

WISPs

Basics & Tools

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WISP Theory Day
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- Introduction

- what is an EFT?

- principles of EFT

- EFT for WISPs

- Hidden photons

- kinetic mixing

- masses via

- Higgs mechanism

- Stückelberg mechanism

- dim 6 couplings

- Axions and axion-like particles

- QCD axion

- PNGB

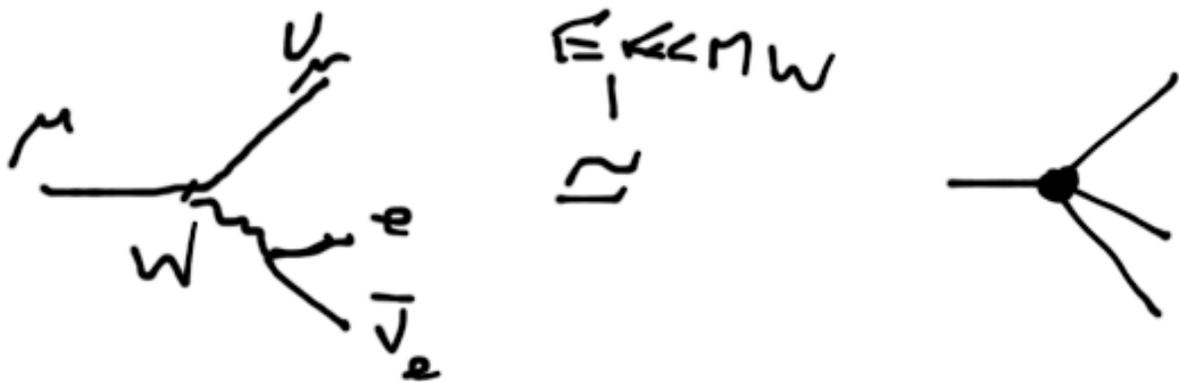
- axion mass and couplings

INTRODUCTION



• At low energies, the effect of heavy particles can be described by effective local interactions between the light particles
 - example:

at 4 fermion interactions describing weak interactions of light quarks and leptons at $E \ll M_W$:



$$\mathcal{L}_{\text{eff}} = -\frac{e^2}{2M_W^2 s_W^2 c_W^2} \left[(\bar{u}_L \gamma_\mu \delta_{ij} u_L) (\bar{e}_L \gamma_\mu \nu_{eL}) + \text{h.c.} \right]$$

6 Euler Heisenberg Effective Lagrangian
 describing photon interactions at
 $\hbar \ll m_e$



$$\mathcal{L}_{EH} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \frac{2\alpha^2}{45m_e^4} \left[\frac{1}{4} (F_{\mu\nu} F^{\mu\nu})^2 + \frac{7}{16} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right]$$

with

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$$

+ $\mathcal{O}\left(\frac{1}{m_e^6}\right)$

$$= \frac{1}{2} (E^2 - B^2) + \frac{2\alpha^2}{45m_e^4} [(E^2 - B^2)^2 + 7(E \cdot B)^2] + \mathcal{O}\left(\frac{1}{m_e^6}\right)$$

These examples: $1/4$ state:

At low energies the effects of heavy particles can be taken into account by

- (Low Energy) effective field theory

EFT

$\hat{=}$ expansion of Lagrangian in terms of local operators that only involve light degrees of freedom.

expansion in derivation of light fields $\hat{=}$ external momenta divided by scale of heavy physics

• Top down: even if more fundamental
high scale theory known, very convinced to
use LEFT

QED:

Fuler Heisenberg E.F. 1929.

QFD:

Fermi Lagrangian

QCD:

Chiral Lagrangian

• Bottom up: even if ultimate theory at high energies/small distance not known, EFT provides systematic way to extrapolate from an existing theory to higher energies.

EFT describes physics at given energy scale E , to a given accuracy ϵ , in terms of QFT with a finite set of parameters

Schematic.

- light scalar
symmetry:

with $\phi \rightarrow -\phi$
dim

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 + \frac{\lambda}{4!}\phi^4 & 4 \\ & + \frac{1}{\Lambda^2}\phi^6 + \frac{1}{\Lambda^2}(\partial\phi)^2\phi^2 & 6 \\ & + \frac{1}{\Lambda^4}\phi^8 + \frac{1}{\Lambda^4}(\partial\phi)^2\phi^4 + \frac{1}{\Lambda^4}(\partial\phi)^4 & 8 \\ & + \dots & \dots \end{aligned}$$

Effect of physics at arbitrary high energies on physics at energy E can be described by tower of interactions with integral mass dimension from two to infinity, beginning with conventional renormalizable interactions, but going on to include non-renormalizable interactions of arbitrary high mass dimensions

Principle, that governs tower
of interactions

- There are a finite number
of parameters which describe
the interactions of each
dimension n

→ example

- The coefficients of each of
the interaction terms of
dimension n is less than

or of the order of $\frac{1}{M^{n-4}}$

for some $E < M$, independent of n

⇒ only finite number of parameters
required to calculate physical quantities
at energy E with accuracy ϵ

- SM as EFT

dim

$$\mathcal{L} = \mathcal{L}_{SM} + \theta G_{\mu\nu} \tilde{G}^{\mu\nu} \quad 4$$

→ strong CP problem

$$+ \frac{\lambda_{ij}}{M} (l_i H^\dagger) (l_j H^\dagger) \quad 5$$

→ Majorana neutrino
masses $m_\nu \sim \frac{\lambda v^2}{M}$

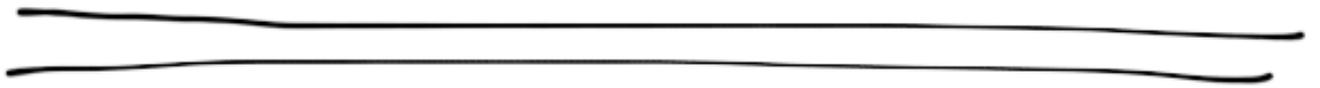
$$+ \frac{C}{M^2} (\text{4-fermion operator}) \quad 6$$

→ FCNC

$b \rightarrow sy, K^0 \rightarrow \mu^+ \mu^-$

+ ...

EFT for WISPs



Hidden Photon

- Apart from photon (A_μ)
new Abelian ($U(1)$) gauge
boson (X_μ), under which
SM particles are unchanged
("singlets")
- Most general EFT for SM
respecting hidden $U(1)$ + hidden
 $U(1)$
gauge invariance:

$$X_\mu \rightarrow X_\mu + \partial_\mu \phi$$

dim

$$\mathcal{L}_{SM} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu}$$

$$- \frac{\lambda}{2} X_{\mu\nu} F^{\mu\nu}$$

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kinetic mixing!

$$+ \frac{1}{M^2} X_{\mu\nu} \left[\bar{q}_L \sigma^{\mu\nu} c_u \tilde{H} u_R + \bar{q}_L \sigma^{\mu\nu} c_d H d_R + \bar{l}_L \sigma^{\mu\nu} c_e H e_R + h.c. \right]$$

6

$$+ \mathcal{O}\left(\frac{1}{M^4}\right)$$

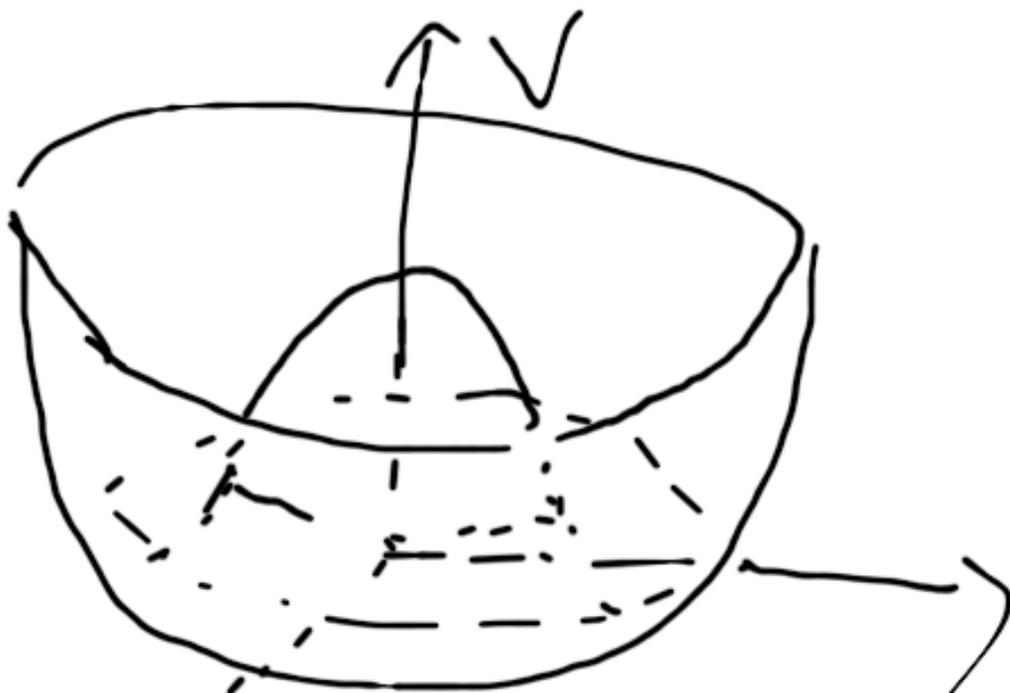
Hidden photon mass

via Higgs mechanism:

add complex scalar field charged under hidden $U(1)$

$$\mathcal{L}_\phi = (\partial_\mu - ie_h X_\mu) \phi^\dagger (\partial^\mu + ie_h X^\mu) \phi + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

$\underbrace{\hspace{10em}}_{-V}$



minimum occurs for

$$\phi \phi|_{\min} = \frac{\mu^2}{\lambda} \equiv v^2$$

parameterize ϕ as:

$$\phi(x) = e^{i\theta(x)/v} \frac{v + \rho(x)}{\sqrt{2}}$$

θ ... massless Goldstone boson

ρ ... massive Higgs boson

$$(\partial_\mu + i e_h X_\mu) \phi$$

$$= (\partial_\mu + i e_h X_\mu) (e^{i\frac{\theta}{2}} \phi')$$

$$= \frac{i}{2} \partial_\mu \theta e^{i\frac{\theta}{2}} \phi' + e^{i\frac{\theta}{2}} \partial_\mu \phi' + i e_h X_\mu e^{i\frac{\theta}{2}} \phi'$$

$$= e^{i\frac{\theta}{2}} \left(\frac{i}{2} \partial_\mu \theta + \partial_\mu + i e_h X_\mu \right) \phi'$$

$$= e^{i\frac{\theta}{2}} \left(\partial_\mu + i e_h \left(X_\mu + \frac{1}{e_h} \partial_\mu \theta \right) \right) \phi'$$

X'_μ

$$= e^{i\frac{\theta}{2}} (\partial_\mu + i e_h X'_\mu) \frac{(v+\varphi)}{\sqrt{2}}$$

$$= e^{i\frac{\theta}{2}} \left(\frac{\partial_\mu \varphi}{\sqrt{2}} + i e_h X'_\mu \frac{(v+\varphi)}{\sqrt{2}} \right)$$

$$\begin{aligned}
& (\partial_\mu - ie_h X_\mu) \phi^* \\
&= (\partial_\mu - ie_h X_\mu) e^{-i\frac{\theta}{v}} \phi' \\
&= e^{-i\frac{\theta}{v}} \left(-\frac{i}{v} \partial_\mu \theta + \partial_\mu - ie_h X_\mu \right) \phi' \\
&= e^{-i\frac{\theta}{v}} \left(\partial_\mu - ie_h \left(X_\mu + \frac{1}{e_h v} \partial_\mu \theta \right) \right) \phi' \\
&= e^{-i\frac{\theta}{v}} \left(\frac{\partial_\mu \rho}{\sqrt{2}} - ie_h X'_\mu \left(\frac{v + \varphi}{\sqrt{2}} \right) \right)
\end{aligned}$$

$$(\partial_\mu - ie_h X_\mu) \phi^* (\partial^\mu + ie_h X^\mu) \phi$$

$$= \left(\frac{\partial_\mu \rho}{\sqrt{2}} - ie_h X_\mu \left(\frac{\nu + \phi}{\sqrt{2}} \right) \right)$$

$$\times \left(\frac{\partial^\mu \rho}{\sqrt{2}} + ie_h X^\mu \left(\frac{\nu + \phi}{\sqrt{2}} \right) \right)$$

$$= \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + e_h^2 X^{\mu 2} \frac{(\nu + \phi)^2}{2}$$

$$= \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + e_h^2 \frac{(\nu + \phi)^2}{2} \left(X + \frac{1}{e\nu} \partial_\mu \theta \right)^2$$

$$= \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + e_h^2 \nu \rho \left(X + \frac{1}{e\nu} \partial_\mu \theta \right)^2$$

$$+ \frac{e_h^2}{2} \rho^2 \left(X + \frac{1}{e\nu} \partial_\mu \theta \right)^2$$

$$+ \frac{e_h^2}{2} \nu^2 \left(X + \frac{1}{e\nu} \partial_\mu \theta \right)^2$$

$$\mu^2 \phi^4 \phi - \lambda (\phi^2 \phi)^2$$

$$= \lambda \phi^4 \phi (v^2 - (\phi^2 \phi)^2)$$

$$= \frac{\lambda}{2} (v+\phi)^2 (v^2 - \frac{1}{2} (v+\phi)^2)$$

$$= -\lambda v^2 \phi^2 - \lambda v \phi^3 - \frac{\lambda}{4} \phi^4 + \frac{\lambda}{4} v^4$$

Thus:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} \\ &+ (\partial_\mu - ie_h X_\mu) \phi^\dagger (\partial^\mu + ie_h X^\mu) \phi \\ &+ \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \\ &= -\frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \underbrace{\frac{e_h^2}{2} v^2}_{m_\phi^2} (X + \frac{1}{e_h v} \partial\theta)^2 \\ &+ \frac{1}{2} \partial\rho\partial\rho - \underbrace{\lambda v^2}_{2m_\rho^2} \rho^2 \\ &+ e_h^2 v \rho (X + \frac{1}{e_h v} \partial\theta)^2 \\ &+ \frac{e_h^2}{2} \rho^2 (X + \frac{1}{e_h v} \partial\theta)^2 \\ &- \lambda v \rho^3 - \frac{\lambda}{4} \rho^4 + \frac{\lambda}{4} v^4 \end{aligned}$$

θ ... massless Goldstone boson

can be eliminated
by gauge transformation

$$X \rightarrow X' = X + \frac{1}{e_h v} \partial_\mu \theta$$

" θ eaten by gauge field
giving it a mass"

$$m_{\delta'} = \frac{e_h v}{\sqrt{2}}$$

ρ ... massive Higgs boson

$$m_\rho = \frac{\sqrt{\lambda} v}{\sqrt{2}}$$

Stückelberg mechanism:

take the limit $m_p \gg m_{\gamma'}$:

Integrate out A_{μ} :

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_p^2 \left(X + \frac{1}{\sqrt{2} m_{\gamma'}} \partial_\mu \theta \right)^2$$

+ ...

↑
power suppressed

• do not need to introduce
Higgs field in first place

to describe massive gauge boson

Stückelberg field θ : shift
symmetry

$$\theta \rightarrow \theta + \text{const.}$$

Axion

- Spin-0 field enjoying approximate shift symmetry

$a \rightarrow a + \text{const}$, only broken by anomalous couplings to gauge fields in SM.

→ solves the strong CP problem; i.e. eliminates

$\theta_0 G \tilde{G}$ term.

most general EFT solving
strong CP problem:

dim

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2} \partial_\mu a \partial^\mu a$$

4

$$+ \frac{a}{f_a} \left[\frac{g_s^2}{32\pi^2} G \tilde{G} + C_{\text{anom}} \frac{g_L^2}{32\pi^2} W \tilde{W} + C_{\text{anom}} \frac{g_Y^2}{32\pi^2} Y \tilde{Y} \right]$$

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$$+ \frac{\lambda a}{f_a} \left[\sum_\psi \bar{\psi}_L \gamma^\mu X_\psi \psi_L + X_H H^\dagger i \overleftrightarrow{D}^\mu H \right]$$

$$+ \mathcal{O} \left(\frac{1}{f_a^2} \right)$$

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- Topological charge fluctuations in QCD vacuum break shift symmetry to discrete subgroup (periodicity) \Rightarrow Potential can be parametrized as:

$$V_{\text{eff}} = \left\langle \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G \tilde{G} \right\rangle$$

$$\sim \frac{m^2}{a} f_a^2 \left(1 - \cos \frac{a}{f_a} \right)$$

- \Rightarrow • $\langle a \rangle = 0$ no strong CP problem

- axion pGB, has a mass:

$$m_a^2 f_a^2 \sim \text{QCD scale}$$

Below EW SB:

$$\mathcal{L} \supset \frac{q}{f_a} \left[C_{aWW} \frac{g_L^2}{32\pi^2} W\tilde{W} + C_{aYY} \frac{g_Y^2}{32\pi^2} Y\tilde{Y} \right]$$



$$\frac{q}{f_a} \underbrace{(C_{aWW} + C_{aYY})}_{\text{coupling to photon}} \frac{e^2}{32\pi^2} F\tilde{F}$$

coupling to
photon

at $g_{\text{eV}} \ll E \ll M_W$

model dependent

- At $E \ll \text{GeV}$, light degrees of freedom no more quarks and gluons, but pions (as Goldstone bosons of chiral symmetry) and baryons.

Have to match to chir. Lagr.

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{tr} (\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \frac{f_\pi^2}{2} \text{tr} (M \Sigma^\dagger) + \text{h.c.} + \dots$$

with $\Sigma = \exp\left(\frac{2i}{f_\pi} T^a \pi^a\right)$

• "kinetic term" invariant under

$$SU(3)_L \times SU(2)_R$$

$$\Sigma \rightarrow L \Sigma R^T$$

• "mass term" breaks symmetry
due to quark masses

$$M = \begin{pmatrix} m_u & & \\ & m_d & \\ & & m_s \end{pmatrix}$$

• $f_\pi = 93 \text{ MeV}$; μ phys. parameters
determined by
meson masses and
interactions.

• action coupling two

$$M \rightarrow M e^{i \frac{a}{f_\pi} X} ; \text{tr} X = 1$$

$$V_{\text{eff}}(a) = \min_{\text{as}} \left(\frac{f_{\pi}^2}{2} \text{tr} (M \Sigma^{\dagger}) + h \epsilon \right)$$

as
 f.d.
 of Σ
 for fixed a

$$\approx \frac{f_{\pi}^2 m_{\pi}^2}{8 f_a^2} \frac{m_u m_d}{(m_u + m_d)^2} a^2 + \dots$$

$\underbrace{\hspace{10em}}_{\frac{1}{2} m_a^2}$

small
a

$m_s \gg m_u, m_d$

$$\Rightarrow \langle a \rangle = 0$$

$$m_a \approx \frac{f_{\pi} m_{\pi}}{f_a} \frac{\sqrt{m_u m_d}}{(m_u + m_d)}$$

- Because of mixing with pion:

⇒ model independent
low energy couplings
to photons:

$$\begin{aligned}
 \mathcal{L} &\supset \frac{a}{f_a} (C_{0WW} + C_{0YY}) \\
 &= \frac{4}{3} \frac{m_u + 4m_d}{m_u + m_d} \left) \frac{e^2}{32\pi^2} F\tilde{F}
 \end{aligned}$$

Suggested reading:

- H. Georgi, "Effective Field Theory",
Ann. Rev. Nucl. Part. Sc. 43 (1993) 205
- D. Kaplan, "Five lectures on effective
field theory",
nucl-th/0510023
- H. Ruesch, "The Stückelberg Field",
hep-th/0304245
- R. Peccei, "The strong CP problem
and axions", hep-ph/0607268
- H. Georgi, D. Kaplan, L. Randall,
"Naturalizing the QCD axion at low energy",
Phys. Lett. B 165 (1986) 73