Little Higgs Models
Concepts and Phenomenology

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Kilian, JR PRD 70 (2004), 015004; Kilian, Rainwater, JR PRD 71 (2005), 015008;
PRD 74 (2006), 095003, Boersma/Godfrey/JR, work in progress

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Outline

Hierarchy Problem
Higgs as Pseudo-Nambu-Goldstone Boson (PNGB)
The Little Higgs mechanism

Generic properties – Examples of Models

Phenomenology
Effective Field Theories
Electroweak Precision Observables
Neutrino masses
LHC pheno – Heavy Quark States
LHC pheno – Heavy Vectors
LHC pheno – Heavy Scalars
Reconstruction of Little Higgs Models

Pseudo-Axions in Little Higgs Models
ZH eta coupling as a discriminator
$T$ parity and Dark Matter

Summary and Conclusions
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Summary and Conclusions
Motivation: Hierarchy Problem

- Effective theories below a scale $\Lambda$ ⇒
- Loop integration cut off at order $\sim \Lambda$:

Problem: Naturally, $m_h \sim \mathcal{O}(\Lambda^2)$:

$$m_h^2 = m_0^2 + \Lambda^2 \times \text{(loop factors)}$$

Light Higgs favoured by EW precision observables ($m_h < 0.5 \text{ TeV}$)

$\blacklozenge$ $m_h \ll \Lambda \iff$ Fine-Tuning !?

$\blacklozenge$ Solutions: Large number of ideas since 1970s
Overview of Solutions since 1970

(1) **New strong interactions**
   - Technicolour: Higgs as a bound state of strongly-interacting partons

(2) **Symmetry for cancellation of quantum corrections:**
   - **Supersymmetry:** Spin-Statistics $\Rightarrow$ corrections from bosons and fermions cancel each other
   - **Little Higgs mechanism:** Global symmetries $\Rightarrow$ corrections from like-statistics particles cancel each other

(3) **Non-trivial Space-time structure eliminates hierarchy:**
   - Large Extra Dimensions: Gravity appears only weak
   - Higgsless models: Components of (higher-dem.) gauge fields
   - Warped Extra Dimensions (Randall-Sundrum): Gravity only weak in our world

(4) **Ignoring the Hierarchy**
   - Anthropic principle: parameters have their values, *because we (can) measure them*
Higgs as Pseudo-Goldstone boson

Nambu-Goldstone Theorem: For each \textit{spontaneously broken global} symmetry generator there is a massless boson in the spectrum.

Old idea: Georgi/Pais, 1974; Georgi/Dimopoulos/Kaplan, 1984

Light Higgs as (Pseudo)-Goldstone boson of a spontaneously broken global symmetry

\[
\pi_i \rightarrow i\theta^a T^a_{ik} \pi_k \quad \Rightarrow \quad \frac{\partial \mathcal{V}}{\partial \pi_i} T^a_{ij} \pi_j = 0 \quad \Rightarrow \quad \frac{\partial^2 \mathcal{V}}{\partial \pi_i \partial \pi_j} \begin{bmatrix} T^a_{jk} f_k + \frac{\partial \mathcal{V}}{\partial \pi_j} \end{bmatrix} F T^a_{ji} = 0
\]

\[
= (m^2)_{ij}
\]

Nonlinear Realization (Example $SU(3) \rightarrow SU(2)$):

\[
\mathcal{V}(\Phi) = \left( f^2 - (\Phi^\dagger \Phi) \right)^2 \Rightarrow \Phi = \exp \left[ \frac{i}{f} \begin{bmatrix} 0 \\ \bar{\pi} \\ \pi_0 \end{bmatrix} \right] \begin{bmatrix} 0 \\ f + \sigma \end{bmatrix} \equiv e^{i\pi} \Phi_0
\]

\[
\bar{\pi} \in \text{fundamental } SU(2) \text{ rep., } \quad \pi_0 \text{ singlet}
\]
Higgs as Pseudo-Goldstone boson

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Light Higgs as (Pseudo)-Goldstone boson of a spontaneously broken global symmetry

Analogous: QCD
- Scale $\Lambda$: chiral symmetry breaking, quarks, $SU(3)_c$
- Scale $v$: pions, kaons, ...

Without Fine-Tuning: experimentally excluded
Higgs as Pseudo-Goldstone boson

**Nambu-Goldstone Theorem:** For each *spontaneously broken global symmetry generator* there is a *massless boson* in the spectrum.

**Old idea:** Georgi/Pais, 1974; Georgi/Dimopoulos/Kaplan, 1984

Light Higgs as *(Pseudo)-Goldstone boson* of a spontaneously broken global symmetry

*Scale* $\Lambda$: global symmetry breaking, new particles, new (gauge) IA

*Scale* $v$: Higgs, $W/Z$, $\ell^{\pm}$, ...
Collective symmetry breaking and 3-scale models


2 different global symmetries; one of them unbroken $\Rightarrow$ Higgs exact Goldstone boson

Coleman-Weinberg: boson masses by radiative corrections, but: $m_H$ only at 2-loop level

$$m_H \sim \frac{g_1}{4\pi} \frac{g_2}{4\pi} \Lambda$$

Scale $\Lambda$: global SB, new IA

Scale $F$: Pseudo-Goldstone bosons, new vectors/fermions

Scale $v$: Higgs, $W/Z$, $\ell^\pm$, ...
Prime Example: Simple Group Model

- enlarged gauge group: $SU(3) \times U(1)$; globally $U(3) \to U(2)$
- Two nonlinear $\Phi$ representations

\[ \mathcal{L} = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2 \]

\[ \Phi_{1/2} = \exp \left[ \pm i \frac{f_{2/1}}{f_{1/2}} \Theta \right] \begin{pmatrix} 0 \\ 0 \\ f_{1/2} \end{pmatrix} \quad \Theta = \frac{1}{\sqrt{f_1^2 + f_2^2}} \begin{pmatrix} \eta & 0 & h^* \\ 0 & \eta & h^T \\ h & h & \eta \end{pmatrix} \]

Coleman-Weinberg mechanism: Radiative generation of potential

\[ \frac{g^2}{16\pi^2} \Lambda^2 (|\Phi_1|^2 + |\Phi_2|^2) \sim \frac{g^2}{16\pi^2} f^2 \]
Prime Example: Simple Group Model

- **enlarged gauge group**: $SU(3) \times U(1)$; globally $U(3) \rightarrow U(2)$
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Coleman-Weinberg mechanism: Radiative generation of potential

\[
\frac{g^2}{16\pi^2} \Lambda^2 \left( |\Phi_1|^2 + |\Phi_2|^2 \right) \sim \frac{g^2}{16\pi^2} f^2
\]

but:

\[
\frac{g^4}{16\pi^2} \log \left( \frac{\Lambda^2}{\mu^2} \right) |\Phi_1^\dagger \Phi_2|^2 \Rightarrow \frac{g^4}{16\pi^2} \log \left( \frac{\Lambda^2}{\mu^2} \right) f^2 (h^\dagger h)
\]
Cancellations of Divergencies in Yukawa sector

\[ \propto \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2(k^2 - m_T^2)} \left\{ \lambda_t^2(k^2 - m_T^2) + k^2 \lambda_T^2 - \frac{m_T}{F} \lambda_T k^2 \right\} \]

Little Higgs global symmetry imposes relation

\[ \frac{m_T}{F} = \frac{\lambda_t^2 + \lambda_T^2}{\lambda_T} \quad \Rightarrow \quad \boxed{\text{Quadratic divergence cancels}} \]

Collective Symm. breaking: \( \lambda_t \propto \lambda_1 \lambda_2 \), \( \lambda_1 = 0 \)

or \( \lambda_2 = 0 \) \( \Rightarrow \) \( SU(3) \rightarrow [SU(3)]^2 \)
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Summary and Conclusions
Generic properties of Little-Higgs models

- Extended global symmetry (extended scalar sector)
- Specific functional form of the potential
- Extended gauge symmetry: $\gamma', Z', W'^\pm$
- New heavy fermions: $T$, but also $U, C, \ldots$

**Product Group Models**
(e.g. Littlest Higgs)

\[ \mathcal{H} \to \mathcal{H}' \]
\[ G_1 \to G'_1 \]
\[ [\mathcal{H}_1, \mathcal{H}_2] \neq 0 \]
\[ G_2 \to G'_1 \]
\[ \mathcal{H}_1 \subset \mathcal{H} \]
\[ \mathcal{H}_2 \subset \mathcal{H} \]
\[ g_1 \neq 0 \]
\[ g_2 \neq 0 \]

**Simple Group Models**
(e.g. Simplest Little Higgs)

\[ \mathcal{H}_1 \to \mathcal{H}'_1 \]
\[ \mathcal{H}_2 \to \mathcal{H}'_2 \]
\[ \mathcal{H}_1 \ni h \in \mathcal{H}_2 \]
\[ \mathcal{H}'_1 \ni h' \in \mathcal{H}'_2 \]
\[ G_{\text{diag}} \to G' \]
Generic properties of Little-Higgs models

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Product Group Models

(e.g. Littlest Higgs)

\[
\mathcal{H} \rightarrow \mathcal{H}'
\]

\[
G_1 \rightarrow G_1'
\]

\[
[H_1, H_2] \neq 0
\]

\[
G_2 \rightarrow G_1'
\]

\[
g_1 \neq 0
\]

\[
g_2 \neq 0
\]

\[
H_1 \subset H \quad H_2 \subset H
\]

Moose Models

(e.g. Minimal Moose Model)

\[
H_1 \quad H_2 \quad H_3 \quad H_4 \quad H_5 \quad \ldots \quad H_n
\]

\[
G_1 \quad G_2 \quad G_3 \quad G_4 \quad \ldots \quad G_n
\]
Little Higgs Models

Plethora of “Little Higgs Models” in 3 categories:

- **Moose Models**
  - Orig. Moose (Arkani-Hamed/Cohen/Georgi, 0105239)
  - Simple Moose (Arkani-Hamed/Cohen/Katz/Nelson/Gregoire/Wacker, 0206020)
  - Linear Moose (Casalbuoni/De Curtis/Dominici, 0405188)

- **Simple (Goldstone) Representation Models**
  - Littlest Higgs (Arkani-Hamed/Cohen/Katz/Nelson, 0206021)
  - Antisymmetric Little Higgs (Low/Skiba/Smith, 0207243)
  - Custodial $SU(2)$ Little Higgs (Chang/Wacker, 0303001)
  - Littlest Custodial Higgs (Chang, 0306034)
  - Little SUSY (Birkedal/Chacko/Gaillard, 0404197)

- **Simple (Gauge) Group Models**
  - Orig. Simple Group Model (Kaplan/Schmaltz, 0302049)
  - Holographic Little Higgs (Contino/Nomura/Pomarol, 0306259)
  - Simplest Little Higgs (Schmaltz, 0407143)
  - Simplest Little SUSY (Roy/Schmaltz, 0509357)
  - Simplest T parity (Kilian/Rainwater/JR/Schmaltz,...)
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Varieties of Particle spectra

\[\mathcal{H} = \frac{SU(5)}{SO(5)}, \mathcal{G} = \frac{[SU(2) \times U(1)]^2}{SU(2) \times U(1)}\]

Arkani-Hamed/Cohen/Katz/Nelson, 2002

\[\mathcal{H} = \frac{SO(6)}{Sp(6)}, \mathcal{G} = \frac{[SU(2) \times U(1)]^2}{SU(2) \times U(1)}\]

Low/Skiba/Smith, 2002

\[\mathcal{H} = \frac{[SU(3)]^2}{[SU(2)]^2}, \mathcal{G} = \frac{SU(3) \times U(1)}{SU(2) \times U(1)}\]

Schmaltz, 2004

\[[SU(4)]^4 \rightarrow [SU(3)]^4\]

Kaplan/Schmaltz, 2003

2HDM, \(h_{1/2}, \Phi_{1,2,3}, \Phi'_{1,2,3}, Z'_1,\ldots, 8, W'_{1,2}^{\pm}, q', \ell' \)

\[Z' \subset \mathcal{H} = [SU(3)]^4 \rightarrow [SU(3)]^4 \]

Kaplan/Schmaltz, 2003

\[X^0/Y^0 \subset \mathcal{H} = [SU(3)]^4 \rightarrow [SU(3)]^4 \]

Schmaltz, 2004

\[W'_{1,2}^{\pm} \subset \mathcal{H} = [SU(3)]^4 \rightarrow [SU(3)]^4 \]

Schmaltz, 2004
Effective Field Theories

How to *clearly* separate effects of heavy degrees of freedom?

Toy model: Two interacting scalar fields $\varphi, \Phi$

$$\mathcal{Z}[j, J] = \int \mathcal{D}[\Phi] \mathcal{D}[\varphi] \exp \left[ i \int dx \left( \frac{1}{2} (\partial \varphi)^2 - \frac{1}{2} \Phi (\Box + M^2) \Phi - \lambda \varphi^2 \Phi - \ldots + J \Phi + j \varphi \right) \right]$$

Low-energy effective theory $\Rightarrow$ integrating out heavy degrees of freedom (DOF) in path integrals, set up Power Counting

Kilian/JR, 2003

Completing the square:

$$\Phi' = \Phi + \frac{\lambda}{M^2} \left( 1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2 \Rightarrow \quad \begin{array}{c} \text{Red} \\ \text{Black} \end{array} \quad \rightarrow \begin{array}{c} \text{Red} \\ \text{Black} \end{array}$$

$$\frac{1}{2} (\partial \Phi)^2 - \frac{1}{2} M^2 \Phi^2 - \lambda \varphi^2 \Phi = -\frac{1}{2} \Phi' (M^2 + \partial^2) \Phi' + \frac{\lambda^2}{2 M^2} \varphi^2 \left( 1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2.$$
Effective Dim. 6 Operators

\[ O^{(I)}_{JJ} = \frac{1}{F^2} \text{tr}[J^{(I)} \cdot J^{(I)}] \]

\[ O'_{h,1} = \frac{1}{F^2} \left( (Dh)^\dagger h \right) \cdot \left( h^\dagger (D^h) \right) - \frac{v^2}{2} |Dh|^2 \]

\[ O'_{hh} = \frac{1}{F^2} (h^\dagger h - v^2/2) (Dh)^\dagger \cdot (Dh) \]

\[ O'_{h,3} = \frac{1}{F^2} \frac{1}{3} (h^\dagger h - v^2/2)^3 \]
\[ \mathcal{O}_{WW}' = -\frac{1}{F^2} \frac{1}{2} (h^\dagger h - v^2/2) \text{tr} W_{\mu\nu} W^{\mu\nu} \]

\[ \mathcal{O}_B = \frac{1}{F^2} \frac{i}{2} (D_\mu h)^\dagger (D_\nu h) B^{\mu\nu} \]

\[ \mathcal{O}_{BB}' = -\frac{1}{F^2} \frac{1}{4} (h^\dagger h - v^2/2) B_{\mu\nu} B^{\mu\nu} \]

\[ \mathcal{O}_{Vq} = \frac{1}{F^2} \bar{q} h (\slashed{D} h) q \]
Oblique Corrections: $S$, $T$, $U$

\[ Z_L \quad Z_L \quad \rightarrow \quad Z_L \quad Z_L \quad \Delta T \sim \Delta \rho \sim \Delta M_Z^2 Z \cdot Z \]

\[ Z_T \quad Z_T \quad \rightarrow \quad Z_T \quad Z_T \quad \Delta S \sim W_0^{\mu \nu} B^{\mu \nu}, \Delta U \sim W_0^{\mu \nu} W_0^{\mu \nu} \]

- All low-energy effects order $v^2/F^2$ (Wilson coefficients)
- Low-energy observables with low-energy input $G_F$, $\alpha$, $M_Z$ affected by non-oblique contributions:

\[
G_F = \frac{1}{v} \quad \rightarrow \quad \frac{1}{v} (1 - \alpha \Delta T + \delta), \\
\delta \equiv -\frac{v^2}{4} \int f_{JJ}^{(3)} \text{ LHM} \quad \rightarrow \quad -\frac{c^4 v^2}{F^2}
\]

- Little Higgs Models: $S_{\text{eff}}$, $T_{\text{eff}}$, $c$, $c'$
- non-oblique flavour-dependent corrections $\Rightarrow$ enforce flavour-dependent EW fit
Constraints on LHM

Constraints from contact IA: \(( f_{JJ}^{(3)}, f_{JJ}^{(1)} ) \) 4.5 TeV \(\lesssim F / c^2 \) 10 TeV \(\lesssim F / c'^2 \)

\[ \Delta S, \Delta T \text{ in the Littlest Higgs model, violation of } \text{Custodial SU(2)}: \]

Csáki et al., 2002; Hewett et al., 2002; Han et al., 2003; Chen/Dawson, 2003; Kilian/JR, 2003

\[ \frac{\Delta S}{8\pi} = - \left[ \frac{c^2(c^2-s^2)}{g^2} + 5 \frac{c'^2(c'^2-s'^2)}{g'^2} \right] \frac{v^2}{F^2} \rightarrow 0 \quad \alpha \Delta T \rightarrow \frac{5}{4} \frac{v^2}{F^2} - \frac{2v^2\lambda_{2\phi}^2}{M_{\phi}^4} \gtrsim \frac{v^2}{F^2} \]

General models

- Triplet sector: (almost) identical to Littlest Higgs (\( \Delta S \) only)
- More freedom in \( U(1) \) sector: (\( \Delta T \))
EW Precision Observables

Higgs mass variable
(Coleman-Weinberg, UV completion)

\[ \Delta S = \frac{1}{12\pi} \ln \frac{m_H^2}{m_0^2} \]

\[ \Delta T = -\frac{3}{16\pi c_w^2} \ln \frac{m_H^2}{m_0^2} \]

Peskin/Takeuchi, 1992; Hagiwara et al., 1992

Making the Higgs heavier reduces amount of fine-tuning
Neutrino masses

Kilian/JR, 2003; del Aguila et al., 2004; Han/Logan/Wang, 2005

⋆ Naturalness does not require cancellation mechanism for light fermions

Lepton-number violating interactions can generate neutrino masses (due to presence of triplet scalars)

Lagrangian invariant under full gauge symmetry

$$\mathcal{L}_N = -g_N F (\bar{L}^c) T \Xi L$$

with $$L = (i \tau^2 \ell_L, 0, 0)^T$$

EWSB: Generation of neutrino masses

$$m_\nu \sim g_N \nu^2 / F$$

Caveat: $$m_\nu$$ too large compared to observations

$$\Rightarrow g_N$$ small, e.g. $$F / \Lambda'$$, where $$\Lambda'$$: scale of lepton number breaking
Heavy Quark States

- EW single dominates QCD pair production: Perelstein/Peskin/Pierce, ’03

![Graph showing production cross-section as a function of scalar mass.](image-url)
Heavy Quark States

- EW single dominates QCD pair production: Perelstein/Peskin/Pierce, ’03

Characteristic branching ratios:

\[ \Gamma(T \rightarrow th) \approx \Gamma(T \rightarrow tZ) \approx \frac{1}{2} \Gamma(T \rightarrow bW^+) \approx \frac{M_T \lambda_T^2}{64\pi}, \quad \Gamma_T \sim 10-50 \text{ GeV} \]

- Proof of \( T \) as EW singlet; but: \( T \rightarrow Z'T', W'b, t\eta! \)

**AIM:** Determination of \( M_T, \lambda_T, \lambda_{T'} \) \( \lambda_{T'} \) indirect (\( T\bar{T}h \) impossible)
$T \rightarrow Zt \rightarrow \ell^+\ell^-\ell\nu b$  

SN-ATLAS-2004-038

- $\not{E}_T > 100$ GeV, $\ell\ell\ell, p_T > 100/30$ GeV, $b, p_T > 30$ GeV
- Bkgd.: $WZ, ZZ, btZ$
- Observation for $M_T \lesssim 1.4$ TeV
\( T \to Wb \to \ell\nu b \) \quad SN-ATLAS-2004-038

- \( \slashed{E}_T > 100 \text{ GeV}, \ell, p_T > 100 \text{ GeV}, \)
  \( b, p_T > 200 \text{ GeV}, \text{max. } jj, p_T > 30 \text{ GeV} \)

- Bkgd.: \( t\bar{t}, Wb\bar{b}, \text{single } t \)

- Observation for \( M_T \lesssim 2.5 \text{ TeV} \)
$T \to th \to \ell\nu bbb$  SN-ATLAS-2004-038

- $\ell, p_T > 100\,\text{GeV}, jjj, p_T > 130\,\text{GeV}$, at least 1 $b$-tag
- Bkgd.: $t\bar{t}, Wb\bar{b}$, single $t$
- Observation for $M_T \lesssim 2.5\,\text{TeV}$
\[ T \rightarrow th \rightarrow \ell\nu bbb \]  

SN-ATLAS-2004-038

- \( \ell, p_T > 100 \text{ GeV}, jjj, p_T > 130 \text{ GeV} \), at least 1 \( b \)-tag
- Bkgd.: \( t\bar{t}, Wb\bar{b}, \) single \( t \)
- Observation for \( M_T \lesssim 2.5 \text{ TeV} \)

Additional heavy quarks (Simple Group Models): \( U, C \) or \( D, S \)

- Large cross section: \( u \) or \( d \) PDF
- Huge final state \( \ell \) charge asymmetry
- Good mass reconstruction
$T \rightarrow th \rightarrow \ell v b b$  SN-ATLAS-2004-038

- $\ell, p_T > 100 \text{ GeV}, jjj, p_T > 130 \text{ GeV}$, at least 1 $b$-tag
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Heavy Vectors
Drell-Yan Production: Tevatron Limits $\sim 500 - 600 \text{ GeV}$

- Dominant decays:
  - Product group: $Z' \rightarrow Zh, WW$
  - $W' \rightarrow Wh, WZ$
  - Simple group: $Z' \rightarrow qq, \quad X \rightarrow fF$

![Graph showing production cross-section and discovery channel for heavy vectors](image_url)
Heavy Vectors

Drell-Yan Production: Tevatron Limits $\sim 500 - 600$ GeV

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![Graph showing cross-section vs. mass](image-url)
Heavy Vectors

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- **Dominant decays:**
  - Product group: $Z' \rightarrow Zh, WW$, $W' \rightarrow Wh, WZ$
  - Simple group: $Z' \rightarrow qq$, $X \rightarrow fF$
- **Discovery channel:** $Z' \rightarrow \ell\ell$, $W' \rightarrow \ell\nu$
- **Resonance widths:** $\Gamma_{Z'} \sim 10 - 50$ GeV, $\Gamma_X \sim 0.1 - 10$ GeV
Heavy Vectors

Drell-Yan Production: Tevatron Limits $\sim 500 - 600$ GeV

- Dominant decays:
  - Product group: $Z' \rightarrow Zh, WW,$ $W' \rightarrow Wh, WZ$
  - Simple group: $Z' \rightarrow qq, \ X \rightarrow fF$
- Discovery channel: $Z' \rightarrow \ell\ell, W' \rightarrow \ell\nu$
- $\Gamma_{Z'} \sim 10 - 50$ GeV, $\Gamma_{X} \sim 0.1 - 10$ GeV

![Graph showing the distribution of events as a function of $m_{ee}$ and $m_T$ for different values of $\cot\theta$.](ATLAS_graph1)

![Graph showing the distribution of events as a function of $m_{ee}$ and $m_T$ for different values of $\cot\theta$.](ATLAS_graph2)
Proof: Sum rule for cancellation of divergences: \( g_{HHVV} + g_{HHV'V'} = 0 \), associated production \( pp \rightarrow V'h \)
Heavy Scalars

Generally: **Large model dependence**
no states  complex singlet  complex triplet

- **Littlest Higgs**, complex triplet:
  \( \Phi^0, \Phi_P, \Phi^\pm, \Phi^{\pm\pm} \)

- **Cleanest channel:** \( q\bar{q} \rightarrow \Phi^{++}\Phi^{--} \rightarrow \ell\ell\ell\ell \):
  Killer: PS

- **WW-Fusion:** \( dd \rightarrow uu\Phi^{++} \rightarrow uuW^+W^+ \)

- 2 hard forward jets, hard close \( \ell^+\ell^+ \)
  \( p_T \)-unbalanced

**Alternative:** **Model-Independent search** in **WW** fusion:

**ILC:** Beyer/Kilian/Krstonosic/Mönig/JR/Schmidt/Schröder, 2006

**LHC:** Kilian/Kobel/Mader/JR/Schumacher/Schumacher
Reconstruction of Little Higgs Models  Kilian/JR, 2003; Han et al., 2005

- Goldstone-boson nature of Higgs boson (nonlinear representation)
- Mechanism for cancellation of $\delta m_H$ quantum corrections

**STRATEGY:**  Kilian/JR, 2003

- **LHC:** $Z', W' \Rightarrow M_{Z'}, M_{W'}$ up to $5 - 6$ TeV
  - ILC: contact terms $\Rightarrow M_{Z'}, M_{W'}$ up to $10 - 20$ TeV
  - Extraction of $F$ and $c \equiv \cos \phi$
- **LHC:** $T \Rightarrow M_T$ and mixing parameters
- **ILC:** Higgsstrahlung and $WW$ fusion (angular distributions/energy spectra) $\Rightarrow$ Higgs couplings/potential
- **ILC/\gamma\gamma:** Higgs decays $\Rightarrow$ Goldstone boson structure
- **ILC/GigaZ:** measurement of $\Delta T \Rightarrow$ contributions of heavy scalars
- **Global fit to LHC/ILC data**
Outline

Hierarchy Problem
  Higgs as Pseudo-Nambu-Goldstone Boson (PNGB)
  The Little Higgs mechanism

Generic properties – Examples of Models

Phenomenology
  Effective Field Theories
  Electroweak Precision Observables
  Neutrino masses
  LHC pheno – Heavy Quark States
  LHC pheno – Heavy Vectors
  LHC pheno – Heavy Scalars
  Reconstruction of Little Higgs Models

Pseudo-Axions in Little Higgs Models
  ZH eta coupling as a discriminator
  $T$ parity and Dark Matter

Summary and Conclusions
Pseudo-Axions in Little Higgs

- gauged $U(1)$ group: $Z'$ ↔ ungauged: $\eta$
- couples to fermions like a pseudoscalar
- $m_\eta \lesssim 400$ GeV
- SM singlet, couplings to SM particles $v/F$ suppressed
- $\eta$ axion-like particle:

$$\text{Anomalous } U(1): \quad \frac{1}{F} \frac{\alpha_s}{8\pi^2} \eta F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$

- $U(1)$ explicitly broken $\Rightarrow$ Axion limits from astroparticle physics not applicable
Classification of Axions in Little Higgs Models

Number of Pseudo-Axions: \( n = g - l \)
Mismatch between global \((g)\) and local rank reduction \((l)\)

**Product Group Models**  Arkani-Hamed, . . .
- Doubling of electroweak gauge group: \( SU(2) \times SU(2) \rightarrow SU(2)_L, U(1) \times U(1) \rightarrow U(1)_Y \) (latter not necessary)  \( \Rightarrow l = 1 \)
  - Littlest Higgs, \( g \): \( SU(5) \rightarrow SO(5) \Rightarrow n = (4 - 2) - 1 = 1 \)
  - Antisymmetric, \( g \): \( Sp(6)/SO(6), n = (3 - 2) - 1 = 0 \)

**Simple Group Models**  Kaplan, Schmaltz, . . .
- Simple gauge group: \( SU(N) \times U(1) \rightarrow SU(2) \times U(1) \Rightarrow l = N - 2 \)
- Higgs is distributed over several global symmetry multiplets
- Simplest Little Higgs, \( g \): \( [SU(3)]^2/[SU(2)]^2 \)  \( n = g - l = 2 - 1 = 1 \)
- Original Simple Group Model, \( g \): \( [SU(4)]^3/[SU(3)^3 \times SU(2)], l: SU(4) \rightarrow SU(2) \)  \( n = g - l = 4 - 2 = 2 \)

**Moose Models**  Arkani-Hamed, . . .
- “Minimal” Moose: \( g \) \( [SU(3)]^4 \rightarrow SU(3), l [SU(3) \times SU(2)]/SU(2) \)  \( n = g - l = 6 - 2 = 4 \)
- 3-site model: \( g \) \( [SU(2)]^4/[SU(2)]^2, l [SU(2)]^2 \rightarrow SU(2), n = 2 - 1 = 1 \)
Z\(H\eta\) coupling as a discriminator

- pseudo-axion: \(\xi = \exp[i\eta/F]\), \(\Sigma = \exp[i\Pi/F]\) non-linear representation of the remaining Goldstone multiplet \(\Pi\)

\[\mathcal{L}_{\text{kin.}} \sim F^2 \text{Tr} \left[ (D^\mu(\xi\Sigma)^\dagger(D_\mu(\xi\Sigma)) \right] = \ldots -2F(\partial_\mu\eta) \text{Im Tr} \left[ (D^\mu\Sigma)^\dagger\Sigma \right] + O(\eta^2)\]

- Use special structure of covariant derivatives:

\[D_\mu\Sigma = \partial_\mu\Sigma + A_a^{1,\mu} \left( T_1^a \Sigma + \Sigma(T_1^a)^T \right) + A_a^{2,\mu} \left( T_2^a \Sigma + \Sigma(T_2^a)^T \right),\]

\[\text{Tr} \left[ (D^\mu\Sigma)^\dagger\Sigma \right] \sim W_\mu^a \text{Tr} \left[ \Sigma^\dagger(T_1^a + T_2^a)\Sigma + (T_1^a + T_2^a)^* \right] = 0.\]

- Little Higgs mechanism cancels this coupling

- Simple Group Models: \(\Phi = \exp[i\Sigma/F]\), \(\zeta = (0, \ldots, 0, F)^T\) VEV directing in the \(N\) direction
\[ \mathcal{L}_{\text{kin.}} \sim F^2 D^\mu (\zeta^\dagger \Phi^\dagger) D_\mu (\Phi \zeta) = \ldots + \frac{i}{F} (\partial_\mu \eta) \zeta^\dagger \left( \Phi^\dagger (D_\mu \Phi) - (D_\mu \Phi^\dagger) \Phi \right) \zeta \]

\[ = \ldots + iF (\partial_\mu \eta) \left( \Phi^\dagger (D_\mu \Phi) - (D_\mu \Phi^\dagger) \Phi \right) \]

\[ \Sigma = \begin{pmatrix} 0 & h \\ h^\dagger & 0 \end{pmatrix}, \quad \mathbf{V}_\mu = \begin{pmatrix} W_\mu & 0 \\ 0 & 0 \end{pmatrix} + \text{heavy vector fields} \]

\[ \mathbf{V}_\mu + \frac{i}{F} [\Sigma, \mathbf{V}_\mu] - \frac{1}{2F^2} [\Sigma, [\Sigma, \mathbf{V}_\mu]] + \ldots \]

\[ = \begin{pmatrix} W_\mu & 0 \\ 0 & 0 \end{pmatrix} + \frac{i}{F} \begin{pmatrix} 0 & -W_\mu h \\ h^\dagger W_\mu & 0 \end{pmatrix} - \frac{1}{2F^2} \begin{pmatrix} hh^\dagger W + Whh^\dagger & 0 \\ 0 & -2h^\dagger Wh \end{pmatrix} + \ldots \]

- 1st term cancels by multiple Goldstone multiplets
- 2nd term cancels by EW symmetry
- 3rd term

\[ (\partial_\mu \eta) h^\dagger W_\mu h \sim vH Z_\mu \partial_\mu \eta . \]
More properties of Pseudo-Axions

- Take e.g. one specific model: Simplest Little Higgs Schmaltz, 2004
- Simple Group Model, two Higgs-triplets with a $\tan \beta$-like mixing angle

- $\tan \beta \sim 1$: heavy Higgs, (very) light pseudoscalar
- Heavy top decays: Kilian/Rainwater/JR, 2006
Discovery of Pseudo-axions

LHC: Gluon fusion, diphoton signal for $m_\eta \gtrsim 200$ GeV, 7$\sigma$ possible

LHC: $T \rightarrow t\eta$

ILC: $e^+e^- \rightarrow t\bar{t}\eta$
Discovery of Pseudo-axions


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![Graph showing $e^- e^+ \rightarrow t\bar{t}b\bar{b}$]

$\sqrt{s} = 800$ GeV

$\int L = 1$ ab$^{-1}$

$g_{tt\eta} = 0.2$

$m_\eta = 50$ GeV

$\#_{\text{evt}}/2$ GeV
Discovery of Pseudo-axions


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$ZH\eta$ coupling
forbidden in Product Group Models

Discriminator of diff. model classes

$gg \rightarrow \left\{ \begin{array}{l}
H \rightarrow Z\eta \rightarrow \ell\ell bb \\
\eta \rightarrow ZH \rightarrow \ell\ell bb, \ell\ell jj
\end{array} \right\}$
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If $ZH\eta$ coupling present: $H\eta$ production in analogy to $HA$:

- Light pseudoaxion, $\eta \rightarrow bb$, final state $Hbb$
- Intermediate range, $\eta \rightarrow gg$, final state $Hjj$
- $\eta \rightarrow ZH$: $ZHH$ final state

More detailed insights from photon collider option
If $ZH\eta$ coupling present: $H\eta$ production in analogy to $HA$:

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More detailed insights from photon collider option
Invisible Higgs decays (?)

- “Invisible decay” $H \rightarrow \eta\eta$ [quite similar to $H \rightarrow \alpha\alpha$ in NMSSM]
  but only due to mixing effects because $U(1)_\eta$ protective symmetry

$$\Gamma_{H \rightarrow \eta\eta} \sim \frac{1}{16\pi} \sqrt{1 - \frac{4m^2_\eta}{m^2_H} \frac{v^5}{F^4}} \sim \frac{15}{(F[\text{TeV}])^4} \text{MeV}$$

- Light Higgs might become invisible at the LHC
  - Not possible in Simplest Little Higgs
  - Possible in other Simple Group Models (together with $\eta$, $A$ mixing)
  - Can become the dominant decay (with BR $\sim .8 - .95$)

- ILC can cover that hole!
  JR, 2007
Pseudo Axions at the Photon Collider

- Photon Collider as precision machine for Higgs physics ($s$ channel resonance, anomaly coupling)

- $S/B$ analogous to LC

- $\eta$ in the $\mu$ model with (almost) identical parameters as $A$ in MSSM

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure}
\caption{Diagram of the process $\gamma\gamma \rightarrow \eta, H$}
\end{figure}

\begin{align*}
\sigma_{\text{eff}}(\gamma\gamma \rightarrow \eta, H) & \quad [\text{pb}]
\end{align*}

\begin{tabular}{c|c|c|c|c|c}
$m_H$ & $m_{\eta}$ & $100$ & $200$ & $300$ & $400$ & $500$ & $600$ & $700$
\hline
0.001 & 0.01 & 0.1 & 1 & 10
\end{tabular}

\begin{align*}
\text{Mühlleitner et al. (2001)}
\end{align*}
Pseudo Axions at the Photon Collider

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- S/B analogous to LC

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  ( $\leftrightarrow$ Mühlleitner et al. (2001) )

\[ \sigma_{\text{eff}}(\gamma\gamma \rightarrow \eta, H) \ [\text{pb}] \]

\[ m_H, m_\eta \ [\text{GeV}] \]
$g_{bb\eta} = 0.4 \cdot g_{bbh}$

<table>
<thead>
<tr>
<th>$m_\eta$</th>
<th>100</th>
<th>130</th>
<th>200</th>
<th>285</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{\gamma\gamma}$ [keV]</td>
<td>0.15</td>
<td>0.27</td>
<td>1.1</td>
<td>3.6</td>
</tr>
</tbody>
</table>

$\gamma\gamma \rightarrow (H, \eta) \rightarrow b\bar{b}$

- $\sqrt{s_{ee}} = 200$ GeV
- $\int L_{ee} = 400$ fb$^{-1}$
- $|\cos(\theta_T)| < 0.76$
- $|p_z| / \sqrt{s_{ee}} < 0.1$

- $m_\eta = 100$ GeV
- 130

$\gamma\gamma \rightarrow (H, \eta) \rightarrow b\bar{b}$

- $\sqrt{s_{ee}} = 500$ GeV
- $\int L_{ee} = 1$ ab$^{-1}$
- $|\cos(\theta_T)| < 0.76$
- $|p_z| / \sqrt{s_{ee}} < 0.04$

- $m_\eta = 200$ GeV
- 130
- 285
$T$ parity and Dark Matter

Cheng/Low, 2003; Hubisz/Meade, 2005

- **$T$ parity**: $T^a \rightarrow T^a$, $X^a \rightarrow -X^a$, automorphism of coset space analogous to $R$ parity in SUSY, KK parity in extra dimensions

- Bounds on $F$ MUCH relaxed, $F \sim 1 \text{ TeV}$

  *but*: Pair production!, typical cascade decays

- Lightest $T$-odd particle (LTP) ⇒ **Candidate for Cold Dark Matter**
T parity and Dark Matter

- **T parity**: \( T^a \rightarrow T^a, \quad X^a \rightarrow -X^a \), automorphism of coset space analogous to \( R \) parity in SUSY, KK parity in extra dimensions
- Bounds on \( F \) MUCH relaxed, \( F \sim 1 \text{ TeV} \)
  
  *but*: Pair production!, typical cascade decays
- Lightest \( T \)-odd particle (LTP) \( \Rightarrow \) Candidate for Cold Dark Matter

Littlest Higgs: \( A' \) LTP

\( W', Z' \sim 650 \text{ GeV}, \quad \Phi \sim 1 \text{ TeV} \)

\( T, T' \sim 0.7-1 \text{ TeV} \)

Annihilation: \( A' A' \rightarrow h \rightarrow WW, ZZ, hh \)

Hubisz/Meade, 2005
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Hubisz/Meade, 2005

$T$ parity Simplest LH: Pseudo-Axion $\eta$ LTP

$Z'$ remains odd: good or bad (?)  Kilian/Rainwater/JR/Schmaltz

$T$ parity might be anomalous (???)  Hill/Hill, 2007
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Little Higgs: Global Symmetry structure stabilizes EW scale

New heavy gauge bosons, scalars, quarks (same spin)

EW precision observables:
Higgs is generically heavy in LHM (Little Big Higgs)

Inclusion of Dark Matter: $T$-parity (anomalies?)

Open questions: UV embedding, GUTs, Flavour
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Little Higgs models generally have extra pseudoscalars

- Maybe the only observable states (before VLHC!)
- Discriminator between Product and Simple Group Models
- LHC has first option: gluon fusion, $T/Z'/W'$ decays
- ILC can cover all regions
- Possible “invisible” Higgs decays
Summary and Conclusions

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Simplest Little Higgs ("$\mu$ Model")

Schmaltz '04, Kilian/Rainwater/JR '04

Field content \((SU(3)_c \times SU(3)_w \times U(1)_X)\) quantum numbers

\[
\begin{align*}
\Phi_{1,2} & : (1, 3)_{-\frac{1}{3}} \\
\Psi_{\ell} & : (1, 3)_{-\frac{1}{3}} \\
\Psi_Q & : (3, 3)_{\frac{1}{3}} \\
d_c & : (\bar{3}, 1)_{\frac{1}{3}} \\
e^c, n^c & : (1, 1)_{1,0}
\end{align*}
\]

Lagrangian \(\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{Yuk}} + \mathcal{L}_{\text{pot}}\): \(\Psi_{Q,L} = (u, d, U)_L, \Psi_{\ell} = (\nu, \ell, N)_L\):

\[
\begin{align*}
\mathcal{L}_{\text{Yuk.}} &= -\lambda_u^u \bar{u}_{1,R} \Phi_1^\dagger \Psi_{T,L} - \lambda_u^d \bar{u}_{2,R} \Phi_2^\dagger \Psi_{T,L} - \frac{\lambda_d}{\Lambda} \epsilon^{ijk} d_R \Phi_1^i \Phi_2^j \Psi_{T,L}^k \\
\mathcal{L}_{\text{pot.}} &= \mu^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}
\end{align*}
\]

Hypercharge embedding \((\text{diag}(1, 1, -2)/(2\sqrt{3}))\):

\[
Y = X - T^8/\sqrt{3}
\]

\[
D_\mu \Phi = \left( \partial_\mu - \frac{1}{3} g_X B^X_\mu \Phi + ig W^w_\mu \right) \Phi
\]