Multi-Step Grand Unification — From Model Building to Pheno

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The Standard Model (SM) – Theorist's View

Renormalizable Quantum Field Theory (only with Higgs!) based on $SU(3)_c \times SU(2)_w \times U(1)_Y$ non-simple gauge group

$$L_L = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L \quad Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad u_R^c \quad d_R^c \quad \ell_R^c \quad [\nu_R^c] \quad \left[\begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \right]$$

Interactions:

Gauge IA (covariant derivatives in kinetic terms):

$$\partial_{\mu} \longrightarrow D_{\mu} = \partial_{\mu} + i \sum_{k} g_{k} V_{\mu}^{a} T^{a}$$

Yukawa IA:

$$Y^{u}\overline{Q}_{L}H_{u}u_{R} + Y^{d}\overline{Q}_{L}H_{d}d_{R} + Y^{e}\overline{L}_{L}H_{d}e_{R} \quad \left[+Y^{n}\overline{L}_{L}H_{u}\nu_{R}\right]$$

Scalar self-IA:

$$(H^{\dagger}H)$$
 $(H^{\dagger}H)^2$



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The group-theoretical bottom line

Things to remember:

Representations of SU(N)

- ▶ fundamental reps. $\phi_i \sim N, \psi^i \sim \overline{N}$, adjoint reps. $A_i^j \sim N^2 1$
- ► SU(N) invariants: contract all indices

 $\phi_i \psi^i \qquad \phi_i A_i^j \psi^j \qquad \epsilon_{ij} \phi_i \xi_j \qquad \epsilon_{ijk} \phi_i \xi_j \eta_k$

Symmetry properties: Young tableaux

The (group-theoretical) essence of the SM:

remember: $Q_{\text{el.}} = T_3 + \frac{Y}{2}$

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		1
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Loose Ends/Deficiencies of the Standard Model

Incompleteness

- Electroweak Symmetry Breaking
- Higgs boson
- Origin of neutrino masses
- Dark Matter: $m_{DM} \sim 100 \, {\rm GeV}$



$$m_h^2 = m_0^2 + \Lambda^2 \times (\text{loop factors})$$

Theoretical Dissatisfaction

- 28 free parameters
- "strange" fractional
 U(1) quantum numbers
- Hierarchy problem



Supersymmetry

connects gauge and space-time symmetries

multiplets with equal-mass fermions and bosons

 \Rightarrow SUSY is broken in Nature stabilizes the hierarchy





- Minimal Supersymmetric Standard Model (MSSM)
- Charginos, Neutralinos, Gluino Sleptons, Squarks, Sneutrinos
- R parity: discrete symmetry
- LSP: Dark matter
- Superpartners have *identical* gauge quantum numbers



Anatomy of MSSM

• Yukawa couplings \Rightarrow Superpotential:

$$\mathcal{W} = \hat{\Phi}_1 \hat{\Phi}_2 \hat{\Phi}_3 \equiv \Phi_1 \Phi_2 \Phi_3 \quad \Longrightarrow \quad (\phi_1 \phi_2)^2, \dots, (\overline{\psi}_1 \psi_2) \phi_3$$

- NB: part of scalar potential from gauge kinetic terms (light Higgs)
- MSSM superpotential

$$\mathcal{W}_{\mathsf{MSSM}} = Y^u u^c Q H_u + Y^d d^c Q H_d + Y^e e^c L H_d + \mu H_u H_d$$

- Ignorance about SUSY breaking shows up as "soft-breaking terms": Gaugino and sparticle masses, trilinear scalar potential terms
- μ problem: EWSB demands $\mu \sim \mathcal{O}(100 \,\text{GeV} 1 \,\text{TeV})$
- Additional SUSY degrees of freedom modify vacuum polarization
 ⇒ Unification of gauge couplings possible



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Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138 (Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world—that all elementary particle forces (strong, weak, and electromagnetic) are different manifestations of the same fundamental interaction involving a single coupling strength, the fine-structure constant. Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously. of the GIM mechanism with the notion of colored quarks' keeps the successes of the quark model and gives an important bonus: Lepton and hadron anomalies cancel so that the theory of weak and electromagnetic interactions is renormalizable.⁵

The next step is to include strong interactions. We assume that strong interactions are mediated by on octet of neutral vector gauge gluons associated with local color SU(3) symmetry, and that there are no fundamental strongly interacting scalar-meson fields.⁶ This insures that



Renormalization group (RG) running of gauge couplings:

$$\frac{dg_a}{d\log\mu} = \frac{g_a^3}{16\pi^2} B_a \qquad \begin{array}{c} \text{SM} \quad B_a = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) \\ \text{MSSM} \quad B_a = \left(\frac{33}{5}, 1, -3\right) \end{array}$$





Renormalization group (RG) running of gauge couplings:





Renormalization group (RG) running of gauge couplings:



H. Georgi and H. D. Politzer. Phys. Rev. D (to be published); D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973), and Phys. Rev. D (to be published).

¹⁶A naive calculation indicates that the vector boson mass must be greater than $10^{15} \text{ GeV} \simeq 10^{-9} \text{ g}!$ Let the reader who finds this hard to swallow double the num-



The prime example: (SUSY) SU(5)

$$SU(5) \xrightarrow{M_X} SU(3)_c \times SU(2)_w \times U(1)_Y \xrightarrow{M_Z} SU(3)_c \times U(1)_{em}$$

 $SU(5)$ has $5^2 - 1 = 24$ generators:

$$\mathbf{24} o \underbrace{(\mathbf{8},\mathbf{1})_0}_{G^eta} \oplus \underbrace{(\mathbf{1},\mathbf{3})_0}_W \oplus \underbrace{(\mathbf{1},\mathbf{1})_0}_B \oplus \underbrace{(\mathbf{3},\mathbf{2})_{\frac{5}{6}}}_{X,Y} \oplus \underbrace{(\overline{\mathbf{3}},\mathbf{2})_{-\frac{5}{6}}}_{ar{X},ar{Y}}$$



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$$A = g \sum_{a=1}^{24} A^{a} \frac{\lambda^{a}}{2} = \frac{g}{\sqrt{2}} \begin{pmatrix} \sqrt{2}G^{a} \frac{\lambda^{a}_{\mathsf{GM}}}{2} & \begin{vmatrix} \bar{X} & \bar{Y} \\ \bar{X} & \bar{X} & \bar{Y} \\ \bar{Y} & Y & Y \end{vmatrix} & \sqrt{2}W^{a} \frac{\sigma}{2} \end{pmatrix}$$

$$- \frac{g}{2\sqrt{15}} B \begin{pmatrix} -2 & & & \\ & -2 & & \\ & & & & +3 \end{pmatrix}$$



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Quantum numbers

- Hypercharge: $\frac{\lambda_{12}}{2} = \sqrt{\frac{3}{5}} \frac{Y}{2}$ $Y = \frac{1}{3} \operatorname{diag}(-2, -2, 3, 3, 3)$ Quantized hypercharges are fixed by non-Abelian generator
- Weak Isospin: $T_{1,2,3} = \lambda_{9,10,11}/2$
- Electric Charge: $Q = T^3 + Y/2 = \text{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0)$
- Prediction for the weak mixing angle (with RGE running): $\alpha^{-1}(M_Z) = 128.91(2), \alpha_s(M_Z) = 0.1176(20), s_w^2(M_Z) = 0.2312(3)$

non-SUSY:
$$s_w^2(M_Z) = \frac{23}{134} + \frac{\alpha(M_Z)}{\alpha_s(M_Z)} \frac{109}{201} \approx 0.207$$



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 (1)
SUSY: $s_w^2(M_Z) = \frac{1}{5} + \frac{\alpha(M_Z)}{\alpha_s(M_Z)}\frac{7}{15} \approx 0.231$ (1)

New Gauge Bosons Two colored EW doublets: $(X, Y), (\bar{X}, \bar{Y})$ with charges $\pm \frac{4}{3}, \pm \frac{1}{3}$



Fermions (Matter Superfields)

The only possible way to group together the matter:

$$\overline{\mathbf{5}} = \boxed{\begin{array}{c} \vdots \\ \end{array}} : \begin{pmatrix} d^{c} \\ d^{c} \\ \ell \\ -\nu_{\ell} \end{pmatrix} \qquad \mathbf{10} = \boxed{\begin{array}{c} \vdots \\ 1 \\ \sqrt{2} \end{array}} \begin{pmatrix} 0 & u^{c} & -u^{c} & | & -u & -d \\ -u^{c} & 0 & u^{c} & | & -u & -d \\ u^{c} & -u^{c} & 0 & | & -u & -d \\ \hline u^{c} & -u^{c} & 0 & | & -u & -d \\ \hline u^{c} & u & u & | & 0 & -e^{c} \\ d & d & d & | & e^{c} & 0 \end{pmatrix}}$$

 $\overline{\mathbf{5}} = \ (\overline{\mathbf{3}}, \mathbf{1})_{\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-1} \qquad \mathbf{10} = \ (\mathbf{3}, \mathbf{2})_{\frac{1}{3}} \oplus (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{4}{3}} \oplus (\mathbf{1}, \mathbf{1})_{2}$

Remarks

- $\blacktriangleright \ \mathbf{2} = \Box = \overline{\mathbf{2}}, \qquad (\mathbf{5} \otimes \mathbf{5})_a = \mathbf{10}, \quad (\mathbf{3} \otimes \mathbf{3})_a = \overline{\mathbf{3}}, \quad (\Box \otimes \Box)_a = \Box$
- Quarks and leptons in the same multiplet
- Fractional charges from tracelessness condition (color!)
- $\blacktriangleright~\overline{5}$ and 10 have equal and opposite anomalies
- ν^c must be SU(5) singlet

Interactions



The doublet-triplet splitting problem

SU(5) breaking: Higgs Σ in adjoint 24 rep.

$$\langle 0 | \mathbf{\Sigma} | 0
angle = w imes {\sf diag}(1,1,1,-rac{3}{2},-rac{3}{2}) \qquad M_X = M_Y = rac{5}{2\sqrt{2}} \, g \, w$$

other breaking mechanisms possible (e.g. orbifold)



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The doublet-triplet splitting problem

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other breaking mechanisms possible (e.g. orbifold) (MS)SM Higgs(es) included in $5\otimes\overline{5}$

$$\mathbf{5} = \Box : \begin{pmatrix} \mathbf{D} \\ D \\ D \\ h^+ \\ h^0 \end{pmatrix} \qquad \mathbf{\overline{5}} = \boxed{\Box} : \begin{pmatrix} \mathbf{D}^c \\ D^c \\ D^c \\ h^- \\ -h^0 \end{pmatrix}$$
$$\mathbf{5} = (\mathbf{3}, \mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{2})_1 \qquad \mathbf{\overline{5}} = (\mathbf{\overline{3}}, \mathbf{1})_{\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-1}$$

- D, D^c coloured triplet Higgses with charges $\pm \frac{1}{3}$
- Doublet-triplet splitting problem



Naive estimate of proton lifetime



Effective 4-fermion operator (analogy to muon decay)

$$\mathcal{L}_{F} = \frac{4G_{F}}{\sqrt{2}} (\overline{\mu}\gamma_{\kappa}\nu_{\mu}) (\overline{\nu}_{e}\gamma^{\kappa}e) \quad \mathcal{L}_{GUT} = \frac{4G_{GUT}}{\sqrt{2}} (\overline{u}\Gamma u) (\overline{e}\Gamma d)$$

$$\frac{G_{F}}{\sqrt{2}} = \frac{g_{2}^{2}}{8M_{W}^{2}} \qquad \frac{G_{GUT}}{\sqrt{2}} = \frac{g^{2}}{8M_{GUT}^{2}}$$

$$\tau(\mu \to e\nu_{\mu}\overline{\nu}_{\mu}) \sim \frac{192\pi^{3}}{G_{F}^{2}m_{\mu}^{5}} \qquad \tau(p \to e^{+}\pi^{0}) \sim \frac{192\pi^{3}}{G_{GUT}^{2}m_{p}^{5}}$$

Proton lifetime for $\alpha(M_{GUT}) \sim 1/24$ and $M_{GUT} \sim 2 \times 10^{16}$ GeV: $\tau(p \rightarrow e^+\pi^0) \sim \frac{M_{GUT}^4}{[\alpha(M_{GUT})]^2 m_p^5} \rightarrow 10^{31\pm 1}$ years Compare: $\tau_p^{SM} \gtrsim 10^{150}$ years (gravity-induced)



Proton decay crucial for "GUT search"

- Tracking calorimeter (SOUDAN) or RICH Cherenkovs
- Super-Kamiokande: 50 kt water RICH
- measure change and time for reconstruction







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Channel	$ au_p$ (10 ³⁰ years)
$p \rightarrow \text{invisible}$	0.21
$p \rightarrow e^+ \pi^0$	1600
$p \rightarrow \mu^+ \pi^0$	473
$p \rightarrow \nu \pi^+$	25
$p \rightarrow \nu K^+$	670
$p \rightarrow e^+ \eta^0$	312
$p ightarrow \mu^+ \eta^0$	126
$p \rightarrow e^+ \rho^0$	75
$p ightarrow \mu^+ ho^0$	110
$p \rightarrow \nu \rho^+$	162
$p ightarrow e^+ \omega^0$	1000
$p ightarrow \mu^+ \omega^0$	117
$p \rightarrow e^+ K^0$	150
$p \to \mu^+ K^0$	1300
$p \rightarrow \nu K^+$	2300
$p \rightarrow e^+ \gamma$	670
$p \rightarrow \mu^+ \gamma$	478

New experiments: HyperK (1 Mt), UNO (650 kt), European project Fréjus (1 Mt)

Precision: 10 years running $\implies 10^{34} - 10^{35}$ years



Why chiral exotics?

Unification verification only with megatons? What about colliders?

- SPA: super precision accurately
- Look for chiral exotics
- Physics beyond MSSM provides handle to GUT scale
- $\mu \ {\rm problem}$
 - NMSSM trick
 - Singlet superfield with TeV-scale VEV

Doublet-triplet splitting problem, Longevity of the proton

- Try to keep D, D^c superfields at TeV scale
- Need mechanism to prevent rapid proton decay
- Need to rearrange running for unification

Flavour problem

Flavour might help protecting the proton



Sketch of a model

Superpotential:

$$\begin{split} \mathcal{W} &= \mathcal{W}_{\text{MSSM}} + \mathcal{W}_D + \mathcal{W}_N \\ \mathcal{W}_{\text{MSSM}} &= Y^u u^c Q H_u + Y^d d^c Q H_d + Y^e e^c L H_d \\ \mathcal{W}_D &= Y^D D u^c e^c + Y^{D^c} D^c Q L \\ \mathcal{W}_S &= Y^{S_H} S H_u H_d + Y^{S_D} S D D^c \end{split}$$

$U(1)_S$ symmetry

- gauged at high energies, radiative breaking of global left-over U(1)
- $\langle 0|S|0 \rangle$ generates μ_H and μ_D

See-saw mechanism ν^c at $\sim M_{\nu} \sim 10^{14} - 10^{15}$ GeV

$$m_{
u} \sim rac{v^2}{M_{
u}} \sim 10^{-1} - 10^{-2} {
m eV}$$

At M_{ν} left-right symmetric model: $SU(2)_L \times SU(2)$

$$Q_R = \begin{pmatrix} u^c \\ d^c \end{pmatrix}, \qquad L_R = \begin{pmatrix} \nu^c \\ \ell^c \end{pmatrix} \qquad H = \begin{pmatrix} H_u \\ H_d \end{pmatrix} \qquad D \qquad D^c$$



RGE running





RGE running





RGE running





Matter-Higgs unification: E_6

▶ Pati-Salam group $SU(4)_c \times SU(2)_L \times SU(2)_R$

$$egin{aligned} \mathbf{Q}_L &= (Q,L) = \ (\mathbf{4},\mathbf{2},\mathbf{1}) \ \mathbf{Q}_R &= ((u^c,d^c),(
u^c,\ell^c)) = \ (\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) \ \mathbf{H} &= (H_u,H_d) = \ (\mathbf{1},\mathbf{2},\mathbf{2}) \ \mathbf{D} &= (D,D^c) = \ (\mathbf{6},\mathbf{1},\mathbf{1}) \ \mathbf{S} &= \ (\mathbf{1},\mathbf{1},\mathbf{1}) \end{aligned}$$



Matter-Higgs unification: E_6

▶ Pati-Salam group $SU(4)_c \times SU(2)_L \times SU(2)_R$

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u^c,\ell^c)) = \ (\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) \ \mathbf{H} &= (H_u,H_d) = \ (\mathbf{1},\mathbf{2},\mathbf{2}) \ \mathbf{D} &= (D,D^c) = \ (\mathbf{6},\mathbf{1},\mathbf{1}) \ \mathbf{S} &= \ (\mathbf{1},\mathbf{1},\mathbf{1}) \end{aligned}$$

- ▶ Be radical: Embed everything in fundamental rep. of E₆
 - fundamental 27 contains exactly the stuff above
 - adjoint: 78
 - Matter and Higgs fields in one big multiplet
 - \Rightarrow 3 generations of S and D
 - ightarrow ightarrow 3 generations of Higgs fields: Higgs (VEV) vs. "unhiggs" (no VEV)
 - Problem of FCNCs: Introduce Z₂ symmetry (*H* parity)



Flavour Symmetry and proton decay

Assume $SU(3)_F$ or $SO(3)_F$ flavour symmetry

- Left-right symmetry: $SU(2)_L \times SU(2)_R$, $SU(3)_c$, $SU(3)_F$
- Diquark couplings vanish identically:

$$DQ_LQ_L = \epsilon^{abc} \epsilon_{\alpha\beta\gamma} \epsilon_{jk} D^a_\alpha \left(Q_L\right)^b_{\beta j} \left(Q_L\right)^c_{\gamma k}$$

- Baryon number is symmetry of the superpotential
- SU(2)_R and SU(3)_F breaking spurions?
 symmetry breaking by condensates linear/bilinear in fundamental reps.:
 D, D^c couple to other quarks only as singlets
- Integrating out heavy fields: baryon number emerges as low-energy symmetry, flavour symmetry not
- Leptoquark couplings possible



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• Toy model: $E_8 \rightarrow E_6 \times SU(3)_F$

- $\blacktriangleright \ 248 = 27_3 \oplus \overline{27}_{\overline{3}} \oplus 78_1 \oplus 1_8$
- Filavour-symmetric Kaluza-Klein tower of mirror mmatter $\overline{27}_{\overline{3}}$ breaks E_8
- mirror-Higgs superfields µ term breaks E₆ to PS, breaks flavour



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A little bit of Pheno

Next step: Provide a viable low-energy spectrum

Extended MSSM Higgs sector

- relaxed Higgs bounds (light pseudoscalars)
- possibly large invisible decay ratio
- ► lightest unhiggs: *H* parity protected dark matter
- dark matter mix: interesting relic abundance (relaxes all neutralino bounds!)
- Pair production of unhiggses/unhiggsinos, cascade decays

(Down-type) Leptoquarks, Leptoquarkinos

- 3 generations at TeV scale
- produced in gluon fusion, single production
- final states: $t\tau, b\nu_{\tau}, \tilde{t}\tau, \dots$
- ▶ if flavor symmetry leaves traces: $gq \rightarrow D\ell$ enhanced, decays $t\mu, te$

Extended neutralino sector like in NMSSM



no Z'

Some Unification needs time



