K. S. Babu, B. Dutta and R. N. Mohapatra, "Solving the strong CP and the SUSY phase problems with parity symmetry," Phys. Rev. D **65**, 016005 (2002).

- Dynamical relaxation

- If $\bar{\theta}$ were a field $\bar{\theta}(x)$ rather than a parameter, then QCD dynamics would lead to $\langle \bar{\theta} \rangle = 0$.



[Peccei, Quinn '77]

- Effective potential of $\bar{\theta}(x)$ can be obtained from chiral Lagrangian
  - Eliminate $\bar{\theta} G \tilde{G}$ in favor of phase of e.g. up quark mass, $m_u \rightarrow m_u e^{i\bar{\theta}}$
  - The low energy dynamics of this theory is described by the chiral Lagrangian:

$$\mathcal{L} = \frac{f_\pi^2}{4} \mathrm{tr}\left[\partial_\mu U \partial^\mu U^{-1}\right] + \frac{f_a^2}{2} \partial_\mu \bar{\theta} \partial^\mu \bar{\theta}$$

$$+ \frac{f_\pi^2}{2} \mu \,\mathrm{tr}\left[M U + \bar{M} U^{-1}\right]$$

with $$U = \exp\left[2i \frac{\tau \pi}{f_\pi}\right] \in SU(2)_F$$

$$\mu = \frac{m_\pi^2}{m_u + m_d}$$

$$M = \begin{pmatrix} m_u \boxed{e^{i\bar{\theta}}} & 0 \\ 0 & m_d \end{pmatrix}$$

$$V(\bar{\theta}) =$$

$$\min_{\substack{U \\ \text{for fixed} \\ \bar{\theta}}} \left[-\frac{f_\pi^2 \mu}{2} \operatorname{tr}\left[M U + \bar{M} U^{-1} \right] \right]$$

$$= \frac{f_\pi^2 m_\pi^2}{2} \frac{m_u m_d}{(m_u + m_d)^2} \bar{\theta}^2$$

$$+ \mathcal{O}(\bar{\theta}^4)$$

$\Rightarrow$ minimum at $\bar{\theta} = 0$

$\Rightarrow$

- Dynamical $\bar{\theta}(x) \equiv \frac{\vec{a}(x)}{f_a}$

wipes out strong problem;

$$\langle \bar{\theta} \rangle = \frac{\langle \bar{a} \rangle}{f_a} = 0$$

- How to realize a dynamical $\bar{\theta}$ parameter?

Add to SM a boson with satisfying shift symmetry

$$a(x) \longrightarrow a(x) + \text{const.}$$

which is only violated by axionic coupling to gluons,

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a}{f_a}\frac{\alpha_s}{8\pi} G\tilde{G}$$

Then the constant $\bar{\theta}$ can be eliminated by

$$\bar{\theta} + \frac{a(x)}{f_a} \equiv \frac{\bar{a}(x)}{f_a} \equiv \bar{\theta}_{eff}(x)$$

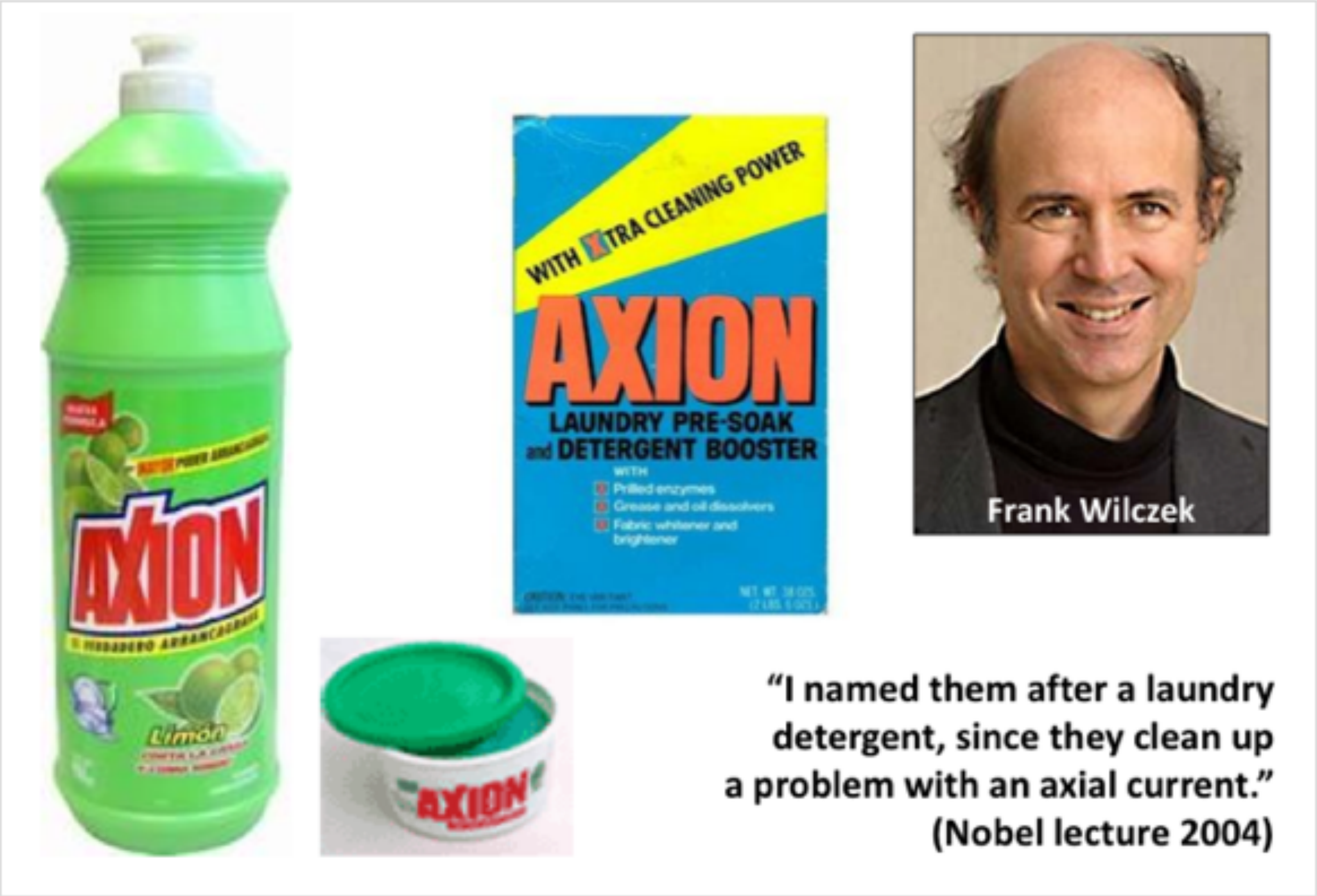
and the QCD dynamics leads to $\langle \bar{a} \rangle = 0$, as demonstrated before.

$a(x)$ … axion field

↑

name of detergent in US

[Wilczek 78]

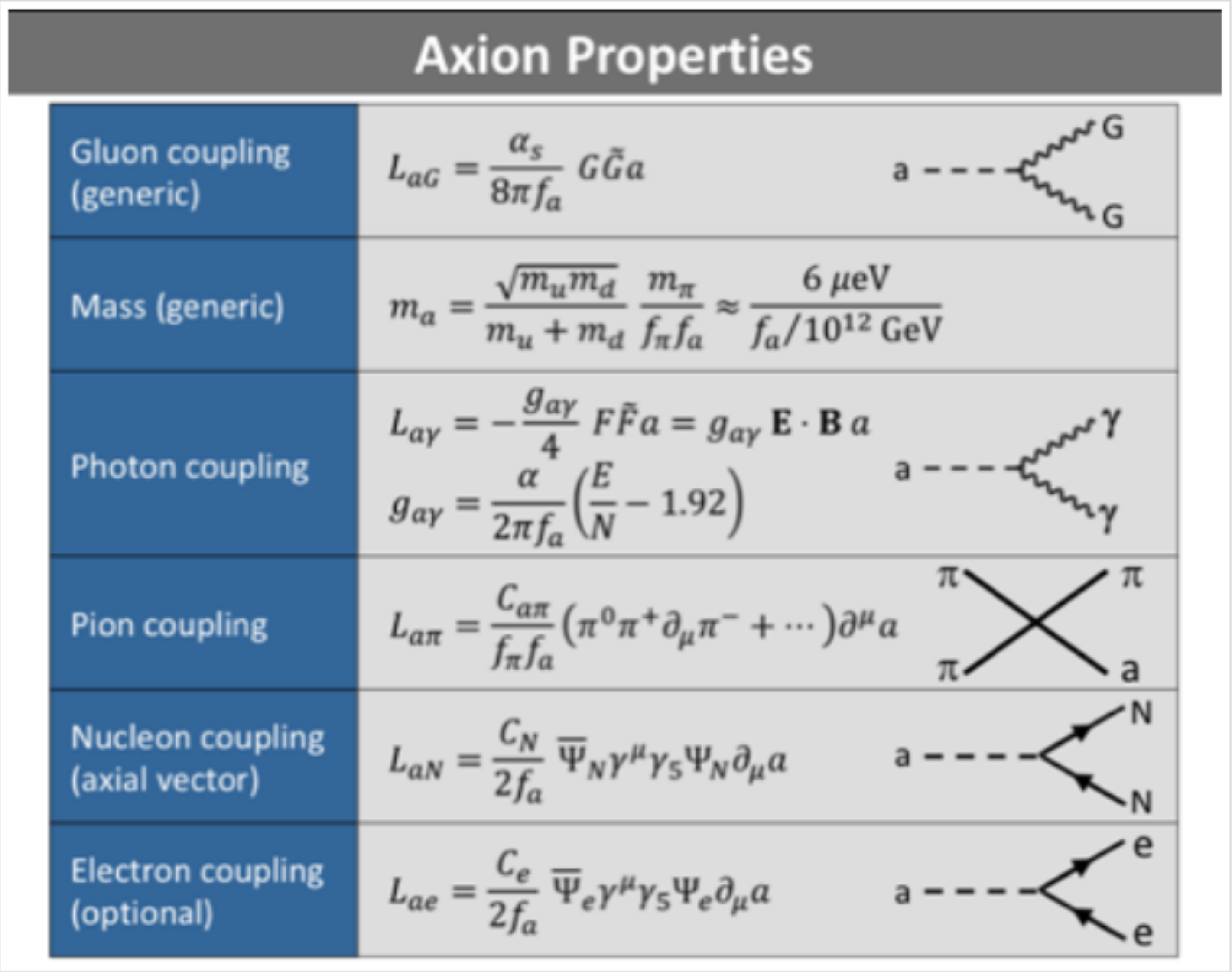


"I named them after a laundry detergent, since they clean up a problem with an axial current."
(Nobel lecture 2004)

- Mass of elementary particle excitation around $\langle \bar{a} \rangle = 0$, the axion, can be read off the quadratic term in V.

$$m_a = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{(m_u + m_d)}$$

[Weinberg 78]



Axion Properties

Gluon coupling (generic)	$L_{aG} = \frac{\alpha_s}{8\pi f_a}\, G\tilde{G}a$	a, G, G
Mass (generic)	$m_a = \frac{\sqrt{m_u m_d}}{m_u + m_d}\,\frac{m_\pi}{f_\pi f_a} \approx \frac{6\ \mu\mathrm{eV}}{f_a/10^{12}\ \mathrm{GeV}}$	
Photon coupling	$L_{a\gamma} = -\frac{g_{a\gamma}}{4}\, F\tilde{F}a = g_{a\gamma}\, \mathbf{E}\cdot\mathbf{B}\, a$ $g_{a\gamma} = \frac{\alpha}{2\pi f_a}\left(\frac{E}{N} - 1.92\right)$	a, γ, γ
Pion coupling	$L_{a\pi} = \frac{C_{a\pi}}{f_\pi f_a}\left(\pi^0\pi^+\partial_\mu\pi^- + \cdots\right)\partial^\mu a$	π, π, π, a
Nucleon coupling (axial vector)	$L_{aN} = \frac{C_N}{2f_a}\, \overline{\Psi}_N\gamma^\mu\gamma_5\Psi_N\partial_\mu a$	a, N, N
Electron coupling (optional)	$L_{ae} = \frac{C_e}{2f_a}\, \overline{\Psi}_e\gamma^\mu\gamma_5\Psi_e\partial_\mu a$	a, e, e

Constraints from astrophysics:

$$f_a \gtrsim 10^{8\div 9}\ \mathrm{GeV}$$ ✓

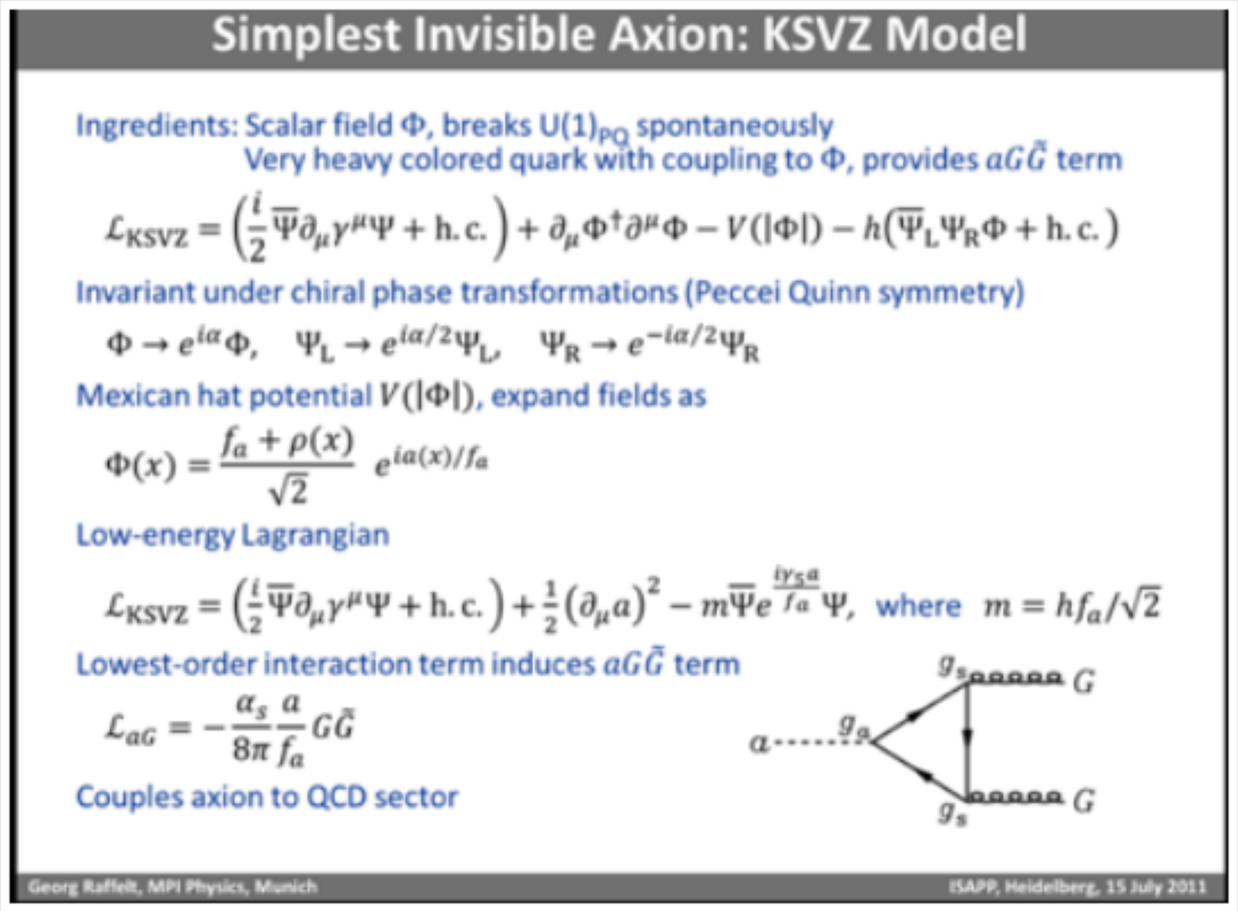
4) Axion from UV completions

- Axion from new Higgs fields.

Postulate new global U(1) symmetry, which is spontaneousley broken at scale f_a through vev of a Higgs field

$$\phi = \frac{f_a + \rho(x)}{\sqrt{2}} e^{i\frac{a(x)}{f_a}}$$

Engineer that Nambu-Goldstone field $a(x)$ has anomalous coupling $\sim aG\tilde{G}$ arising from triangle graph $\rightarrow$ a is axion.



Simplest Invisible Axion: KSVZ Model

Ingredients: Scalar field Φ, breaks $U(1)_{PQ}$ spontaneously
Very heavy colored quark with coupling to Φ, provides $aG\tilde{G}$ term

$$\mathcal{L}_{\rm KSVZ} = \left(\frac{i}{2}\overline{\Psi}\partial_\mu\gamma^\mu\Psi + {\rm h.c.}\right) + \partial_\mu\Phi^\dagger\partial^\mu\Phi - V(|\Phi|) - h\left(\overline{\Psi}_{\rm L}\Psi_{\rm R}\Phi + {\rm h.c.}\right)$$

Invariant under chiral phase transformations (Peccei Quinn symmetry)

$$\Phi \to e^{i\alpha}\Phi, \quad \Psi_{\rm L} \to e^{i\alpha/2}\Psi_{\rm L}, \quad \Psi_{\rm R} \to e^{-i\alpha/2}\Psi_{\rm R}$$

Mexican hat potential $V(|\Phi|)$, expand fields as

$$\Phi(x) = \frac{f_a + \rho(x)}{\sqrt{2}}\, e^{ia(x)/f_a}$$

Low-energy Lagrangian

$$\mathcal{L}_{\rm KSVZ} = \left(\frac{i}{2}\overline{\Psi}\partial_\mu\gamma^\mu\Psi + {\rm h.c.}\right) + \frac{1}{2}\left(\partial_\mu a\right)^2 - m\overline{\Psi}e^{\frac{i\gamma_5 a}{f_a}}\Psi, \quad \text{where} \quad m = hf_a/\sqrt{2}$$

Lowest-order interaction term induces $aG\tilde{G}$ term

$$\mathcal{L}_{aG} = -\frac{\alpha_s}{8\pi}\frac{a}{f_a}G\tilde{G}$$

Couples axion to QCD sector

Georg Raffelt, MPI Physics, Munich

ISAPP, Heidelberg, 15 July 2011

Kim

Shifman, Vainstein, Zakharov

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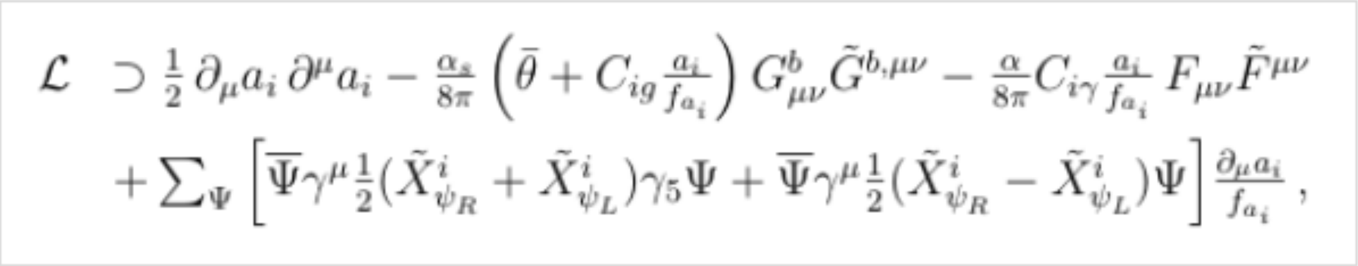
- Axions from string theory:

- 4D low energy EFT predicts natural candidates for axion
- axions and ALPs arise as KK zero modes of anti-symmetric form fields bel. to massless spectrum of bosonic string
- axionic couplings to gauge fields $a G\tilde{G}$, $a F\tilde{F}$, ..., also predicted from dimensional reduction of higher dimensional action to four dimensions.

- Often an <u>axiverse</u> (#axions ~ #cycles): axion + many axion-like particles (ALPs):



$$\begin{aligned}\mathcal{L} \;&\supset \tfrac{1}{2}\,\partial_\mu a_i\,\partial^\mu a_i - \tfrac{\alpha_s}{8\pi}\left(\bar\theta + C_{ig}\tfrac{a_i}{f_{a_i}}\right) G^b_{\mu\nu}\tilde{G}^{b,\mu\nu} - \tfrac{\alpha}{8\pi}C_{i\gamma}\tfrac{a_i}{f_{a_i}}\,F_{\mu\nu}\tilde{F}^{\mu\nu} \\ &+ \textstyle\sum_\Psi \left[\overline{\Psi}\gamma^\mu\tfrac{1}{2}(\tilde{X}^i_{\psi_R} + \tilde{X}^i_{\psi_L})\gamma_5\Psi + \overline{\Psi}\gamma^\mu\tfrac{1}{2}(\tilde{X}^i_{\psi_R} - \tilde{X}^i_{\psi_L})\Psi\right]\tfrac{\partial_\mu a_i}{f_{a_i}}\,,\end{aligned}$$

- $f_a \sim f_{a_i} \sim 10^{9 \div 16}$ GeV

Review:

AR, 1209.2299

5) Axion Cold Dark Matter ($f_a \gtrsim 10^9\,\text{GeV}$)

- For $f_a > T_{RH}$,

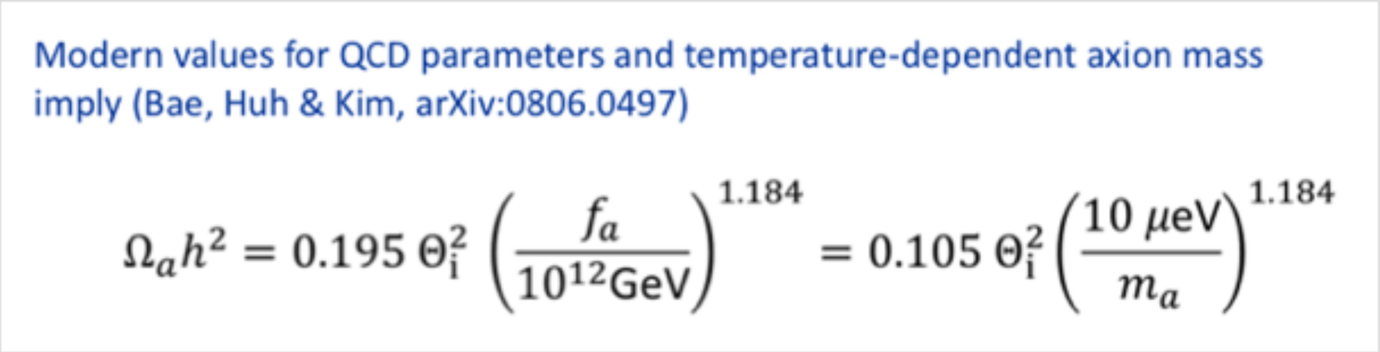
axion produced non-thermally via vacuum realignment mechanism:

Preskill et al
Dine, Fischler
Abbott, Sikivie] 83

$$\ddot{a} + 3H(T)\dot{a} - m_a^2(T) = 0$$

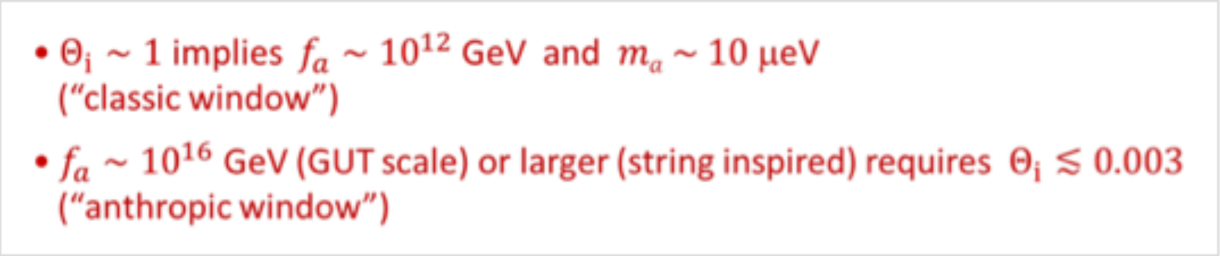
- **At early times**, where $H(T) \gtrsim m_a(T)$, i.e. $T_{RH} > T \gtrsim 1\,\text{GeV}$, axion field sits fixed at $a_i = \theta_i f_a$

- At late time, when $m_a(T) \gtrsim 3H(T)$, axion field starts quickly oscillating $\hat{=}$ coherent state of non-relativistic particles $\hat{=}$ Cold Dark Matter



Modern values for QCD parameters and temperature-dependent axion mass imply (Bae, Huh & Kim, arXiv:0806.0497)

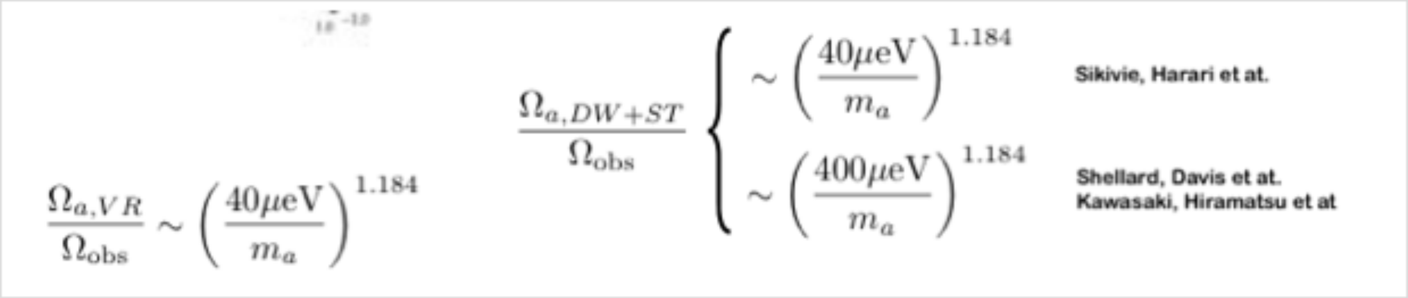
$$\Omega_a h^2 = 0.195\,\Theta_\mathrm{i}^2 \left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{1.184} = 0.105\,\Theta_\mathrm{i}^2 \left(\frac{10\ \mu\mathrm{eV}}{m_a}\right)^{1.184}$$



- $\Theta_\mathrm{i} \sim 1$ implies $f_a \sim 10^{12}$ GeV and $m_a \sim 10$ μeV ("classic window")
- $f_a \sim 10^{16}$ GeV (GUT scale) or larger (string inspired) requires $\Theta_\mathrm{i} \lesssim 0.003$ ("anthropic window")

- For $T_{RH} > f_a$:

axions produced nonthermally via:
- vacuum realignment
- string decay
- domain wall decay



$$\frac{\Omega_{a,VR}}{\Omega_{\rm obs}} \sim \left(\frac{40\mu\text{eV}}{m_a}\right)^{1.184}$$

$$\frac{\Omega_{a,DW+ST}}{\Omega_{\rm obs}} \begin{cases} \sim \left(\frac{40\mu\text{eV}}{m_a}\right)^{1.184} & \text{Sikivie, Harari et at.} \\ \sim \left(\frac{400\mu\text{eV}}{m_a}\right)^{1.184} & \text{Shellard, Davis et at. Kawasaki, Hiramatsu et at} \end{cases}$$

Axion can be dominant part of CDM for

$$10\,\mu\text{eV} \lesssim m_a \lesssim \text{meV}$$

Literature:

- Review on $d_n(\bar{\theta})$:

  Pospelov, Ritz, Ann. Phys. 318 (2005) 119 [hep-ph/0504321]

- Review on axion and ALPs:

  A R, 1210.5081