$pp \to b\bar{b}b\bar{b}$ at NLO with GOLEM and WHIZARD

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Loopfest VIII in Madison

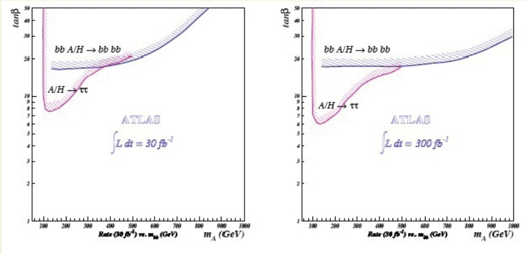
[Freiburg Market Square]

Overview

LHC heavy Higgs search

- In 2HDM the Higgses (especially A^0) tend to decay dominantly to b quarks (BR up to 89 %)
- Process under consideration: $pp \to bbH \to bb\,bb + X$

  [Dai/Gunion/Vega, 1994/6], [Balazs/Diaz-Cruz/He/Tait/Yuan, 1999], [ATLAS/CMS TDR], [Kao/Mangano/Shankar/Sayre/Wang, 2009]
- Usage of a 3-jet trigger: $p_T > 70$ GeV (CMS), 80 GeV ATLAS



- Result Kao et al.: associated production $pp \to Hb \to bbb$ might be better
- BUT: Depends crucially on normalization and shapes of b jet distributions
- Explicit calculations of the actual K factors are needed.

$b\bar{b}H \to b\bar{b}b\bar{b}$

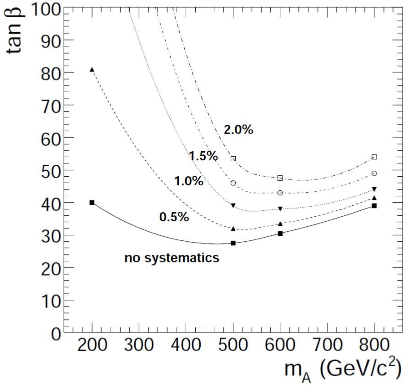
Effect of background uncertainty on discovery reach: (2σ contours)

Big improvements possible by NLO background calculation

in combination with other channels:

- $b\bar{b}\tau^+\tau^-$
- $b\bar{b}\mu^+\mu^-$
- associated bH production, $H \to bb$

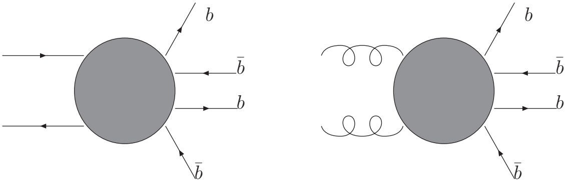
Good prospects for heavy Higgses



[Fig.: CMS Physics TDR]

The $q\bar{q} \to b\bar{b}b\bar{b}$ Process – Structure of the Amplitude

- $pp \to b\bar{b}b\bar{b}$ irreducible background for SUSY Higgs searches.
- 4-b (and 4-jet) are on the Experimentalists' Wish List. [Les Houches 2007]



Complete NLO cross-section can be written as

$$\sigma^{\text{NLO}} = \int_N d\sigma^{\text{LO}} + \int_N \left(d\sigma^{\mathcal{V}} + \int_1 d\sigma^{\mathcal{A}} \right)_{\varepsilon=0} + \int_{N+1} \left(d\sigma^{\mathcal{R}} - d\sigma^{\mathcal{A}} \right)_{\varepsilon=0}$$

- leading order contribution
- virtual corrections
- real emission
- subtraction terms [Catani,Seymour]

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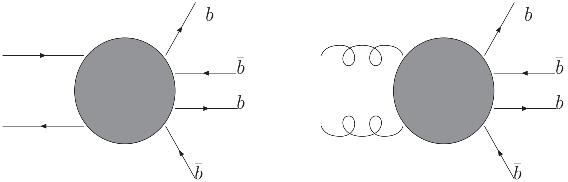

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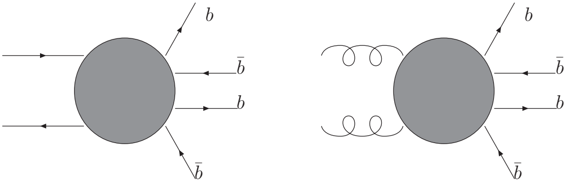

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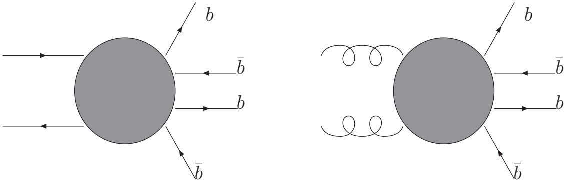

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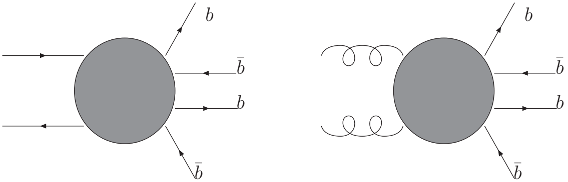

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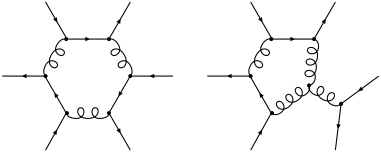
This talk: mainly $\int_{N+1} (d\sigma^{\mathcal{R}} - d\sigma^{\mathcal{A}})$

The Structure of the Virtual Corrections

- based on Feynman diagrams
- High complexity due to pentagons and hexagons
- Improved reduction method for tensor integrals



Using helicity projections:

$$d\sigma^{\mathcal{V}} = d\Phi^{(N)} \sum_{\{\lambda\}} \left(\mathcal{M}^{\text{LO}}(\{\lambda\},\{p\})^\dagger \otimes \mathcal{M}^{\mathcal{V}}(\{\lambda\},\{p\}) + \text{h.c.} \right) F_J^{(N)}$$

$$\mathcal{M}_i^{\mathcal{V}} = c(\{\lambda\},\{p\}) \sum_{\text{diagrams}} \sum_{\mathcal{F}} \text{tr}[\{\lambda\}]\{\not{p}\}_{i,\mathcal{F}} \mathcal{F}(S)$$

$$\mathcal{F}(S) \in \left\{ A^{N',r}(j_1,\ldots;S), B^{N',r}(j_1,\ldots;S), C^{N',r}(j_1,\ldots;S) \right\}$$

- Form factors $\mathcal{F}(S)$ from tensor integrals
- Coefficients products of Dirac traces $\text{tr}[\{\lambda\}]\{\not{p}\}_{i,\mathcal{F}}$
- Non-trivial color structure $\Rightarrow$ Color flow decomposition

Golem95: Tensor Reduction and Numerical Evaluation

Reduction of the tensor integrals:

- One of the main problems in (one) loop calculations:
- Either avoid it. . . [e.g.: Unitarity methods]
- . . . or do it in a smart way.
- Mixed algebraic/numerical approach: [Binoth, Guillet, Heinrich, Pilon, Reiter (arXiv:0810.0992)]
  - Reduction of 1- and 2-point integrals: trivial
  - Reduction of $(N \geq 5)$ point integrals: always reduced to 3- and 4-point
- What about 3- and 4-point integrals?

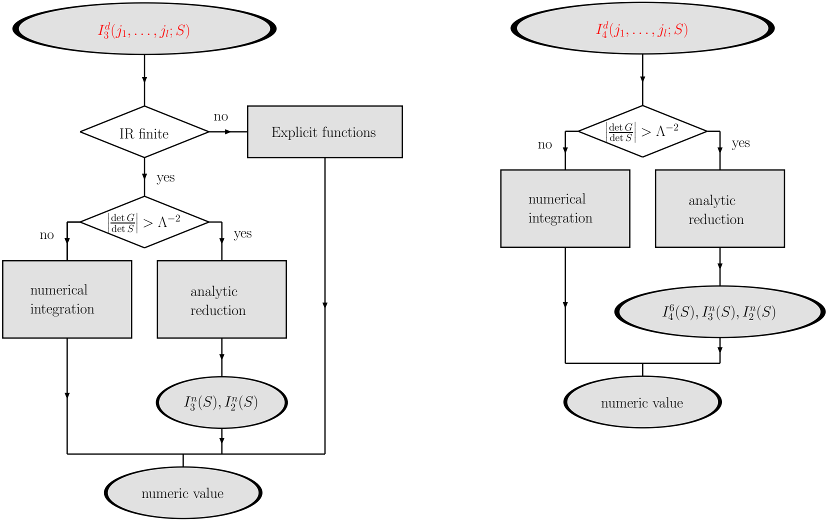
Use of extended set of basis functions

$$A^{N,r}(S), B^{N,r}(S), C^{N,r}(S) \rightsquigarrow I_N^d(j_1, \ldots, j_l; S)$$

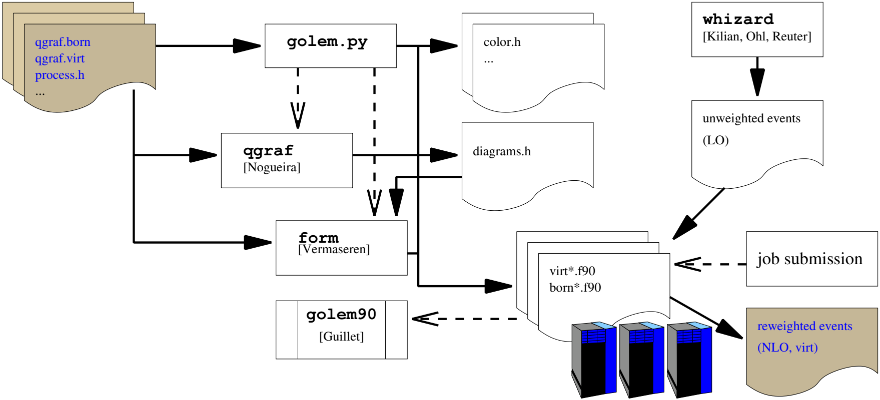
$$I_N^d(j_1, \ldots, j_l; S) = (-1)^N \Gamma(N-d/2) \times \int_0 \mathrm{d}z_1 \cdots \mathrm{d}z_N \, \delta(1-\sum z_i) \frac{z_{j_1} \cdots z_{j_l}}{(-\frac{1}{2} z_i S_{ik} z_k - i\delta)^{N-d/2}}$$

- Required for $N \in \{3, 4\}$ only
- Reduction and numerical calculation possible

The GOLEM algorithm



The GOLEM–WHIZARD Setup



The implementation in WHIZARD

- Acronym: **W**, **HI**ggs, **Z**, **A**nd **R**espective **D**ecays (deprecated)
- Authors/location: Freiburg/Siegen/Würzburg W. Kilian, T. Ohl, JR + PhDs
- Current version: 1.93 (2.0.0 α)
  `http://projects.hepforge.org/whizard` and
  `http://whizard.event-generator.org`
- Reference [arXiv: 0708.4233 [hep-ph]]
- Languages: `O'Caml` and `FORTRAN 95` (`FORTRAN 2003` in v2)
- parton shower ($p_\perp$ ordered) and analytic (v2.0.0α) (S. Schmidt)
- no hadronization
- underlying event: preliminary version (H.-W. Boschmann)
- Arbitrary processes: a generator generator (O'Mega)
- BSM: cf. next page
- 2.0 features: ME/PS matching, cascades, new versatile user interface and syntax, WHIZARD as a shared library

WHIZARD – Overview over BSM Models

Very high level of Complexity:

- $e^+e^- \to t\bar{t}H \to b\bar{b}b\bar{b}jj\ell\nu$ (110,000 diagrams)
- $e^+e^- \to ZHH \to ZWWWW \to bb + 8j$ (12,000,000 diagrams)
- $pp \to \ell\ell + nj, n = 0, 1, 2, 3, 4, \ldots$ (2,100,000 diagrams with 4 jets + flavors)
- $pp \to \tilde{\chi}_1^0\tilde{\chi}_1^0 bbbb$ (32,000 diagrams, 22 color flows, $\sim 10,000$ PS channels)
- $pp \to VVjj \to jj\ell\ell\nu\nu$ incl. anomalous TGC/QGC
- Test case $gg \to 9g$ (224,000,000 diagrams, matched by PHEGAS)

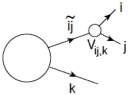
MODEL TYPE	with CKM matrix	trivial CKM
QED with e, μ, τ, γ	–	QED
QCD with d, u, s, c, b, t, g	–	QCD
Standard Model	SM_CKM	SM
SM with anomalous couplings	SM_ac_CKM	SM_ac
SM with K matrix	—	SM_KM
MSSM	MSSM_CKM	MSSM
MSSM with gravitinos	—	MSSM_Grav
NMSSM	—	NMSSM
extended SUSY models	—	PSSSM
Littlest Higgs	—	Littlest
Littlest Higgs with ungauged $U(1)$	—	Littlest_Eta
Littlest Higgs with T parity	—	Littlest_Tpar
Simplest Little Higgs (anomaly-free)	—	Simplest
Simplest Little Higgs (universal)	—	Simplest_univ
UED	—	UED
SUSY Xdim. (inoff.)	—	SED
Noncommutative SM (inoff.)	—	NCSM
SM with Z'	—	Zprime
SM with gravitino and photino	—	GravTest
Augmentable SM template	—	Template

easy to implement new models

The unintegrated dipoles

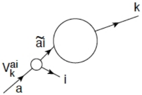
- Use Born $pp \to b\bar{b}b\bar{b}$ and real corr. $pp \to b\bar{b}b\bar{b}g$ in one module
- Cancellation happens at level of helicity matrix elements

FF Dipoles



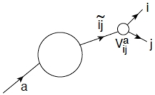

```
subroutine ff_dipole (v_ijk,y_ijk,p_ij,pp_k,p_i,p_j,p_k)
  type(momentum), intent(in) :: p_i, p_j, p_k
  type(momentum), intent(out) :: p_ij, pp_k
  real(kind=omega_prec), intent(out) :: y_ijk
  real(kind=omega_prec) :: z_i
  real(kind=omega_prec), intent(out) :: v_ijk
  z_i   = (p_i*p_k) / ((p_k*p_j) + (p_k*p_i))
  y_ijk = (p_i*p_j) / ((p_i*p_j) + (p_i*p_k) + (p_j*p_k))
  p_ij  = p_i + p_j - y_ijk/(1.0_omega_prec - y_ijk) * p_k
  pp_k  = (1.0/(1.0_omega_prec - y_ijk)) * p_k
  v_ijk = 8.0_omega_prec * PI * CF * &
       (2.0 / (1.0 - z_i*(1.0 - y_ijk)) - (1.0 + z_i))
end subroutine ff_dipole
```

IF Dipoles



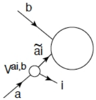

```
subroutine if_dipole (v_kja,u_j,p_aj,pp_k,p_k,p_j,p_a)
  type(momentum), intent(in) :: p_k, p_j, p_a
  type(momentum), intent(out) :: p_aj, pp_k
  real(kind=omega_prec), intent(out) :: u_j
  real(kind=omega_prec) :: x_kja
  real(kind=omega_prec), intent(out) :: v_kja
  u_j   = (p_a*p_j) / ((p_a*p_j) + (p_a*p_k))
  x_kja = ((p_a*p_k) + (p_a*p_j) - (p_j*p_k)) &
       / ((p_a*p_j) + (p_a*p_k))
  p_aj  = x_kja * p_a
  pp_k  = p_k + p_j - (1.0_omega_prec - x_kja) * p_a
  v_kja = 8.0_omega_prec * PI * CF * &
       (2.0 / (1.0 - x_kja + u_j) - (1.0 + x_kja)) / x_kja
end subroutine if_dipole
```

FI Dipoles




```
subroutine fi_dipole (v_ija,x_ija,p_ij,pp_a,p_i,p_j,p_a)
  type(momentum), intent(in) :: p_i, p_j, p_a
  type(momentum), intent(out) :: p_ij, pp_a
  real(kind=omega_prec), intent(out) :: x_ija
  real(kind=omega_prec) :: z_i
  real(kind=omega_prec), intent(out) :: v_ija
  z_i   = (p_i*p_a) / ((p_a*p_j) + (p_a*p_i))
  x_ija = ((p_i*p_a) + (p_j*p_a) - (p_i*p_j)) &
       / ((p_i*p_a) + (p_j*p_a))
  p_ij  = p_i + p_j - (1.0_omega_prec - x_ija) * p_a
  pp_a  = x_ija * p_a
  v_ija = 8.0_omega_prec * PI * CF * &
       (2.0 / (1.0 - z_i + (1.0 - x_ija)) - (1.0 + z_i)) / x_ija
end subroutine fi_dipole
```

II Dipoles




```
subroutine ii_dipole (v_jab,v_j,pp,p_aj,p_a,p_b,p_j)
  type(momentum), intent(in) :: p_a, p_b, p_j
  type(momentum) :: k, kk
  type(momentum), intent(out) :: p_aj
  type(momentum), dimension(:), intent(inout) :: pp
  real(kind=omega_prec), intent(inout) :: v_j
  real(kind=omega_prec) :: x_jab
  real(kind=omega_prec), intent(out) :: v_jab
  integer :: i
  x_jab = ((p_a*p_b) - (p_a*p_j) - (p_b*p_j)) / (p_a*p_b)
  v_j = (p_a*p_j) / (p_a * p_b)
  p_aj  = x_jab * p_a
  k     = p_a + p_b - p_j
  kk    = p_aj + p_b
  do i = 3,6
     pp(i) = pp(i) - 2.0*((k+kk)*pp(i))/((k+kk)*(k+kk)) * (k+kk) + &
          (2.0 * (k*pp(i)) / (k*k)) * kk
  end do
  v_jab = 8.0_omega_prec * PI * CF * &
       (2.0 / (1.0 - x_jab) - (1.0 + x_jab)) / x_jab
end subroutine ii_dipole
```

Phase space slicing

- To reduce unnecessary calls in irrelevant phase space regions: use $0 \leq \alpha \leq 1$ phase-space slicing [Nagy, hep-ph/0307268]
- Separates soft-/collinear PS regions from main bulk of PS
- $\alpha = 1$ corresponds to complete Catani-Seymour

$$\begin{aligned} D_{ij,k} &\to D_{ij,k} \times \theta(y_{ij,k} < \alpha) \\ D_k^{ai} &\to D_k^{ai} \times \theta(u_i < \alpha) \\ D_{ij}^{a} &\to D_{ij}^{a} \times \theta(1 - x_{ij,a} < \alpha) \\ D^{ai,b} &\to D^{ai,b} \times \theta(\bar{v}_i < \alpha) \end{aligned}$$

$$K_i \to K_i(\alpha) = K_i - \mathbf{T}_i^2 \log^2 \alpha + \gamma_i(\alpha - 1 - \log \alpha)$$

Modifications in flavor kernels: $+$- $\to (1-\alpha)$-distributions

- Stabilizes numerical integrations and speeds things up!

The integrated dipoles (aka as flavor kernels)

$$
\begin{aligned}
\int_0^1 dx\, \sigma^{NLO}_{a=u,b=\bar{u}}(x) = & \int_0^1 dx \int d\Phi(x\,p_a, p_b) \sum_{c_1c_2=1}^{6} \mathcal{W}^{c_1c_2}_{a'=u,b=\bar{u}}(xp_a, p_b, p_3, \ldots, p_6) \\
& \langle c_1 | \mathbf{K}^{a=u,a'=u}(x) + \mathbf{P}^{a=u,a'=u}(xp_a, x, \mu_F^2) | c_2 \rangle \\
& + \int_0^1 dx \int d\Phi(p_a, x\,p_b) \sum_{cd=1}^{6} \mathcal{W}^{c_1c_2}_{a=u,b'=\bar{u}}(p_a, x\,p_b, p_3, \ldots, p_6) \\
& \langle c_1 | \mathbf{K}^{b=\bar{u},b'=\bar{u}}(x) + \mathbf{P}^{b=\bar{u},b'=\bar{u}}(xp_b, x, \mu_F^2) | c_2 \rangle
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{K}^{a=u,a'=u}(x) = & \frac{\alpha_s}{2\pi} \Big\{ \bar{K}^{qq}(x) + \sum_i \mathbf{T}_i \cdot \mathbf{T}_a \frac{\gamma_i}{\mathbf{T}_i^2} \Big[\Big(\frac{1}{1-x} \Big)_+ + \delta(1-x) \Big] \\
& - \frac{\mathbf{T}_{b=\bar{u}} \cdot \mathbf{T}_{a'=u}}{\mathbf{T}^2_{a'=u}} \tilde{K}^{qq}(x) \Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{P}^{a=u,a'=u}(xp_{a=u}, x; \mu_F^2) = & \frac{\alpha_s}{2\pi} P^{qq}(x) \frac{1}{\mathbf{T}^2_{a'=u}} \Big[\sum_i \mathbf{T}_i \cdot \mathbf{T}_{a'=u} \ln \frac{\mu_F^2}{2xp_a \cdot p_i} \\
& + \mathbf{T}_b \cdot \mathbf{T}_{a'=u} \ln \frac{\mu_F^2}{2xp_a \cdot p_b} \Big]
\end{aligned}
$$

$$
\begin{aligned}
P^{qq}(x) &= C_F \left(\frac{1+x^2}{1-x} \right)_+ \\
P^{qq}_{\rm reg}(x) &= -C_F(1+x) \\
\bar{K}^{qq}(x) &= C_F \Big[\Big(\frac{2}{1-x} \log \frac{1-x}{x} \Big)_+ - (1+x) \log \frac{1-x}{x} + (1-x) - \delta(1-x)(5-\pi^2) \Big] \\
\tilde{K}^{qq}(x) &= P^{qq}_{\rm reg}(x) \log(1-x) + C_F \Big[\Big(\frac{2}{1-x} \log(1-x) \Big)_+ - \frac{\pi^2}{3} \delta(1-x) \Big]
\end{aligned}
$$

- implemented in WHIZARD as color-off-diagonal structure function
- specific phase space mapping for delta functions and $+$-distributions

Consistency Checks and Tests

VIRTUAL PART:

- Comparison diagram by diagram for single phase space points by two independent codes:

	Our Code	Alternative
Diagram Generation	QGraf	Feynarts
Simplification	Form	Form, Maple
Representation	numerical	analytic
	form factors	basis integrals
Numerical Evaluation	Fortran95	Maple

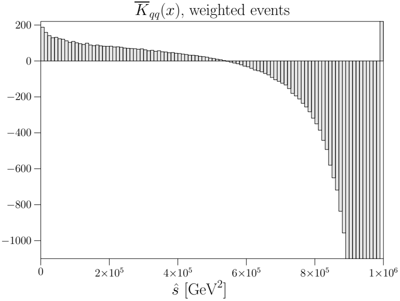
- Cancellation of infrared poles
- Independent analytic calculation (for up to 3-point diagrams)

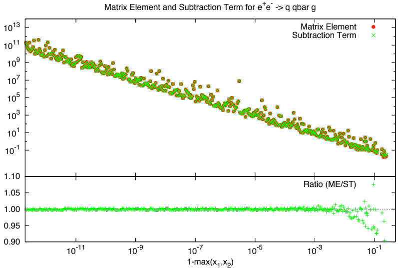
REAL PART:

- Independence of unintegrated dipoles from cut on unresolved parton ($p_T \lesssim 20$ GeV, then breakdown of soft-/collinear approx.)
- Explicit check with `MadDipole` ([R. Frederix et al.]) (complete agreement within 32bit accuracy)
- Cancellation of PS-slicing parameter α dependent terms (between flavor kernels, insertion operator, unintegrated dipoles, range $10^{-5} < \alpha < 0.1$)

More Checks and Tests...

- Checks for integrated dipoles: structure function approach vs. Born reweighted externally event by event
- Explicit checks of bits and pieces:
  (e.g. Monte Carlo over flavor kernels)





- Automatization: $e^+e^- \to q\bar{q}$, $e^+e^- \to q\bar{q}g$ [Schmidt/Guffanti/JR]
- Ratio of $|\mathcal{M}|^2$s
- collinear approx. breaks down at $x_{\max} \sim 0.99$ (as expected)
- numerically amazingly stable

Results: $u\bar{u} \to b\bar{b}b\bar{b}$ (virtual)

Results are based on

- 200,000 unweighted LO-events, generated from WHIZARD
- reweighted with local K-factor
- cuts: $p_T > 30\,\text{GeV}$, $\eta < 2.5$, $\Delta R > 0.4$
- $\mu_R = \mu_F = \sum p_T/4$, CTEQ6.5

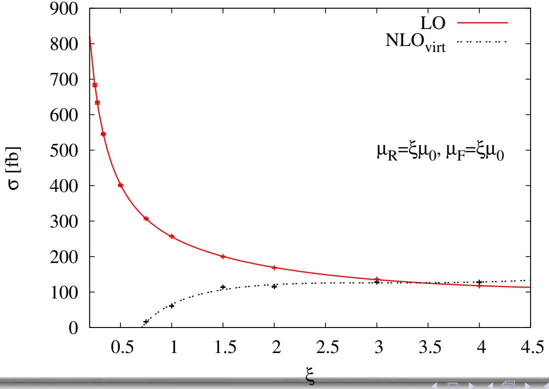
$$\langle O \rangle_{\text{LO}} = \frac{\sigma^{\text{LO}}}{N} \sum_E O(E)$$

$$\langle O \rangle_{\mathcal{V}} = \frac{\sigma^{\text{LO}}}{N} \sum_E \frac{d\sigma^{\text{LO}} + d\sigma^{\mathcal{V}} + \int_1 d\sigma^{\mathcal{A}}}{d\sigma^{\text{LO}}} O(E)$$

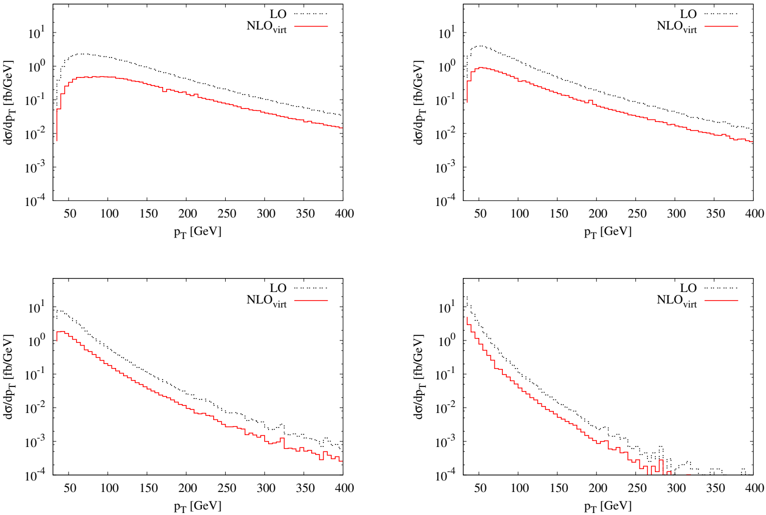
$E \in$ unweighted events

Results: $u\bar{u} \to b\bar{b}b\bar{b}$ (virtual)

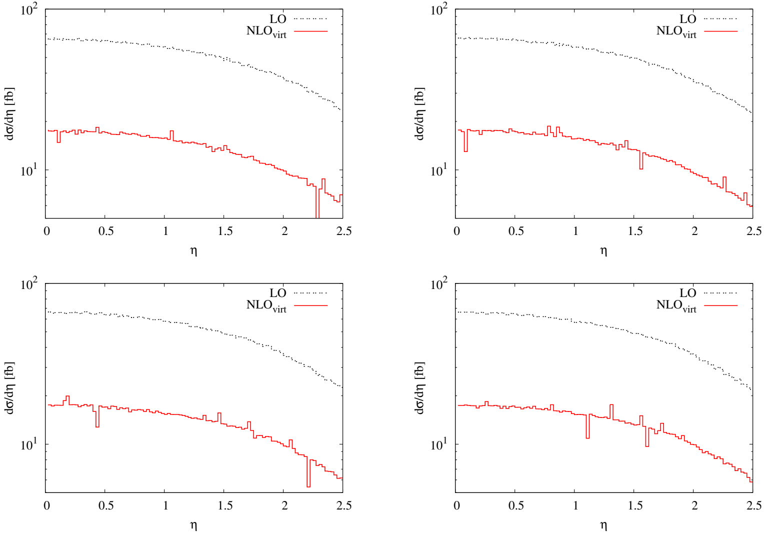
- Numerical evaluation very stable:
  1. cancellation of IR-poles
     all points passed test
  2. "suspicious" points (i.e. $K > 10$)
     2.4‰ failed, re-evaluated with quadruple precision
- K-factor contribution: 0.78
- Usual scale variation plot (Note: NLO is only virtual part, prelim.)



p_T Distributions: $u\bar{u} \to b\bar{b}b\bar{b}$ (virtual)



Rapidity Distributions: $u\bar{u} \to b\bar{b}b\bar{b}$ (virtual)



Note about statistics and run time

- **Virtual part:** $\mathcal{O}$(1s) per phase space point.
  (includes all helicities, all colors: no hidden cost)

  200k points took 60 CPUh.

  Cluster principle: go parallel (for both virtual and real parts)
  - Problem is “Embarrassingly Parallel”
  - Edinburgh Compute and Data Facility (ECDF)
  - 200k points take 12 minutes (on 345 nodes).
- **Real part:** less severe, but phase space integration is an issue
  - Unintegrated dipoles: $\mathcal{O}$ (1/2 day) for per mil precision ($\mathcal{O}$ (1 ms) per phase space point
  - Integrated dipoles: really fast

Numbers for single computers: (Intel Xeon®, QuadCore 2.4 GHz)

Conclusion and Outlook

- 4 b final state interesting signature for BSM models
- Exp. sensitivity analyses depend crucially on NLO SM bkgd. predictions
- Semi-automated, Feynman diagram-based generator by combining NLO calculator GOLEM and event generator WHIZARD
- GOLEM95: application of Fortran95 library for reduction of tensor integrals
- Phase-space generation/integration and dipole subtraction terms in WHIZARD
- Efficient method: Event-by-event reweighting of LO momenta with NLO amplitude

Currently in progress:

- Final number for the NLO cross section (within $\mathcal{O}$ (1 week)
- Complete automatization of WHIZARD dipole terms
- full $gg \to b\bar{b}b\bar{b}$ amplitude (virtual part ready, debugging phase)
- Further (long term) targets:
  - Generality (e.g. masses, particle content)
  - Easier setup, more automation
  - Improved performance
  - Matching to showers