

Little Higgs Models

Concepts and Phenomenology

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Kilian, JR **PRD 70** (2004), 015004; Kilian, Rainwater, JR **PRD 71** (2005), 015008;
PRD 74 (2006), 095003, Boersma/Godfrey/JR, work in progress

Theorie-Seminar RWTH Aachen, December 6th, 2007

Outline

Hierarchy Problem

- Higgs as Pseudo-Nambu-Goldstone Boson (PNGB)
- The Little Higgs mechanism

Generic properties – Examples of Models

Phenomenology

- Effective Field Theories
- Electroweak Precision Observables
- Neutrino masses
- LHC pheno – Heavy Quark States
- LHC pheno – Heavy Vectors
- LHC pheno – Heavy Scalars
- Reconstruction of Little Higgs Models

Pseudo-Axions in Little Higgs Models

- ZH eta coupling as a discriminator
- T parity and Dark Matter

Summary and Conclusions

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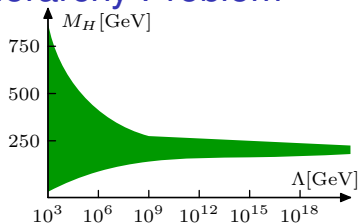
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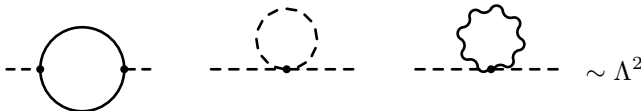
Summary and Conclusions

Hierarchy Problem



Motivation: **Hierarchy Problem**

- ▶ Effective theories below a scale $\Lambda \Rightarrow$
- ▶ Loop integration cut off at order $\sim \Lambda$:



Problem: Naturally, $m_h \sim \mathcal{O}(\Lambda^2)$:

$$m_h^2 = m_0^2 + \Lambda^2 \times (\text{loop factors})$$

◇ *Light* Higgs favoured by EW precision observables ($m_h < 0.5 \text{ TeV}$)

- ▶ $m_h \ll \Lambda \Leftrightarrow$ **Fine-Tuning !?**
- ▶ **Solutions:** Large number of ideas since 1970s

Overview of Solutions since 1970

(1) New strong interactions

- Technicolour: Higgs as a bound state of strongly-interacting partons

(2) Symmetry for cancellation of quantum corrections:

- **Supersymmetry**: Spin-Statistics \Rightarrow corrections from bosons and fermions cancel each other
- **Little Higgs mechanism**: Global symmetries \Rightarrow corrections from like-statistics particles cancel each other

(3) Non-trivial Space-time structure eliminates hierarchy:

- Large Extra Dimensions: Gravity appears only weak
- Higgsless models: components of (higher-dem.) gauge fields
- Warped Extra Dimensions (Randall-Sundrum): Gravity only weak in our world

(4) Ignoring the Hierarchy

- Anthropic principle: parameters have their values, *because we* (can) measure them

Higgs as Pseudo-Goldstone boson

Nambu-Goldstone Theorem: For each *spontaneously broken global symmetry generator* there is a **massless boson** in the spectrum.

Old idea:

Georgi/Pais, 1974; Georgi/Dimopoulos/Kaplan, 1984

Light Higgs as **(Pseudo)-Goldstone boson** of a spontaneously broken global symmetry

$$\pi_i \rightarrow i\theta^a T_{ik}^a \pi_k \Rightarrow \frac{\partial \mathcal{V}}{\partial \pi_i} T_{ij}^a \pi_j = 0 \Rightarrow \underbrace{\frac{\partial^2 \mathcal{V}}{\partial \pi_i \partial \pi_j} \Big|_F}_{=(m^2)_{ij}} T_{jk}^a f_k + \underbrace{\frac{\partial \mathcal{V}}{\partial \pi_j} \Big|_F}_{=0} T_{ji}^a = 0$$

Nonlinear Realization (Example $SU(3) \rightarrow SU(2)$):

$$\mathcal{V}(\Phi) = \left(f^2 - (\Phi^\dagger \Phi) \right)^2 \Rightarrow \Phi = \exp \left[\frac{i}{f} \left(\begin{array}{c|c} 0 & \vec{\pi} \\ \hline \vec{\pi}^\dagger & \pi_0 \end{array} \right) \right] \left(\begin{array}{c} 0 \\ f + \sigma \end{array} \right) \equiv e^{i\pi} \Phi_0$$

$\vec{\pi} \in$ fundamental $SU(2)$ rep., π_0 singlet

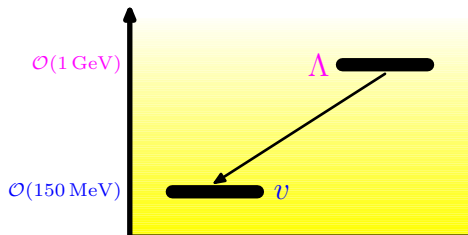
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Light Higgs as (Pseudo)-Goldstone boson of a spontaneously broken global symmetry



Analogous: QCD

Scale Λ : chiral symmetry breaking, quarks, $SU(3)_c$

Scale v : pions, kaons, ...

Without Fine-Tuning: experimentally excluded

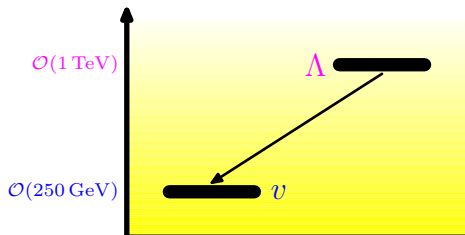
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Scale Λ : global symmetry breaking, new particles, new (gauge) IA

Scale v : Higgs, W/Z , ℓ^\pm , ...

Without Fine-Tuning: experimentally excluded

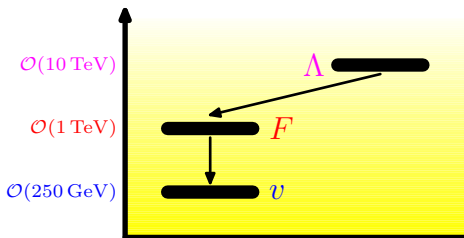
Collective symmetry breaking and 3-scale models

Collective symmetry breaking: Arkani-Hamed/Cohen/Georgi/Nelson/..., 2001

2 different global symmetries; one of them unbroken \Rightarrow Higgs
exact Goldstone boson

Coleman-Weinberg: boson masses by radiative corrections, but: m_H only at 2-loop level

$$m_H \sim \frac{g_1}{4\pi} \frac{g_2}{4\pi} \Lambda$$



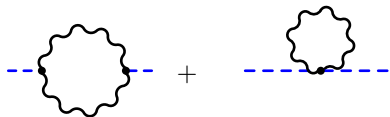
Scale Λ : global SB, new IA
Scale F : Pseudo-Goldstone bosons, new vectors/fermions
Scale v : Higgs, W/Z , ℓ^\pm , ...

Prime Example: Simple Group Model

- ▶ enlarged gauge group: $SU(3) \times U(1)$; globally $U(3) \rightarrow U(2)$
- ▶ **Two** nonlinear Φ representations $\mathcal{L} = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2$

$$\Phi_{1/2} = \exp \left[\pm i \frac{f_{2/1}}{f_{1/2}} \Theta \right] \begin{pmatrix} 0 \\ 0 \\ f_{1/2} \end{pmatrix} \quad \Theta = \frac{1}{\sqrt{f_1^2 + f_2^2}} \begin{pmatrix} \eta & 0 & h^* \\ 0 & \eta & \\ h^T & & \eta \end{pmatrix}$$

Coleman-Weinberg mechanism: Radiative generation of potential



The diagram shows two Feynman diagrams representing radiative corrections to a scalar self-energy. The first diagram is a tadpole diagram with a scalloped loop. The second diagram is a tadpole diagram with a scalloped loop and a dashed line loop. The sum of these diagrams is equated to a mathematical expression.

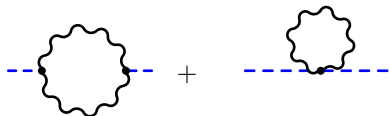
$$= \frac{g^2}{16\pi^2} \Lambda^2 (|\Phi_1|^2 + |\Phi_2|^2) \sim \frac{g^2}{16\pi^2} f^2$$

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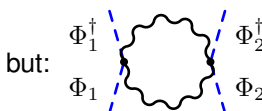
Coleman-Weinberg mechanism: Radiative generation of potential



The diagram shows two Feynman diagrams representing radiative corrections to the potential. The first diagram is a tadpole diagram with a dashed external line and a loop of a scalar field. The second diagram is a self-energy diagram with a dashed external line and a loop of a scalar field. The sum of these diagrams is equated to an expression involving the gauge coupling g , the scale Λ , and the vacuum expectation values f_1 and f_2 .

$$= \frac{g^2}{16\pi^2} \Lambda^2 (|\Phi_1|^2 + |\Phi_2|^2) \sim \frac{g^2}{16\pi^2} f^2$$

but:



The diagram shows a tadpole diagram with a dashed external line and a loop of a scalar field. The diagram is equated to an expression involving the gauge coupling g , the scale Λ , and the vacuum expectation values f_1 and f_2 .

$$= \frac{g^4}{16\pi^2} \log \left(\frac{\Lambda^2}{\mu^2} \right) |\Phi_1^\dagger \Phi_2|^2 \Rightarrow \frac{g^4}{16\pi^2} \log \left(\frac{\Lambda^2}{\mu^2} \right) f^2 (h^\dagger h)$$

Cancellations of Divergencies in Yukawa sector

$$\propto \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2(k^2 - m_T^2)} \left\{ \lambda_t^2(k^2 - m_T^2) + k^2 \lambda_T^2 - \frac{m_T}{F} \lambda_T k^2 \right\}$$

Little Higgs global symmetry imposes relation

$$\frac{m_T}{F} = \frac{\lambda_t^2 + \lambda_T^2}{\lambda_T}$$

 \Rightarrow

Quadratic divergence cancels

Collective Symm. breaking: $\lambda_t \propto \lambda_1 \lambda_2$, $\lambda_1 = 0$
 or $\lambda_2 = 0 \Rightarrow SU(3) \rightarrow [SU(3)]^2$

$$\sim \frac{\lambda_1^2 \lambda_2^2}{16\pi^2} \log \left(\frac{\Lambda^2}{\mu^2} \right) |\Phi_1^\dagger \Phi_2|^2$$

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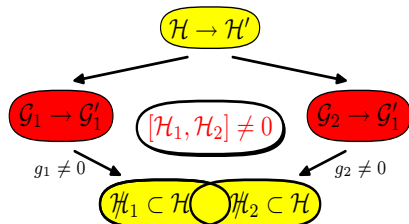
Summary and Conclusions

Generic properties of Little-Higgs models

- Extended global symmetry (extended scalar sector)
- **Specific functional form of the potential**
- Extended gauge symmetry:
 γ', Z', W'^{\pm}
- New heavy fermions: T , but also U, C, \dots

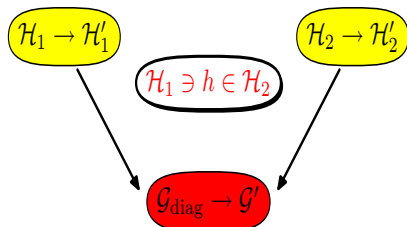
Product Group Models

(e.g. Littlest Higgs)



Simple Group Models

(e.g. Simplest Little Higgs)

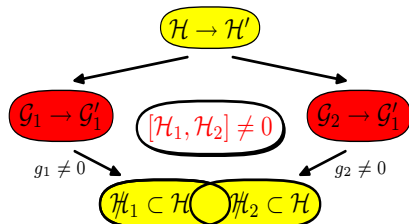


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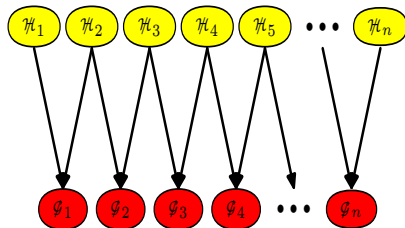
Product Group Models

(e.g. Littlest Higgs)



Moose Models

(e.g. Minimal Moose Model)



Little Higgs Models

Plethora of “Little Higgs Models” in 3 categories:

▶ Moose Models

- ▶ Orig. Moose (Arkani-Hamed/Cohen/Georgi, 0105239)
- ▶ Simple Moose (Arkani-Hamed/Cohen/Katz/Nelson/Gregoire/Wacker, 0206020)
- ▶ Linear Moose (Casalbuoni/De Curtis/Dominici, 0405188)

▶ Simple (Goldstone) Representation Models

- ▶ Littlest Higgs (Arkani-Hamed/Cohen/Katz/Nelson, 0206021)
- ▶ Antisymmetric Little Higgs (Low/Skiba/Smith, 0207243)
- ▶ Custodial $SU(2)$ Little Higgs (Chang/Wacker, 0303001)
- ▶ Littlest Custodial Higgs (Chang, 0306034)
- ▶ Little SUSY (Birkedal/Chacko/Gaillard, 0404197)

▶ Simple (Gauge) Group Models

- ▶ Orig. Simple Group Model (Kaplan/Schmaltz, 0302049)
- ▶ Holographic Little Higgs (Contino/Nomura/Pomarol, 0306259)
- ▶ Simplest Little Higgs (Schmaltz, 0407143)
- ▶ Simplest Little SUSY (Roy/Schmaltz, 0509357)
- ▶ Simplest T parity (Kilian/Rainwater/JR/Schmaltz,...)

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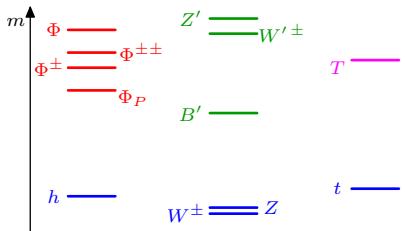
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Varieties of Particle spectra

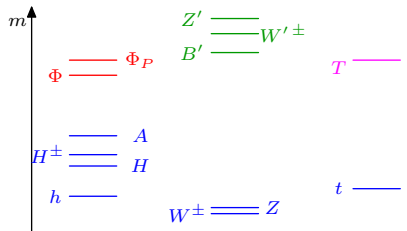
$$\mathcal{H} = \frac{SU(5)}{SO(5)}, \mathcal{G} = \frac{[SU(2) \times U(1)]^2}{SU(2) \times U(1)}$$

Arkani-Hamed/Cohen/Katz/Nelson, 2002



$$\mathcal{H} = \frac{SO(6)}{Sp(6)}, \mathcal{G} = \frac{[SU(2) \times U(1)]^2}{SU(2) \times U(1)}$$

Low/Skiba/Smith, 2002



$$\mathcal{H} = \frac{[SU(3)]^2}{[SU(2)]^2}, \mathcal{G} = \frac{SU(3) \times U(1)}{SU(2) \times U(1)} \Rightarrow$$

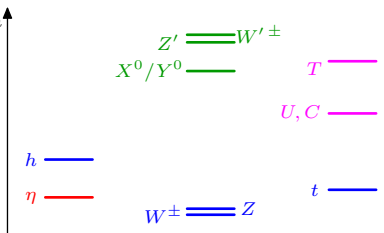
Schmaltz, 2004

$$[SU(4)]^4 \rightarrow [SU(3)]^4$$

Kaplan/Schmaltz, 2003

2HDM, $h_{1/2}$, $\Phi'_{1,2,3}$, $\Phi'_{P 1,2,3}$,

$Z'_{1,\dots,8}$, $W'_{1,2}$, q' , ℓ'



Effective Field Theories



How to *clearly* separate effects of **heavy degrees of freedom**?

Toy model: Two interacting scalar fields φ, Φ

$$\mathcal{Z}[j, J] = \int \mathcal{D}[\Phi] \mathcal{D}[\varphi] \exp \left[i \int dx \left(\frac{1}{2} (\partial\varphi)^2 - \frac{1}{2} \Phi (\square + M^2) \Phi - \lambda \varphi^2 \Phi - \dots + J\Phi + j\varphi \right) \right]$$

Low-energy effective theory \Rightarrow integrating out **heavy degrees of freedom (DOF)** in path integrals, set up **Power Counting** Kilian/JR, 2003

Completing the square:

$$\Phi' = \Phi + \frac{\lambda}{M^2} \left(1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2 \Rightarrow$$

$$\frac{1}{2} (\partial\Phi)^2 - \frac{1}{2} M^2 \Phi^2 - \lambda \varphi^2 \Phi = -\frac{1}{2} \Phi' (M^2 + \partial^2) \Phi' + \frac{\lambda^2}{2M^2} \varphi^2 \left(1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2.$$

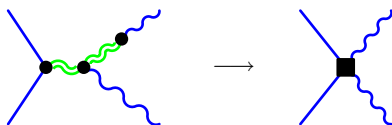
Effective Dim. 6 Operators

$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{F^2} \text{tr}[J^{(I)} \cdot J^{(I)}]$$

$$\mathcal{O}'_{h,1} = \frac{1}{F^2} ((Dh)^\dagger h) \cdot (h^\dagger (Dh)) - \frac{v^2}{2} |Dh|^2$$

$$\mathcal{O}'_{hh} = \frac{1}{F^2} (h^\dagger h - v^2/2) (Dh)^\dagger \cdot (Dh)$$

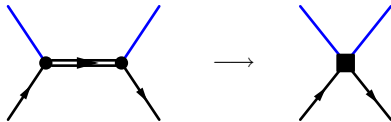
$$\mathcal{O}'_{h,3} = \frac{1}{F^2} \frac{1}{3} (h^\dagger h - v^2/2)^3$$



$$\mathcal{O}'_{WW} = -\frac{1}{F^2} \frac{1}{2} (h^\dagger h - v^2/2) \text{tr} W_{\mu\nu} W^{\mu\nu}$$

$$\mathcal{O}_B = \frac{1}{F^2} \frac{i}{2} (D_\mu h)^\dagger (D_\nu h) B^{\mu\nu}$$

$$\mathcal{O}'_{BB} = -\frac{1}{F^2} \frac{1}{4} (h^\dagger h - v^2/2) B_{\mu\nu} B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{F^2} \bar{q} h (\not{D} h) q$$

Oblique Corrections: S, T, U

$$\Delta T \sim \Delta\rho \sim \Delta M_Z^2 Z \cdot Z$$

$$\Delta S \sim W^0_{\mu\nu} B^{\mu\nu}, \Delta U \sim W^0_{\mu\nu} W^{0\mu\nu}$$

-
- ◇ All low-energy effects order v^2/F^2 (Wilson coefficients)
 - ◇ Low-energy observables with low-energy input G_F, α, M_Z affected by **non-oblique** contributions:

$$G_F = \frac{1}{v} \longrightarrow \frac{1}{v} (1 - \alpha\Delta T + \delta),$$

$$\delta \equiv -\frac{v^2}{4} f_{JJ}^{(3)} \xrightarrow{\text{LHM}} -\frac{c^4 v^2}{F^2}$$

$$S_{\text{eff}} = \Delta S$$

$$T_{\text{eff}} = \Delta T - \frac{1}{\alpha}\delta$$

$$U_{\text{eff}} = [\Delta U = 0] + \frac{4s_w^2}{\alpha}\delta$$

- ▶ Little Higgs Models: $S_{\text{eff}}, T_{\text{eff}}, c, c'$
- ▶ non-oblique flavour-dependent corrections \Rightarrow enforce **flavour-dependent EW fit**

Constraints on LHM

Constraints from **contact IA**: $(f_{JJ}^{(3)}, f_{JJ}^{(1)})$ $4.5 \text{ TeV} \lesssim F/c^2$ $10 \text{ TeV} \lesssim F/c'^2$

◇ **Constraints evaded** $\iff c, c' \ll 1$
 B', Z', W'^{\pm} superheavy ($\mathcal{O}(\Lambda)$) *decouple from fermions*

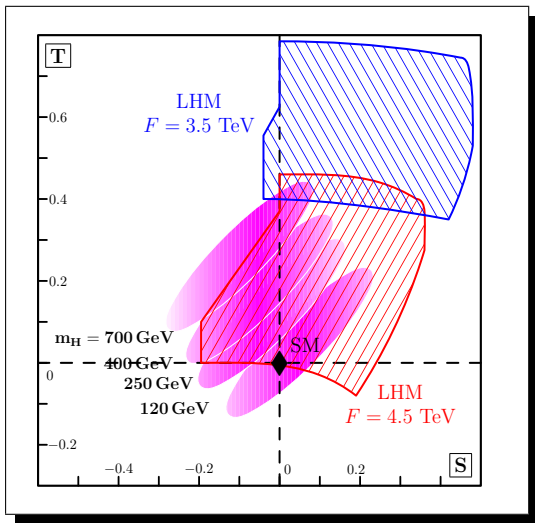
$\Delta S, \Delta T$ in the **Littlest Higgs model**, violation of **Custodial SU(2)**: Csáki
 et al., 2002; Hewett et al., 2002; Han et al., 2003; Chen/Dawson, 2003; Kilian/JR, 2003

$$\frac{\Delta S}{8\pi} = - \left[\frac{c^2(c^2 - s^2)}{g^2} + 5 \frac{c'^2(c'^2 - s'^2)}{g'^2} \right] \frac{v^2}{F^2} \rightarrow 0 \quad \alpha \Delta T \rightarrow \frac{5}{4} \frac{v^2}{F^2} - \frac{2v^2 \lambda_{2\phi}^2}{M_\phi^4} \gtrsim \frac{v^2}{F^2}$$

General models

- ▶ Triplet sector: (almost) identical to Littlest Higgs (ΔS only)
- ▶ More freedom in $U(1)$ sector: (ΔT)

EW Precision Observables



Higgs mass *variable*
(Coleman-Weinberg,
UV completion)

$$\Delta S = \frac{1}{12\pi} \ln \frac{m_H^2}{m_0^2}$$

$$\Delta T = -\frac{3}{16\pi c_w^2} \ln \frac{m_H^2}{m_0^2}$$

Peskin/Takeuchi, 1992; Hagiwara et al., 1992

Making the Higgs heavier reduces amount of fine-tuning

Neutrino masses

Kilian/JR, 2003; del Aguila et al., 2004; Han/Logan/Wang, 2005

- ★ *Naturalness does not require cancellation mechanism for light fermions*

Lepton-number violating interactions can generate **neutrino masses**
(due to presence of triplet scalars)

Lagrangian invariant under **full gauge symmetry**

$$\mathcal{L}_N = -g_N F (\bar{L}^c)^T \Xi L \quad \text{with} \quad L = (i\tau^2 \ell_L, 0, 0)^T$$

EWSB: Generation of neutrino masses $m_\nu \sim g_N v^2 / F$

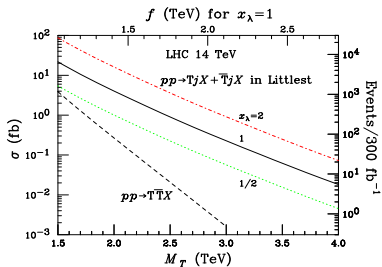


Caveat: m_ν too large compared to observations

$\Rightarrow g_N$ small, e.g. F/Λ' , where Λ' : scale of lepton number breaking

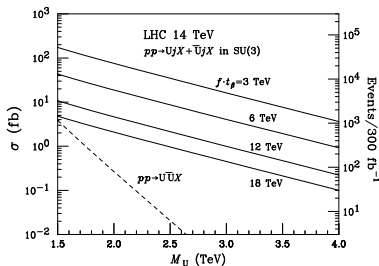
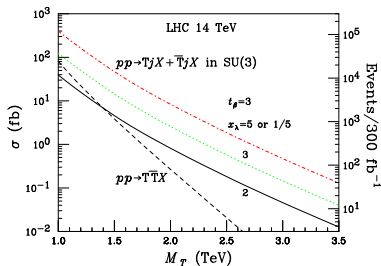
Heavy Quark States

- EW single dominates QCD pair production: Perelstein/Peskin/Pierce, '03



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- Characteristic branching ratios :

$$\Gamma(T \rightarrow th) \approx \Gamma(T \rightarrow tZ) \approx \frac{1}{2} \Gamma(T \rightarrow bW^+) \approx \frac{M_T \lambda_T^2}{64\pi}, \quad \Gamma_T \sim 10-50 \text{ GeV}$$

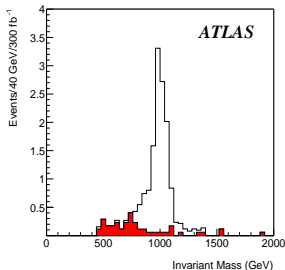
- Proof of T as EW singlet; but: $T \rightarrow Z'T, W'b, t\eta$!

AIM: Determination of $M_T, \lambda_T, \lambda_{T'}$

$\lambda_{T'}$ indirect ($T\bar{T}h$ impossible)

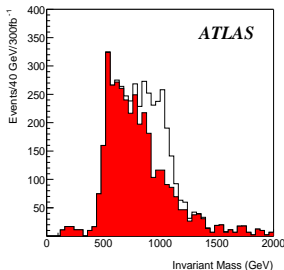
$T \rightarrow Zt \rightarrow \ell^+ \ell^- \ell \nu b$ SN-ATLAS-2004-038

-
- ▶ $\cancel{E}_T > 100 \text{ GeV}$, $ll\ell, p_T > 100/30 \text{ GeV}$,
 $b, p_T > 30 \text{ GeV}$
 - ▶ Bkgd.: WZ, ZZ, btZ
 - ▶ Observation for $M_T \lesssim 1.4 \text{ TeV}$



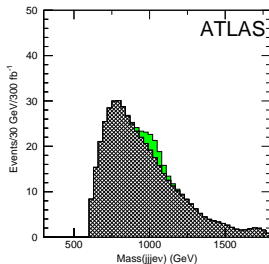
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-
- ▶ $\cancel{E}_T > 100 \text{ GeV}$, $\ell, p_T > 100 \text{ GeV}$,
 $b, p_T > 200 \text{ GeV}$, max. $jj, p_T > 30 \text{ GeV}$
 - ▶ Bkgd.: $t\bar{t}$, $Wb\bar{b}$, single t
 - ▶ Observation for $M_T \lesssim 2.5 \text{ TeV}$



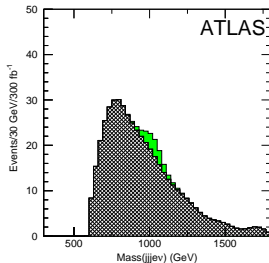
$T \rightarrow th \rightarrow \ell\nu bbb$ SN-ATLAS-2004-038

- ▶ $\ell, p_T > 100 \text{ GeV}, jjj, p_T > 130 \text{ GeV}$,
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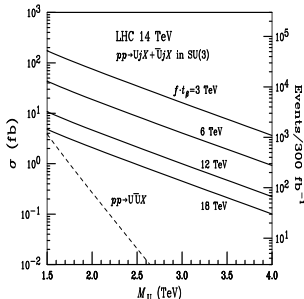
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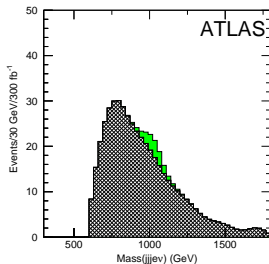
Additional heavy quarks (Simple Group Models): U, C or D, S Han et al., 05

- ▶ Large cross section: u or d PDF
- ▶ Huge final state ℓ charge asymmetry
- ▶ Good mass reconstruction



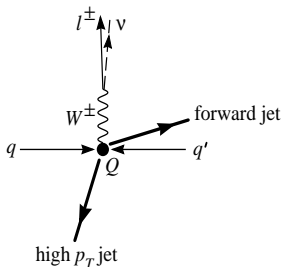
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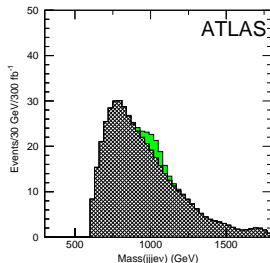
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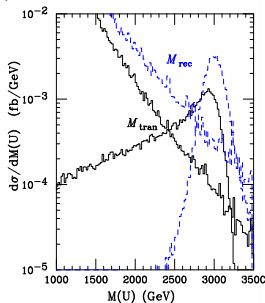
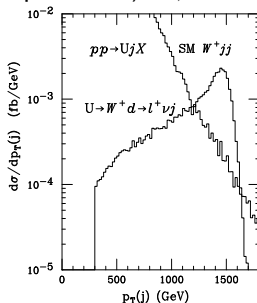
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Heavy Vectors

Drell-Yan Production:

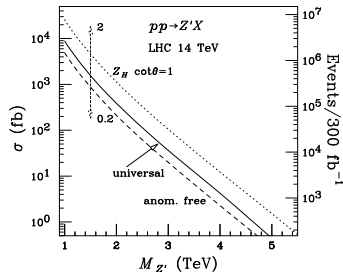
Tevatron Limits $\sim 500 - 600$ GeV

► **Dominant decays:**

Product group: $Z' \rightarrow Zh, WW,$

$W' \rightarrow Wh, WZ$

Simple group: $Z' \rightarrow qq, \quad X \rightarrow fF$



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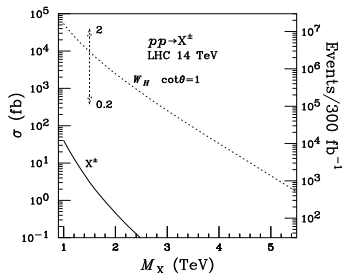
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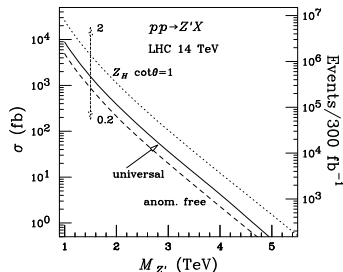


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- ▶ $\Gamma_{Z'} \sim 10 - 50$ GeV, $\Gamma_X \sim 0.1 - 10$ GeV

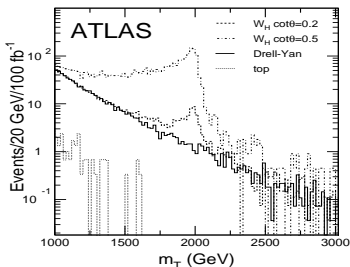
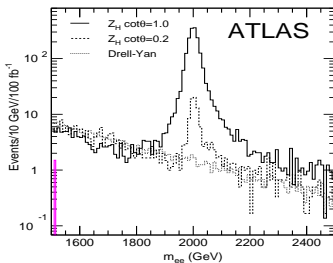
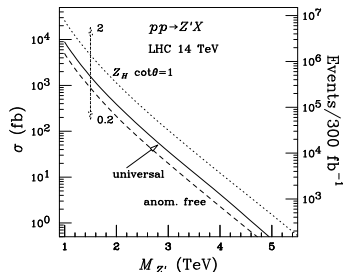


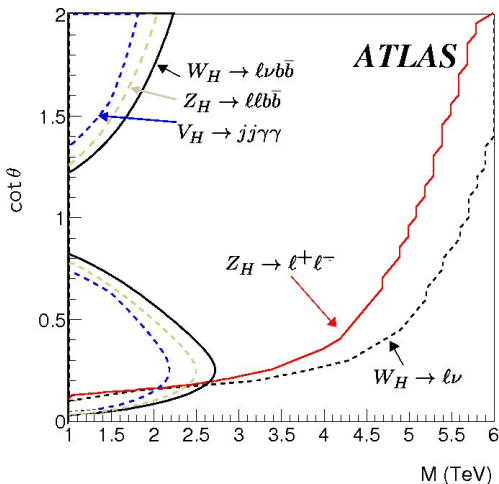
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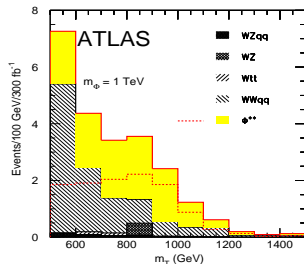
Proof: Sum rule for cancellation of divergences: $g_{HHVV} + g_{HHV'V'} = 0$,
 associated production $pp \rightarrow V'h$

Heavy Scalars

Generally: **Large model dependence**

no states complex singlet **complex triplet**

- ▶ **Littlest Higgs**, complex triplet:
 $\Phi^0, \Phi_P, \Phi^\pm, \Phi^{\pm\pm}$
- ▶ Cleanest channel: $q\bar{q} \rightarrow \Phi^{++}\Phi^{--} \rightarrow llll$:
 Killer: PS
- ▶ WW -Fusion: $dd \rightarrow uu\Phi^{++} \rightarrow uuW^+W^+$
- ▶ 2 hard forward jets, hard close l^+l^+
 p_T -unbalanced



Alternative: Model-Independent search in WW fusion:

ILC: Beyer/Kilian/Krstonosic/Mönig/JR/Schmidt/Schröder, 2006

LHC: Kilian/Kobel/Mader/JR/Schumacher/Schumacher

Reconstruction of Little Higgs Models

Kilian/JR, 2003; Han et al., 2005

- ◇ Goldstone-boson nature of Higgs boson (nonlinear representation)
- ◇ Mechanism for cancellation of δm_H quantum corrections

STRATEGY:

Kilian/JR, 2003

- ▶ LHC: $Z', W' \Rightarrow M_{Z'}, M_{W'}$ up to 5 – 6 TeV
ILC: contact terms $\Rightarrow M_{Z'}, M_{W'}$ up to 10 – 20 TeV
Extraction of F and $c \equiv \cos \phi$
- ▶ LHC: $T \Rightarrow M_T$ and mixing parameters
- ▶ ILC: Higgsstrahlung and WW fusion (angular distributions/energy spectra) \Rightarrow Higgs couplings/potential
- ▶ ILC/ $\gamma\gamma$: Higgs decays \Rightarrow Goldstone boson structure
- ▶ ILC/GigaZ: measurement of $\Delta T \Rightarrow$ contributions of heavy scalars
- ▶ Global fit to LHC/ILC data

Outline

Hierarchy Problem

- Higgs as Pseudo-Nambu-Goldstone Boson (PNGB)
- The Little Higgs mechanism

Generic properties – Examples of Models

Phenomenology

- Effective Field Theories
- Electroweak Precision Observables
- Neutrino masses
- LHC pheno – Heavy Quark States
- LHC pheno – Heavy Vectors
- LHC pheno – Heavy Scalars
- Reconstruction of Little Higgs Models

Pseudo-Axions in Little Higgs Models

- ZH eta coupling as a discriminator
- T parity and Dark Matter

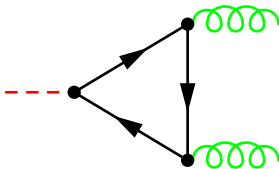
Summary and Conclusions

Pseudo-Axions in Little Higgs

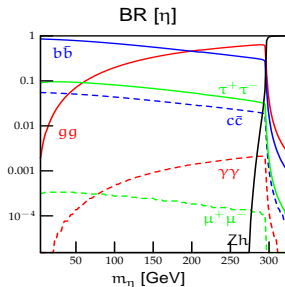
Kilian/Rainwater/JR, 2004, 2006; JR, 2007

- gauged $U(1)$ group: Z' \longleftrightarrow ungauged: η
- couples to fermions like a pseudoscalar
- $m_\eta \lesssim 400$ GeV
- SM singlet, couplings to SM particles v/F suppressed
- η axion-like particle:

Anomalous $U(1)$: - - -



$$\longrightarrow \frac{1}{F} \frac{\alpha_s}{8\pi^2} \eta F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$



- $U(1)$ explicitly broken \Rightarrow Axion limits from astroparticle physics not applicable

Classification of Axions in Little Higgs Models

Number of Pseudo-Axions: $n = g - l$

Mismatch between global (g) and local rank reduction (l)

Product Group Models Arkani-Hamed, ...

- ▶ Doubling of electroweak gauge group: $SU(2) \times SU(2) \rightarrow SU(2)_L$,
 $U(1) \times U(1) \rightarrow U(1)_Y$ (latter not necessary) $\Rightarrow l = 1$
 - ▶ Littlest Higgs, $g: SU(5) \rightarrow SO(5) \Rightarrow n = (4 - 2) - 1 = 1$
 - ▶ antisymmetric, $g: Sp(6)/SO(6)$, $n = (3 - 2) - 1 = 0$

Simple Group Models Kaplan, Schmaltz, ...

- ▶ Simple gauge group: $SU(N) \times U(1) \rightarrow SU(2) \times U(1) \Rightarrow l = N - 2$
- ▶ Higgs is distributed over several global symmetry multiplets
- ▶ Simplest Little Higgs, $g: [SU(3)]^2/[SU(2)]^2$ $n = g - l = 2 - 1 = 1$
- ▶ Original Simple Group Model, $g: [SU(4)]^3/[SU(3)^3 \times SU(2)]$,
 $l: SU(4) \rightarrow SU(2)$ $n = g - l = 4 - 2 = 2$

Moose Models Arkani-Hamed, ...

- ▶ "Minimal" Moose: $g [SU(3)]^4 \rightarrow SU(3)$, $l [SU(3) \times SU(2)]/SU(2)$
 $n = g - l = 6 - 2 = 4$
- ▶ 3-site model: $g [SU(2)]^4/[SU(2)]^2$, $l [SU(2)]^2 \rightarrow SU(2)$, $n = 2 - 1 = 1$

$ZH\eta$ coupling as a discriminator

Kilian/Rainwater/JR, 2006

- ▶ pseudo-axion: $\xi = \exp[i\eta/F]$, $\Sigma = \exp[i\Pi/F]$ non-linear representation of the remaining Goldstone multiplet Π

$$\mathcal{L}_{\text{kin.}} \sim F^2 \text{Tr} [(D^\mu(\xi\Sigma)^\dagger)(D_\mu(\xi\Sigma))] = \dots - 2F(\partial_\mu\eta) \text{Im Tr} [(D^\mu\Sigma)^\dagger\Sigma] + O(\eta^2)$$

- ▶ Use special structure of covariant derivatives:

$$D_\mu\Sigma = \partial_\mu\Sigma + A_{1,\mu}^a (T_1^a\Sigma + \Sigma(T_1^a)^T) + A_{2,\mu}^a (T_2^a\Sigma + \Sigma(T_2^a)^T),$$

$$\text{Tr} [(D^\mu\Sigma)^\dagger\Sigma] \sim W_\mu^a \text{Tr} [\Sigma^\dagger(T_1^a + T_2^a)\Sigma + (T_1^a + T_2^a)^*] = 0.$$

- ▶ Little Higgs mechanism cancels this coupling
- ▶ Simple Group Models: $\Phi = \exp[i\Sigma/F]$, $\zeta = (0, \dots, 0, F)^T$ VEV directing in the N direction

$$\begin{aligned}\mathcal{L}_{\text{kin.}} &\sim F^2 D^\mu (\zeta^\dagger \Phi^\dagger) D_\mu (\Phi \zeta) = \dots + \frac{i}{F} (\partial_\mu \eta) \zeta^\dagger (\Phi^\dagger (D_\mu \Phi) - (D_\mu \Phi^\dagger) \Phi) \zeta \\ &= \dots + iF (\partial_\mu \eta) (\Phi^\dagger (D_\mu \Phi) - (D_\mu \Phi^\dagger) \Phi)_{N,N} .\end{aligned}$$

$$\Sigma = \begin{pmatrix} 0 & h \\ h^\dagger & 0 \end{pmatrix}, \quad \mathbb{V}_\mu = \begin{pmatrix} \mathbb{W}_\mu & 0 \\ 0 & 0 \end{pmatrix} + \text{heavy vector fields}$$

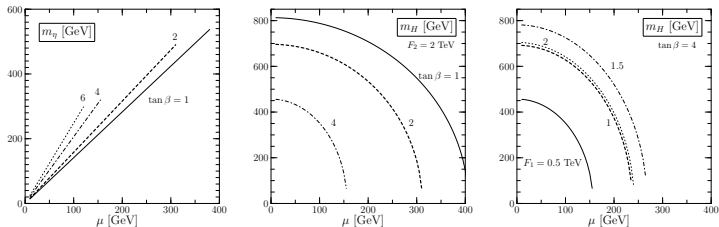
$$\begin{aligned}&\mathbb{V}_\mu + \frac{i}{F} [\Sigma, \mathbb{V}_\mu] - \frac{1}{2F^2} [\Sigma, [\Sigma, \mathbb{V}_\mu]] + \dots \\ &= \begin{pmatrix} \mathbb{W}_\mu & 0 \\ 0 & 0 \end{pmatrix} + \frac{i}{F} \begin{pmatrix} 0 & -\mathbb{W}_\mu h \\ h^\dagger \mathbb{W}_\mu & 0 \end{pmatrix} - \frac{1}{2F^2} \begin{pmatrix} hh^\dagger \mathbb{W} + \mathbb{W} h h^\dagger & 0 \\ 0 & -2h^\dagger \mathbb{W} h \end{pmatrix} + \dots\end{aligned}$$

- ▶ 1st term cancels by multiple Goldstone multiplets
- ▶ 2nd term cancels by EW symmetry
- ▶ 3rd term

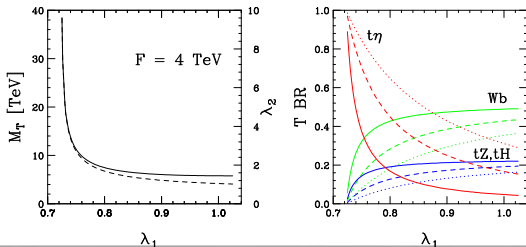
$$(\partial^\mu \eta) h^\dagger \mathbb{W}_\mu h \sim v H Z_\mu \partial^\mu \eta .$$

More properties of Pseudo-Axions

- ▶ Take e.g. one specific model: Simplest Little Higgs Schmaltz, 2004
- ▶ Simple Group Model, two Higgs-triplets with a $\tan\beta$ -like mixing angle



- ▶ $\tan\beta \sim 1$: heavy Higgs, (very) light pseudoscalar
- ▶ Heavy top decays: Kilian/Rainwater/JR, 2006



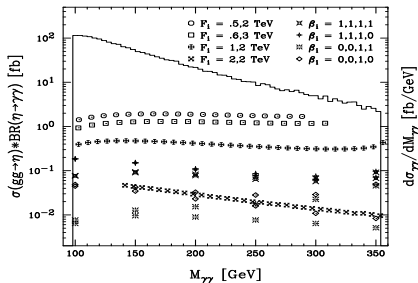
Discovery of Pseudo-axions

Kilian/Rainwater/JR, 2004, 2006

LHC: Gluon fusion, diphoton
signal for $m_\eta \gtrsim 200$ GeV, 7σ
possible

LHC: $T \rightarrow t\eta$

ILC: $e^+e^- \rightarrow t\bar{t}\eta$



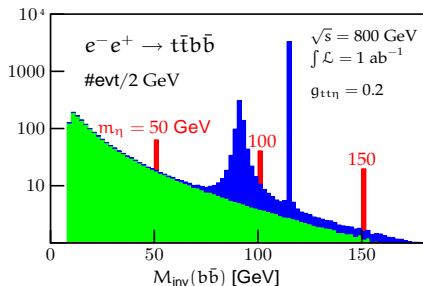
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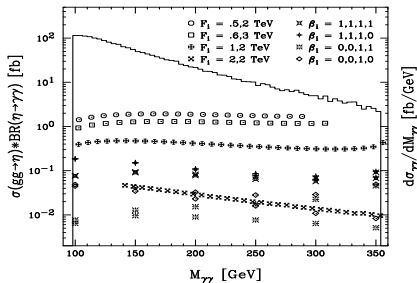
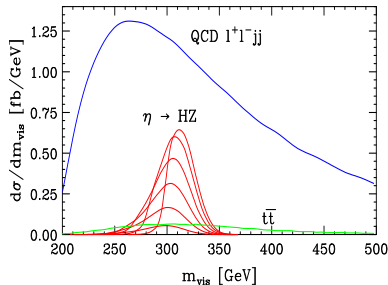
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ZHη coupling

forbidden in Product Group Models

Discriminator of diff. model classes

$$gg \rightarrow \left\{ \begin{array}{ll} H \rightarrow Z\eta & \rightarrow llbb \\ \eta \rightarrow ZH & \rightarrow llbb, llljj \end{array} \right\}$$

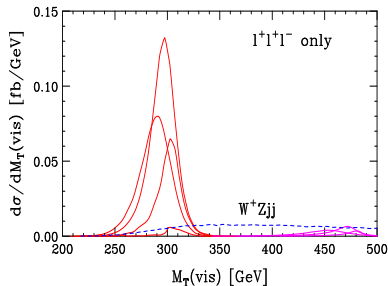
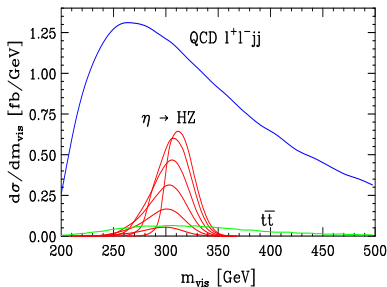
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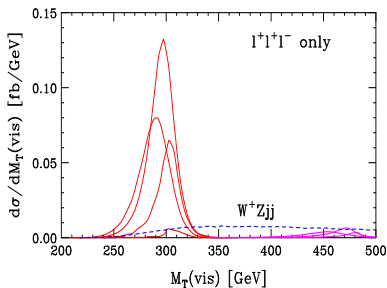
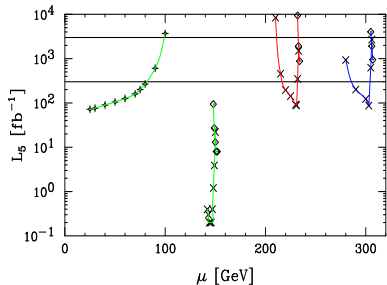
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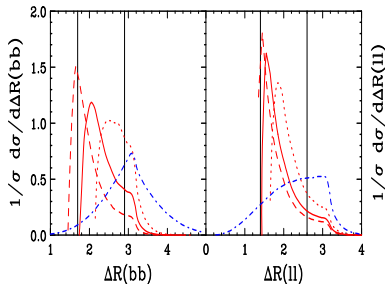
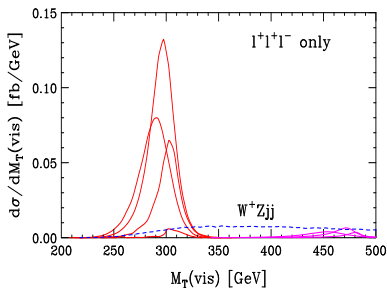
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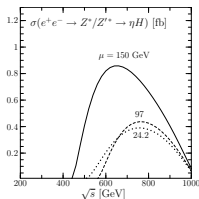
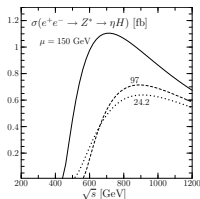
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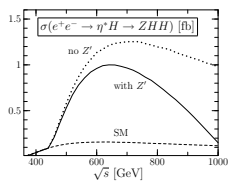
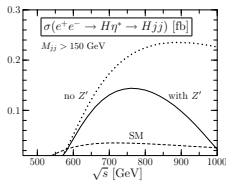
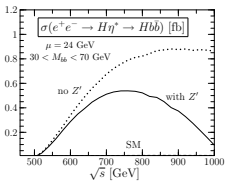
η pheno at ILC

Kilian/Rainwater/JR, 2006

If $ZH\eta$ coupling present: $H\eta$ production in analogy to HA :



- ▶ Light pseudoaxion, $\eta \rightarrow bb$, final state Hbb
- ▶ Intermediate range, $\eta \rightarrow gg$, final state Hjj
- ▶ $\eta \rightarrow ZH$: ZHH final state

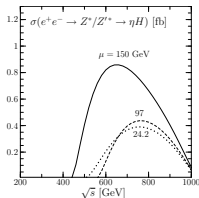
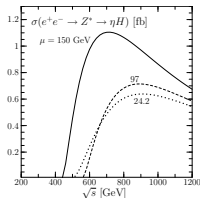


More detailed insights from photon collider option

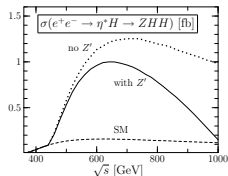
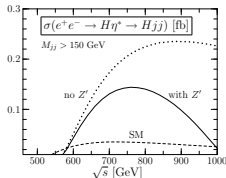
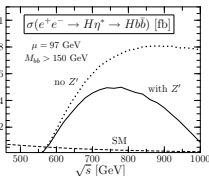
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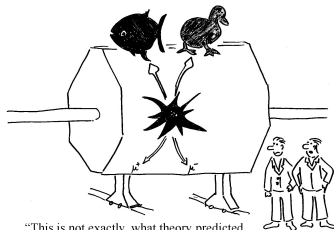
Invisible Higgs decays (?)

- ▶ “Invisible decay” $H \rightarrow \eta\eta$ [quite similar to $H \rightarrow aa$ in NMSSM]
but only due to mixing effects because
 $U(1)_\eta$ protective symmetry

$$\Gamma_{H \rightarrow \eta\eta} \sim \frac{1}{16\pi} \sqrt{1 - \frac{4m_\eta^2}{m_H^2}} \frac{v^5}{F^4} \sim \frac{15}{(F [\text{TeV}])^4} \text{ MeV}$$

- ▶ Light Higgs might become invisible at the LHC $H \rightarrow \eta\eta \rightarrow jjjj$
 - ▶ Not possible in Simplest Little Higgs
 - ▶ Possible in other Simple Group Models (together with η, A mixing)
 - ▶ Can become the dominant decay (with BR $\sim .8 - .95$)
- ▶ ILC can cover that hole!

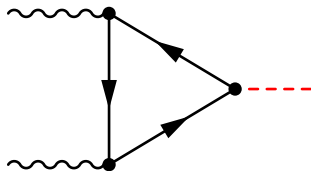
JR, 2007



“This is not exactly, what theory predicted for the Higgs decay!”

Pseudo Axions at the Photon Collider

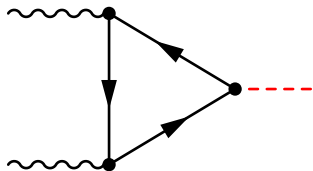
- ▶ **Photon Collider** as precision machine for Higgs physics (s channel resonance, anomaly coupling)



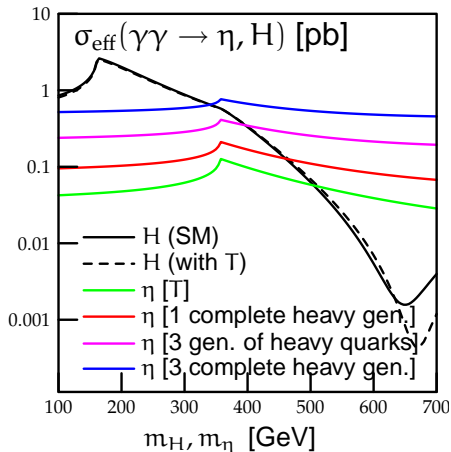
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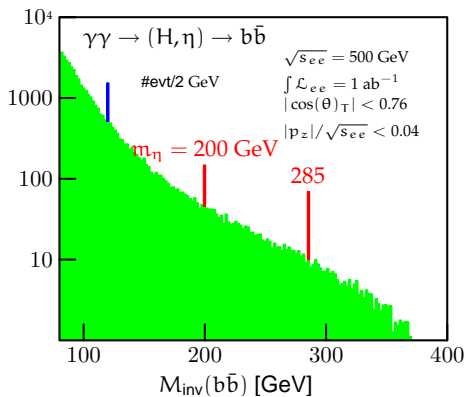
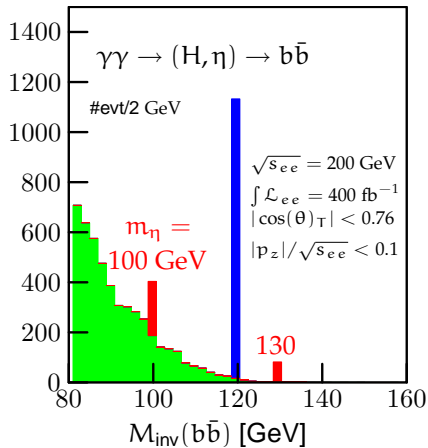


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$$g_{bb\eta} = 0.4 \cdot g_{bbh}$$

m_η	100	130	200	285
$\Gamma_{\gamma\gamma}$ [keV]	0.15	0.27	1.1	3.6



T parity and Dark Matter

Cheng/Low, 2003; Hubisz/Meade, 2005

- ▶ T parity: $T^a \rightarrow T^a$, $X^a \rightarrow -X^a$, automorphism of coset space analogous to R parity in SUSY, KK parity in extra dimensions
- ▶ Bounds on F MUCH relaxed, $F \sim 1 \text{ TeV}$
but: Pair production!, typical **cascade decays**
- ▶ Lightest T -odd particle (LTP) \Rightarrow **Candidate for Cold Dark Matter**

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Littlest Higgs: A' LTP

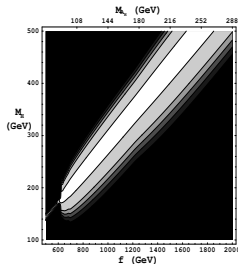
$W', Z' \sim 650$ GeV, $\Phi \sim 1$ TeV

$T, T' \sim 0.7$ -1 TeV

Annihilation: $A' A' \rightarrow h \rightarrow WW, ZZ, hh$

Hubisz/Meade, 2005

0/10/50/70/100



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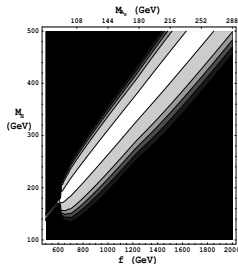
- ▶ T parity Simplest LH: **Pseudo-Axion η LTP**

Z' remains odd: good or bad (?)

Kilian/Rainwater/JR/Schmaltz

- ▶ T parity might be anomalous (???)

Hill/Hill, 2007



Outline

Hierarchy Problem

- Higgs as Pseudo-Nambu-Goldstone Boson (PNGB)
- The Little Higgs mechanism

Generic properties – Examples of Models

Phenomenology

- Effective Field Theories
- Electroweak Precision Observables
- Neutrino masses
- LHC pheno – Heavy Quark States
- LHC pheno – Heavy Vectors
- LHC pheno – Heavy Scalars
- Reconstruction of Little Higgs Models

Pseudo-Axions in Little Higgs Models

- ZH eta coupling as a discriminator
- T parity and Dark Matter

Summary and Conclusions

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Little Higgs: Global Symmetry structure stabilizes EW scale

New heavy gauge bosons, scalars, quarks (same spin)

EW precision observables:

Higgs is generically heavy in LHM (Little Big Higgs)

Inclusion of Dark Matter: T -parity (anomalies?)

Open questions: UV embedding, GUTs, Flavour

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Little Higgs models generally have extra pseudoscalars

- ▶ **Maybe the only observable states (before VLHC!)**
- ▶ Discriminator between Product and Simple Group Models
- ▶ LHC has first option: gluon fusion, $T/Z'/W'$ decays
- ▶ ILC can cover all regions
- ▶ Possible “invisible” Higgs decays

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Simplest Little Higgs (“ μ Model”)

Schmaltz '04, Kilian/Rainwater/JR '04

Field content ($SU(3)_c \times SU(3)_w \times U(1)_X$ quantum numbers)

$$\begin{array}{lll}
 \Phi_{1,2} & : (1, 3)_{-\frac{1}{3}} & \Psi_\ell & : (1, 3)_{-\frac{1}{3}} & u_{1,2}^c & : (\bar{3}, 1)_{-\frac{2}{3}} \\
 \Psi_Q & : (3, 3)_{\frac{1}{3}} & d^c & : (\bar{3}, 1)_{\frac{1}{3}} & e^c, n^c & : (1, 1)_{1,0}
 \end{array}$$

Lagrangian $\mathcal{L} = \mathcal{L}_{\text{kin.}} + \mathcal{L}_{\text{Yuk.}} + \mathcal{L}_{\text{pot.}}$ $\Psi_{Q,L} = (u, d, U)_L, \Psi_\ell = (\nu, \ell, N)_L$:

$$\begin{aligned}
 \mathcal{L}_{\text{Yuk.}} = & -\lambda_1^u \bar{u}_{1,R} \Phi_1^\dagger \Psi_{T,L} - \lambda_2^u \bar{u}_{2,R} \Phi_2^\dagger \Psi_{T,L} - \frac{\lambda^d}{\Lambda} \epsilon^{ijk} \bar{d}_R^b \Phi_1^i \Phi_2^j \Psi_{T,L}^k \\
 & - \lambda^n \bar{n}_{1,R} \Phi_1^\dagger \Psi_{Q,L} - \frac{\lambda^e}{\Lambda} \epsilon^{ijk} \bar{e}_R^i \Phi_1^j \Phi_2^k \Psi_{Q,L} + \text{h.c.},
 \end{aligned}$$

$$\mathcal{L}_{\text{pot.}} = \mu^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}$$

Hypercharge embedding (diag(1, 1, -2)/(2 $\sqrt{3}$)):

$$Y = X - T^8/\sqrt{3} \qquad D_\mu \Phi = (\partial_\mu - \frac{1}{3} g_X B_\mu^X \Phi + ig W_\mu^w) \Phi$$