

The Big Deal with the Little Higgs

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Outline

Hierarchy Problem, Goldstone-Bosons and Little Higgs

- Higgs as Pseudo-Goldstone Boson
- Nambu-Goldstone Bosons
- The Little Higgs mechanism

Examples of Models

Phenomenology

- For example: Littlest Higgs
- Neutrino masses
- Effective Field Theories
- Electroweak Precision Observables
- Direct Searches
- Reconstruction of Little Higgs Models
- Pseudo Axions in LHM
- T parity and Dark Matter

Conclusions



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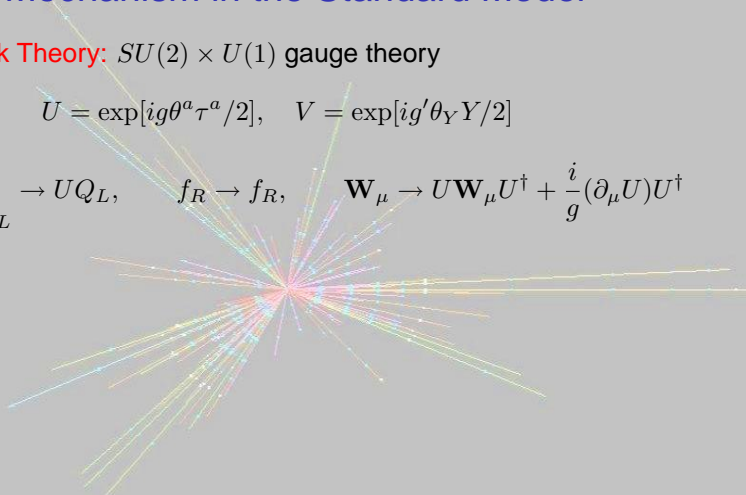


The Higgs Mechanism in the Standard Model

Electroweak Theory: $SU(2) \times U(1)$ gauge theory

$$U = \exp[ig\theta^a \tau^a / 2], \quad V = \exp[ig'\theta_Y Y / 2]$$

$$Q_L \equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \rightarrow U Q_L, \quad f_R \rightarrow f_R, \quad \mathbf{W}_\mu \rightarrow U \mathbf{W}_\mu U^\dagger + \frac{i}{g} (\partial_\mu U) U^\dagger$$



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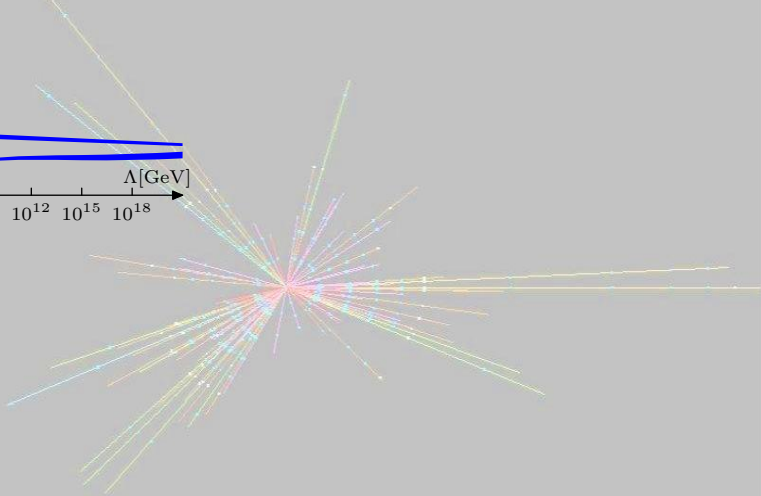
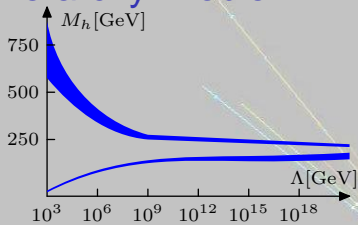
Problem: Mass terms for W, Z and fermions not gauge invariant

- ▶ **Solution:** Introduction of a field which *absorbs the mismatch of transformation laws*: **Higgs field**
- ▶ **Spontaneous symmetry breaking:** Higgs gets a Vacuum Expectation value (VEV):

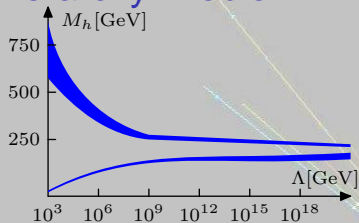
$$\mathcal{V}(\Phi) = -\mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \Rightarrow \Phi \rightarrow \exp[i\pi/v] \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

$$|D_\mu \Phi|^2 \rightarrow \frac{1}{2} M_W^2 W_a^2, \quad -Y_d \bar{Q}_L \Phi d_R \rightarrow -m_d \bar{d}_L d_R$$

Hierarchy Problem

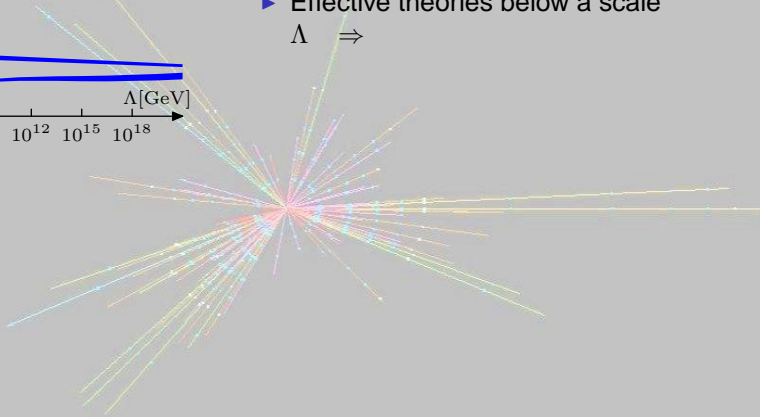


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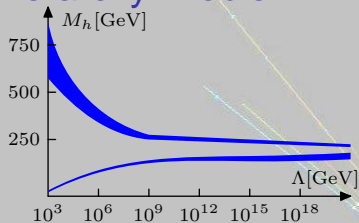


Motivation: Hierarchy Problem

- ▶ Effective theories below a scale $\Lambda \Rightarrow$

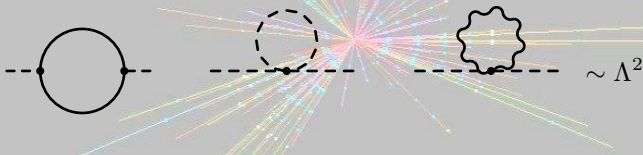


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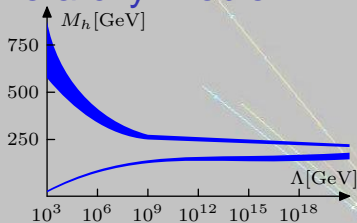


Motivation: **Hierarchy Problem**

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- ▶ Loop integration cut off at order $\sim \Lambda$:



Hierarchy Problem



Motivation: Hierarchy Problem

- ▶ Effective theories below a scale $\Lambda \Rightarrow$
- ▶ Loop integration cut off at order $\sim \Lambda$:



Problem: Naturally, $m_h \sim \mathcal{O}(\Lambda^2)$:

$$m_h^2 = m_0^2 + \Lambda^2 \times (\text{loop factors})$$

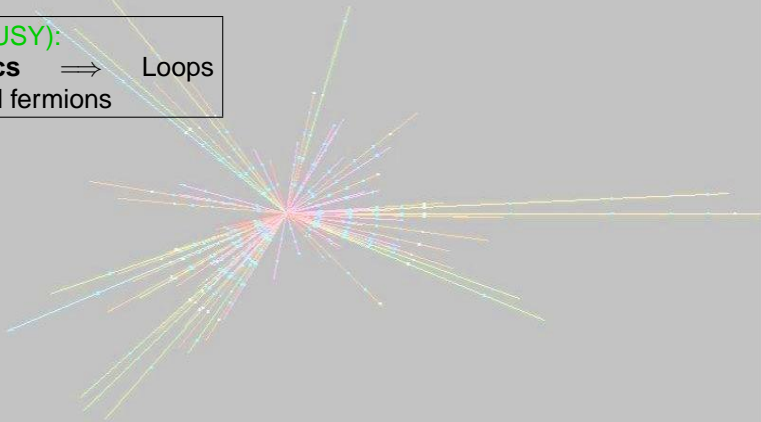
- ◇ *Light Higgs* favoured by EW precision observables ($m_h < 0.5 \text{ TeV}$)

- ▶ $m_h \ll \Lambda \Leftrightarrow$ **Fine-Tuning !?**
- ▶ **Solution:** Mechanism for **eliminating loop contributions**

Higgs as Pseudo-Goldstone Boson

Invent (approximate) symmetry to protect particle mass

Traditional (SUSY):
Spin-Statistics \Rightarrow Loops
of bosons and fermions

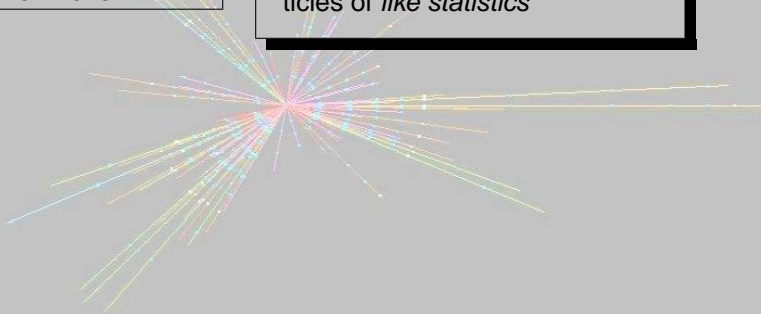


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Old Idea:

Georgi/Pais, 1974; Georgi/Dimopoulos/Kaplan, 1984

Light Higgs as **Pseudo-Goldstone boson** \Leftrightarrow spontaneously broken (approximate) *global* symmetry; non-linear sigma model

■ w/o Fine-Tuning: $v \sim \Lambda/4\pi$

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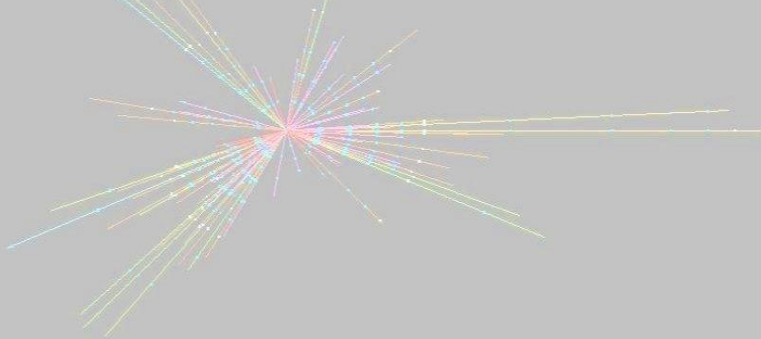
■ w/o Fine-Tuning: $v \sim \Lambda/4\pi$

New Ingredient: Arkani-Hamed/Cohen/Georgi/..., 2001

Collective Symmetry Breaking eliminates quadratic divergences @ 1-loop level \Rightarrow **3-scale model**

The Nambu-Goldstone-Theorem

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$$\pi_i \rightarrow i\theta^a T_{ik}^a \pi_k \quad \Rightarrow \quad \frac{\partial \mathcal{V}}{\partial \pi_i} T_{ij}^a \pi_j = 0 \quad \Rightarrow \quad \underbrace{\frac{\partial^2 \mathcal{V}}{\partial \pi_i \partial \pi_j} \Big|_f}_{=(m^2)_{ij}} T_{jk}^a f_k + \underbrace{\frac{\partial \mathcal{V}}{\partial \pi_j} \Big|_v}_{=0} T_{ji}^a = 0$$

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Nonlinear Realization (Example $SU(3) \rightarrow SU(2)$):

$$\mathcal{V}(\Phi) = (f^2 - (\Phi^\dagger \Phi))^2 \Rightarrow \Phi = \exp \left[\frac{i}{f} \left(\begin{array}{c|c} 0 & \vec{\pi} \\ \hline \vec{\pi}^\dagger & \pi_0 \end{array} \right) \right] \begin{pmatrix} 0 \\ f + \sigma \end{pmatrix} \equiv e^{i\pi} \Phi_0$$

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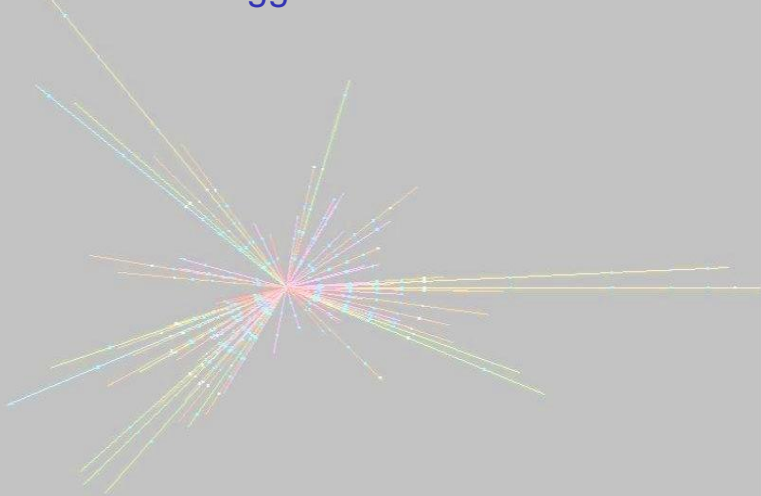
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$$\Phi \rightarrow U_2 \Phi = (U_2 \Phi U_2^\dagger) U_2 \Phi_0 = e^{i(U_2 \pi U_2^\dagger)} \Phi_0 \quad U_2 = \begin{pmatrix} \hat{U}_2 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad \boxed{\vec{\pi} \rightarrow \hat{U}_2 \vec{\pi}, \quad \pi_0 \rightarrow \pi_0} \quad \vec{\pi} \in \text{fundamental } SU(2) \text{ rep.}, \pi_0 \text{ singlet}$$

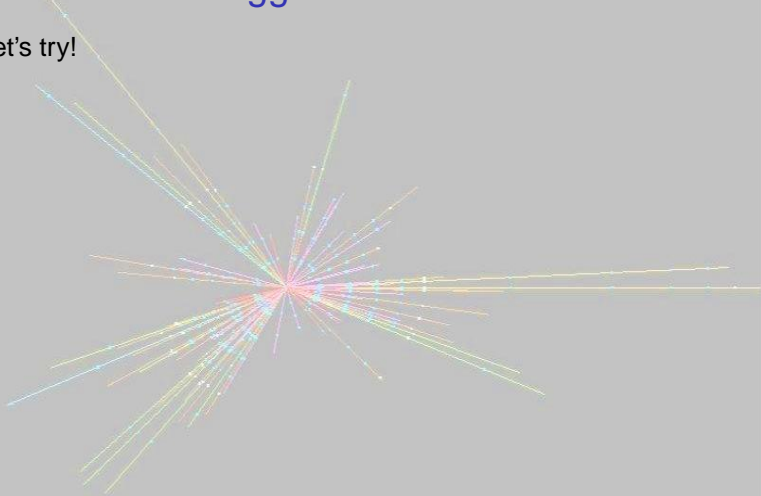
Construction of a Little Higgs model

- ▶ $\vec{\pi} \equiv h$??



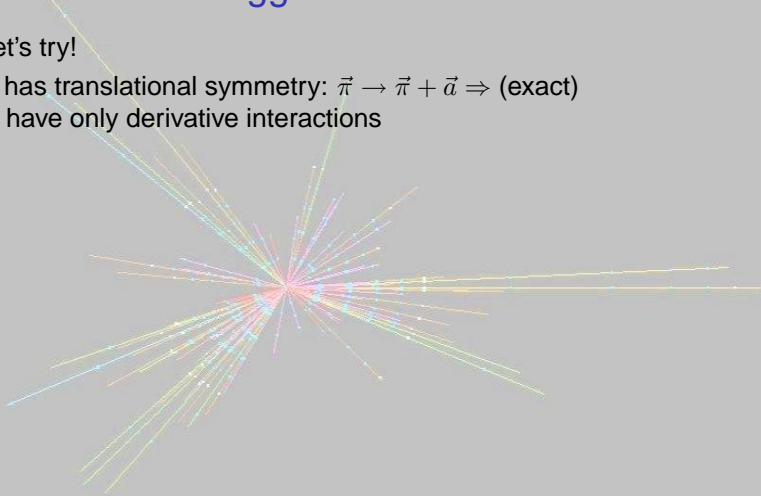
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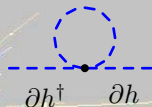
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- ▶ Lagrangian has translational symmetry: $\vec{\pi} \rightarrow \vec{\pi} + \vec{a} \Rightarrow$ (exact)
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- ▶ Gauge and Yukawa interactions?
- ▶ Expanding the kinetic term:

$$f^2 |\partial\Phi|^2 = |\partial h|^2 + \frac{1}{f^2} (h^\dagger h) |\partial h|^2 + \dots$$

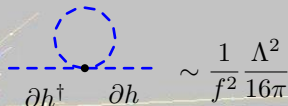


$$\sim \frac{1}{f^2} \frac{\Lambda^2}{16\pi}$$

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→ Theory becomes strongly interacting at $\Lambda = 4\pi f$.

- ▶ Bad news Easy attempts: no potential or quadratic divergences again

Collective Symmetry breaking: Two ways of model building:

- ▶
 1. simple **Higgs representation**, doubled **gauge group**
 2. simple **gauge group**, doubled **Higgs representation**

Prime Example: Simple Group Model

- ▶ enlarged gauge group: $SU(3) \times U(1)$; globally $U(3) \rightarrow U(2)$
- ▶ **Two** nonlinear Φ representations $\mathcal{L} = |D_\mu \Phi_1|^2 + |D_\mu \Phi_2|^2$

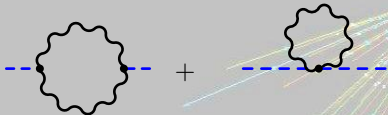
$$\Phi_{1/2} = \exp \left[\pm i \frac{f_{2/1}}{f_{1/2}} \Theta \right] \begin{pmatrix} 0 \\ 0 \\ f_{1/2} \end{pmatrix} \quad \Theta = \frac{1}{\sqrt{f_1^2 + f_2^2}} \begin{pmatrix} \eta & 0 & h^* \\ 0 & \eta & \\ h^T & & \eta \end{pmatrix}$$

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Coleman-Weinberg mechanism: Radiative generation of potential



The diagram shows two Feynman diagrams representing radiative corrections to the potential. The first diagram is a tadpole diagram with a dashed line on the left and a loop on the right. The second diagram is a self-energy diagram with a dashed line on the left and a loop on the right. The diagrams are summed and equated to the radiatively generated potential.


$$= \frac{g^2}{16\pi^2} \Lambda^2 (|\Phi_1|^2 + |\Phi_2|^2) \sim \frac{g^2}{16\pi^2} f^2$$

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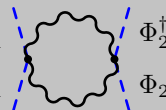
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but:



The diagram shows a tadpole diagram with a dashed line entering from the left and exiting to the right, and a loop of a scalar field. The loop is connected to the external lines by dashed lines, representing the insertion of the scalar field into the loop.

$$= \frac{g^4}{16\pi^2} \log \left(\frac{\Lambda^2}{\mu^2} \right) |\Phi_1^\dagger \Phi_2|^2 \Rightarrow \frac{g^4}{16\pi^2} \log \left(\frac{\Lambda^2}{\mu^2} \right) f^2 (h^\dagger h)$$

Yukawa interactions and heavy Top

Simplest Little Higgs (“ μ Model”)

Schmaltz (2004), Kilian/Rainwater/JR (2004)

Field content ($SU(3)_c \times SU(3)_w \times U(1)_X$ quantum numbers)

$$\Phi_{1,2} : (1, 3)_{-\frac{1}{3}}$$

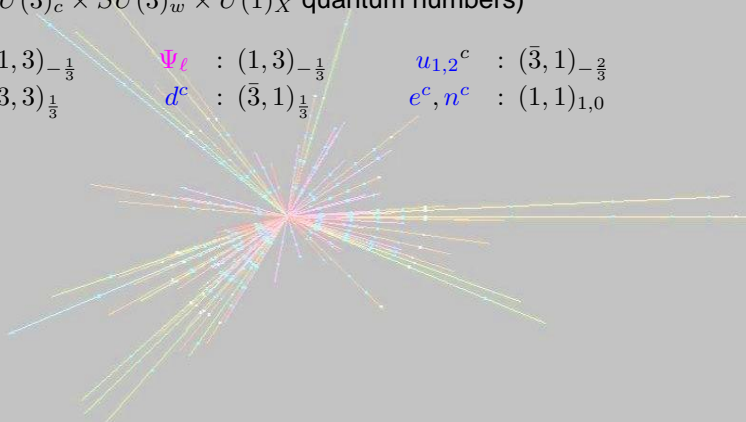
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 \mathcal{L}_{\text{Yuk.}} = & -\lambda_1^u \bar{u}_{1,R} \Phi_1^\dagger \Psi_{T,L} - \lambda_2^u \bar{u}_{2,R} \Phi_2^\dagger \Psi_{T,L} - \frac{\lambda^d}{\Lambda} \epsilon^{ijk} \bar{d}_R^b \Phi_1^i \Phi_2^j \Psi_{T,L}^k \\
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Hypercharge embedding

(remember: $\text{diag}(1, 1, -2)/(2\sqrt{3})$):

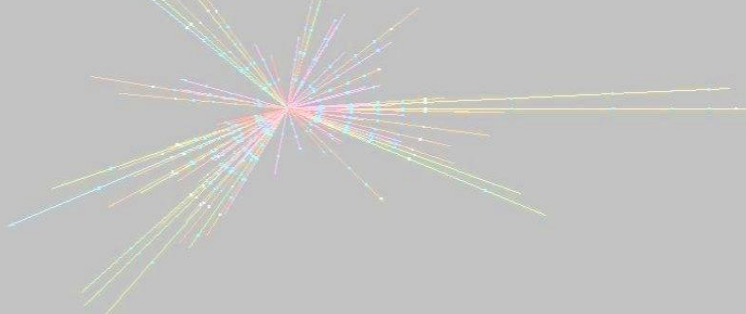
$$Y = X - T^8/\sqrt{3}$$

$$D_\mu \Phi = (\partial_\mu - \frac{1}{3} g_X B_\mu^X \Phi + ig W_\mu^w) \Phi$$

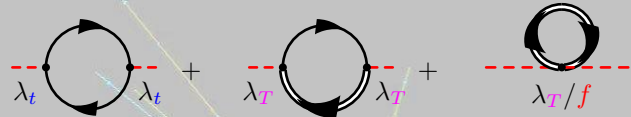


Cancellations of Divergencies in Yukawa sector

$$\lambda_t \text{ (tadpole with fermion loop)} + \lambda_T \text{ (tadpole with scalar loop)} + \lambda_T/f \text{ (tadpole with scalar loop)}$$



Cancellations of Divergencies in Yukawa sector



$$\propto \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k^2 - m_T^2)} \left\{ \lambda_t^2 (k^2 - m_T^2) + k^2 \lambda_T^2 - \frac{m_T}{F} \lambda_T k^2 \right\}$$

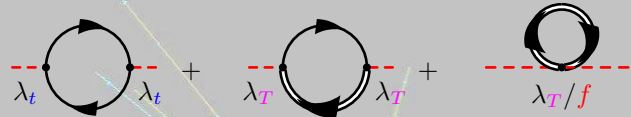
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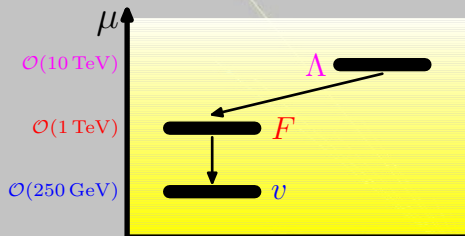


Quadratic divergence cancels

**Collective Symm. break-
ing:** $\lambda_t \propto \lambda_1 \lambda_2$, $\lambda_1 = 0$
or $\lambda_2 = 0 \Rightarrow SU(3) \rightarrow$
 $[SU(3)]^2$

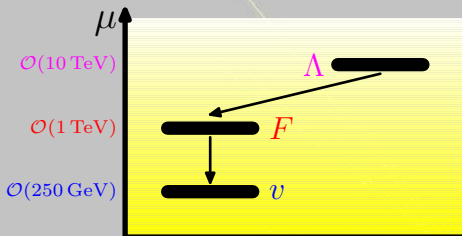
$$\sim \frac{\lambda_1^2 \lambda_2^2}{16\pi^2} \log \left(\frac{\Lambda^2}{\mu^2} \right) |\Phi_1^\dagger \Phi_2|^2$$

Scales and Masses



- ◇ Scale Λ : global SB, new dynamics, UV embedding
- ◇ Scale F : Pseudo-Goldstone bosons, new vector bosons and fermions
- ◇ Scale v : Higgs, W^\pm , Z , l^\pm , ...

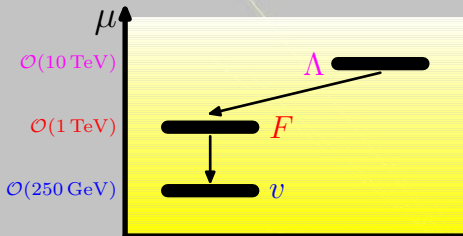
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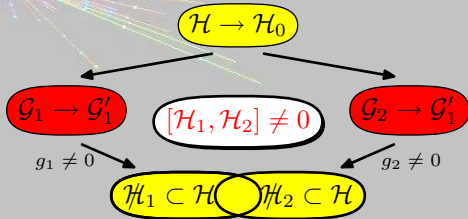
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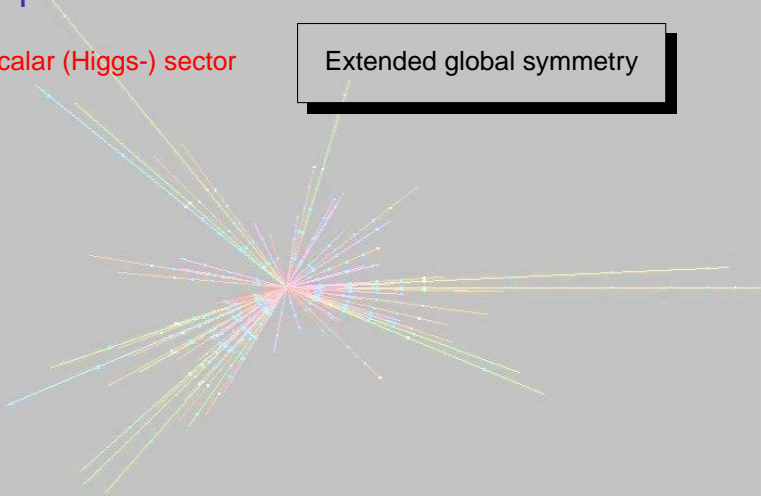


$$M_H \sim g_1 g_2 \Lambda / 16\pi^2$$

Generic properties

- ▶ Extended scalar (Higgs-) sector

Extended global symmetry



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Extended global symmetry

- ▶ **Specific form of the potential:**

$$\mathcal{V}(h, \Phi) = C_1 F^2 \left| \Phi + \frac{i}{F} h \otimes h \right|^2 + C_2 F^2 \left| \Phi - \frac{i}{F} h \otimes h \right|^2$$

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- ▶ Extended Gauge Sector : B', Z', W'^{\pm} (Hypercharge singlets & triplets)

- ▶ Extended top sector: new heavy quarks, t, t' loops $\Rightarrow M_h^2 < 0$
 \Rightarrow EWSB

Outline

Hierarchy Problem, Goldstone-Bosons and Little Higgs

- Higgs as Pseudo-Goldstone Boson
- Nambu-Goldstone Bosons
- The Little Higgs mechanism

Examples of Models

Phenomenology

- For example: Littlest Higgs
- Neutrino masses
- Effective Field Theories
- Electroweak Precision Observables
- Direct Searches
- Reconstruction of Little Higgs Models
- Pseudo Axions in LHM
- T parity and Dark Matter

Conclusions



Little Higgs Models

Plethora of “Little Higgs Models” in 3 categories:

▶ Moose Models

- ▶ Orig. Moose (Arkani-Hamed/Cohen/Georgi, 0105239)
- ▶ Simple Moose (Arkani-Hamed/Cohen/Katz/Nelson/Gregoire/Wacker, 0206020)
- ▶ Linear Moose (Casalbuoni/De Curtis/Dominici, 0405188)

▶ Simple (Goldstone) Representation Models

- ▶ Littlest Higgs (Arkani-Hamed/Cohen/Katz/Nelson, 0206021)
- ▶ Antisymmetric Little Higgs (Low/Skiba/Smith, 0207243)
- ▶ Custodial $SU(2)$ Little Higgs (Chang/Wacker, 0303001)
- ▶ Littlest Custodial Higgs (Chang, 0306034)
- ▶ Little SUSY (Birkedal/Chacko/Gaillard, 0404197)

▶ Simple (Gauge) Group Models

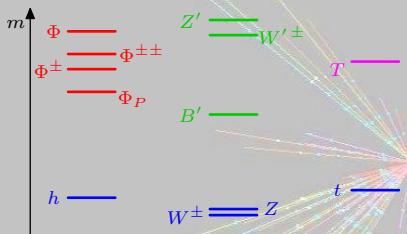
- ▶ Orig. Simple Group Model (Kaplan/Schmaltz, 0302049)
- ▶ Holographic Little Higgs (Contino/Nomura/Pomarol, 0306259)
- ▶ Simplest Little Higgs (Schmaltz, 0407143)
- ▶ Simplest Little SUSY (Roy/Schmaltz, 0509357)
- ▶ Simplest T parity (Kilian/Rainwater/JR/Schmaltz,...)



Varieties of Particle spectra

$$\mathcal{H} = \frac{SU(5)}{SO(5)}, \mathcal{G} = \frac{[SU(2) \times U(1)]^2}{SU(2) \times U(1)}$$

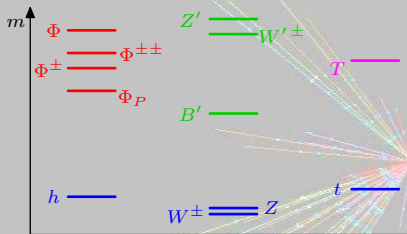
Arkani-Hamed/Cohen/Katz/Nelson, 2002



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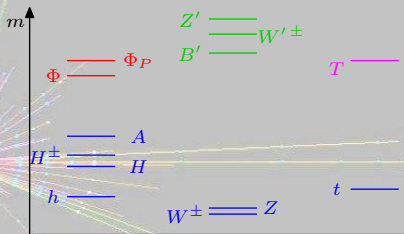
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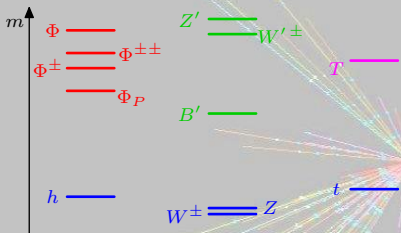
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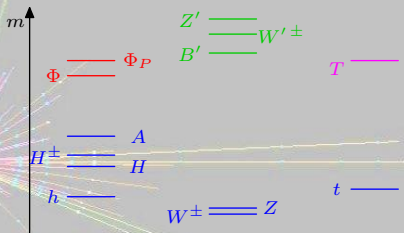
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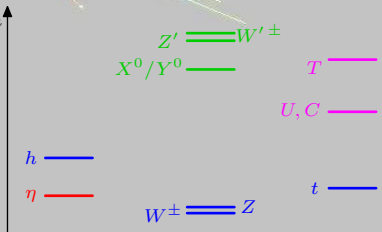
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Low/Skiba/Smith, 2002



$$\mathcal{H} = \frac{[SU(3)]^2}{[SU(2)]^2}, \mathcal{G} = \frac{SU(3) \times U(1)}{SU(2) \times U(1)}$$

Schmaltz, 2004

 \Rightarrow


$$[SU(4)]^4 \rightarrow [SU(3)]^4$$

Kaplan/Schmaltz, 2003

2HDM, $h_{1/2}$, $\Phi'_{1,2,3}$, $\Phi'_{P 1,2,3}$,
 $Z'_{1,\dots,8}$, $W'_{1,2}$, q' , l'

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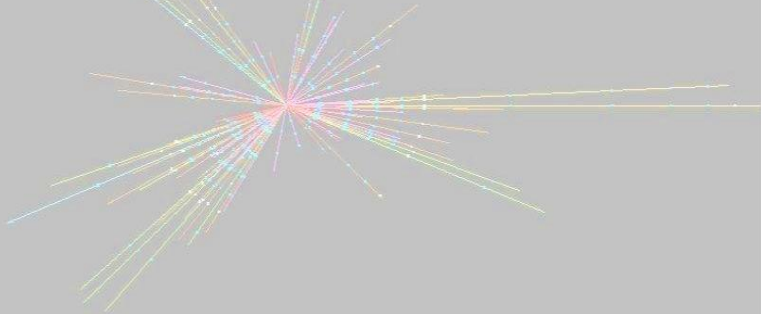


The Littlest Higgs Model Setup

Symmetry breaking:

$$\begin{array}{lll} SU(5) & \rightarrow & SO(5) \quad (\text{global}) \\ [SU(2) \times U(1)]^2 & \rightarrow & SU(2)_L \times U(1)_Y \quad (\text{local}) \end{array}$$

1 heavy triplet X_μ ,
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The unbroken Lagrangian:

$$\mathcal{L} = \mathcal{L}_0^{(3)} + \mathcal{L}_0^{(1)} + \mathcal{L}_0^G.$$

$$\mathcal{L}_0^{(3)} = -\frac{1}{2g_1^2} \text{Tr} \mathbf{A}_{1\mu\nu} \mathbf{A}_1^{\mu\nu} - \frac{1}{2g_2^2} \text{Tr} \mathbf{A}_{2\mu\nu} \mathbf{A}_2^{\mu\nu} - 2 \text{tr} A_1^\mu J^{(3)}{}_\mu$$

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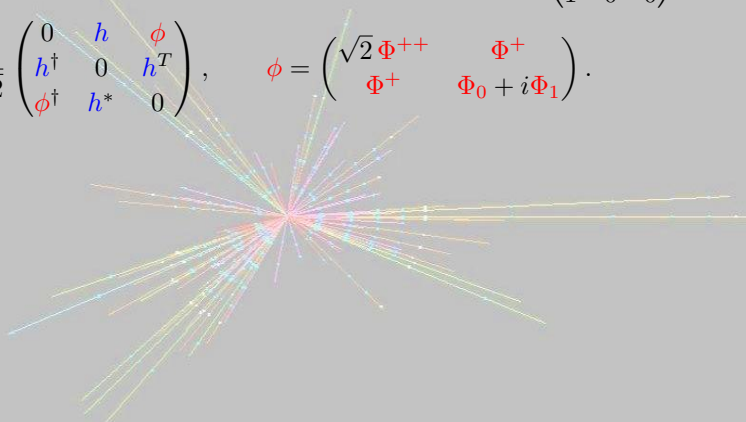
Triplet current: $J^{(3)}_\mu = J_\mu^{(3),a} \frac{\tau^a}{2}$

Singlet current: $J^{(1)}_\mu$

Couplings not unique, but: (Anomaly cancellation!)

$$\mathcal{L}_0^G = \frac{F^2}{8} \text{Tr}(D_\mu \Xi)(D^\mu \Xi)^*, \quad \Xi = \left(\exp \frac{2i}{F} \Pi \right) \Xi_0, \quad \Xi_0 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\Pi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & h & \phi \\ h^\dagger & 0 & h^T \\ \phi^\dagger & h^* & 0 \end{pmatrix}, \quad \phi = \begin{pmatrix} \sqrt{2} \Phi^{++} & \Phi^+ \\ \Phi^+ & \Phi_0 + i\Phi_1 \end{pmatrix}.$$



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Chiral fields: $\mathbf{Q}_R: b_R, t_R, T_R$, and $\mathbf{Q}_L: q_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}, T_L$,

$$\mathcal{L}_t = \sum_Q \bar{Q} i \not{D} Q + \mathcal{L}_Y - \lambda_2 F (\bar{T}_L T_R + \text{h.c.}).$$

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Use a global $SU(3)$ subsymmetry

$$\chi_L = \begin{pmatrix} i\tau^2 T_L & iq_L & 0 \\ -iq_L^T & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{L}_Y = \lambda_1 F \bar{t}_R \text{Tr} [\Xi^* (iT_2^2) \Xi \hat{\chi}_L] + \text{h.c.}$$

Neutrino masses

Kilian/JR, 2003; del Aguila et al., 2004; Han/Logan/Wang, 2005

- ★ *Naturalness does not require cancellation mechanism for light fermions*

Lepton-number violating interactions can generate **neutrino masses**
(due to presence of triplet scalars)

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Caveat: m_ν too large compared to observations

$\Rightarrow g_N$ small, e.g. F/Λ' , where Λ' : scale of lepton number breaking

Heavy Vector Fields

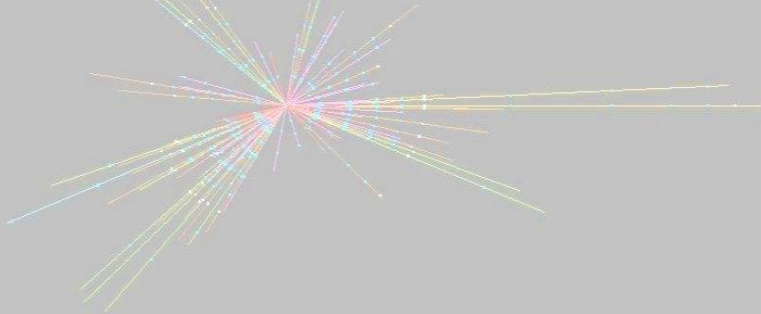
Mixing of the **gauge fields**:

$$A_1^\mu = W^\mu + g_X c^2 X^\mu,$$

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Expand the Goldstone Lagrangian

$$\begin{aligned} \mathcal{L}_0^G = & M_X^2 \operatorname{tr} X \cdot X + g_X \frac{c^2 - s^2}{2} \operatorname{tr}[X \cdot V^{(3)}] + \frac{1}{2} M_Y^2 Y \cdot Y + g_Y \frac{c'^2 - s'^2}{4} Y \cdot V^{(1)} \\ & + \frac{1}{2} \operatorname{tr}(D_\mu \phi)^\dagger (D^\mu \phi) + (D_\mu h)^\dagger (D^\mu h) - \frac{1}{6F^2} \operatorname{tr}[V^{(3)} \cdot V^{(3)}] + \dots, \end{aligned}$$

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Heavy vector masses:

$$M_X = g_X F/2$$

$$M_Y = g_Y F/(2\sqrt{5})$$

Higgs current

$$V_\mu = i [h(D_\mu h)^\dagger - (D_\mu h)h^\dagger]$$

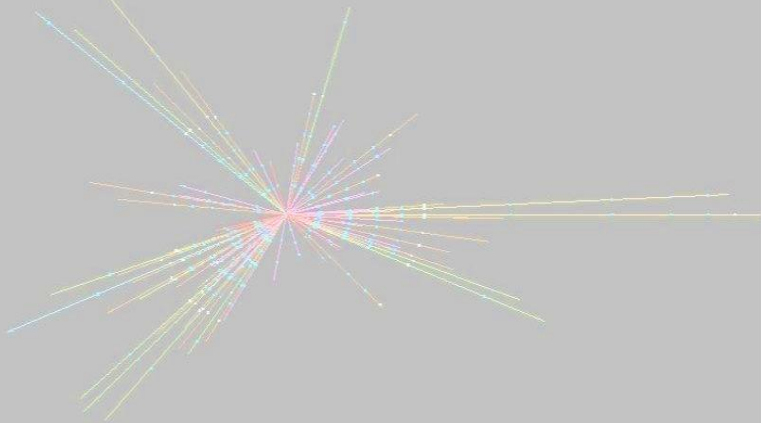
$$\mathbf{1}_0 : V^{(1)} = \operatorname{tr} V, \quad \mathbf{3}_0 : V^{(3)} = V - \frac{1}{2} \operatorname{tr} V.$$



Effective Field Theories



How to *clearly* separate effects of **heavy degrees of freedom**?



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Toy model: Two interacting scalar fields φ, Φ

$$\mathcal{Z}[j, J] = \int \mathcal{D}[\Phi] \mathcal{D}[\varphi] \exp \left[i \int dx \left(\frac{1}{2} (\partial\varphi)^2 - \frac{1}{2} \Phi (\square + M^2) \Phi - \lambda \varphi^2 \Phi - \dots + J\Phi + j\varphi \right) \right]$$

Low-energy effective theory \Rightarrow integrating out **heavy degrees of freedom (DOF)** in path integrals, set up **Power Counting**

Effective Field Theories



How to *clearly* separate effects of **heavy degrees of freedom**?

Toy model: Two interacting scalar fields φ, Φ

$$\mathcal{Z}[j, J] = \int \mathcal{D}[\Phi] \mathcal{D}[\varphi] \exp \left[i \int dx \left(\frac{1}{2} (\partial\varphi)^2 - \frac{1}{2} \Phi (\square + M^2) \Phi - \lambda \varphi^2 \Phi - \dots + J\Phi + j\varphi \right) \right]$$

Low-energy effective theory \Rightarrow integrating out **heavy degrees of freedom (DOF)** in path integrals, set up **Power Counting**

Completing the square:

$$\Phi' = \Phi + \frac{\lambda}{M^2} \left(1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2 \Rightarrow$$

$$\frac{1}{2} (\partial\Phi)^2 - \frac{1}{2} M^2 \Phi^2 - \lambda \varphi^2 \Phi = -\frac{1}{2} \Phi' (M^2 + \partial^2) \Phi' + \frac{\lambda^2}{2M^2} \varphi^2 \left(1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2.$$

Integrating out

Integrating out the X and Y vector fields \Rightarrow

$$\mathcal{L}^{(1)} + \mathcal{L}^{(3)} = \mathcal{L}_{g,gf}^{\text{EW}} + f_{JJ}^{(3)} \text{tr}[J^{(3)} \cdot J^{(3)}] + f_{VJ}^{(3)} \text{tr}[V^{(3)} \cdot J^{(3)}] + f_{VV}^{(1)} \text{tr}[V^{(3)} \cdot V^{(3)}] \\ + f_{JJ}^{(1)} J^{(1)} \cdot J^{(1)} + f_{VJ}^{(1)} V^{(1)} \cdot J^{(1)} + f_{VV}^{(1)} V^{(1)} \cdot V^{(1)}$$

In the Littlest Higgs e.g.

$$f_{VV}^{(3)} = -\frac{1}{6F^2} \left(1 + \frac{3}{2}(c^2 - s^2)^2 \right)$$

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Coleman-Weinberg potential of the scalar fields @ 1-loop:

$$\mathcal{L}_0^{CW} = -\frac{1}{2} M_\phi^2 \text{tr}[\phi\phi^\dagger] + \mu^2 (h^\dagger h) - \lambda_4 (h^\dagger h)^2 - \lambda_{2\phi i} (h^\dagger \phi h^* - h^T \phi^\dagger h) \\ - \lambda_{2\phi\phi} \text{tr}[(\phi\phi^\dagger)(hh^\dagger)] - \lambda_{4\phi i} (h^\dagger h) (h^\dagger \phi h^* - h^T \phi^\dagger h) - \lambda_6 (h^\dagger h)^3$$

Sensitive to UV completion, dimensionless parameters k and k' . EWSB \Rightarrow
Constraints on k, k'

Integrate out the heavy scalar (**Power counting!**)

$$\phi' = \phi - \frac{2i\lambda_{2\phi}}{M_\phi^2} \left(1 + \frac{D^2}{M_\phi^2} + \frac{2\lambda_{2\phi\phi}}{M_\phi^2} hh^\dagger \right)^{-1} \times \left(1 + \frac{\lambda_{4\phi}}{\lambda_{2\phi}} h^\dagger h \right) hh^T$$

Higgs mass up to order v^4/F^2

$$m_H^2 = 2\lambda_4^{\text{eff}} v^2$$

$$= -2v^2 \left(\frac{e^2}{s_w^2 c^2} + \frac{e^2}{c_w^2 c'^2} \right) k \cdot \frac{\left(\frac{e^2}{s_w^2 s^2} + \frac{e^2}{c_w^2 s'^2} \right) k + \frac{\lambda_t^2}{c_t^2} k'}{\left(\frac{e^2}{s_w^2 s^2 c^2} + \frac{e^2}{c_w^2 s'^2 c'^2} \right) k + \frac{\lambda_t^2}{c_t^2} k'}$$

(Remember $\mu^2 = m_H^2/2$)

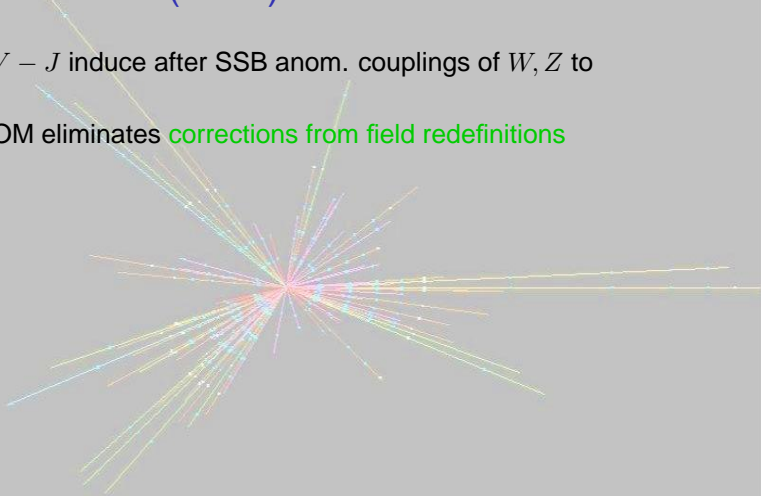
EWSB

$$\lambda_4^{\text{eff}} > 0 \Rightarrow$$

$$\frac{\lambda_{2\phi}^2}{M_\phi^4} < \frac{1}{8F^2}$$

Equations of Motion (EOM)

- ▶ Couplings $V - J$ induce after SSB anom. couplings of W, Z to fermions
- ▶ Applying EOM eliminates **corrections from field redefinitions**



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Custodial- $SU(2)$ Conserving Terms

$$0 = \text{tr}[V^{(3)}_{\mu} \frac{\delta \mathcal{L}}{\delta W_{\mu}}] = \text{tr}[V^{(3)} \cdot V^{(3)}] - \frac{2}{g^2} \text{tr}[V^{(3)}_{\mu} D_{\nu} W^{\mu\nu}] - 2 \text{tr}[V^{(3)} \cdot J^{(3)}]$$

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$$\mathcal{O}_{VW} = \text{tr} V^{(3)}_{\mu\nu} W^{\mu\nu}, \quad \mathcal{O}_{JJ}^{(3)} = \text{tr} J^{(3)} \cdot J^{(3)}, \quad \mathcal{O}_{VV}^{(3)} = \text{tr} V^{(3)} \cdot V^{(3)}$$

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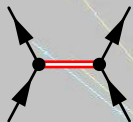
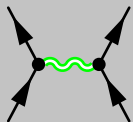
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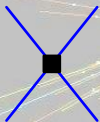
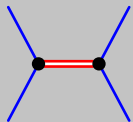
Custodial- $SU(2)$ Violating Terms

$$\mathcal{L}^{(1)} = \mathcal{L}_{g,gf}^{\text{EW}} + f_{JJ}^{(1)} \mathcal{O}_{JJ}^{(1)} + f_{VB} \mathcal{O}_{VB} + f_{VV}^{(1)} \mathcal{O}_{VV}^{(1)}$$

Effective Dim. 6 Operators

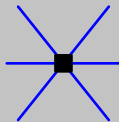
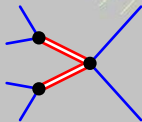
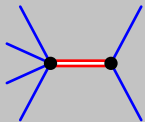


$$\mathcal{O}_{JJ}^{(I)} = \frac{1}{F^2} \text{tr}[J^{(I)} \cdot J^{(I)}]$$

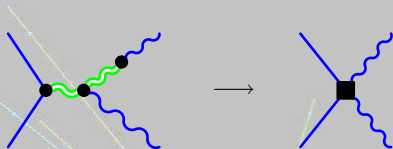


$$\mathcal{O}'_{h,1} = \frac{1}{F^2} ((Dh)^\dagger h) \cdot (h^\dagger (Dh)) - \frac{v^2}{2} |Dh|^2$$

$$\mathcal{O}'_{hh} = \frac{1}{F^2} (h^\dagger h - v^2/2) (Dh)^\dagger \cdot (Dh)$$



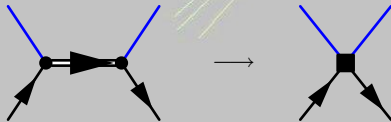
$$\mathcal{O}'_{h,3} = \frac{1}{F^2} \frac{1}{3} (h^\dagger h - v^2/2)^3$$



$$\mathcal{O}'_{WW} = -\frac{1}{F^2} \frac{1}{2} (h^\dagger h - v^2/2) \text{tr} W_{\mu\nu} W^{\mu\nu}$$

$$\mathcal{O}_B = \frac{1}{F^2} \frac{i}{2} (D_\mu h)^\dagger (D_\nu h) B^{\mu\nu}$$

$$\mathcal{O}'_{BB} = -\frac{1}{F^2} \frac{1}{4} (h^\dagger h - v^2/2) B_{\mu\nu} B^{\mu\nu}$$



$$\mathcal{O}_{Vq} = \frac{1}{F^2} \bar{q} h (\not{D} h) q$$

Oblique Corrections: S, T, U

◇ All low-energy effects order v^2/F^2

◇ Low-energy observables parameterized by $\Delta S, \Delta T$, 2 parameters for **contact interactions** (no ΔU here)

$\mathcal{O}_{VW}, \mathcal{O}_{VB} \Rightarrow$ Change in gauge couplings g and g'

$$g = \frac{e}{s_w} [1 + M_W^2 (f_{VW} + 2f_{VB})]$$

$$g' = \frac{e}{c_w} [1 + [M_Z^2 - M_W^2] (f_{VW} + 2f_{VB})]$$

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S parameter ($\mathcal{O}_{VW}, \mathcal{O}_{VB}$)

$$\Delta S = 8\pi v^2 (f_{VW} + 2f_{VB})$$

$SU(2)_c$ -violating sector ($\mathcal{O}'_{h,1}$)

$$\alpha \Delta T = \Delta M_W^2 / M_W^2 = -2v^2 f_{VV}^{(1)} - \frac{2v^2 \lambda_{2\phi}^2}{M_\phi^4}$$

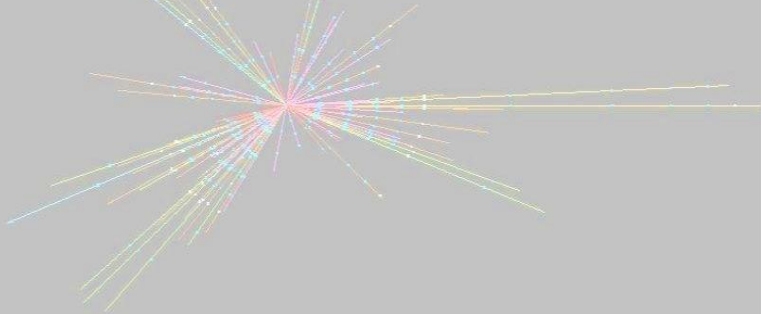
Shift in physical **vector masses**:

$$M_W^2 = \left(\frac{ev}{2s_w} \right)^2 (1 + x)$$

$$x = \alpha \left(\Delta S / (4s_w^2) + \Delta T \right)$$

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Four-Fermion Interactions

Very low energies \Rightarrow Fermi theory

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G_F - M_Z - α **scheme** (muon decay, LEP I, Bhabha), define the parameters \hat{v}_0 and \hat{s}_0 by

$$\hat{v}_0 = (\sqrt{2} G_F)^{-1/2} \quad \text{and} \quad M_Z = \frac{e \hat{v}_0}{2 \hat{s}_0 \hat{c}_0} \quad \Rightarrow$$

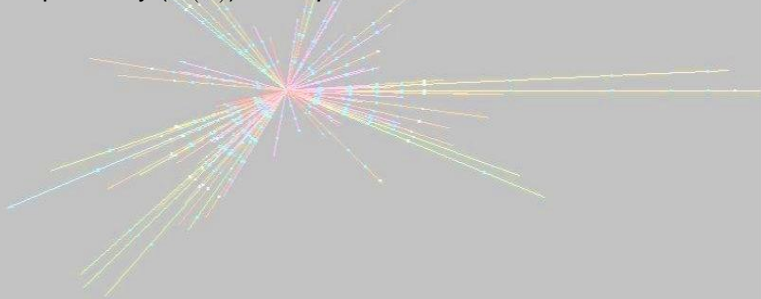
$$s_w^2 = \hat{s}_0^2 \left(1 + \frac{\hat{c}_0^2}{\hat{c}_0^2 - \hat{s}_0^2} (y+z) \right)$$

$$c_w^2 = \hat{c}_0^2 \left(1 - \frac{\hat{s}_0^2}{\hat{c}_0^2 - \hat{s}_0^2} (y+z) \right)$$

Constraints on LHM

Constraints from **contact IA**: $(f_{JJ}^{(3)}, f_{JJ}^{(1)}) \quad c^2 \lesssim F/4.5 \text{ TeV} \quad c'^2 \lesssim F/10 \text{ TeV}$

◇ **Constraints evaded** $\iff c, c' \ll 1$
 B', Z', W', \pm superheavy ($\mathcal{O}(\Lambda)$) *decouple from fermions*



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$\Delta S, \Delta T$ in the **Littlest Higgs model**, violation of **Custodial SU(2)**: Csáki et al., 2002; Hewett et al., 2002; Han et al., 2003; Kilian/JR, 2003

◇ **Mixing of (Z, B', Z') and (W^{\pm}, W'^{\pm})**

$$\frac{\Delta S}{8\pi} = - \left[\frac{c^2(c^2 - s^2)}{g^2} + 5 \frac{c'^2(c'^2 - s'^2)}{g'^2} \right] \frac{v^2}{F^2} \rightarrow 0 \quad \alpha \Delta T \rightarrow \frac{5}{4} \frac{v^2}{F^2} - \frac{2v^2 \lambda_{2\phi}^2}{M_\phi^4} \gtrsim \frac{v^2}{F^2}$$

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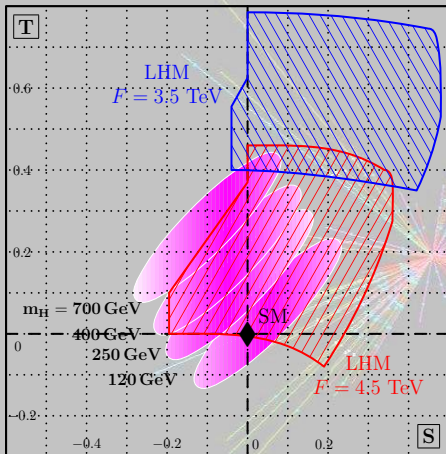
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General models

- ▶ Triplet sector: (almost) identical to Littlest Higgs (ΔS only)
- ▶ More freedom in $U(1)$ sector: (ΔT)

EW Precision Observables



Higgs mass *variable*
(Coleman-Weinberg,
UV completion)

$$\Delta S = \frac{1}{12\pi} \ln \frac{m_H^2}{m_0^2}$$

$$\Delta T = -\frac{3}{16\pi c_w^2} \ln \frac{m_H^2}{m_0^2}$$

Peskin/Takeuchi, 1992; Hagiwara et al., 1992

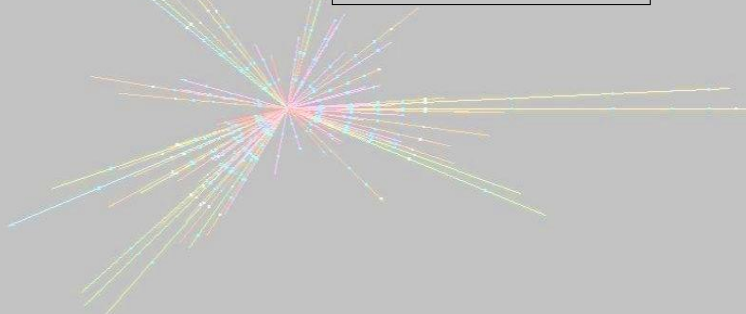
Making the Higgs heavier reduces amount of fine-tuning

Direct Searches

► **Heavy Gauge Bosons:**

Resonance in $e^+e^- \rightarrow f\bar{f}$, Drell-Yan \Rightarrow Tevatron: $M_{B'} \gtrsim 650 \text{ GeV}$

Detection:

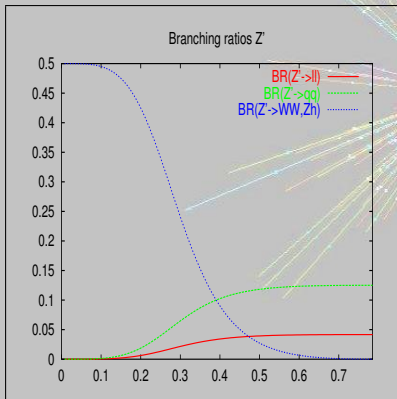


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Total width Z' , fixed $M_{Z'}$:

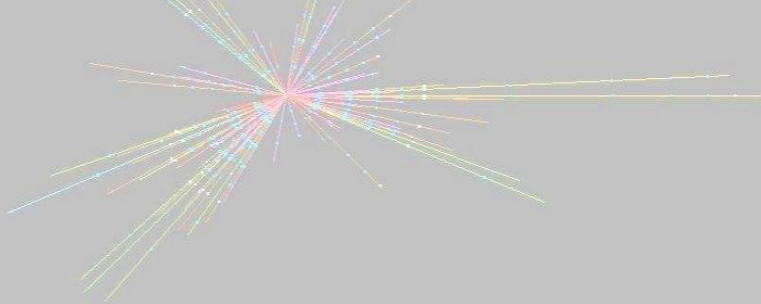
$$\Gamma_{Z'} = \frac{g^2}{96\pi} (\cot^2 2\phi + 24 \tan^2 \phi) M_{Z'}$$

Determination of \mathbf{F} , c , s

$\mathcal{O}(10^2)$ lepton events
@ LHC w. 300 fb^{-1}

Heavy Scalars:

- ◇ Φ_P : $e^+e^-, q\bar{q} \rightarrow \Phi_P h, \Phi_P \rightarrow hZ_L$
- ◇ Φ^\pm : $e^+e^-, q\bar{q} \rightarrow \Phi^+ W_L^-, \Phi^\pm \rightarrow W_L Z_L$
- ◇ $\Phi^{\pm\pm}$: $e^-e^- \rightarrow \nu\nu\Phi^{--}, \Phi^{\pm\pm} \rightarrow W_L^- W_L^-$
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Heavy Quarks:

T production @ LHC: $bq \rightarrow Tq'$

Decay $T \rightarrow W_L^+ b, th, tZ$ Perelstein/Peskin/Pierce, 2003

Total cross section

$$\Gamma_T = m_T \lambda_T^2 / (16\pi)$$

BRs (limit $g \rightarrow 0$):

$$\Gamma(T \rightarrow th) \approx \Gamma(T \rightarrow tZ) \approx \frac{1}{2} \Gamma(T \rightarrow bW^+) \approx \frac{m_T \lambda_T^2}{64\pi}$$

Determination of m_T, λ_T

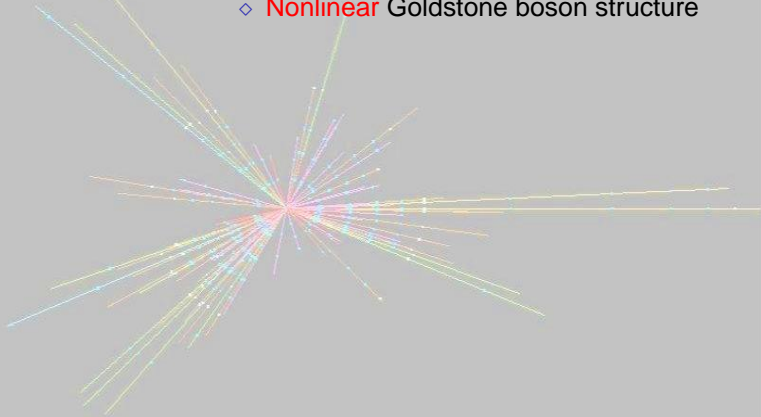
Reconstruction of LHM



*How to unravel the structure
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Kilian/JR, 2003; Han et al., 2005

- ◇ **Symmetry structure**
⇒ Quadr. Div. Cancell.
- ◇ **Nonlinear** Goldstone boson structure



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SIGNALS:

- ▶ Anom. **Triple Gauge Couplings**: WWZ , $WW\gamma$
- ▶ Anom. **Higgs Coupl.**: $H(H)WW$, $H(H)ZZ$
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Vectors:

- ▶ Direct Search (LHC) M_V, F, c, c'
- ▶ ILC: Contact Terms $e^+e^- \rightarrow \ell^+\ell^-, [\nu\bar{\nu}\gamma] \Rightarrow M_{B'} \lesssim 10[5] \text{ TeV}$
- ▶ Higgsstr., WW fusion: $HZff$, $HWff$ angular distr./energy dependence $\Rightarrow f_{VJ}^{(1/3)}$
- ▶ Check from TGC (ILC: per mil precision), **GigaZ** $\Rightarrow f_{JJ}^{(3)}$

Combining \Rightarrow **Determination of all coefficients in the gauge sector**

▶ $\Delta T, f_{VV}^{(1)}, B'$ known $\Rightarrow (\lambda_{2\phi}/M_\phi^2)^2$

▶ Higgsstr., WW fusion \Rightarrow Higgs coupl., $f_{VV}^{(3)}$

▶ Higgs BRs $\Rightarrow f_{VV}^{(1)}, f_{VV}^{(3)}$; (take care of t)

$f_{VV}^{(3)}$ **Goldstone contr.** \Rightarrow **Evidence for nonlinear nature**

▶ HH production $\Rightarrow f_{h,3}$ (difficult!)

Scalars:

Affected by **scalars**
and **vectors**

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▶ $t\bar{t}$ production $\Rightarrow f_{Vq}, v_t, a_t$; accuracy 1-2 %

▶ tbW from t decays, single t production $g_{ttH}/g_{bbH} \Rightarrow anom.$
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▶ tbW from t decays, single t production $g_{ttH}/g_{bbH} \Rightarrow anom.$
Yukawa coupl. $\Rightarrow f_{hq}$, nonlin. structure w. $\sim 2.5\%$ accuracy

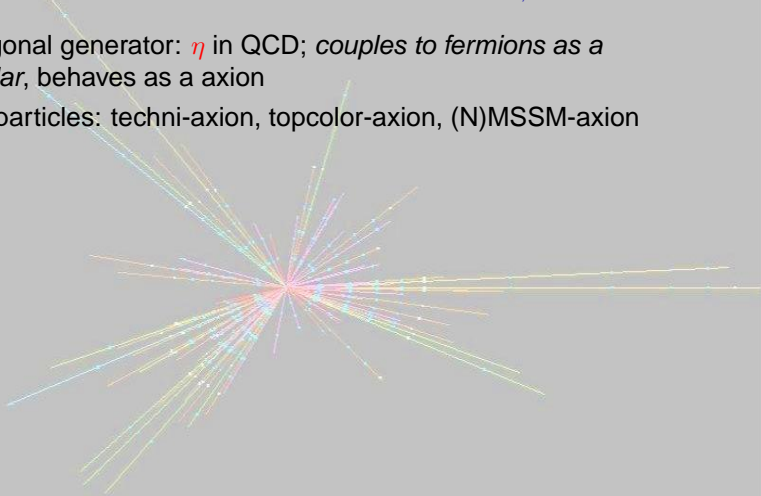
Top:

Include all observables in a combined fit if Little Higgs signals are found (sufficient data from LHC and ILC)

Pseudo Axions in LHM

Kilian/Rainwater/JR, 2004

- ▶ broken diagonal generator: η in QCD; *couples to fermions as a pseudoscalar*, behaves as a axion
- ▶ analogous particles: techni-axion, topcolor-axion, (N)MSSM-axion



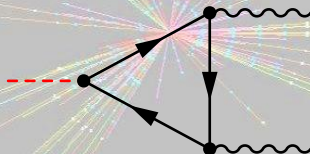
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QCD-(PQ) axion:
$$\mathcal{L}_{\text{Ax.}} = \frac{1}{\Lambda} \frac{\alpha_s}{8\pi^2} A_g \eta G_{\mu\nu} \tilde{G}^{\mu\nu}, \quad \tilde{G}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$$

Anomalous $U(1)_\eta$:



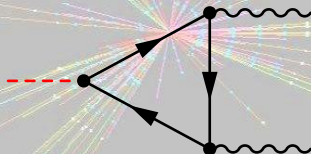
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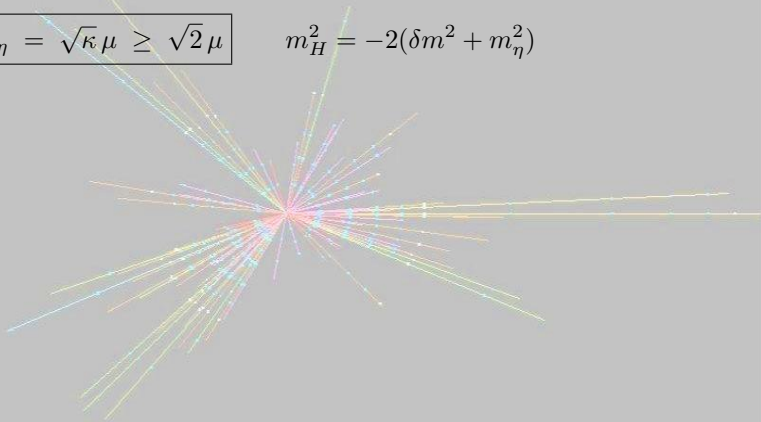


- ▶ **explicit symmetry breaking** $\Rightarrow m_\eta$ and $g_{\eta\gamma\gamma}$ independent \Rightarrow axion bounds *not applicable*
- ▶ *no new hierarchy problem* $\Rightarrow m_\eta \lesssim v \sim 250 \text{ GeV}$
- ▶ η EW singlet, couplings an to SM particles v/F suppressed

Example: Simple Group Model

Scalar Potential: $\mu\Phi_1^\dagger\Phi_2 + \text{h.c.} + \text{Coleman-Weinberg pot.}$:

$$m_\eta = \sqrt{\kappa}\mu \geq \sqrt{2}\mu \quad m_H^2 = -2(\delta m^2 + m_\eta^2)$$

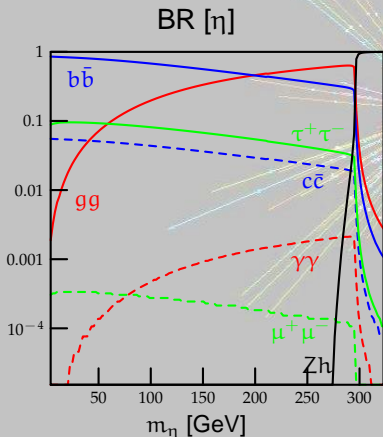


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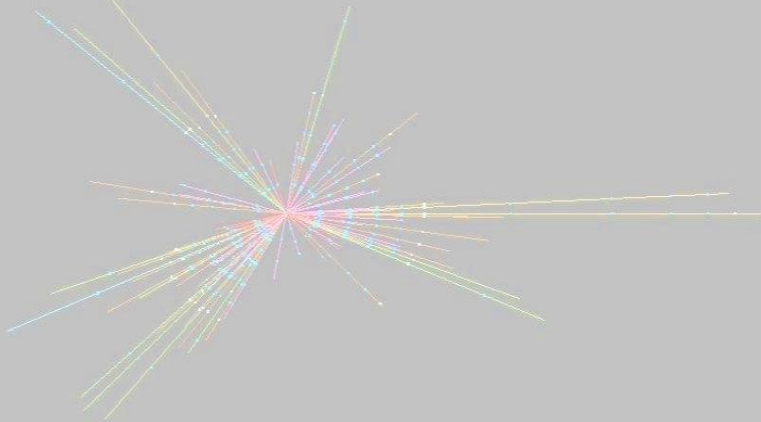
new Higgs decays ($H \rightarrow Z\eta$, $H \rightarrow \eta\eta$)

$\text{BR}(H \rightarrow \eta\eta) < 10^{-4}$ [$\sim 5-10\%$ OSG]

m_H [GeV]	m_η [GeV]	BR($Z\eta$)
341	223	0.1 %
375	193	0.5 %
400	167	0.8 %
422	137	1.0 %
444	96	1.2 %
464	14	1.4 %

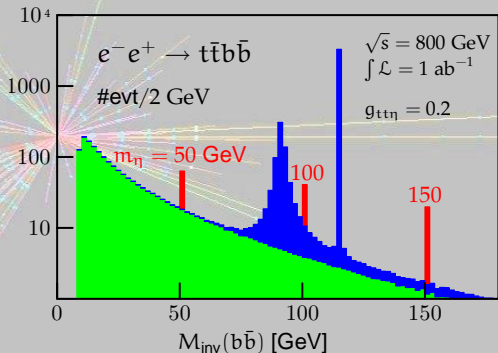
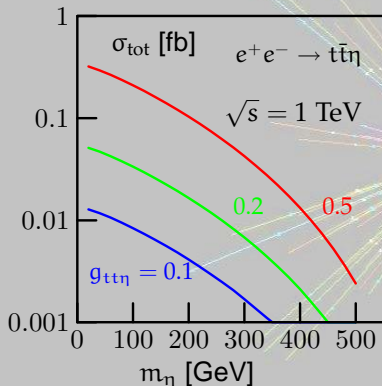
Pseudo Axions at LHC and ILC

- ▶ **LHC: Gluon Fusion** (axial $U(1)_\eta$ anomaly), Peak in diphoton spectrum



Pseudo Axions at LHC and ILC

- ▶ **LHC: Gluon Fusion** (axial $U(1)_\eta$ anomaly), Peak in diphoton spectrum
- ▶ **ILC: associated production** **Problem:** Cross section vs. bkgd.



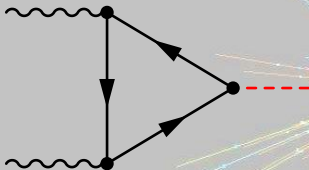
Possibility: $Z^* \rightarrow H\eta$ (analogous to A in 2HDM)

Kilian/JR/Rainwater (in prep.)

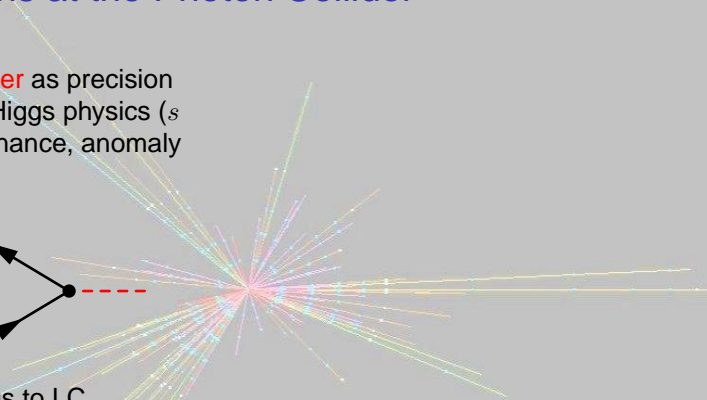


Pseudo Axions at the Photon Collider

- ▶ **Photon Collider** as precision machine for Higgs physics (s channel resonance, anomaly coupling)

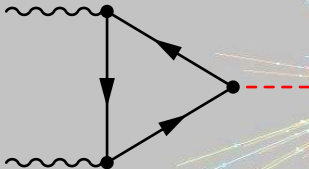


- ▶ S/B analogous to LC
- ▶ η in the μ model with (almost) identical parameters as A in MSSM
(\hookrightarrow Mühlleitner et al. (2001))

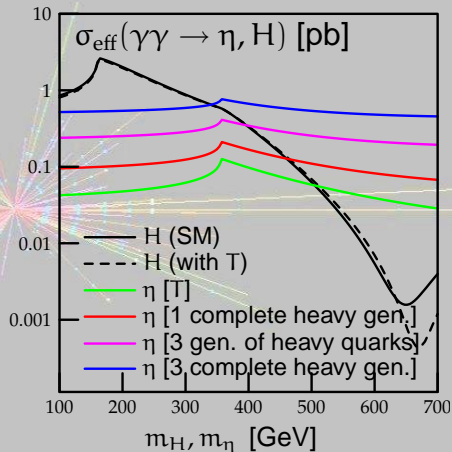


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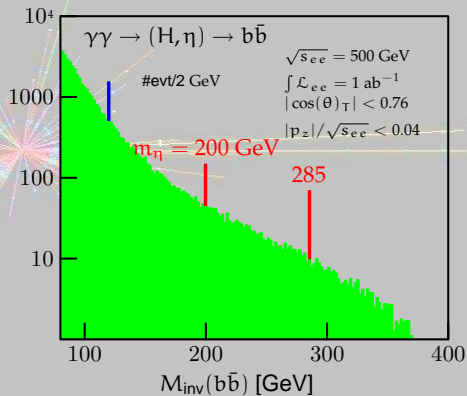
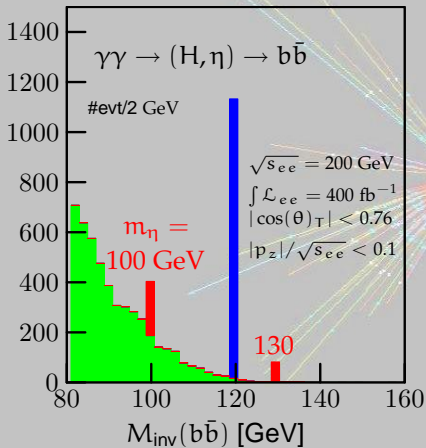


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$$g_{bb\eta} = 0.4 \cdot g_{bbh}$$

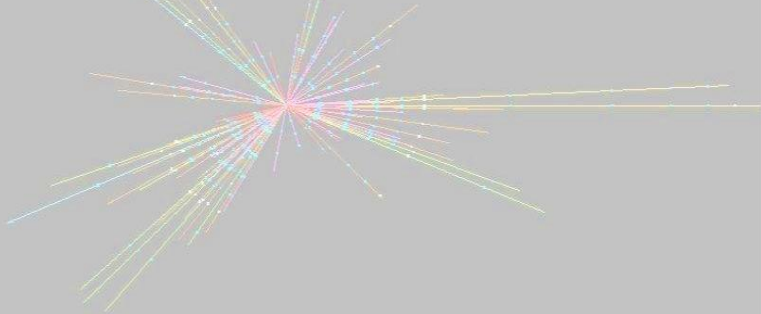
m_η	100	130	200	285
$\Gamma_{\gamma\gamma}$ [keV]	0.15	0.27	1.1	3.6



T parity and Dark Matter

Cheng/Low, 2003; Hubisz/Meade, 2005

- ▶ T parity: $T^a \rightarrow T^a$, $X^a \rightarrow -X^a$, automorphism of coset space
- ▶ analogous to R parity in SUSY, KK parity
- ▶ Bounds on f relaxed, *but*: pair production!
- ▶ Lightest T -odd particle (LTP) \Rightarrow Candidate for Cold Dark Matter



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Littlest Higgs: A' LTP

$W', Z' \sim 650$ GeV

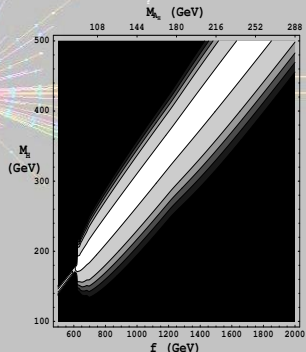
$\Phi \sim 1$ TeV

$T, T' \sim 0.7$ -1 TeV

Annihilation:

$A'A' \rightarrow h \rightarrow WW, ZZ, hh$

0/10/50/70/100



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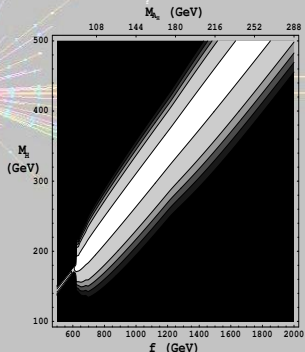
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- ▶ T parity Simple Group model: Pseudo-Axion η LTP

Kilian/Rainwater/JR/Schmaltz

Outline

Hierarchy Problem, Goldstone-Bosons and Little Higgs

- Higgs as Pseudo-Goldstone Boson
- Nambu-Goldstone Bosons
- The Little Higgs mechanism

Examples of Models

Phenomenology

- For example: Littlest Higgs
- Neutrino masses
- Effective Field Theories
- Electroweak Precision Observables
- Direct Searches
- Reconstruction of Little Higgs Models
- Pseudo Axions in LHM
- T parity and Dark Matter

Conclusions



Conclusions

Little Higgs elegant alternative to SUSY Gauge/Global Symmetry
structure stabilizes EW scale

- ▶ Generics: new heavy **gauge bosons**, **scalars**, **quarks**

Little Higgs in accord w EW precision observ. **w/o Fine Tuning** ($M_H!$)

- ▶ New developments: **Pseudo-Axions**, **T-parity**, LH Dark Matter



UV embedding, **GUT**, **Flavor** ?

Clear experimental signatures:

direct search [Gauge & Top sector, LHC (ILC)] \longleftrightarrow

precision observables [Gauge, Scalar, Top sector ILC (LHC)]

Strategy for Reconstruction by *Complementarity* of ILC & LHC