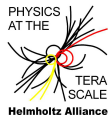


Simplified Models for Vector Boson Scattering

Jürgen R. Reuter

DESY Hamburg



Alboreanu/Kilian/JRR, **JHEP 0811** (2008) 010;
Beyer/Kilian/Krstonošić/Mönig/JRR/Schmitt/Schröder, **EPJC 48** (2006), 353;
JRR/Kilian/Sekulla, arXiv:1307.8170 and in prep.

LCWS 2013, Tokyo, Nov. 13th, 2013

Motivation

- Light Higgs boson found

Motivation

- Light Higgs boson found
- SM-like (clear from EWPO)

Motivation

- Light Higgs boson found
- SM-like (clear from EWPO)
- Mediator of EWSB found

Motivation

- Light Higgs boson found
- SM-like (clear from EWPO)
- Mediator of EWSB found
- Mechanism of EWSB still poorly understood:
 - ▶ single Higgs field vs. Higgs sector
 - ▶ Higgs potential: stable vs. metastable vs. unstable !?
 - ▶ Higgs self-coupling vs. Higgs field scattering
 - ▶ Importance of longitudinal EW gauge bosons

Motivation

- Light Higgs boson found
- SM-like (clear from EWPO)
- Mediator of EWSB found
- **Mechanism of EWSB still poorly understood:**
 - ▶ single Higgs field vs. Higgs sector
 - ▶ Higgs potential: stable vs. metastable vs. unstable !?
 - ▶ Higgs self-coupling vs. Higgs field scattering
 - ▶ Importance of longitudinal EW gauge bosons
- Deviations from the SM: where? what? how?
- **Anomalous Triple Gauge Couplings:** dibosons

Motivation

- Light Higgs boson found
- SM-like (clear from EWPO)
- Mediator of EWSB found
- **Mechanism of EWSB still poorly understood:**
 - ▶ single Higgs field vs. Higgs sector
 - ▶ Higgs potential: stable vs. metastable vs. unstable !?
 - ▶ Higgs self-coupling vs. Higgs field scattering
 - ▶ Importance of longitudinal EW gauge bosons
- Deviations from the SM: where? what? how?
- **Anomalous Triple Gauge Couplings:** dibosons
- **Anomalous Quartic Gauge Couplings:** tribosons, VV scattering
- Hot topic: Snowmass BNL 04/13, SM@LHC Freiburg 04/13, LHCEWWG 04/13, Snowmass 07/13, Dresden workshop 10/13

Extensions of the SM

- ▶ Lagrangian of the EW SM (no fermions/QCD here):

$$\mathcal{L}_{EW} = -\frac{1}{2} \text{tr} [W_{\mu\nu} W^{\mu\nu}] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_\mu \Phi)^\dagger (D^\mu \Phi) + \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2$$

with building blocks:

$$D_\mu = \partial_\mu + \frac{i}{2} g \tau^I W_\mu^I + \frac{i}{2} g' B_\mu$$

$$W_{\mu\nu} = \frac{i}{2} g \tau^I (\partial_\mu W_\nu^I - \partial_\nu W_\mu^I + g \epsilon_{IJK} W_\mu^J W_\nu^K)$$

$$B_{\mu\nu} = \frac{i}{2} g' (\partial_\mu B_\nu - \partial_\nu B_\mu)$$

- ▶ Any EFT has higher-dimensional operators:

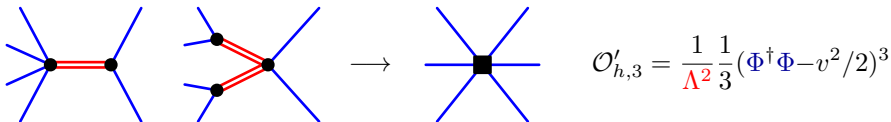
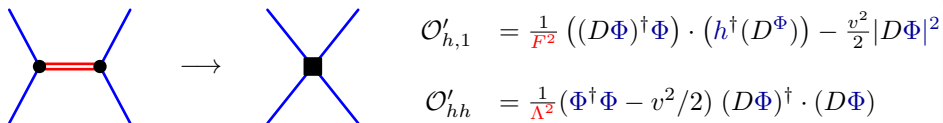
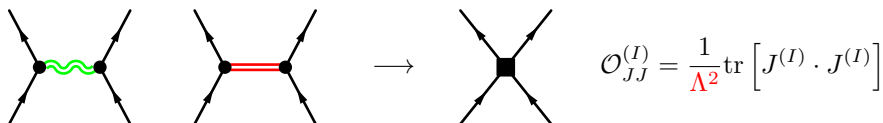
Weinberg, 1979

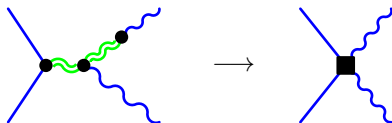
$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \left[\frac{a_i}{\Lambda} \mathcal{O}_i^{(5)} + \frac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + \frac{e_i}{\Lambda^4} \mathcal{O}_i^{(8)} \dots \right]$$

- ▶ without more fundamental theory \Rightarrow no clue on the scale (neither on the coefficients)

Effective EW Dim. 6 Operators

Hagiwara/Hikasa/Peccei/Zeppenfeld, 1987; Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993

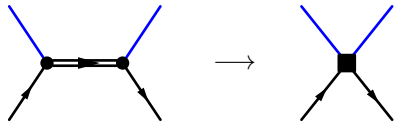




$$\mathcal{O}_{\Phi W} = -\frac{1}{\Lambda^2} \frac{1}{2} (\Phi^\dagger \Phi - v^2/2) \text{tr} [W_{\mu\nu} W^{\mu\nu}]$$

$$\mathcal{O}_B = \frac{1}{\Lambda^2} \frac{i}{2} (D_\mu \Phi)^\dagger B^{\mu\nu} (D_\nu \Phi)$$

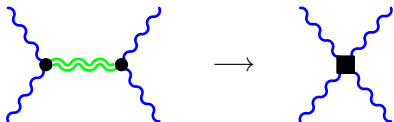
$$\mathcal{O}_{\Phi B} = -\frac{1}{\Lambda^2} \frac{1}{4} (\Phi^\dagger \Phi - v^2/2) B_{\mu\nu} B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{\Lambda^2} \bar{q} h (\not{D} h) q$$

Effective Dim. 8 Operators

Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993



$$\begin{aligned}\mathcal{O}_\lambda &= \frac{i}{\Lambda^4} \text{tr} [W_{\mu\nu} \times W^{\nu\rho} (\Phi^\dagger \frac{\vec{\sigma}}{2} [D_\rho, D^\mu] \Phi)] \\ \mathcal{O}_\kappa &= (D^\mu \Phi)^\dagger (D^\nu \Phi) (\Phi^\dagger [D_\mu, D_\nu] \Phi)\end{aligned}$$

- ▶ operators linked through e.o.m.
- ▶ SM: 59 independent operators (1 fermion gen.)

Grzadkowski/Iskrzynski/Misiak/Rosiek, 2010

Buchmüller/Wyler, 1986;

- ▶ Renormalization mixes operators
- ▶ Beware of power counting

Classification of Operators (I): Dim 6

(always v^2 subtracted)

- Dimension-6 operators (CP-conserving)

$$\mathcal{O}_{WWW} = \text{Tr}[W_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}]$$

$$\mathcal{O}_W = (D_{\mu}\Phi)^{\dagger} W^{\mu\nu} (D_{\nu}\Phi)$$

$$\mathcal{O}_B = (D_{\mu}\Phi)^{\dagger} B^{\mu\nu} (D_{\nu}\Phi)$$

$$\mathcal{O}_{\partial\Phi} = \partial_{\mu} (\Phi^{\dagger}\Phi) \partial^{\mu} (\Phi^{\dagger}\Phi)$$

$$\mathcal{O}_{\Phi W} = (\Phi^{\dagger}\Phi) \text{Tr}[W^{\mu\nu} W_{\mu\nu}]$$

$$\mathcal{O}_{\Phi B} = (\Phi^{\dagger}\Phi) B^{\mu\nu} B_{\mu\nu}$$

- Dimension-6 operators (CP-violating)

$$\mathcal{O}_{\widetilde{W}W} = \Phi^{\dagger} \widetilde{W}_{\mu\nu} W^{\mu\nu} \Phi$$

$$\mathcal{O}_{\widetilde{B}B} = \Phi^{\dagger} \widetilde{B}_{\mu\nu} B^{\mu\nu} \Phi$$

$$\mathcal{O}_{\widetilde{W}WW} = \text{Tr}[\widetilde{W}_{\mu\nu} W^{\nu\rho} W_{\rho}^{\mu}]$$

$$\mathcal{O}_{\widetilde{W}} = (D_{\mu}\Phi)^{\dagger} \widetilde{W}^{\mu\nu} (D_{\nu}\Phi)$$

	ZWW	AWW	HWW	HZZ	HZA	HAA	WWWW	ZZWW	ZAWW	AAWW
\mathcal{O}_{WWW}	✓	✓					✓	✓	✓	✓
\mathcal{O}_W	✓	✓	✓	✓	✓		✓	✓	✓	
\mathcal{O}_B	✓	✓		✓	✓					
$\mathcal{O}_{\Phi d}$			✓	✓						
$\mathcal{O}_{\Phi W}$			✓	✓	✓	✓				
$\mathcal{O}_{\Phi B}$				✓	✓	✓				
$\mathcal{O}_{\widetilde{W}WW}$	✓	✓					✓	✓	✓	✓
$\mathcal{O}_{\widetilde{W}}$	✓	✓	✓	✓	✓					
$\mathcal{O}_{\widetilde{W}W}$			✓	✓	✓	✓				
$\mathcal{O}_{\widetilde{B}B}$				✓	✓	✓				

Classification of Operators (II): Dim 8

(always v^2 subtracted)

- Dimension-8 operators (only $D_\mu \Phi$)

$$\mathcal{O}_{S,0} = \left[(D_\mu \Phi)^\dagger D_\nu \Phi \right] \times \left[(D^\mu \Phi)^\dagger D^\nu \Phi \right],$$

$$\mathcal{O}_{S,1} = \left[(D_\mu \Phi)^\dagger D^\mu \Phi \right] \times \left[(D_\nu \Phi)^\dagger D^\nu \Phi \right],$$

- Dimension-8 operators (only field strength/mixed)

$$\mathcal{O}_{T,0} = \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \cdot \text{Tr} [W_{\alpha\beta} W^{\alpha\beta}], \quad \mathcal{O}_{M,0} = \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \cdot \left[(D_\beta \Phi)^\dagger D^\beta \Phi \right]$$

$$\mathcal{O}_{T,1} = \text{Tr} [W_{\alpha\nu} W^{\mu\beta}] \cdot \text{Tr} [W_{\mu\beta} W^{\alpha\nu}], \quad \mathcal{O}_{M,1} = \text{Tr} [W_{\mu\nu} W^{\nu\beta}] \cdot \left[(D_\beta \Phi)^\dagger D^\mu \Phi \right]$$

$$\mathcal{O}_{T,2} = \text{Tr} [W_{\alpha\mu} W^{\mu\beta}] \cdot \text{Tr} [W_{\beta\nu} W^{\nu\alpha}], \quad \mathcal{O}_{M,2} = [B_{\mu\nu} B^{\mu\nu}] \cdot \left[(D_\beta \Phi)^\dagger D^\beta \Phi \right],$$

$$\mathcal{O}_{T,5} = \text{Tr} [W_{\mu\nu} W^{\mu\nu}] \cdot B_{\alpha\beta} B^{\alpha\beta}, \quad \mathcal{O}_{M,3} = [B_{\mu\nu} B^{\nu\beta}] \cdot \left[(D_\beta \Phi)^\dagger D^\mu \Phi \right],$$

$$\mathcal{O}_{T,6} = \text{Tr} [W_{\alpha\nu} W^{\mu\beta}] \cdot B_{\mu\beta} B^{\alpha\nu}, \quad \mathcal{O}_{M,4} = \left[(D_\mu \Phi)^\dagger W_{\beta\nu} D^\mu \Phi \right] \cdot B^{\beta\nu},$$

$$\mathcal{O}_{T,7} = \text{Tr} [W_{\alpha\mu} W^{\mu\beta}] \cdot B_{\beta\nu} B^{\nu\alpha}, \quad \mathcal{O}_{M,5} = \left[(D_\mu \Phi)^\dagger W_{\beta\nu} D^\nu \Phi \right] \cdot B^{\beta\mu},$$

$$\mathcal{O}_{T,8} = B_{\mu\nu} B^{\mu\nu} B_{\alpha\beta} B^{\alpha\beta}, \quad \mathcal{O}_{M,6} = \left[(D_\mu \Phi)^\dagger W_{\beta\nu} W^{\beta\nu} D^\mu \Phi \right],$$

$$\mathcal{O}_{T,9} = B_{\alpha\mu} B^{\mu\beta} B_{\beta\nu} B^{\nu\alpha}, \quad \mathcal{O}_{M,7} = \left[(D_\mu \Phi)^\dagger W_{\beta\nu} W^{\beta\mu} D^\nu \Phi \right],$$

Classification of Operators (III)

	WWWW	WWZZ	ZZZZ	WWAZ	WWAA	ZZZA	ZZAA	ZAAA	AAAA
$\mathcal{O}_{S,0/1}$	✓	✓	✓						
$\mathcal{O}_{M,0/1/6/7}$	✓	✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{M,2/3/4/5}$		✓	✓	✓	✓	✓	✓		
$\mathcal{O}_{T,0/1/2}$	✓	✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,5/6/7}$		✓	✓	✓	✓	✓	✓	✓	✓
$\mathcal{O}_{T,8/9}$			✓			✓	✓	✓	✓

- ▶ Dim. 8 operators generate aQGCs, but not aTGCs
- ▶ generate neutral quartics
- ▶ Redundancy of the operators:
 - Equations of motion: $D_\mu W^{\mu\nu} = \Phi^\dagger (D^\nu \Phi) - (D^\nu \Phi)^\dagger \Phi + \dots$
 - Gauge symmetry structure: $[D_\mu, D_\nu] \Phi \propto W_{\mu\nu} \Phi$
 - Integration by parts (up to total derivatives)
 - Leads to relations like:

$$\begin{aligned} \mathcal{O}_B &= \mathcal{O}_{\tilde{W}} + \frac{1}{2} \mathcal{O}_{WW} - \frac{1}{2} \mathcal{O}_{BB} \\ \mathcal{O}_{BW} &= -2 \mathcal{O}_W - \mathcal{O}_{WW} \\ \mathcal{O}_{\partial W} &= -4 \mathcal{O}_{WWW} + \text{gauge-fermion operators} \end{aligned}$$

Classification of Operators (IV)

Originally for heavy Higgses or Higgsless models

$$\mathcal{L}_\chi \ni \beta_1 \mathcal{L}'_0 + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v} \sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2} \sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \dots \quad \alpha_i^{(n)} = \frac{v^{n-4}}{\Lambda^{n-4}} C_i^n$$

$$\mathcal{L}'_0 = \frac{v^2}{4} \text{tr} [\mathbf{T} \mathbf{V}_\mu] \text{tr} [\mathbf{T} \mathbf{V}^\mu]$$

$$\mathcal{L}_1 = \text{tr} [\mathbf{B}_{\mu\nu} \mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_2 = \text{itr} [\mathbf{B}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_3 = \text{itr} [\mathbf{W}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_4 = \text{tr} [\mathbf{V}_\mu \mathbf{V}_\nu] \text{tr} [\mathbf{V}^\mu \mathbf{V}^\nu]$$

$$\mathcal{L}_5 = \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] \text{tr} [\mathbf{V}_\nu \mathbf{V}^\nu]$$

$$\mathcal{L}_6 = \text{tr} [\mathbf{V}_\mu \mathbf{V}_\nu] \text{tr} [\mathbf{T} \mathbf{V}^\mu] \text{tr} [\mathbf{T} \mathbf{V}^\nu]$$

$$\mathcal{L}_7 = \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] \text{tr} [\mathbf{T} \mathbf{V}_\nu] \text{tr} [\mathbf{T} \mathbf{V}^\nu]$$

$$\mathcal{L}_8 = \frac{1}{4} \text{tr} [\mathbf{T} \mathbf{W}_{\mu\nu}] \text{tr} [\mathbf{T} \mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_9 = \frac{i}{2} \text{tr} [\mathbf{T} \mathbf{W}_{\mu\nu}] \text{tr} [\mathbf{T} [\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_{10} = \frac{1}{2} (\text{tr} [\mathbf{T} \mathbf{V}_\mu] \text{tr} [\mathbf{T} \mathbf{V}^\mu])^2$$

Classification of Operators (IV)

Originally for heavy Higgses or Higgsless models

$$\mathcal{L}_X \ni \beta_1 \mathcal{L}'_0 + \sum_i \alpha_i \mathcal{L}_i + \frac{1}{v} \sum_i \alpha_i^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^2} \sum_i \alpha_i^{(6)} \mathcal{L}^{(6)} + \dots \quad \alpha_i^{(n)} = \frac{v^{n-4}}{\Lambda^{n-4}} C_i^n$$

$$\mathcal{L}'_0 = \frac{v^2}{4} \text{tr}[\mathbf{T}\mathbf{V}_\mu] \text{tr}[\mathbf{T}\mathbf{V}^\mu]$$

$$\mathcal{L}_1 = \text{tr}[\mathbf{B}_{\mu\nu} \mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_2 = \text{itr}[\mathbf{B}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_3 = \text{itr}[\mathbf{W}_{\mu\nu} [\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_4 = \text{tr}[\mathbf{V}_\mu \mathbf{V}_\nu] \text{tr}[\mathbf{V}^\mu \mathbf{V}^\nu]$$

$$\mathcal{L}_5 = \text{tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \text{tr}[\mathbf{V}_\nu \mathbf{V}^\nu]$$

$$\mathcal{L}_6 = \text{tr}[\mathbf{V}_\mu \mathbf{V}_\nu] \text{tr}[\mathbf{T}\mathbf{V}^\mu] \text{tr}[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_7 = \text{tr}[\mathbf{V}_\mu \mathbf{V}^\mu] \text{tr}[\mathbf{T}\mathbf{V}_\nu] \text{tr}[\mathbf{T}\mathbf{V}^\nu]$$

$$\mathcal{L}_8 = \frac{1}{4} \text{tr}[\mathbf{T}\mathbf{W}_{\mu\nu}] \text{tr}[\mathbf{T}\mathbf{W}^{\mu\nu}]$$

$$\mathcal{L}_9 = \frac{i}{2} \text{tr}[\mathbf{T}\mathbf{W}_{\mu\nu}] \text{tr}[\mathbf{T}[\mathbf{V}^\mu, \mathbf{V}^\nu]]$$

$$\mathcal{L}_{10} = \frac{1}{2} (\text{tr}[\mathbf{T}\mathbf{V}_\mu] \text{tr}[\mathbf{T}\mathbf{V}^\mu])^2$$

Indirect info on new physics in β_1, α_i, \dots (Flavor physics only in M)

Electroweak precision observables (LEP I/II, SLC):

$$\Delta S = -16\pi\alpha_1$$

$$\Delta T = 2\beta_1/\alpha_{\text{QED}}$$

$$\Delta U = -16\pi\alpha_8$$

$$\alpha_1 = 0.0026 \pm 0.0020$$

$$\beta_1 = -0.00062 \pm 0.00043$$

$$\alpha_8 = -0.0044 \pm 0.0026$$

Classification of approaches

Remarks:

- ▶ EFT approach leads to new interaction vertices
- ▶ Coupling constants are EFT Lagrangian parameters
- ▶ Framework for higher-order corrections straightforward (though rarely needed)
- ▶ Threshold/soft-collinear resummation \Rightarrow momentum-dependent couplings/form factors
- ▶ Anomalous couplings understood as effective vertices/vertex functions
- ▶ Nevertheless: Lagrangian for new physics reconstructable
- ▶ Parameterize new physics effects as new resonances/particles

Classification of approaches

- Switch diff. operator bases (dep. on vertex):

Snowmass EW White Paper

$$\text{for the WWWW-Vertex: } \alpha_4 = \frac{f_{S,0}}{\Lambda^4} \frac{v^4}{8}$$

$$\alpha_4 + 2 \cdot \alpha_5 = \frac{f_{S,1}}{\Lambda^4} \frac{v^4}{8}$$

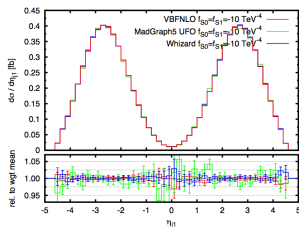
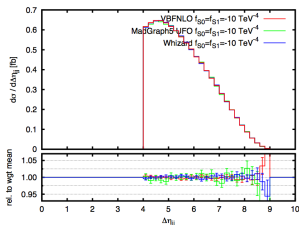
$$\text{for the WWZZ-Vertex: } \alpha_4 = \frac{f_{S,0}}{\Lambda^4} \frac{v^4}{16}$$

$$\alpha_5 = \frac{f_{S,1}}{\Lambda^4} \frac{v^4}{16}$$

for the ZZZZ-Vertex:

$$\alpha_4 + \alpha_5 = \left(\frac{f_{S,0}}{\Lambda^4} + \frac{f_{S,1}}{\Lambda^4} \right) \frac{v^4}{16}$$

- Full agreement among generators: VBF@NLO, WHIZARD, Madgraph



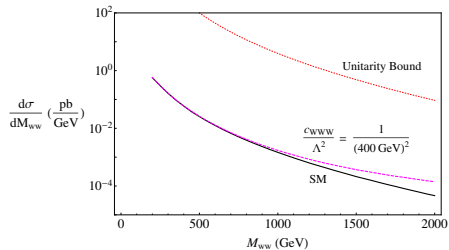
Classification of approaches

- Translation between operator bases:

Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993; Wudka,

1994; Degrande/Greiner/Kilian/Mattelaer/Mebane/Stelzer/Willenbrock/Zhang, 2012

$$\begin{aligned}
 \Delta g_1^\gamma &= 0 \\
 \Delta g_1^Z &= c_W \frac{m_Z^2}{2\Lambda^2} \\
 \Delta \kappa_\gamma &= (c_W + c_B) \frac{m_W^2}{2\Lambda^2} \\
 \Delta \kappa_Z &= \delta_Z + (c_W - c_B \tan^2 \theta_W) \frac{m_W^2}{2\Lambda^2} \\
 \lambda_\gamma &= \lambda_Z = c_{WWW} \frac{3g^2 m_W^2}{2\Lambda^2}
 \end{aligned}$$



- Effective Field Theory description valid, **if**
 - ▶ $\hat{s} \ll \Lambda^2$: new physics out of direct LHC reach
 - ▶ Operator coefficients rather smallish, e.g. $c_{WWW} \lesssim 1$
 - ▶ No large logarithms in the game (resummation)
- Relation $\Delta g_1^Z = \Delta \kappa_Z + \tan^2 \theta_W \Delta \kappa_\gamma$ invalidated by dim 8 operators

Unique way of operator assignment?

- ▶ Usage of different measurements: $W\gamma$, WZ production: $WW\gamma$ vs. WWZ
- ▶ VVV and VBS to access the highest possible energies

Unique way of operator assignment?

- ▶ Usage of different measurements: $W\gamma$, WZ production: $WW\gamma$ vs. WWZ
- ▶ VVV and VBS to access the highest possible energies
- ▶ Answer: **NO UNIQUE WAY!**

Unique way of operator assignment?

- ▶ Usage of different measurements: $W\gamma$, WZ production: $WW\gamma$ vs. WWZ
- ▶ VVV and VBS to access the highest possible energies
- ▶ Answer: **NO UNIQUE WAY!**
- ▶ But: at e^+e^- machines, gauge-fermion operators can be rotated away
- ▶ At LHC this is not possible! [Buchalla et al., 1302.6481](#)

Unique way of operator assignment?

- ▶ Usage of different measurements: $W\gamma$, WZ production: $WW\gamma$ vs. WWZ
- ▶ VVV and VBS to access the highest possible energies
- ▶ Answer: **NO UNIQUE WAY!**
- ▶ But: at e^+e^- machines, gauge-fermion operators can be rotated away
- ▶ At LHC this is not possible! Buchalla et al., 1302.6481
- ▶ **There is no common operator basis for $V + \text{jets}$, VV , VVV and VBS at LHC**
- ▶ Incoherent sum of channels at LHC prevent eliminating operators!

Unique way of operator assignment?

- ▶ Usage of different measurements: $W\gamma$, WZ production: $WW\gamma$ vs. WWZ
- ▶ VVV and VBS to access the highest possible energies
- ▶ Answer: **NO UNIQUE WAY!**
- ▶ But: at e^+e^- machines, gauge-fermion operators can be rotated away
- ▶ At LHC this is not possible! Buchalla et al., 1302.6481
- ▶ **There is no common operator basis for V + jets, VV , VVV and VBS at LHC**
- ▶ Incoherent sum of channels at LHC prevent eliminating operators!
- ▶ **Similar to B physics: observables process [decay] specific**

(Integrating out) Resonances

Operator coefficients \Rightarrow new physics scale Λ : $\alpha_i = v^k / \Lambda^k$

- ▶ Operator normalization is arbitrary
- ▶ Power counting can be intricate

New physics in electroweak sector:

- ▶ Narrow resonances \Rightarrow particles
- ▶ Wide resonances \Rightarrow continuum

$SU(2)_c$ custodial symmetry (weak isospin, broken by hypercharge
 $g' \neq 0$ and fermion masses)

	$J = 0$	$J = 1$	$J = 2$
$I = 0$	σ^0 (Higgs ?)	ω^0 (γ'/Z' ?)	f^0 (Graviton ?)
$I = 1$	π^\pm, π^0 (2HDM ?)	ρ^\pm, ρ^0 (W'/Z' ?)	a^\pm, a^0
$I = 2$	$\phi^{\pm\pm}, \phi^\pm, \phi^0$ (Higgs triplet ?)	—	$t^{\pm\pm}, t^\pm, t^0$

- ▶ $I = 0$: resonant in W^+W^- and ZZ scattering
- ▶ $I = 1$: resonant in W^+Z and W^-Z scattering
- ▶ $I = 2$: resonant in W^+W^+ and W^-W^- scattering

accounts for **weakly and strongly interacting models**

Integrating out resonances

- ▶ Simplest example: scalar singlet σ :

$$\mathcal{L}_\sigma = -\frac{1}{2} [\sigma(M_\sigma^2 + \partial^2)\sigma - g_\sigma v \sigma \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] - h_\sigma \text{tr} [\mathbf{TV}_\mu] \text{tr} [\mathbf{TV}^\mu]]$$

- ▶ Effective Lagrangian $\mathcal{L}_\sigma^{\text{eff}} = \frac{v^2}{8M_\sigma^2} \left[g_\sigma \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] + h_\sigma \text{tr} [\mathbf{TV}_\mu] \text{tr} [\mathbf{TV}^\mu] \right]^2$
- ▶ leads to **anomalous quartic couplings (aQGCs)**

$$\alpha_5 = g_\sigma^2 \left(\frac{v^2}{8M_\sigma^2} \right) \quad \alpha_7 = 2g_\sigma h_\sigma \left(\frac{v^2}{8M_\sigma^2} \right) \quad \alpha_{10} = 2h_\sigma^2 \left(\frac{v^2}{8M_\sigma^2} \right)$$

Resonance	σ	ϕ	ρ	f	a
$\Gamma[g^2 M^2 / (64\pi v^2)]$	6	1	$\frac{4}{3} \left(\frac{v^2}{M^2} \right)$	$\frac{1}{5}$	$\frac{1}{30}$
$\Delta\alpha_4[(16\pi\Gamma/M)(v^4/M^4)]$	0	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{5}{2}$	$-\frac{5}{8}$
$\Delta\alpha_5[(16\pi\Gamma/M)(v^4/M^4)]$	$\frac{1}{12}$	$-\frac{1}{12}$	$-\frac{3}{4}$	$-\frac{5}{8}$	$\frac{35}{8}$

shifts

Example: a Scalar Resonance

[Not counting the observed Higgs with $M = 126$ GeV.]

A higher-mass scalar resonance σ^0 will have

- ▶ Mass M_σ .
- ▶ Coupling to the Higgs sector (Higgs and longitudinal W/Z):

$$g_L^\sigma (D_\mu \Phi)^\dagger (D^\mu \Phi) \sigma$$

- ▶ Coupling to the gauge sector (transversal W/Z):

$$g_T^\sigma \text{tr} [\mathbf{W}^{\mu\nu} \mathbf{W}_{\mu\nu}] \sigma$$

The width Γ_σ gets contributions from both couplings.

Possible Origin: 2HDM isosinglet

A 2HDM is a renormalizable model (no higher-dimensional terms needed).
If revamped to the form of Minimal SM + higher-D operators:

$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad [\text{tree}], \quad g_T^\sigma = O\left(\frac{1}{4\pi M_\sigma}\right) \quad [\text{loop}]$$

Possible Origin: something else

In a strongly interacting model, the hierarchy could be different:

$$g_L^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad [\text{tree}], \quad g_T^\sigma = O\left(\frac{1}{M_\sigma}\right) \quad [\text{tree}]$$

Simplified Models for VBS (and VVV)

- ▶ Rise of amplitude (anomalous coupling) may be Taylor expansion of a resonance
- ▶ We have no idea which resonances exist and where they come from.
- ▶ Including a resonance in the model, there still may be further sources for anomalous couplings (further resonances, $A_{\text{nonres}}(s)$, deviation from the Breit-Wigner shape, etc.)
- ▶ Beyond the resonance, the amplitude may eventually rise and need unitarization again.

Consequence:

- ▶ Resonances in all accessible spin/isospin channels
- ▶ Couplings to the Higgs and gauge sectors are unrelated and arbitrary
- ▶ Still include anomalous couplings
- ▶ Unitarization (later)

Vector Boson Scattering

Beyer et al., hep-ph/0604048

1 TeV, 1 ab^{-1} , full $6f$ final states, 80 % e_R^- , 60 % e_L^+ polarization, binned likelihood

Contributing channels: $WW \rightarrow WW, WW \rightarrow ZZ, WZ \rightarrow WZ, ZZ \rightarrow ZZ$

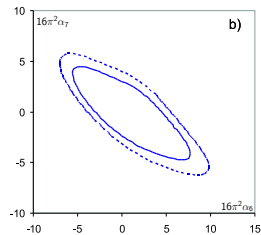
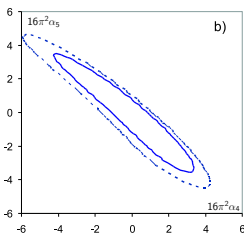
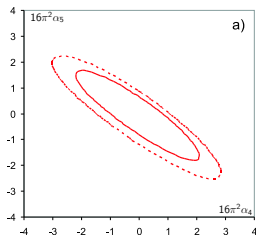
Process	Subprocess	σ [fb]
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q \bar{q} q \bar{q}$	$WW \rightarrow WW$	23.19
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q \bar{q} q \bar{q}$	$WW \rightarrow ZZ$	7.624
$e^+e^- \rightarrow \nu \bar{\nu} q \bar{q} q \bar{q}$	$V \rightarrow VVV$	9.344
$e^+e^- \rightarrow \nu e q \bar{q} q \bar{q}$	$WZ \rightarrow WZ$	132.3
$e^+e^- \rightarrow e^+e^- q \bar{q} q \bar{q}$	$ZZ \rightarrow ZZ$	2.09
$e^+e^- \rightarrow e^+e^- q \bar{q} q \bar{q}$	$ZZ \rightarrow W^+W^-$	414.
$e^+e^- \rightarrow b \bar{b} X$	$e^+e^- \rightarrow t \bar{t}$	331.768
$e^+e^- \rightarrow q \bar{q} q \bar{q}$	$e^+e^- \rightarrow W^+W^-$	3560.108
$e^+e^- \rightarrow q \bar{q} q \bar{q}$	$e^+e^- \rightarrow ZZ$	173.221
$e^+e^- \rightarrow e \nu q \bar{q}$	$e^+e^- \rightarrow e \nu W$	279.588
$e^+e^- \rightarrow e^+e^- q \bar{q}$	$e^+e^- \rightarrow e^+e^- Z$	134.935
$e^+e^- \rightarrow X$	$e^+e^- \rightarrow q \bar{q}$	1637.405

$SU(2)_c$ conserved case, all channels

coupling	σ^-	σ^+
$16\pi^2 \alpha_4$	-1.41	1.38
$16\pi^2 \alpha_5$	-1.16	1.09

$SU(2)_c$ broken case, all channels

coupling	σ^-	σ^+
$16\pi^2 \alpha_4$	-2.72	2.37
$16\pi^2 \alpha_5$	-2.46	2.35
$16\pi^2 \alpha_6$	-3.93	5.53
$16\pi^2 \alpha_7$	-3.22	3.31
$16\pi^2 \alpha_{10}$	-5.55	4.55



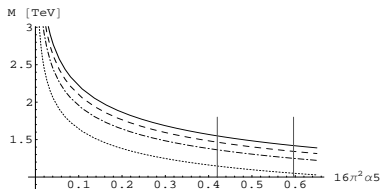
Interpretation as limits on resonances

Beyer et al., hep-ph/0604048

Consider the width to mass ratio, $f_\sigma = \Gamma_\sigma/M_\sigma$

$SU(2)$ conserving scalar singlet

$$M_\sigma = v \left(\frac{4\pi f_\sigma}{3\alpha_5} \right)^{\frac{1}{4}}$$

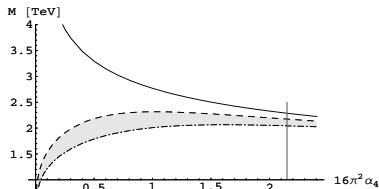


$f = 1.0$ (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)

$SU(2)$ broken vector triplet

needs input from TGC covariance matrix

$$M_{\rho^\pm} = v \left(\frac{12\pi\alpha_4 f_{\rho^\pm}}{\alpha_4^2 + 2(\alpha_2^\lambda)^2 + s_w^2 (\alpha_4^\lambda)^2 / (2c_w^2)} \right)^{\frac{1}{4}}$$



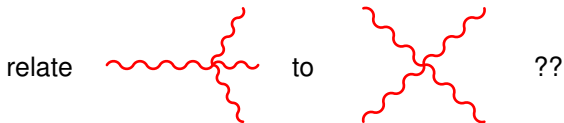
upper/lower limit from λ_Z , grey area: magnetic moments

**Final
result:**

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.55	—	1.95
1	—	2.49	—
2	3.29	—	4.30

Spin	$I = 0$	$I = 1$	$I = 2$
0	1.39	1.55	1.95
1	1.74	2.67	—
2	3.00	3.01	5.84

And Triple Vector Boson Production?



Yes, the same Feynman graphs (in the SM), but. . .

TVBP: one external $W/Z/\gamma$ is always far off-shell. Unitarization has to proceed differently, and a different set of (anomalous) couplings contributes. This is particularly true for resonances.

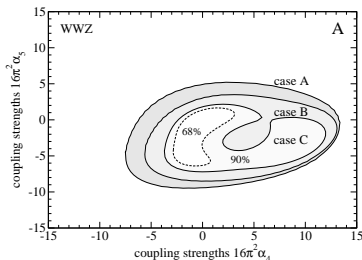
⇒ This is important physics which should be treated **independently** w.r.t. VBS processes. Don't just combine the results!

ILC Results: Triboson production

Beyer et al., hep-ph/0604048

$e^+e^- \rightarrow WWZ/ZZZ$, dep. on $(\alpha_4 + \alpha_6)$, $(\alpha_5 + \alpha_7)$, $\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, 1 ab^{-1} , full 6-fermion final states, SIMDET fast simulation

Observables: M_{WW}^2 , M_{WZ}^2 , $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e_R^- , C) 80% e_R^- , 60% e_L^+

$16\pi^2 \times$	WWZ			ZZZ no pol.	best
	no pol.	e^- pol.	both pol.		
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \rightarrow 6$ jets

Veto against $E_{\text{mis}}^2 + p_{\perp, \text{mis}}^2$

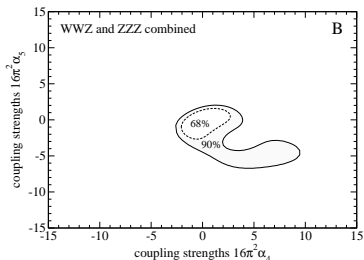
No angular correlations yet

ILC Results: Triboson production

Beyer et al., hep-ph/0604048

$e^+e^- \rightarrow WWZ/ZZZ$, dep. on $(\alpha_4 + \alpha_6)$, $(\alpha_5 + \alpha_7)$, $\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})$

Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR

1 TeV, 1 ab^{-1} , full 6-fermion final states, SIMDET fast simulation

Observables: M_{WW}^2 , M_{WZ}^2 , $\sphericalangle(e^-, Z)$

A) unpol., B) 80% e_R^- , C) 80% e_R^- , 60% e_L^+

$16\pi^2 \times$	WWZ			ZZZ	best
	no pol.	e^- pol.	both pol.	no pol.	
$\Delta\alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta\alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta\alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta\alpha_5^-$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays

Durham jet algorithm

Bkgd. $t\bar{t} \rightarrow 6 \text{ jets}$

Veto against $E_{\text{mis}}^2 + p_{\perp, \text{mis}}^2$

No angular correlations yet

Unitarity of Amplitudes

UV-incomplete theories could violate unitarity

Cross section:
$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2$$

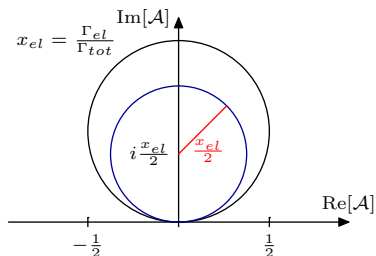
Optical Theorem (Unitarity of the S(cattering) Matrix):

$$\sigma_{\text{tot}} = \text{Im} [\mathcal{M}_{ii}(t=0)] / s \quad t = -s(1 - \cos\theta)/2$$

Partial wave amplitudes:
$$\mathcal{M}(s, t, u) = 32\pi \sum_{\ell} (2\ell + 1) \mathcal{A}_{\ell}(s) P_{\ell}(\cos\theta)$$

Assuming only elastic scattering:

$$\sigma_{\text{tot}} = \sum_{\ell} \frac{32\pi(2\ell+1)}{s} |\mathcal{A}_{\ell}|^2 \stackrel{!}{=} \sum_{\ell} \frac{32\pi(2\ell+1)}{s} \text{Im} [\mathcal{A}_{\ell}] \Rightarrow \boxed{|\mathcal{A}_{\ell}|^2 = \text{Im} [\mathcal{A}_{\ell}]}$$



Argand circle

$$\boxed{|\mathcal{A}(s) - \frac{i}{2}| = \frac{1}{2}}$$

Resonance:
$$\mathcal{A}(s) = \frac{-M\Gamma_{\text{el}}}{s - M^2 + iM\Gamma_{\text{tot}}}$$

Counterclockwise circle, **radius** $\frac{x_{el}}{2}$

Pole at $s = M^2 - iM\Gamma_{\text{tot}}$

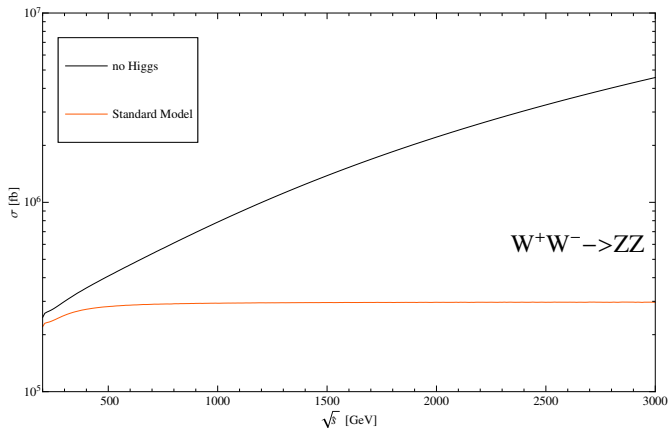
SM: VBS Amplitude unitarized by Higgs

Higgsless Model

$$\mathcal{A}_{WWZZ} = \frac{s}{v^2} + \mathcal{O}(1)$$

SM

$$\mathcal{A}_{WWZZ} = \frac{s}{v^2} - \frac{1}{v^2} \frac{s^2}{s - M_h^2} + \mathcal{O}(1)$$



Unitarity Bound for α_4 AQGC

Bounds for α_4

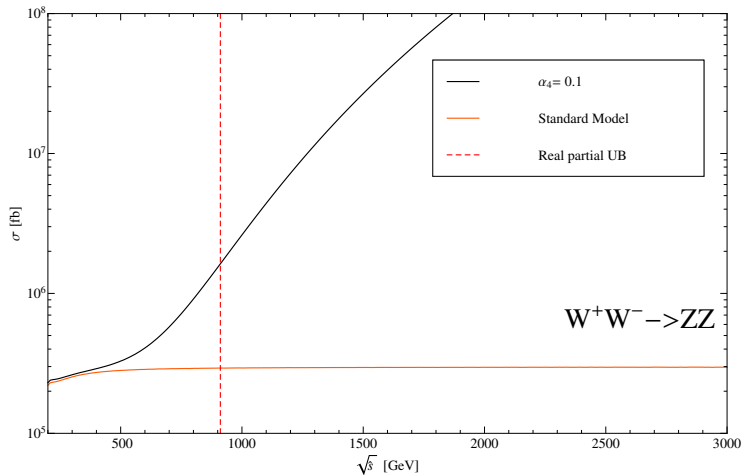
$$\ell = 0 : \sqrt{s} \leq \left(\frac{6\pi}{\alpha_4} \right)^{\frac{1}{4}} v \approx \frac{0.5 \text{ TeV}}{\sqrt[4]{\alpha_4}}$$

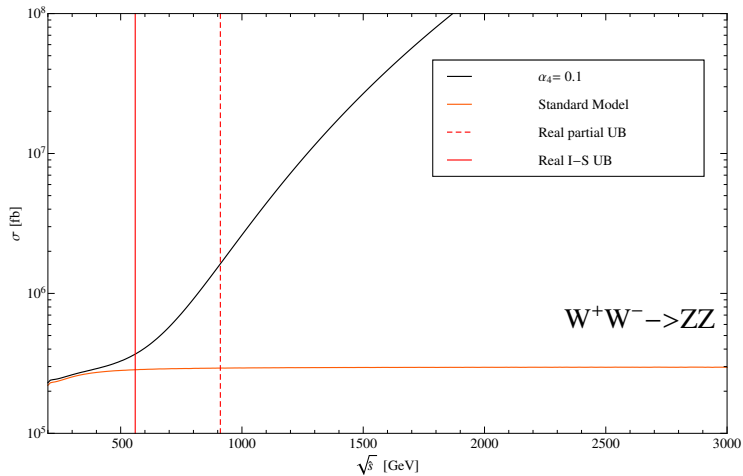
$$\ell = 2 : \sqrt{s} \leq \left(\frac{60\pi}{\alpha_4} \right)^{\frac{1}{4}} v \approx \frac{0.9 \text{ TeV}}{\sqrt[4]{\alpha_4}}$$

- ▶ Bound **depends** on coupling α_4
- ▶ Use strongest bound

α_4 AQGC contribution to
 $WW \rightarrow ZZ$

$$\mathcal{A}(s, t, u) = 4\alpha_4 \frac{t^2 + u^2}{v^4}$$





Cut-Off Method (a.k.a. “Event Clipping”)

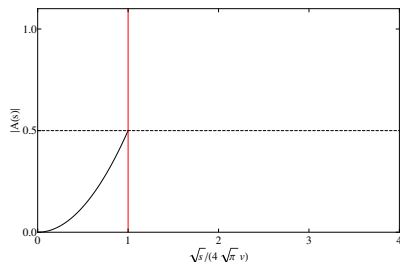
Cut-Off function

$$\Theta(\Lambda_C^2 - s)$$

- ▶ Naive prevention of Unitarity violation
- ▶ No continuous transition at Λ_C
- ▶ Ignore any interesting physics above Unitary bound
- ▶ **Better: Use observables, which do not conflict unitarity condition**

Cut-Off energy Λ_C

Λ_C equates unitarity bounds
(often 0th partial wave)



Form Factor

Form Factor

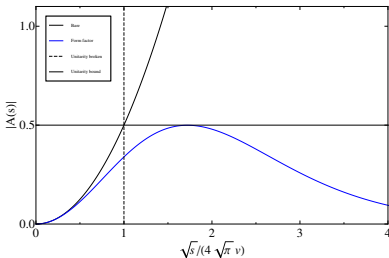
$$\frac{1}{\left(1 + \frac{s}{\Lambda_{FF}^2}\right)^n}$$

- ▶ Use Form Factor to suppress breaking of unitarity
- ▶ Can be generally used for arbitrary anomalous operator
- ▶ Need "Fine Tuning"

Parameters

n Chosen to prevent breaking of Unitarity

Λ_{FF} Calculate highest possible value that satisfy real Unitarity bound (0th partial wave)

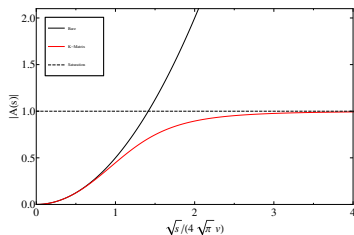
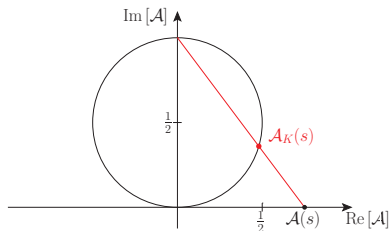


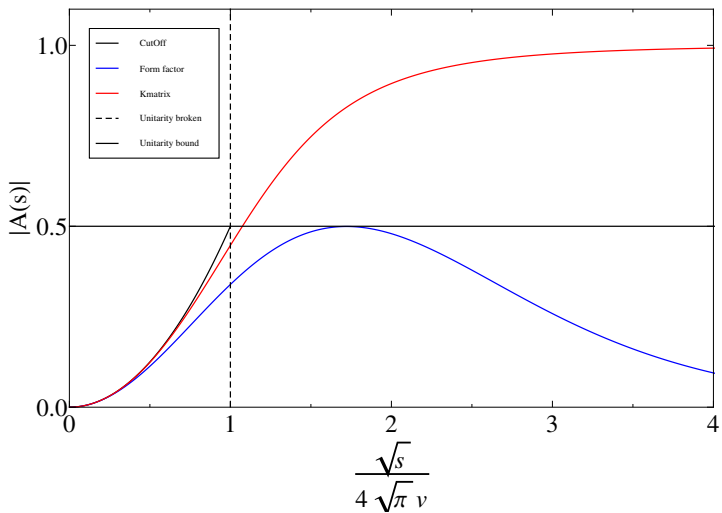
K-Matrix

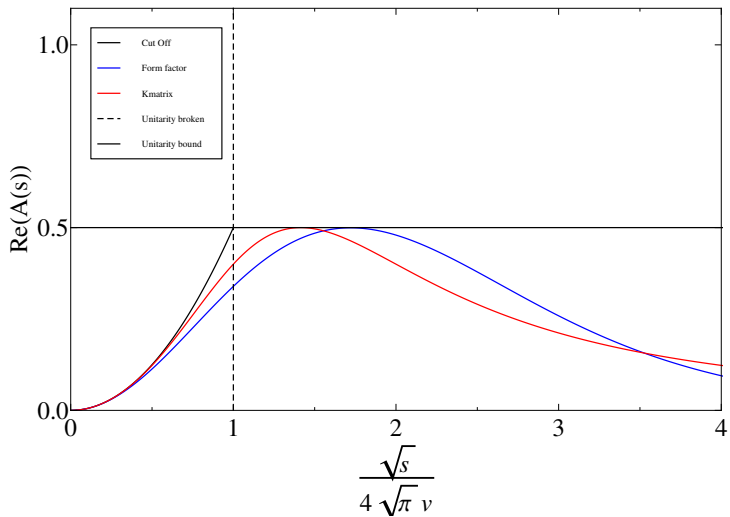
K-Matrix Unitarisation

$$\begin{aligned} \mathcal{A}_K(s) &= \frac{1}{\operatorname{Re}\left(\frac{1}{\mathcal{A}(s)}\right) - i} \\ &= \frac{\mathcal{A}(s)}{1 - i\mathcal{A}(s)} \quad \text{if } \mathcal{A}(s) \in \mathbb{R} \end{aligned}$$

- ▶ Projection of elastic amplitudes onto Argand-Circle
- ▶ At high energies the amplitude saturates
- ▶ Is usable for complex amplitudes
- ▶ Not dependent on additional parameters







"Comparison"

- ▶ Which Unitarisation scheme provides the best description?
- All of them:
Unitarisation schemes are an arbitrary way to guarantee Unitarity

Form Factor

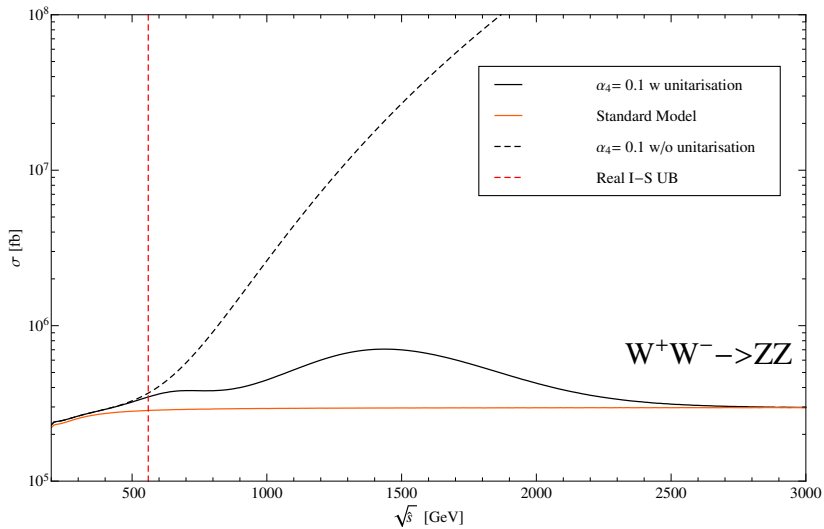
- ▶ Suppression of amplitude to get below Unitarity bound

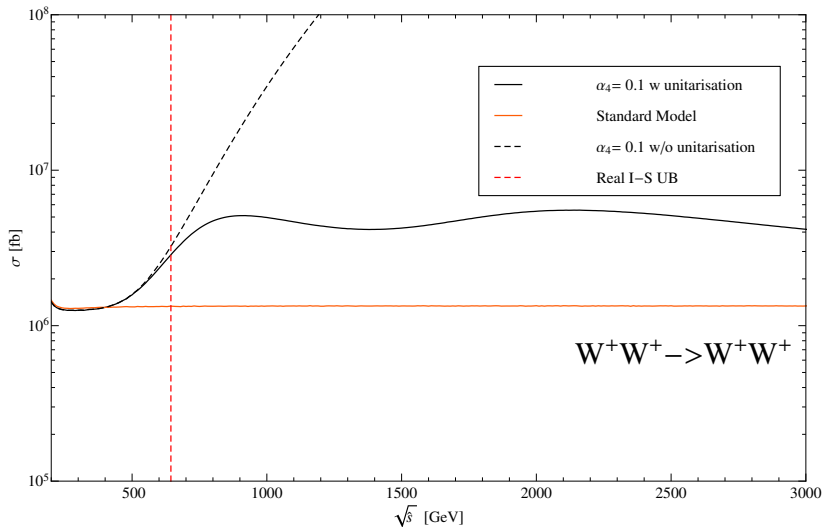
MC Generate less events than possible

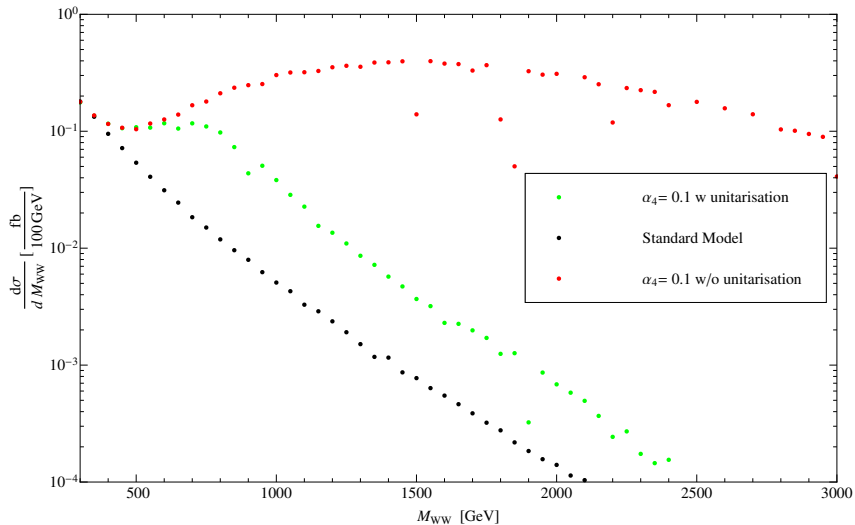
K-Matrix

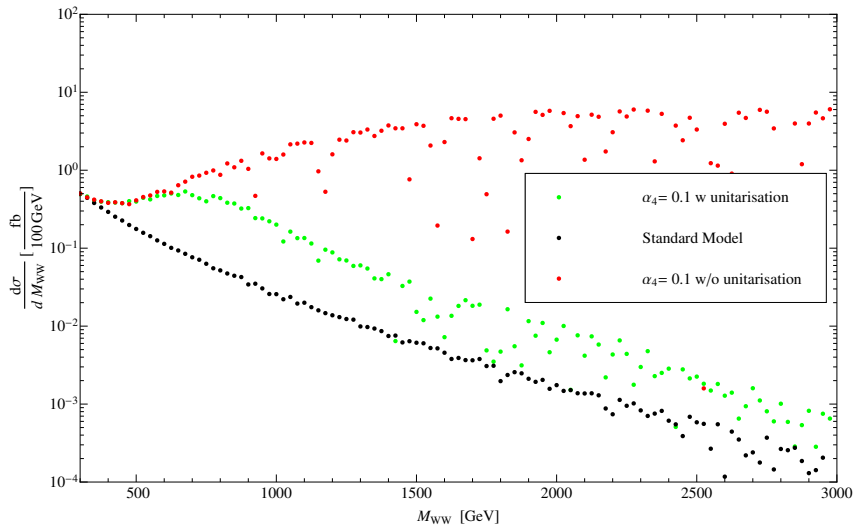
- ▶ Saturation of amplitude to achieve Unitarity

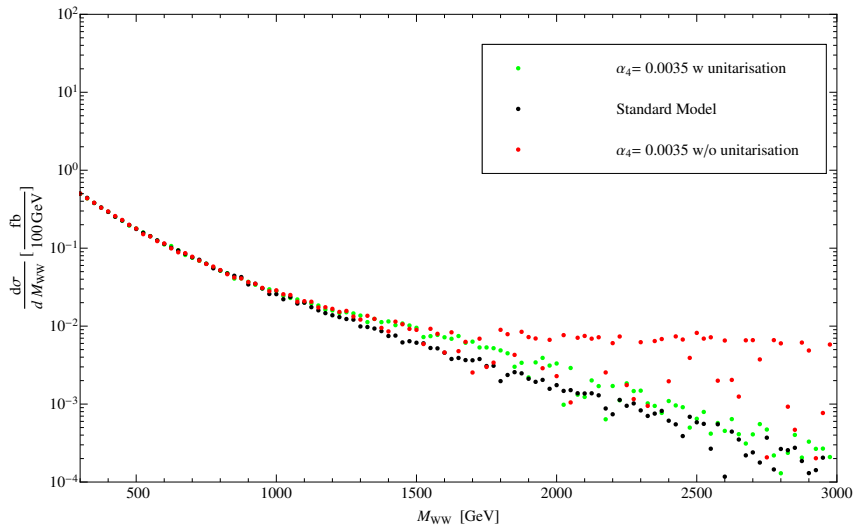
MC Generate maximal possible number of events





$PP \rightarrow e^+e^+\nu_e\nu_e + jj$ at $s = 8$ TeV

$PP \rightarrow e^+e^+\nu_e\nu_e + jj$ at $s = 14$ TeV

$PP \rightarrow e^+e^+\nu_e\nu_e + jj$ at $s = 14$ TeV

Summary/Conclusions

- ▶ Triple/Quartic gauge couplings measured either
 - via diboson production
 - via triple boson production
 - via vector boson scattering
- ▶ Unify LHC and ILC descriptions
- ▶ SM deviations in EW effective Lagrangian (SM + higher-dim. op.)
- ▶ Want to set model independent limits AQGC ($\alpha_4, \frac{f_{S,0}}{\Lambda_{NP}^4}$)
- ▶ We do not know at which scale Λ_{NP} the EFT breaks down
- ▶ But: Energy range for testing AQGC is bound by Unitarity
- ▶ Unitarisation scheme:
Number of events generated by MC fulfill unitarity condition
- ▶ Unitarisation introduces model dependence
- ▶ Unified description for different channels difficult/impossible

Summary/Conclusions

- ▶ VBS Simplified Models: EFT approach for low-energy regime, unitarized by form factors in resonance scheme at high energies
- ▶ interpreted as resonances coupled to EW bosons
- ▶ Approach includes/generalizes standard EFT ansatz
- ▶ Issue of unitarity (PDFs help – but kill energy reach)
- ▶ Photon-induced processes: better sensitivity, but higher constraints!
- ▶ Sensitivity rises with number of intermediate states:
 - LHC sensitivity limited in pure EW sector: $\sim 1 - X \text{ TeV}$ (???)
 - ILC : $1.5 - 6 \text{ TeV}$

Always get the correct ellipses...



Backup Slides:

Anomalous triple and quartic gauge couplings

$$\mathcal{L}_{TGC} = ie \left[g_1^\gamma A_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu} \right) + \kappa^\gamma W_\mu^- W_\nu^+ A^{\mu\nu} + \frac{\lambda^\gamma}{M_W^2} W_\mu^-{}^\nu W_{\nu\rho}^+ A^{\rho\mu} \right] \\ + ie \frac{c_w}{s_w} \left[g_1^Z Z_\mu \left(W_\nu^- W^{+\mu\nu} - W_\nu^+ W^{-\mu\nu} \right) + \kappa^Z W_\mu^- W_\nu^+ Z^{\mu\nu} + \frac{\lambda^Z}{M_W^2} W_\mu^-{}^\nu W_{\nu\rho}^+ Z^{\rho\mu} \right]$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_w^2 - s_w^2}$, $g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0$$

$$\Delta \kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2} \alpha_3$$

$$\Delta \kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0$$

$$\Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2} \alpha_3$$

$$\Delta g_1^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2 \alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4}(\alpha_4 + \alpha_6)$$

$$\Delta g_2^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2[\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})]$$

Anomalous triple and quartic gauge couplings

$$\begin{aligned}
 \mathcal{L}_{QGC} = & e^2 \left[g_1^{\gamma\gamma} A^\mu A^\nu W_\mu^- W_\nu^+ - g_2^{\gamma\gamma} A^\mu A_\mu W^{-\nu} W_\nu^+ \right] \\
 & + e^2 \frac{c_w}{s_w} \left[g_1^{\gamma Z} A^\mu Z^\nu \left(W_\mu^- W_\nu^+ + W_\mu^+ W_\nu^- \right) - 2g_2^{\gamma Z} A^\mu Z_\mu W^{-\nu} W_\nu^+ \right] \\
 & + e^2 \frac{c_w^2}{s_w^2} \left[g_1^{ZZ} Z^\mu Z^\nu W_\mu^- W_\nu^+ - g_2^{ZZ} Z^\mu Z_\mu W^{-\nu} W_\nu^+ \right] \\
 & + \frac{e^2}{2s_w^2} \left[g_1^{WW} W^{-\mu} W^{+\nu} W_\mu^- W_\nu^+ - g_2^{WW} \left(W^{-\mu} W_\mu^+ \right)^2 \right] + \frac{e^2}{4s_w^2 c_w^4} h^{ZZ} (Z^\mu Z_\mu)^2
 \end{aligned}$$

SM values: $g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1$, $\lambda^{\gamma,Z} = 0$ and $\delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_w^2 - s_w^2}$, $g_{1/2}^{VV'} = 1$, $h^{ZZ} = 0$

$$\Delta g_1^\gamma = 0$$

$$\Delta \kappa^\gamma = g^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2} \alpha_3$$

$$\Delta \kappa^Z = \delta_Z - g'^2(\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2(\alpha_9 - \alpha_8)$$

$$\Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0$$

$$\Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4}(\alpha_5 + \alpha_7)$$

$$\Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2} \alpha_3$$

$$\Delta g_1^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) + g^2 \alpha_4$$

$$\Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4}(\alpha_4 + \alpha_6)$$

$$\Delta g_2^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2(\alpha_9 - \alpha_8) - g^2(\alpha_4 + 2\alpha_5)$$

$$h^{ZZ} = g^2 [\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})]$$

Parameters

$$\mathcal{L}_\sigma = -\frac{g_\sigma v}{2} \text{Tr} [\mathbf{V}_\mu \mathbf{V}^\mu] \sigma$$

$$\mathbf{V}_\mu = -ig \mathbf{W}_\mu + ig' \mathbf{B}_\mu$$

$$\mathbf{W}_\mu = W_\mu^a \frac{\tau^a}{2}$$

$$\mathbf{B}_\mu = W_\mu^a \frac{\tau^3}{2}$$

$$\mathcal{L}_\phi = \frac{g_\phi v}{4} \text{Tr} \left[\left(\mathbf{V}_\mu \otimes \mathbf{V}^\mu - \frac{\tau^{aa}}{6} \text{Tr} [\mathbf{V}_\mu \mathbf{V}^\mu] \right) \phi \right]$$

$$\phi = \sqrt{2} (\phi^{++} \tau^{++} + \phi^+ \tau^+ + \phi^0 \tau^0 + \phi^- \tau^- + \phi^{--} \tau^{--})$$

$$\tau^{++} = \tau^+ \otimes \tau^+$$

$$\tau^+ = \frac{1}{2} (\tau^+ \otimes \tau^3 + \tau^3 \otimes \tau^+)$$

$$\tau^0 = \frac{1}{\sqrt{6}} (\tau^3 \otimes \tau^3 - \tau^+ \otimes \tau^- - \tau^- \otimes \tau^+)$$

SM Lagrangian

$$\begin{aligned}
 \mathcal{L}_{\min} = & -\frac{1}{2} \text{tr} [\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}] - \frac{1}{2} \text{tr} [\mathbf{B}_{\mu\nu} \mathbf{B}^{\mu\nu}] && W^\pm, Z \\
 & + (\partial_\mu \phi)^\dagger \partial^\mu \phi - V(\phi) && h \\
 & + \frac{v^2}{4} \text{tr} [(\mathbf{D}_\mu \Sigma)^\dagger (\mathbf{D}^\mu \Sigma)] && w^\pm, z \\
 & - \frac{g_h v}{2} \text{tr} [\mathbf{V}^\mu \mathbf{V}_\mu] h
 \end{aligned}$$

Vector Bosons

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + ig [\mathbf{W}_\mu, \mathbf{W}_\nu]$$

$$\mathbf{B}_{\mu\nu} = \partial_\mu \mathbf{B}_\nu - \partial_\nu \mathbf{B}_\mu$$

$$\mathbf{W}_\mu = W_\mu^a \frac{\tau^a}{2} \quad \mathbf{B}_\mu = B_\mu \frac{\tau^3}{2}$$

$$\mathbf{D}_\mu = \partial_\mu + ig \mathbf{W}_\mu - ig' \mathbf{B}_\mu$$

Higgs Sector

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}$$

$$\Sigma = \exp \left[-\frac{i}{v} w^a \tau^a \right]$$

$$\mathbf{V}_\mu = \Sigma (\mathbf{D}_\mu \Sigma)$$

Unitary Gauge

- ▶ Goldstone bosons are absorbed by vector bosons as longitudinal degrees of freedom
- ▶ $w^a \equiv 0 \rightarrow \Sigma \equiv 1$
- ▶ $\mathbf{D}_\mu = \partial_\mu - \mathbf{V}_\mu = \partial_\mu + \frac{ig}{2} \left(\sqrt{2}(W^+ \tau^+ + W^- \tau^-) + \frac{1}{c_w} Z \tau^3 \right)$

$$\begin{aligned} \mathcal{L}_{\min} = & -\frac{1}{2} \text{tr} [\mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}] - \frac{1}{2} \text{tr} [\mathbf{B}_{\mu\nu} \mathbf{B}^{\mu\nu}] \\ & + \underbrace{(\partial_\mu \phi)^\dagger \partial^\mu \phi - \frac{v^2}{4} \text{tr} [\mathbf{V}^\mu \mathbf{V}_\mu] - \frac{g_h v}{2} \text{tr} [\mathbf{V}^\mu \mathbf{V}_\mu] h - V(\phi)}_{\stackrel{\text{g}_h=1}{\cong} (\mathbf{D}_\mu \phi)^\dagger \mathbf{D}^\mu \phi} \end{aligned}$$

- ▶ Coincides with known SM parametrisation

Gaugeless Limit

- ▶ $g \rightarrow 0, g' \rightarrow 0$
- ▶ $\mathbf{D}_\mu = \partial_\mu$
- ▶ $\mathbf{V}_\mu = \Sigma (\partial_\mu \Sigma)$

$$\begin{aligned}\mathcal{L}_{\min} = & -\frac{1}{2}\text{tr}[\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}] - \frac{1}{2}\text{tr}[\mathbf{B}_{\mu\nu}\mathbf{B}^{\mu\nu}] \\ & + (\partial_\mu\phi)^\dagger\partial^\mu\phi - V(\phi) \\ & + \frac{v^2}{4}\text{tr}[(\partial_\mu\Sigma)^\dagger(\partial^\mu\Sigma)] - \frac{g_h v}{2}\text{tr}[\mathbf{V}^\mu\mathbf{V}_\mu] h\end{aligned}$$

- ▶ Decoupling of Higgs Sector

Isospin decomposition

- ▶ Lowest order chiral Lagrangian (incl. anomalous couplings)

$$\mathcal{L} = -\frac{v^2}{4} \text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu] + \alpha_4 \text{tr} [\mathbf{V}_\mu \mathbf{V}_\nu] \text{tr} [\mathbf{V}^\mu \mathbf{V}^\nu] + \alpha_5 (\text{tr} [\mathbf{V}_\mu \mathbf{V}^\mu])^2$$

- ▶ Leads to the following amplitudes: $s = (p_1 + p_2)^2$ $t = (p_1 - p_3)^2$ $u = (p_1 - p_4)^2$

$$\boxed{\mathcal{A}(s, t, u) =:}$$

$$\begin{aligned} \mathcal{A}(w^+ w^- \rightarrow zz) &= \frac{s}{v^2} + 8\alpha_5 \frac{s^2}{v^4} + 4\alpha_4 \frac{t^2 + u^2}{v^4} \\ \mathcal{A}(w^+ z \rightarrow w^+ z) &= \frac{t}{v^2} + 8\alpha_5 \frac{t^2}{v^4} + 4\alpha_4 \frac{s^2 + u^2}{v^4} \\ \mathcal{A}(w^+ w^- \rightarrow w^+ w^-) &= -\frac{u}{v^2} + (4\alpha_4 + 2\alpha_5) \frac{s^2 + t^2}{v^4} + 8\alpha_4 \frac{u^2}{v^4} \\ \mathcal{A}(w^+ w^+ \rightarrow w^+ w^+) &= -\frac{s}{v^2} + 8\alpha_4 \frac{s^2}{v^4} + 4(\alpha_4 + 2\alpha_5) \frac{t^2 + u^2}{v^4} \\ \mathcal{A}(zz \rightarrow zz) &= 8(\alpha_4 + \alpha_5) \frac{s^2 + t^2 + u^2}{v^4} \end{aligned}$$

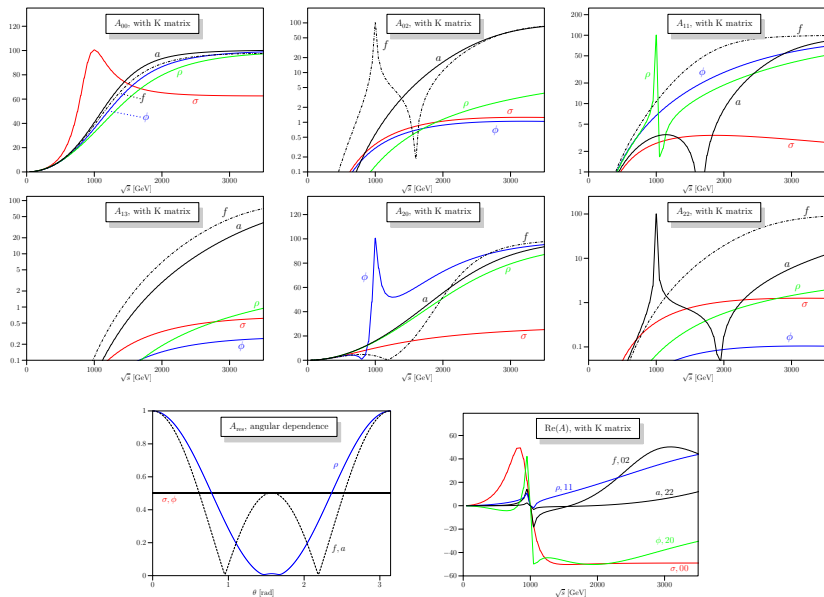
- ▶ (Clebsch-Gordan) Decomposition into isospin eigenamplitudes

$$\mathcal{A}(I = 0) = 3\mathcal{A}(s, t, u) + \mathcal{A}(t, s, u) + \mathcal{A}(u, s, t)$$

$$\mathcal{A}(I = 1) = \mathcal{A}(t, s, u) - \mathcal{A}(u, s, t)$$

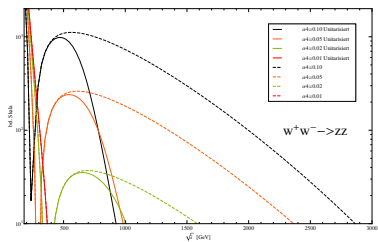
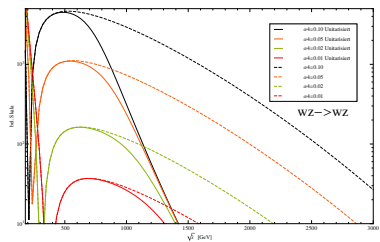
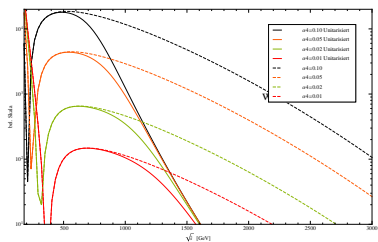
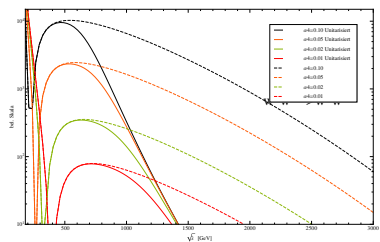
$$\mathcal{A}(I = 2) = \mathcal{A}(t, s, u) + \mathcal{A}(u, s, t)$$

Eigenamplitudes



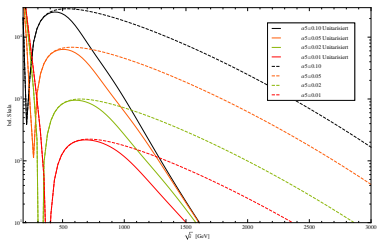
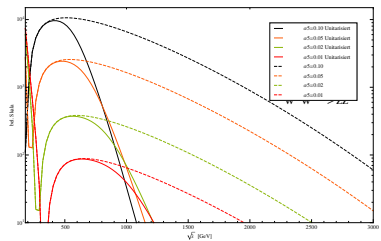
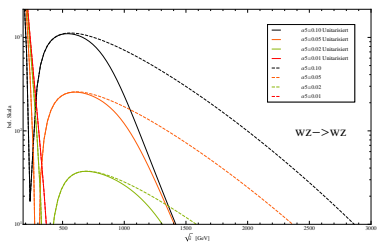
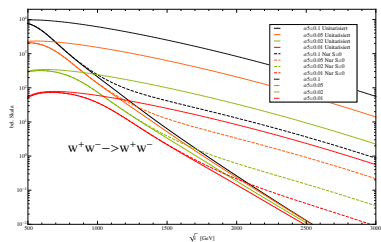
aQGCs: α_4

Kilian/JRR/Sekulla, 2013

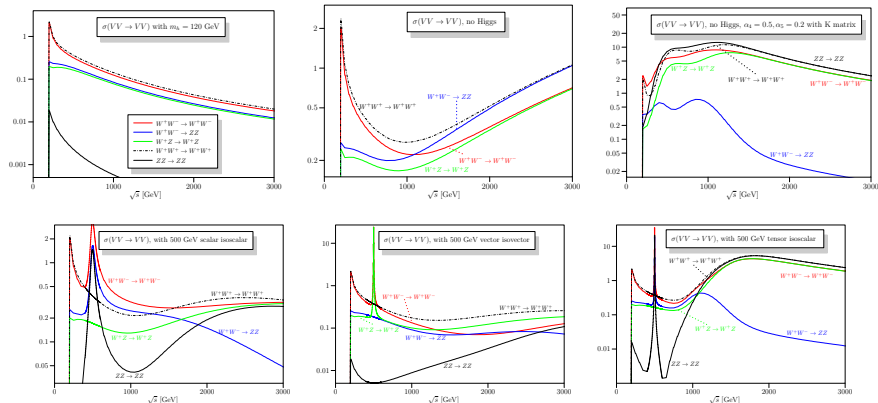


aQGCs: α_5

Kilian/JRR/Sekulla, 2013



“Partonic” cross sections



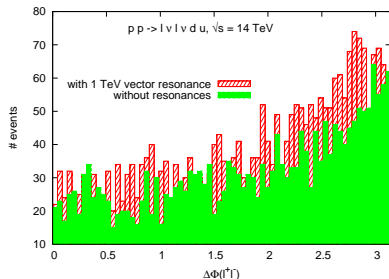
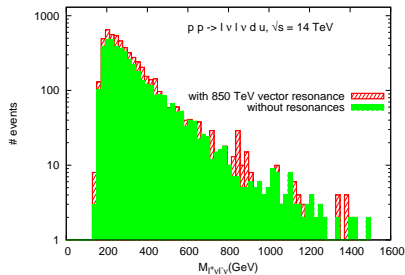
- ▶ $\sigma(VV \rightarrow VV)$ in nb $M_R = 500$ GeV
- ▶ all amplitudes K-matrix unitarized
- ▶ Cut of 15° around the beam axis

LHC Example: Vector Isovector

Alboteanu/Kilian/JRR,

2008

- ▶ Example: 850 GeV vector resonance, coupling $g_\rho = 1$
- ▶ (Theory) Cuts:
 - $p_\perp(\ell\nu) > 30$ GeV
 - $|\delta R(\ell\nu)| < 1.5$
 - $\theta(u/d) > 0.5^\circ$
- ▶ Integrated luminosity: 225 fb^{-1}
- ▶ Discriminator: angular correlations $\Delta\phi(\ell\ell)$
- ▶ Ongoing ATLAS study
 - More kinematic observables
 - Comparison and validation phase
 - first reproduce SM
 - then anom. couplings/BSM resonances



More about K-Matrix

Example

$$\mathcal{A}(s) = \frac{-c}{s - M^2} \rightarrow \frac{-c}{s - M^2 + iM\Gamma_1}$$

$$\mathcal{A}(s) = -\frac{M}{v^2} \frac{s}{s - M^2} \rightarrow -\frac{M}{v^2} \frac{s}{s - M^2 + iM\Gamma_2} \frac{s}{M^2}$$

- ▶ Amplitudes satisfying Unitarity are invariant under Kmatrix-Unitarisation
 - ▶ Transforms simple pole amplitude into Breit-Wigner form
- ⇒ Alternate Implementation of Dyson resummation for s-particle exchange
- ▶ Unitarisation of Pade and IAM (inverse-amplitude method) coincides with K-matrix Unitarisation