(Supersymmetric) Grand Unification

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Uppsala, 15. May 2008

Literature

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The Standard Model (SM) – Theorist's View

Renormalizable Quantum Field Theory (only with Higgs!) based on $SU(3)_c \times SU(2)_w \times U(1)_Y$ *non-simple* gauge group

$$L_L = \begin{pmatrix} \nu \\ \ell \end{pmatrix}_L \quad Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L \quad u^c_R \quad d^c_R \quad \ell^c_R \quad [\nu^c_R] \quad \left[\begin{pmatrix} h^+ \\ h^0 \end{pmatrix} \right]$$

Interactions:

- Gauge IA (covariant derivatives in kinetic terms):

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu + i \sum_k g_k V^a_\mu T^a$$

- Yukawa IA:

$$Y^u \overline{Q}_L H_u u_R + Y^d \overline{Q}_L H_d d_R + Y^e \overline{L}_L H_d e_R \quad \left[+ Y^n \overline{L}_L H_u \nu_R \right]$$

- Scalar self-IA:

$$(H^\dagger H) \qquad (H^\dagger H)^2$$

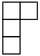
The group-theoretical bottom line

Things to remember:

Representations of $SU(N)$

- fundamental reps. $\phi_i \sim N$, $\psi^i \sim \overline{N}$, adjoint reps. $A_i^j \sim N^2 - 1$
- $SU(N)$ invariants: contract all indices

  $\phi_i \psi^i \qquad \phi_i A_i^j \psi^j \qquad \epsilon_{ij} \phi_i \xi_j \qquad \epsilon_{ijk} \phi_i \xi_j \eta_k$

- [Symmetry properties: Young tableaux ]

The (group-theoretical) essence of the SM:

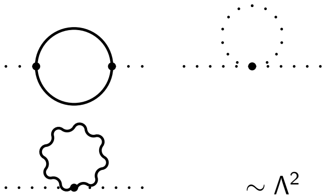
remember: $Q_{\text{el.}} = T_3 + \frac{Y}{2}$

Q_L	u_R^c	d_R^c	L_L	e_R^c	H_d	H_d	ν_R^c
$(\mathbf{2},\mathbf{3})_{\frac{1}{3}}$	$(\mathbf{1},\overline{\mathbf{3}})_{-\frac{4}{3}}$	$(\mathbf{1},\overline{\mathbf{3}})_{\frac{2}{3}}$	$(\mathbf{2},\mathbf{1})_{-1}$	$(\mathbf{1},\mathbf{1})_{2}$	$(\mathbf{2},\mathbf{1})_{1}$	$(\mathbf{2},\mathbf{1})_{-1}$	$(\mathbf{1},\mathbf{1})_{0}$

Loose Ends/Deficiencies of the Standard Model

Incompleteness

- Electroweak Symmetry Breaking
- Higgs boson
- Origin of neutrino masses
- Dark Matter: $m_{DM} \sim 100\,\text{GeV}$

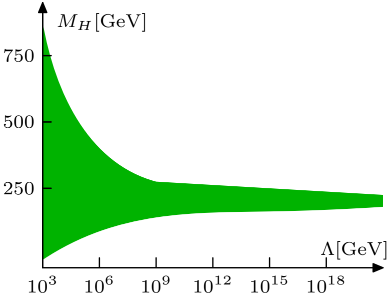


$\sim \Lambda^2$

$$m_h^2 = m_0^2 + \Lambda^2 \times (\text{loop factors})$$

Theoretical Dissatisfaction

- 28 free parameters
- "strange" fractional $U(1)$ quantum numbers
- Hierarchy problem



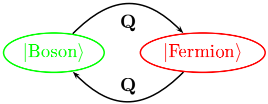
Supersymmetry

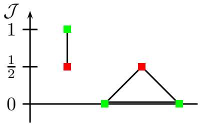
connects gauge and space-time symmetries

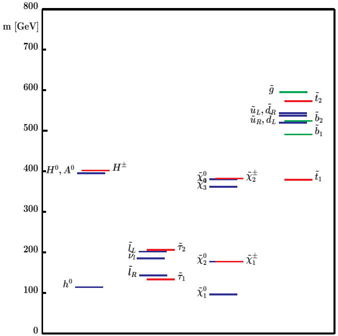
multiplets with equal-mass fermions and bosons

$\Rightarrow$ SUSY is broken in Nature

stabilizes the hierarchy







- Minimal Supersymmetric Standard Model (MSSM)
- Charginos, Neutralinos, Gluino Sleptons, Squarks, Sneutrinos
- R parity: discrete symmetry
- LSP: Dark matter
- Superpartners have *identical* gauge quantum numbers

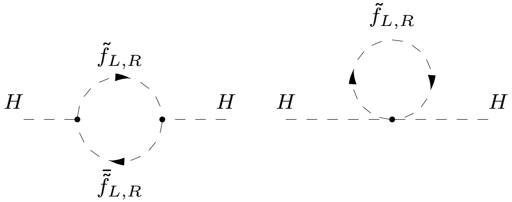
Supersymmetry:

Symmetry between fermions and bosons

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$
$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

Effectively: SM particles have SUSY partners (e.g. $f_{L,R} \to \tilde{f}_{L,R}$)

SUSY: additional contributions from scalar fields:



$$\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2 \int d^4k \left(\frac{1}{k^2 - m_{\tilde{f}_L}^2} + \frac{1}{k^2 - m_{\tilde{f}_R}^2} \right) + \text{terms without quadratic div.}$$

for $\Lambda \to \infty$: $\Sigma_H^{\tilde{f}} \sim N_{\tilde{f}}\, \lambda_{\tilde{f}}^2\, \Lambda^2$

$\Rightarrow$ quadratic divergences cancel for

$$N_{\tilde{f}_L} = N_{\tilde{f}_R} = N_f$$
$$\lambda^2_{\tilde{f}} = \lambda^2_f$$

complete correction vanishes if furthermore

$$m_{\tilde{f}} = m_f$$

Soft SUSY breaking: $m^2_{\tilde{f}} = m^2_f + \Delta^2, \quad \lambda^2_{\tilde{f}} = \lambda^2_f$

$$\Rightarrow \Sigma_H^{f+\tilde{f}} \sim N_f \, \lambda^2_f \, \Delta^2 + \ldots$$

$\Rightarrow$ correction stays acceptably small if mass splitting is of weak scale

$\Rightarrow$ realized if mass scale of SUSY partners

$$M_{\text{SUSY}} \lesssim 1\text{TeV}$$

$\Rightarrow$ SUSY at TeV scale provides attractive solution of hierarchy problem

Anatomy of MSSM

- Yukawa couplings $\Rightarrow$ Superpotential:

$$\mathcal{W} = \hat{\Phi}_1 \hat{\Phi}_2 \hat{\Phi}_3 \equiv \Phi_1 \Phi_2 \Phi_3 \quad \Longrightarrow \quad (\phi_1 \phi_2)^2, \ldots, \left(\overline{\psi}_1 \psi_2\right) \phi_3$$

- NB: part of scalar potential from gauge kinetic terms (light Higgs)
- MSSM superpotential

$$\mathcal{W}_{\text{MSSM}} = Y^u u^c Q H_u + Y^d d^c Q H_d + Y^e e^c L H_d + \mu H_u H_d$$

- Ignorance about SUSY breaking shows up as “soft-breaking terms”: Gaugino and sparticle masses, trilinear scalar potential terms
- μ problem: EWSB demands $\mu \sim \mathcal{O}(100\,\text{GeV} - 1\,\text{TeV})$
- Additional SUSY degrees of freedom modify vacuum polarization
  $\Rightarrow$ Unification of gauge couplings possible

(Gauge) Unification and the running of couplings

VOLUME 32, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1974

Unity of All Elementary-Particle Forces

Howard Georgi* and S. L. Glashow
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138
(Received 10 January 1974)

Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world—that all elementary particle forces (strong, weak, and electromagnetic) are different manifestations of the same fundamental interaction involving a single coupling strength, the fine-structure constant. Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously.

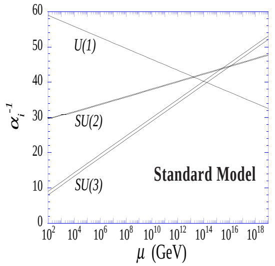
of the GIM mechanism with the notion of colored quarks[4] keeps the successes of the quark model and gives an important bonus: Lepton and hadron anomalies cancel so that the theory of weak and electromagnetic interactions is renormalizable.[5]

The next step is to include strong interactions. We assume that *strong interactions are mediated by an octet of neutral vector gauge gluons* associated with local color SU(3) symmetry, and that there are no fundamental strongly interacting scalar-meson fields.[6] This insures that

(Gauge) Unification and the running of couplings

Renormalization group (RG) running of gauge couplings:

$$\frac{dg_a}{d\log\mu} = \frac{g_a^3}{16\pi^2} B_a \qquad \begin{array}{rl} \text{SM} & B_a = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) \\ \text{MSSM} & B_a = \left(\frac{33}{5}, 1, -3\right) \end{array}$$



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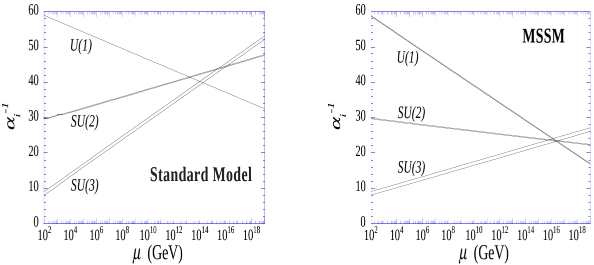

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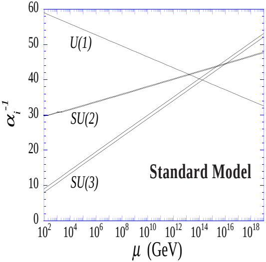

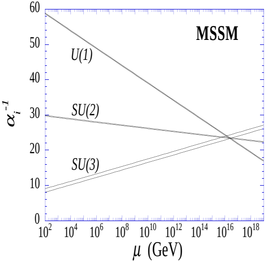

VOLUME 32, NUMBER 8 PHYSICAL REVIEW LETTERS 25 FEBRUARY 1974

H. Georgi and H. D. Politzer, Phys. Rev. D (to be published); D. J. Gross and F. Wilczek, Phys. Rev. D 8, 3633 (1973), and Phys. Rev. D (to be published).

[16]A naive calculation indicates that the vector boson mass must be greater than 10^{15} GeV $\simeq 10^{-9}$ g! Let the reader who finds this hard to swallow double the num-

The prime example: (SUSY) $SU(5)$

$$SU(5) \underset{M_X}{\longrightarrow} SU(3)_c \times SU(2)_w \times U(1)_Y \underset{M_Z}{\longrightarrow} SU(3)_c \times U(1)_{em}$$

$SU(5)$ has $5^2 - 1 = 24$ generators:

$$\mathbf{24} \longrightarrow \underbrace{(\mathbf{8},\mathbf{1})_0}_{G_\alpha^\beta} \oplus \underbrace{(\mathbf{1},\mathbf{3})_0}_{W} \oplus \underbrace{(\mathbf{1},\mathbf{1})_0}_{B} \oplus \underbrace{(\mathbf{3},\mathbf{2})_{\frac{5}{3}}}_{X,Y} \oplus \underbrace{(\bar{\mathbf{3}},\mathbf{2})_{-\frac{5}{3}}}_{\bar{X},\bar{Y}}$$

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$$A = g \sum_{a=1}^{24} A^a \frac{\lambda^a}{2} = \frac{g}{\sqrt{2}} \left(\begin{array}{ccc|cc} \multicolumn{3}{c|}{\sqrt{2} G^a \frac{\lambda^a_{\text{GM}}}{2}} & \bar{X} & \bar{Y} \\ & & & \bar{X} & \bar{Y} \\ & & & \bar{X} & \bar{Y} \\ \hline X & X & X & \multicolumn{2}{c}{\sqrt{2} W^a \frac{\sigma}{2}} \\ Y & Y & Y & & \end{array} \right)$$

$$- \frac{g}{2\sqrt{15}} B \left(\begin{array}{ccc|cc} -2 & & & & \\ & -2 & & \multicolumn{2}{c}{0} \\ & & -2 & & \\ \hline & 0 & & +3 & \\ & & & & +3 \end{array} \right)$$

Quantum numbers

- Hypercharge: $\frac{\lambda_{12}}{2} = \sqrt{\frac{3}{5}} \frac{Y}{2} \qquad Y = \frac{1}{3}\,\mathrm{diag}(-2,-2,3,3,3)$

  Quantized hypercharges are fixed by non-Abelian generator

- Weak Isospin: $T_{1,2,3} = \lambda_{9,10,11}/2$

- Electric Charge: $Q = T^3 + Y/2 = \mathrm{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0)$

- Prediction for the weak mixing angle (with RGE running):

  $\alpha^{-1}(M_Z) = 128.91(2), \alpha_s(M_Z) = 0.1176(20), s_w^2(M_Z) = 0.2312(3)$

$$\text{non-SUSY: } s_w^2(M_Z) \;=\; \frac{23}{134} + \frac{\alpha(M_Z)}{\alpha_s(M_Z)} \frac{109}{201} \;\approx 0.207$$

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$$\text{SUSY:}\quad s_w^2(M_Z) = \frac{1}{5} + \frac{\alpha(M_Z)}{\alpha_s(M_Z)}\frac{7}{15} \approx 0.231$$

New Gauge Bosons Two colored EW doublets:
(X,Y), $(\bar{X},\bar{Y})$ with charges $\pm\frac{4}{3},\pm\frac{1}{3}$

Exercise 10: SUSY GUTs and $\sin^2\theta_W$

The renormalization group equations for the coupling constants g_i, $i = 1, 2, 3$ are given by:

$$\frac{\mathrm{d}\alpha_i}{\mathrm{d}\,\ln\mu} = b_i \frac{\alpha_i^2}{2\pi}. \tag{1}$$

In the SM, the coefficients b_i for the gauge groups SU(i) are:

$$\begin{aligned} b_3 &= -11 + \frac{4}{3}N_\mathrm{g}, \\ b_2 &= -\frac{22}{3} + \frac{4}{3}N_\mathrm{g} + \frac{1}{6}N_\mathrm{H}, \\ b_1 &= \frac{20}{9}N_\mathrm{g} + \frac{1}{6}N_\mathrm{H}, \end{aligned}$$

where N_g is the number of generations and N_H the number of Higgs doublets.

a) Why has b_1 a different form as b_2 and b_3? Show that the solution of (1) is:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu_0)} - \frac{b_i}{4\pi}\ln\left(\frac{\mu^2}{\mu_0^2}\right) \tag{2}$$

b) In a Grand Unified Theory (GUT) we should have the following relation at the GUT M_X :

$$\sqrt{5/3}g_1(M_X) = g_2(M_X) = g_3(M_X) = g_\mathrm{GUT}. \tag{3}$$

In a GUT like $SU(5) \stackrel{M_X}{\rightarrow} SU(3)_c \times SU(2)_L \times U(1)_Y$, d^c, L are together in a multiplet ($\bar{\mathbf{5}}$), as well as u^c, e^c, Q ($\mathbf{10}$). Demand that tr $\left[T^a T^b\right] = \frac{1}{2}\delta^{ab}$ to find the normalization of hypercharge.

Furthermore, we have:

$$\alpha_1 = \frac{\alpha(\mu)}{c_{\mathrm{W}}^2(\mu)} \qquad \alpha_2 = \frac{\alpha(\mu)}{s_{\mathrm{W}}^2(\mu)} \tag{4}$$

c) For the scale $\mu_0 = M_X$ take the the GUT scale. Express $s_{\mathrm{W}}^2(\mu)$ as a function of $\alpha(\mu)$ ans $\ln(M_{\mathrm{GUT}}/\mu)$. Replace the logarithm by a corresponding relation containing $\alpha(\mu)$ and $\alpha_3(\mu) \equiv \alpha_s(\mu)$. Keep the b_i explicite.

c) Wo do you get with $\alpha(M_Z) \approx 1/128$ and $\alpha_s(M_Z) \approx 0.12$ for $s_{\mathrm{W}}^2(M_Z)$, M_X and α_{GUT}? Experimentally, the weak mixing angle has the value $s_{\mathrm{W}}^2 = 0.2312(3)$.

d) Due to the additional particle content of the MSSM the coefficients of the renormalization group equations for the coupling constants are changed with respect to the SM. In the MSSM they are:

$$\begin{aligned} b_3 &= -9 + 2N_{\mathrm{g}}, \\ b_2 &= -6 + 2N_{\mathrm{g}} + \frac{1}{2}N_{\mathrm{H}}, \\ b_1 &= \frac{10}{3}N_{\mathrm{g}} - \frac{1}{2}N_{\mathrm{H}}, \end{aligned}$$

Repeat the above calculations for the MSSM. What changes?

Fermions (Matter Superfields)

The only possible way to group together the matter:

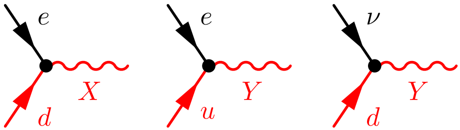
$$\overline{\mathbf{5}} = \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array} : \quad \begin{pmatrix} d^c \\ d^c \\ d^c \\ \ell \\ -\nu_\ell \end{pmatrix} \qquad \mathbf{10} = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array} : \quad \frac{1}{\sqrt{2}} \left(\begin{array}{ccc|cc} 0 & u^c & -u^c & -u & -d \\ -u^c & 0 & u^c & -u & -d \\ u^c & -u^c & 0 & -u & -d \\ \hline u & u & u & 0 & -e^c \\ d & d & d & e^c & 0 \end{array} \right)$$

$$\overline{\mathbf{5}} = (\overline{\mathbf{3}}, \mathbf{1})_{\frac{2}{3}} \oplus (\mathbf{1}, \mathbf{2})_{-1} \qquad \mathbf{10} = (\mathbf{3}, \mathbf{2})_{\frac{1}{3}} \oplus (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{4}{3}} \oplus (\mathbf{1}, \mathbf{1})_{2}$$

Remarks

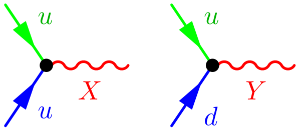
- $\mathbf{2} = \Box = \overline{\mathbf{2}}, \qquad (\mathbf{5} \otimes \mathbf{5})_a = \mathbf{10}, \quad (\mathbf{3} \otimes \mathbf{3})_a = \overline{\mathbf{3}}, \quad (\Box \otimes \Box)_a = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$
- Quarks and leptons in the same multiplet
- Fractional charges from tracelessness condition (color!)
- $\overline{\mathbf{5}}$ and $\mathbf{10}$ have equal and opposite anomalies
- ν^c must be $SU(5)$ singlet

Interactions



Leptoquark couplings

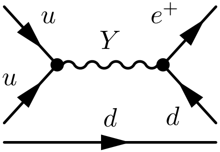
(and SUSY vertices)



Diquark couplings

(and SUSY vertices)

Vector bosons induce e.g. decay $p \to e^+ \pi^0$



The doublet-triplet splitting problem

$SU(5)$ breaking: Higgs $\mathbf{\Sigma}$ in adjoint $\mathbf{24}$ rep.

$$\langle 0|\mathbf{\Sigma}|0\rangle = w \times \text{diag}(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}) \qquad M_X = M_Y = \frac{5}{2\sqrt{2}}\, g\, w$$

other breaking mechanisms possible (e.g. orbifold)

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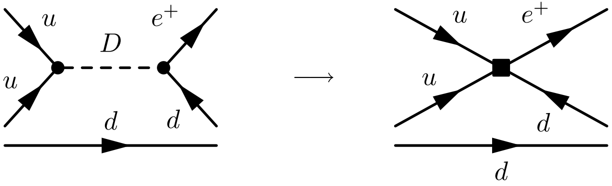
(MS)SM Higgs(es) included in $\mathbf{5} \otimes \overline{\mathbf{5}}$

$$\mathbf{5} = \square : \quad \begin{pmatrix} D \\ D \\ D \\ h^+ \\ h^0 \end{pmatrix} \qquad \overline{\mathbf{5}} = \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \\ \hline \end{array} : \quad \begin{pmatrix} D^c \\ D^c \\ D^c \\ h^- \\ -h^0 \end{pmatrix}$$

$$\mathbf{5} = (\mathbf{3},\mathbf{1})_{-\frac{2}{3}} \oplus (\mathbf{1},\mathbf{2})_1 \qquad \overline{\mathbf{5}} = (\overline{\mathbf{3}},\mathbf{1})_{\frac{2}{3}} \oplus (\mathbf{1},\mathbf{2})_{-1}$$

- D, D^c coloured triplet Higgses with charges $\pm\frac{1}{3}$
- also induces proton decay $\qquad m_H \sim 100\,\text{GeV},\ m_D \sim 10^{16}\,\text{GeV}$
- **Doublet-triplet splitting problem**

Naive estimate of proton lifetime



Effective 4-fermion operator (analogy to muon decay)

$$\mathcal{L}_F = \frac{4G_F}{\sqrt{2}}(\overline{\mu}\gamma_\kappa\nu_\mu)(\overline{\nu}_e\gamma^\kappa e) \qquad \mathcal{L}_{GUT} = \frac{4G_{GUT}}{\sqrt{2}}(\overline{u}\Gamma u)(\overline{e}\Gamma d)$$

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2} \qquad \frac{G_{GUT}}{\sqrt{2}} = \frac{g^2}{8M_{GUT}^2}$$

$$\tau(\mu \to e\nu_\mu\bar{\nu}_\mu) \sim \frac{192\pi^3}{G_F^2 m_\mu^5} \qquad \tau(p \to e^+\pi^0) \sim \frac{192\pi^3}{G_{GUT}^2 m_p^5}$$

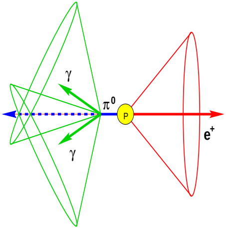
Proton lifetime for $\alpha(M_{GUT}) \sim 1/24$ and $M_{GUT} \sim 2 \times 10^{16}$ GeV:

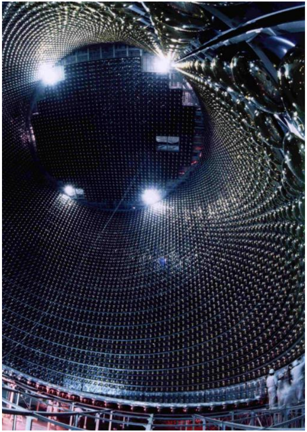
$$\tau(p \to e^+\pi^0) \sim \frac{M_{GUT}^4}{[\alpha(M_{GUT})]^2 m_p^5} \to 10^{31\pm 1} \text{ years}$$

Compare: $\tau_p^{SM} \gtrsim 10^{150}$ years (gravity-induced)

Proton decay crucial for “GUT search”

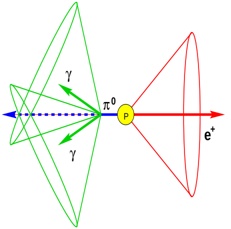
- Tracking calorimeter (SOUDAN) or RICH Cherenkovs
- Super-Kamiokande: 50 kt water RICH
- measure change and time for reconstruction





Proton decay crucial for "GUT search"

- Tracking calorimeter (SOUDAN) or RICH Cherenkovs
- Super-Kamiokande: 50 kt water RICH
- measure change and time for reconstruction



Channel	τ_p (10^{30} years)
$p \to$ invisible	0.21
$p \to e^+\pi^0$	1600
$p \to \mu^+\pi^0$	473
$p \to \nu\pi^+$	25
$p \to \nu K^+$	670
$p \to e^+\eta^0$	312
$p \to \mu^+\eta^0$	126
$p \to e^+\rho^0$	75
$p \to \mu^+\rho^0$	110
$p \to \nu\rho^+$	162
$p \to e^+\omega^0$	1000
$p \to \mu^+\omega^0$	117
$p \to e^+K^0$	150
$p \to \mu^+K^0$	1300
$p \to \nu K^+$	2300
$p \to e^+\gamma$	670
$p \to \mu^+\gamma$	478

New experiments:
HyperK (1 Mt), UNO (650 kt), European project Fréjus (1 Mt)

Precision: 10 years running $\Longrightarrow 10^{34} - 10^{35}$ years

SuperK abuse: Neutrino masses

- Long time no see: no proton decay until now
- Looking for neutrino appearance and disappearance events
- Neutrino oscillations discovered (end of 1990s):

$$P(\nu_i \leftrightarrow \nu_j) \propto \text{products of mixing matrices} \sin^2\left(\frac{m_i^2 - m_j^2}{2E} \Delta L\right)$$

- See-saw mechanism points to a high scale:

  Majorana mass term M_R for ν^c not forbidden by SM symmetries

$$(\nu_L, \nu_R) \begin{pmatrix} 0 & v \\ v & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix} \tag{5}$$

  Neutrino masses: $m_\nu \sim Y_\nu \frac{v^2}{M_R}$ with $M_R \sim 10^{14} - 10^{15}$ GeV

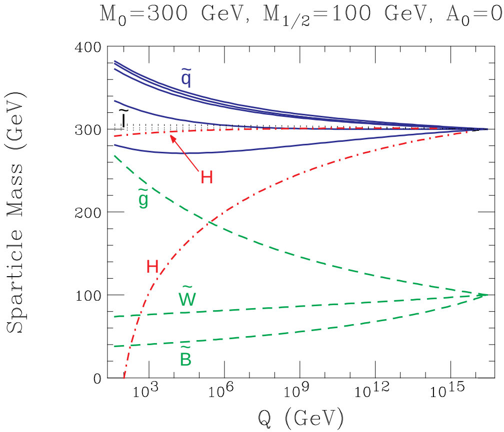
Yukawa coupling unification and all that...

MSSM: negative sign of m_H comes for free provided that ...

– assume GUT scale (as motivated by coupling constant unification)

– take universal input parameters at the GUT scale (see mSUGRA below)

– run down to the electroweak scale with RGEs



Exactly one parameter turns negative: the "μ" in the Higgs potential

But this only works if

$m_t = 150 \ldots 200$ GeV

and $M_{\text{SUSY}} \approx 1$ TeV

Sfermion Systematics

- Off-diagonal element prop. to mass of partner quark ($\tan\beta \equiv v_u/v_d$)

  $\Rightarrow$ mixing important in stop sector (sbottom sector for large $\tan\beta$)

  Taken into account also for stau sector
- Mixing makes (often) $\tilde\tau_1$ lightest slepton (NLSP), $\tilde t_1$ lightest squark
- gauge invariance $\Rightarrow$ relation between $m_{\tilde t_1}, m_{\tilde t_2}, \theta_{\tilde t}, m_{\tilde b_1}, m_{\tilde b_2}, \theta_{\tilde b}$
- No right-handed sneutrinos (GUT-scale ?)
- Characteristics from renormalization group equations:

$$
\begin{aligned}
m^2_{\tilde d_L} &= m_0^2 + K_3 + K_2 + \tfrac{1}{36} K_1 + \Delta D_{\tilde d_L} \\
m^2_{\tilde u_L} &= m_0^2 + K_3 + K_2 + \tfrac{1}{36} K_1 + \Delta D_{\tilde u_L} \\
m^2_{\tilde u_R} &= m_0^2 + K_3 + \tfrac{4}{9} K_1 + \Delta D_{\tilde u_R} \\
m^2_{\tilde d_R} &= m_0^2 + K_3 + \tfrac{1}{9} K_1 + \Delta D_{\tilde d_R} \\
m^2_{\tilde e_L} &= m_0^2 + K_2 + \tfrac{1}{4} K_1 + \Delta D_{\tilde e_L} \\
m^2_{\tilde \nu} &= m_0^2 + K_2 + \tfrac{1}{4} K_1 + \Delta D_{\tilde \nu} \\
m^2_{\tilde e_L} &= m_0^2 + K_1 + \Delta D_{\tilde e_R}
\end{aligned}
$$

$$K_1 \approx 0.15\, m_{1/2}^2$$

$$K_2 \approx 0.5\, m_{1/2}^2$$

$$K_3 \approx (4.5 - 6.5)\, m_{1/2}^2$$

The gluino

Color octet fermion $\Rightarrow$ cannot mix with any other MSSM state

Pure QCD interactions

GUT-inspired models imply at any RG scale (up to tiny 2-loop corrections):

$$M_3 = \frac{\alpha_s}{\alpha} \sin^2 \theta_W \, M_2 = \frac{3}{5} \frac{\alpha_s}{\alpha} \cos^2 \theta_W \, M_1$$

Translates to mass relations near TeV scale:

$$\boxed{M_3 : M_2 : M_1 \approx 6 : 2 : 1}$$

Crucial for SUSY detection:

- Proof its fermion nature
- Proof its Majorana nature
- Proof its octet nature

The Question of Anomalies

- Anomaly: violation of a symmetry of the classical Lagrangian by quantum effects

  Global symmetries: most welcome $\pi \to \gamma\gamma$, $m_{\eta'}$

  Local symmetry: a catastratophe (breakdown of very definition of the theory!)

  - SM/MSSM anomaly-free accidentally: lepton and quark contributions cancel (in each gen.)
  - Ameliorated slightly in $SU(5)$: $\overline{\mathbf{5}}$ and $\mathbf{10}$ cancel

Why chiral exotics?

Unification verification only with megatons? What about colliders?

- SPA: super precision accurately
- Look for chiral exotics
- Physics beyond MSSM provides handle to GUT scale

μ problem

- NMSSM trick
- Singlet superfield with TeV-scale VEV

Doublet-triplet splitting problem, Longevity of the proton

- Try to keep D, D^c superfields at TeV scale
- Need mechanism to prevent rapid proton decay
- Need to rearrange running for unification

Flavour problem

- Flavour might help protecting the proton

Sketch of a model

Superpotential: $\mathcal{W} = \mathcal{W}_{\text{MSSM}} + \mathcal{W}_D + \mathcal{W}_N$

$$\begin{aligned} \mathcal{W}_{\text{MSSM}} &= Y^u u^c Q H_u + Y^d d^c Q H_d + Y^e e^c L H_d \\ \mathcal{W}_D &= Y^D D u^c e^c + Y^{D^c} D^c Q L \\ \mathcal{W}_S &= Y^{S_H} S H_u H_d + Y^{S_D} S D D^c \end{aligned}$$

$U(1)_S$ symmetry

- gauged at high energies, radiative breaking of global left-over $U(1)$
- $\langle 0|S|0\rangle$ generates μ_H and μ_D

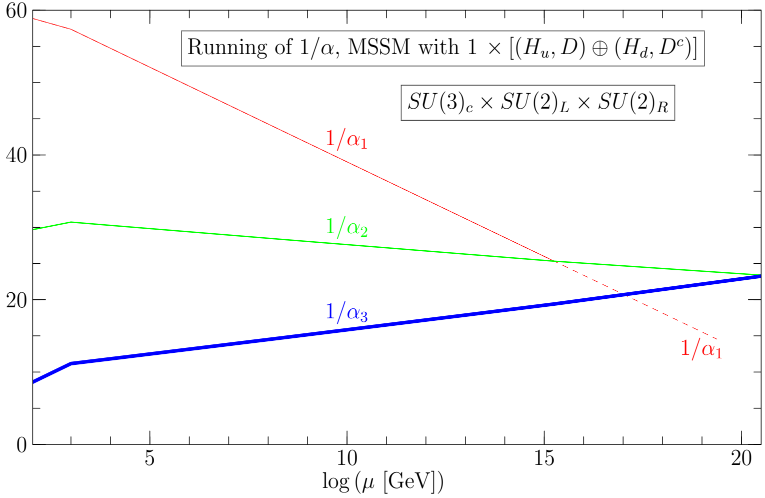
See-saw mechanism ν^c at $\sim M_\nu \sim 10^{14} - 10^{15}$ GeV

$$m_\nu \sim \frac{v^2}{M_\nu} \sim 10^{-1} - 10^{-2}\,\text{eV}$$

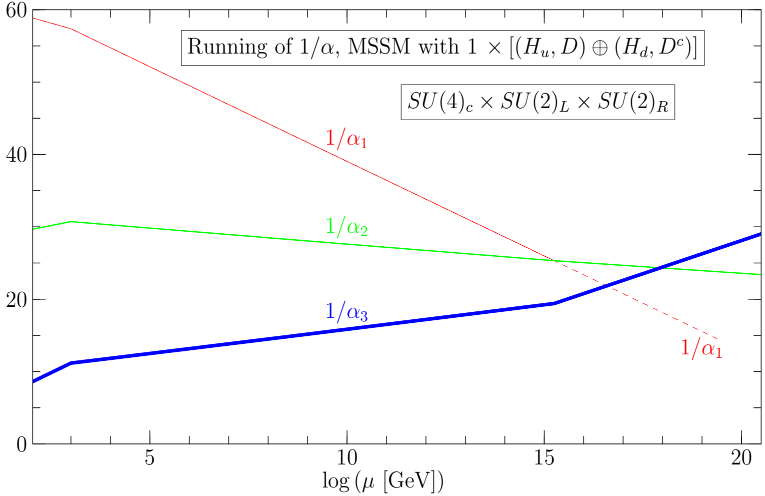
At M_ν left-right symmetric model: $SU(2)_L \times SU(2)$

$$Q_R = \begin{pmatrix} u^c \\ d^c \end{pmatrix}, \qquad L_R = \begin{pmatrix} \nu^c \\ \ell^c \end{pmatrix} \qquad H = \begin{pmatrix} H_u \\ H_d \end{pmatrix} \qquad D \qquad D^c$$

RGE running


Running of $1/\alpha$, MSSM with $1 \times [(H_u, D) \oplus (H_d, D^c)]$
$SU(3)_c \times SU(2)_L \times SU(2)_R$
$1/\alpha_1$
$1/\alpha_2$
$1/\alpha_3$
$1/\alpha_1$
60
40
20
0
5
10
15
20
$\log(\mu$ [GeV])

RGE running



Running of $1/\alpha$, MSSM with $1 \times [(H_u, D) \oplus (H_d, D^c)]$
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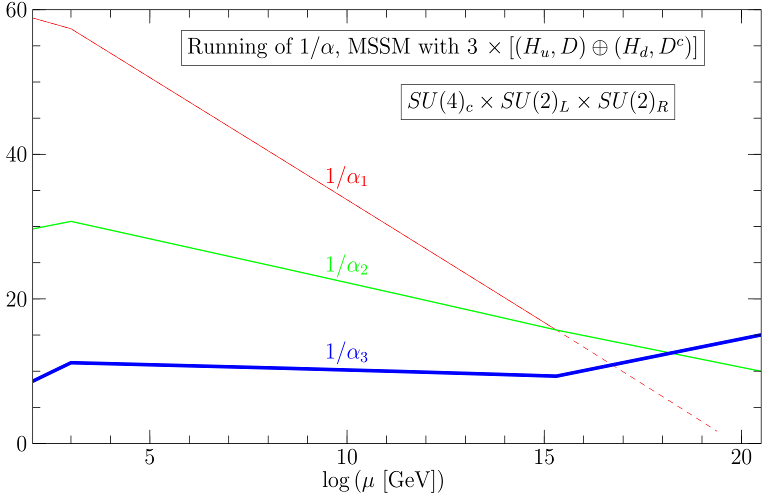

Running of $1/\alpha$, MSSM with $3 \times [(H_u, D) \oplus (H_d, D^c)]$
$SU(4)_c \times SU(2)_L \times SU(2)_R$
$1/\alpha_1$
$1/\alpha_2$
$1/\alpha_3$
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40
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15
20
$\log(\mu\ [\mathrm{GeV}])$

Matter-Higgs unification: E_6

- **Pati-Salam group** $SU(4)_c \times SU(2)_L \times SU(2)_R$

$$\begin{aligned} \mathbf{Q}_L = (Q, L) &= \ (\mathbf{4}, \mathbf{2}, \mathbf{1}) \\ \mathbf{Q}_R = ((u^c, d^c), (\nu^c, \ell^c)) &= \ (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \\ \mathbf{H} = (H_u, H_d) &= \ (\mathbf{1}, \mathbf{2}, \mathbf{2}) \\ \mathbf{D} = (D, D^c) &= \ (\mathbf{6}, \mathbf{1}, \mathbf{1}) \\ \mathbf{S} &= \ (\mathbf{1}, \mathbf{1}, \mathbf{1}) \end{aligned}$$

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$$
\begin{aligned}
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\mathbf{D} = (D, D^c) = &\ (\mathbf{6}, \mathbf{1}, \mathbf{1}) \\
\mathbf{S} = &\ (\mathbf{1}, \mathbf{1}, \mathbf{1})
\end{aligned}
$$

- Be radical: Embed everything in **fundamental rep. of** E_6
  - fundamental $\mathbf{27}$ contains exactly the stuff above
  - adjoint: $\mathbf{78}$
  - Matter and Higgs fields in one big multiplet
  - $\Rightarrow$ 3 generations of S and D
  - $\Rightarrow$ 3 generations of Higgs fields: Higgs (VEV) vs. “unhiggs” (no VEV)
  - Problem of FCNCs: Introduce Z_2 symmetry (H parity)

Flavour Symmetry and proton decay

Assume $SU(3)_F$ **or** $SO(3)_F$ **flavour symmetry**

- Left-right symmetry: $SU(2)_L \times SU(2)_R$, $SU(3)_c$, $SU(3)_F$
- Diquark couplings vanish identically:

$$DQ_LQ_L = \epsilon^{abc}\epsilon_{\alpha\beta\gamma}\epsilon_{jk}D^a_\alpha \left(Q_L\right)^b_{\beta j} \left(Q_L\right)^c_{\gamma k}$$

- Baryon number is symmetry of the superpotential
- $SU(2)_R$ and $SU(3)_F$ breaking spurions?
  symmetry breaking by condensates linear/bilinear in fundamental reps.:
  D, D^c *couple to other quarks only as singlets*
- Integrating out heavy fields: baryon number emerges as low-energy symmetry, flavour symmetry not
- Leptoquark couplings possible

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- Toy model: $E_8 \to E_6 \times SU(3)_F$
  - $\mathbf{248} = \mathbf{27_3} \oplus \overline{\mathbf{27}}_{\overline{\mathbf{3}}} \oplus \mathbf{78_1} \oplus \mathbf{1_8}$
  - flavour-symmetric Kaluza-Klein tower of mirror mmatter $\overline{\mathbf{27}}_{\overline{\mathbf{3}}}$ breaks E_8
  - mirror-Higgs superfields μ term breaks E_6 to PS, breaks flavour

A little bit of Pheno

Next step: Provide a viable low-energy spectrum

Extended MSSM Higgs sector

- relaxed Higgs bounds (light pseudoscalars)
- possibly large invisible decay ratio
- lightest unhiggs: H parity protected dark matter
- dark matter mix: interesting relic abundance (relaxes all neutralino bounds!)
- Pair production of unhiggses/unhiggsinos, cascade decays

(Down-type) **Leptoquarks, Leptoquarkinos**

- 3 generations at TeV scale
- produced in gluon fusion, single production
- final states: $t\tau, b\nu_\tau, \tilde{t}\tau, \ldots$
- if flavor symmetry leaves traces: $gq \to D\ell$ enhanced, decays $t\mu, te$

Extended neutralino sector like in NMSSM

no Z'

Summary

- Grand Unification: Old theoretical idea (35 yrs.)
  All interactions in Nature essentially one (gravity?)
- Experimental hint from LEP (LHC?)
- High-scale recently supported by neutrino masses (see-saw?)
- Prediction: proton decay (lifetime?)
  No experimental evidence: $\tau_p > 10^{33}$ yrs.
  Search will become longer $\equiv$ better: 10^{35} yrs.
- Decay of heavy particles: source of missing CP violation
- Stabilization of large scale difference:
  Strong interactions disfavored (?)
  Supersymmetry (Unification seems perfect)
- "Quasi-natural" breaking chain: $E_8 \to E_6 \to SO(10) \to SU(5) \to G_{SM}$
- Lots of open questions:
  FLAVOR !!!!!
  μ probl., doublet-triplet splitting probl.
  Yukawa coupling pattern
  GUT breaking: Higgs vs. boundary conditions

Some Unification needs time