



# Matching Threshold & Continuum for exclusive top pairs



Jürgen R. Reuter, DESY



in collaboration with: Fabian Bach, Bijan Chokoufè Nejad,  
Andre H. Hoang, Wolfgang Kilian, Thomas Teubner,  
Maximilian Stahlhofen, Christian Weiss  
arXiv:1712.02220 [sent to JHEP]



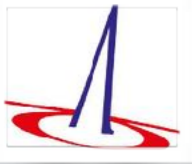


# Top Threshold/Continuum at lepton colliders

- LC top threshold scan best-known method to measure top quark mass,  $\Delta M \sim 30\text{-}70$  MeV
- LC continuum top production best-known method to measure top couplings

error source	$\Delta m_t^{\text{PS}}$ [MeV]
stat. error (200 fb <sup>-1</sup> )	13
theory (NNNLO scale variations, PS scheme)	40
parametric ( $\alpha_s$ , current WA)	35
non-resonant contributions (such as single top)	< 40
residual background / selection efficiency	10 – 20
luminosity spectrum uncertainty	< 10
beam energy uncertainty	< 17
combined theory & parametric	30 – 50
combined experimental & backgrounds	25 – 50
total (stat. + syst.)	40 – 75

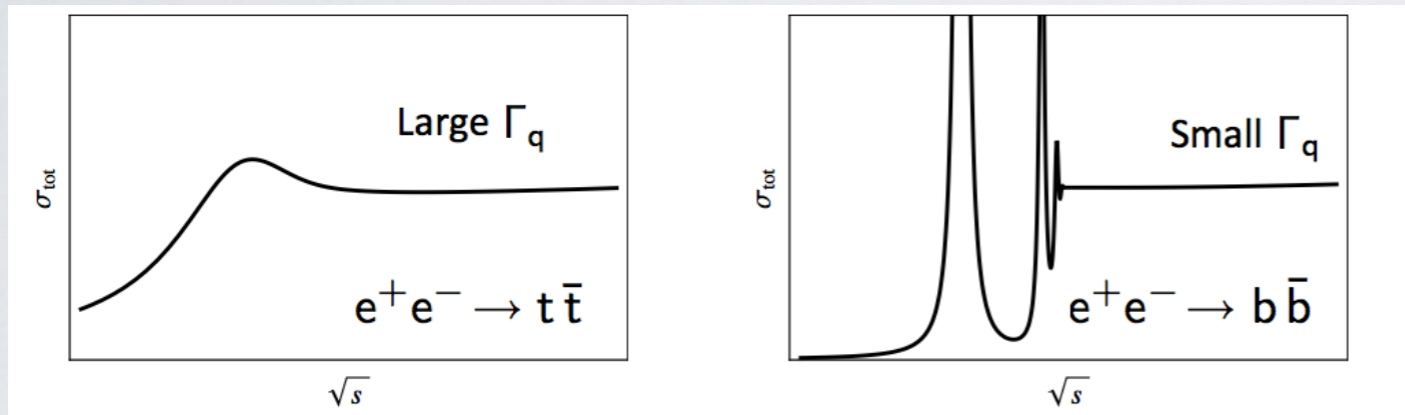
from [1702.05333](#)



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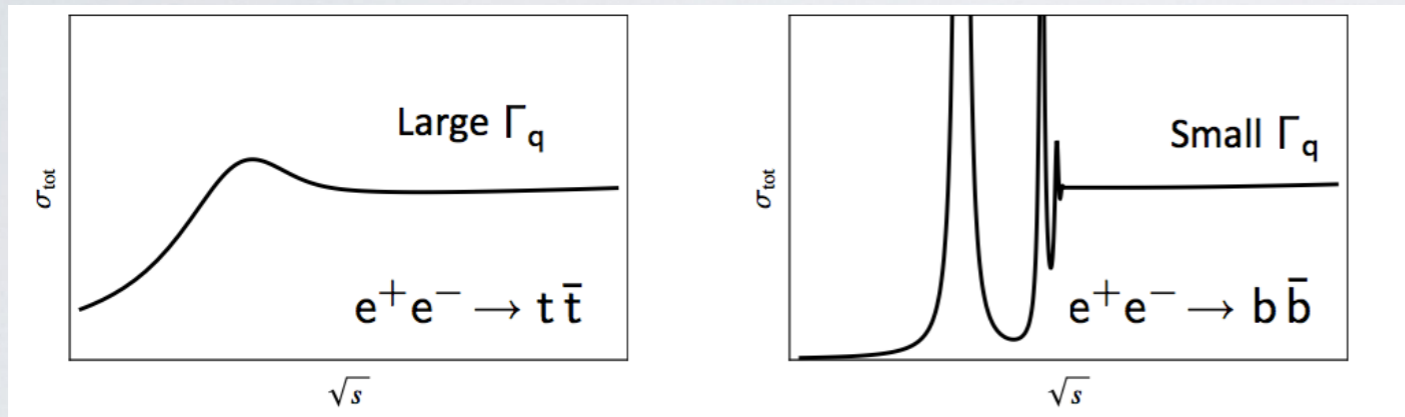
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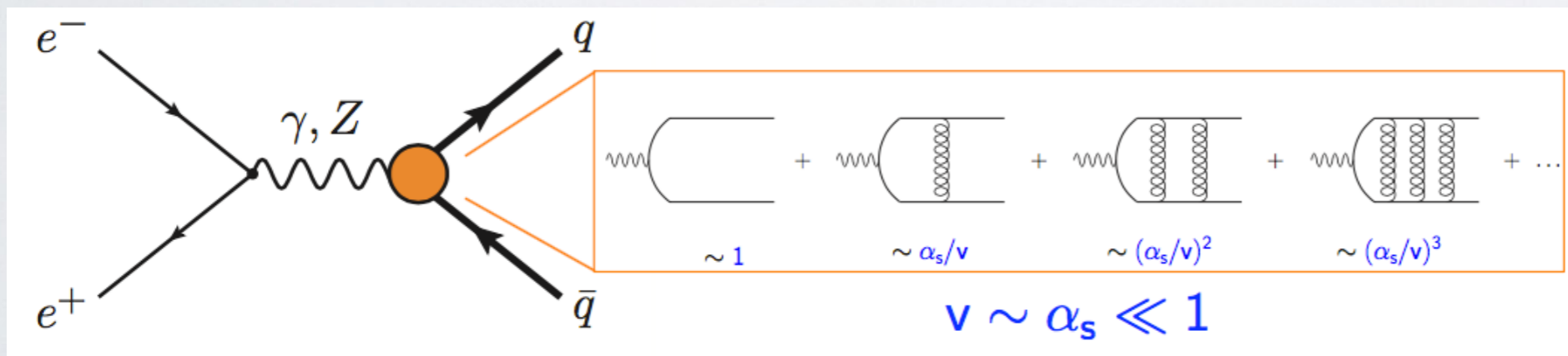
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Threshold region: top velocity  $v \sim \alpha_s \ll 1$  non-relativistic EFT: (v)NRQCD from 1702.05333



Continuum region: “standard” fixed-order QCD

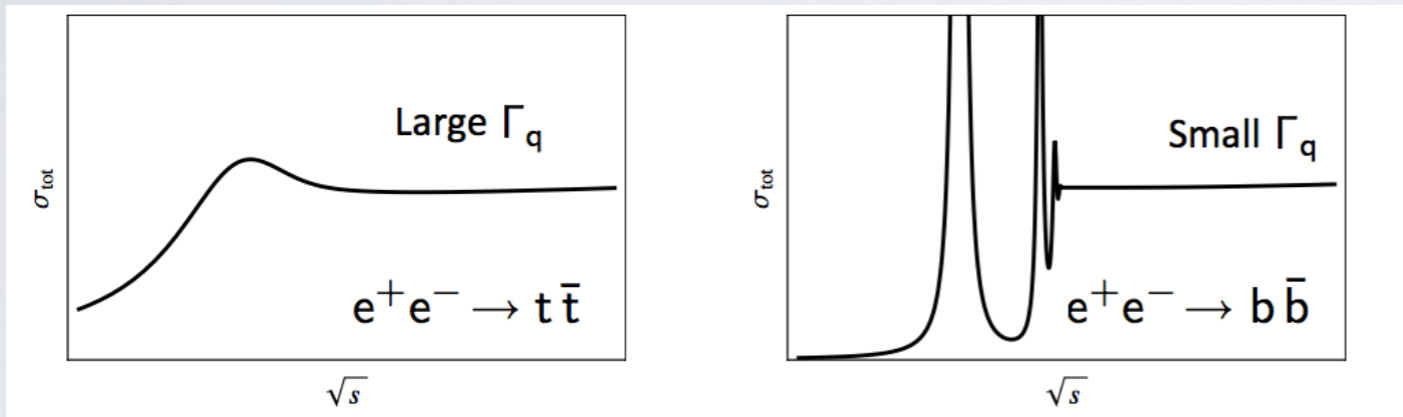




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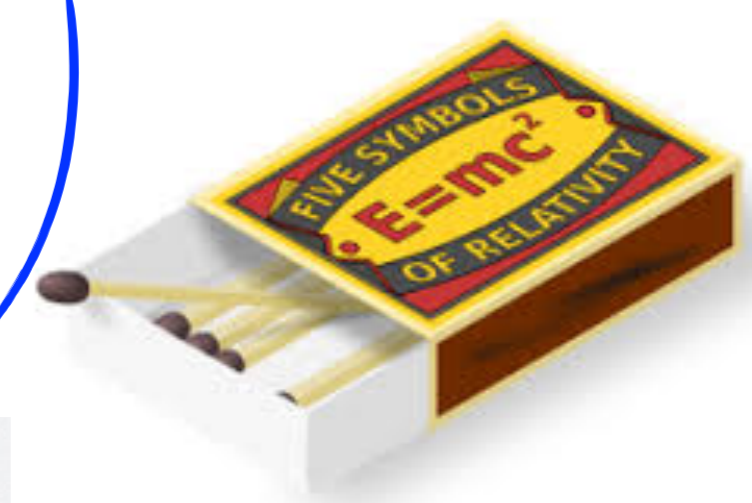
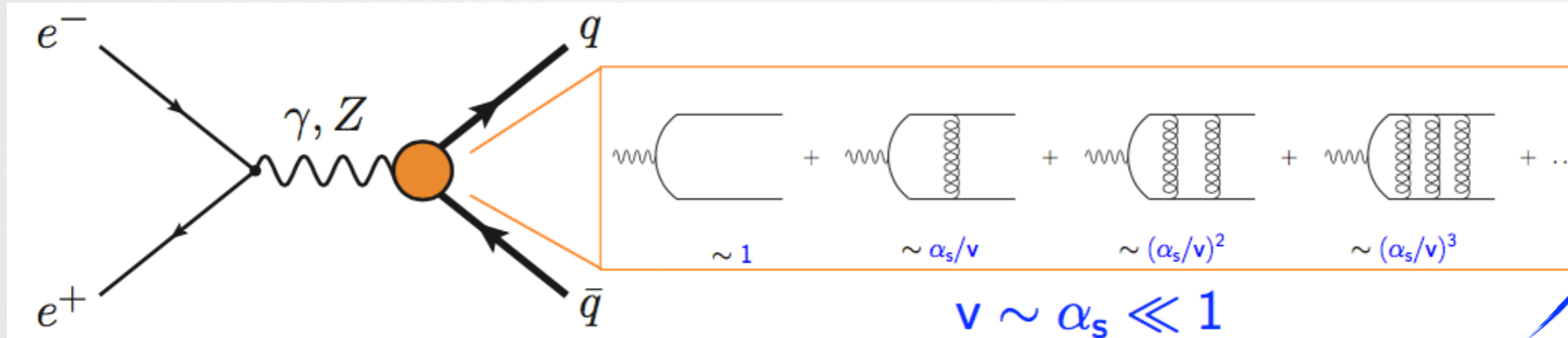
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Continuum region: “standard” fixed-order QCD

MATCH





# WHIZARD: our NLO MC framework

**WHIZARD v2.6.2** (13.12.2017)

<http://whizard.hepforge.org>

<whizard@desy.de>

**WHIZARD Team:** *Wolfgang Kilian, Thorsten Ohl, JRR*

*Simon Braß/Vincent Rothe/Christian Schwinn/Marco Sekulla/So Young Shim/Florian Staub/Pascal Stienemeier/  
Zhijie Zhao + 2 Master*

## PUBLICATIONS

General WHIZARD reference: EPJ C71 (2011) 1742, arXiv:0708.4241

0' Mega (ME generator): LC-TOOL (2001) 040; arXiv:hep-ph/0102195

VAMP (MC integrator): CPC 120 (1999) 13; arXiv:hep-ph/9806432

CIRCE (beamstrahlung): CPC 101 (1997) 269; arXiv:hep-ph/9607454

Parton shower: JHEP 1204 (2012) 013; arXiv:1112.1039

Color flow formalism: JHEP 1210 (2012) 022; arXiv:1206.3700

NLO capabilities: JHEP 1612 (2016) 075; arXiv:1609.03390

Parallelization of MEs: CPC 196 (2015) 58; arXiv:1411.3834

POWHEG matching: EPS-HEP (2015) 317; arXiv:1510.02739

```
alpha_power = 2
alphas_power = 0
```

```
process eett = e1,E1 => t, tbar
{ nlo_calculation = "full" }
```

FKS subtraction [Frixione/Kunszt/Signer, '95]

Resonance-aware treatment  
[Ježo/Nason, 1509.09071]

**NLO QCD final validation phase**

**NLO EW work in progress**

## Working NLO interfaces to:

★ GoSam [N. Greiner, G. Heinrich... et al.]

★ **OpenLoops** [J. Lindert, P. Maierhöfer, S. Pozzorini et al.]

★ Recola [A. Denner, L. Hofer, J.-N. Lang, S. Uccirati]



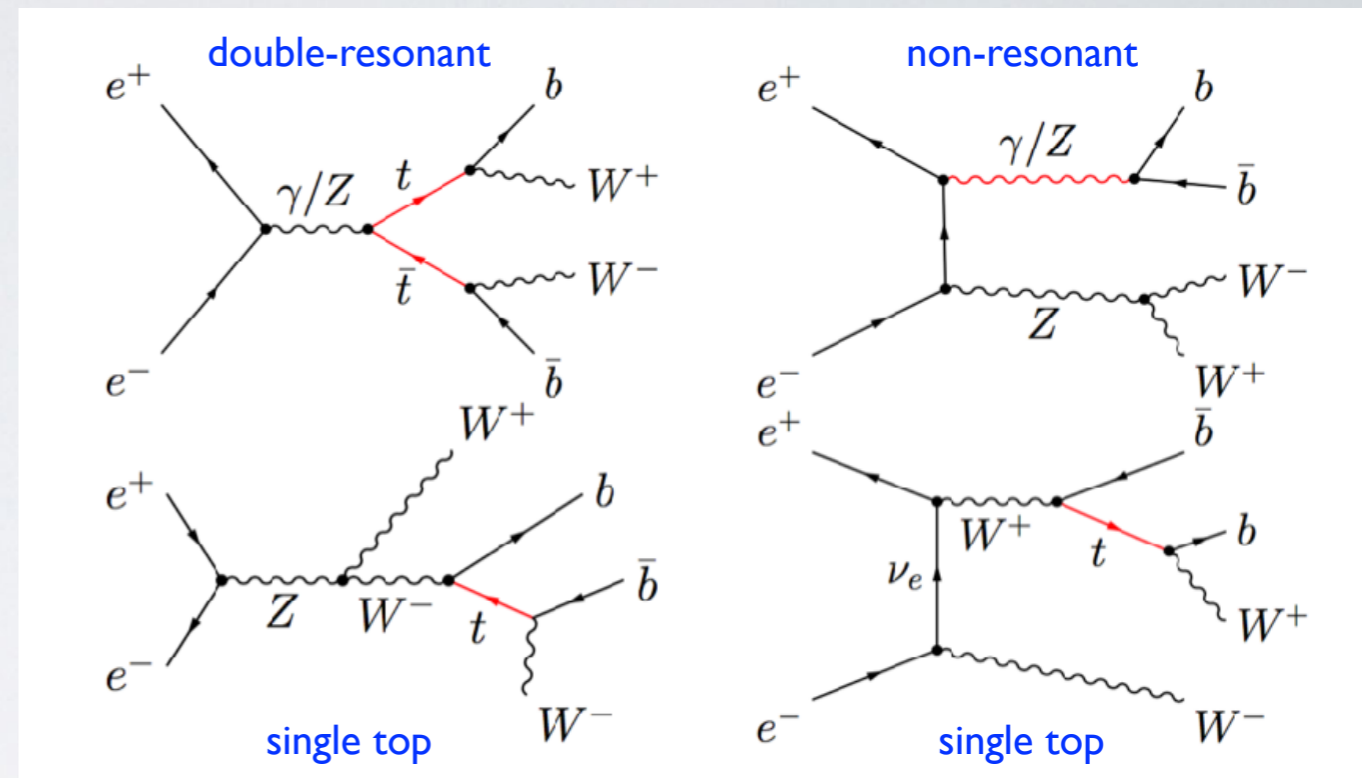
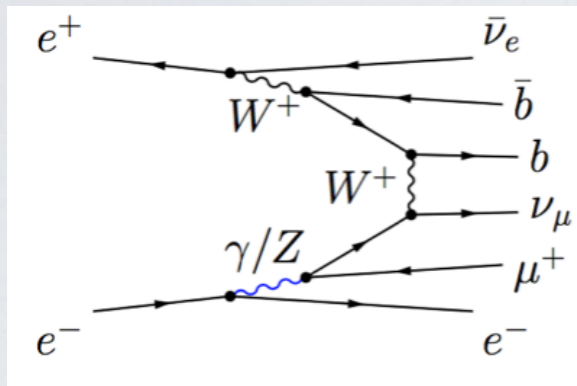


# $e^+e^- \rightarrow tt$ off-shell in the continuum

● Cross checks for  $2 \rightarrow 2$  and  $2 \rightarrow 4$  with Sherpa & Munich

● Using massive  $b$  quarks:  
no cuts necessary for  $e^+e^- \rightarrow W^+W^-bb$

● Full process  $e^+e^- \rightarrow \mu^+\nu_\mu e^-\nu_e bb$  exhibits  
Coulomb singularity:



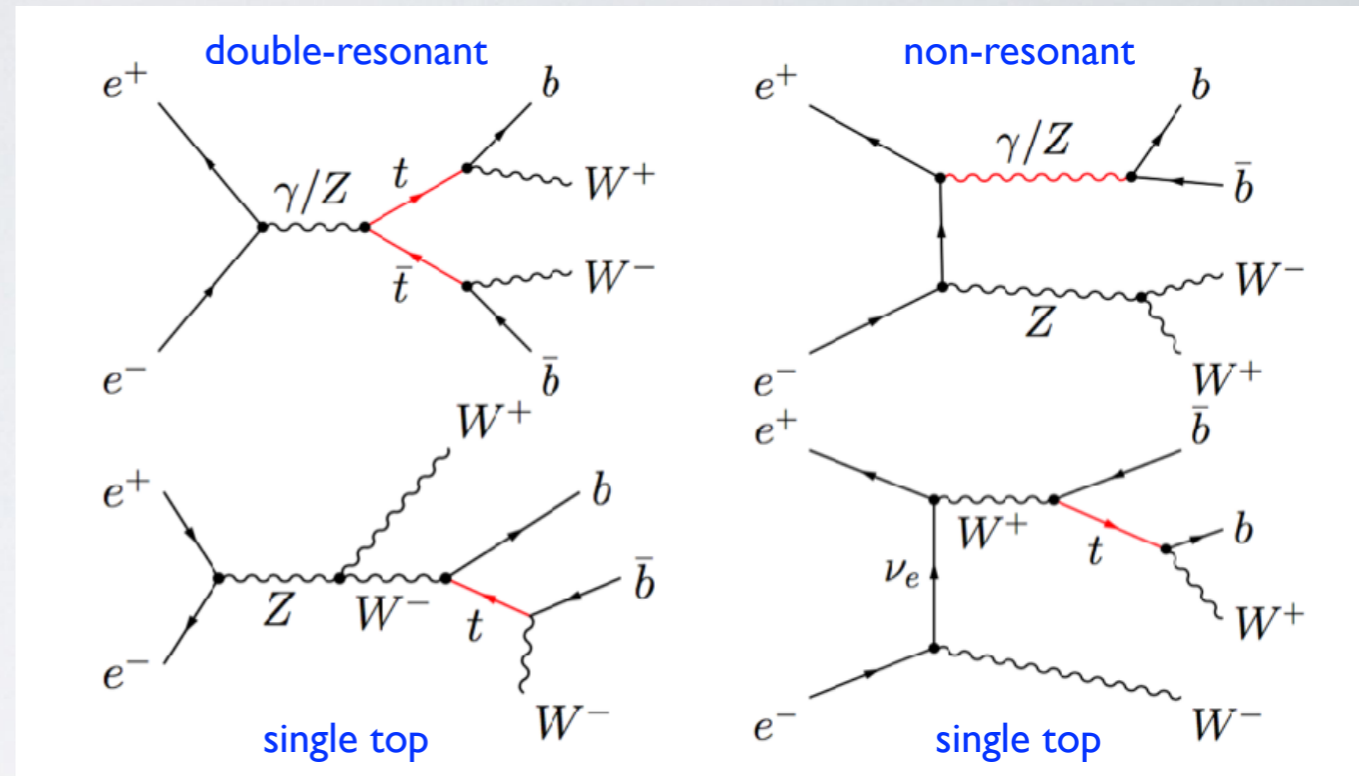
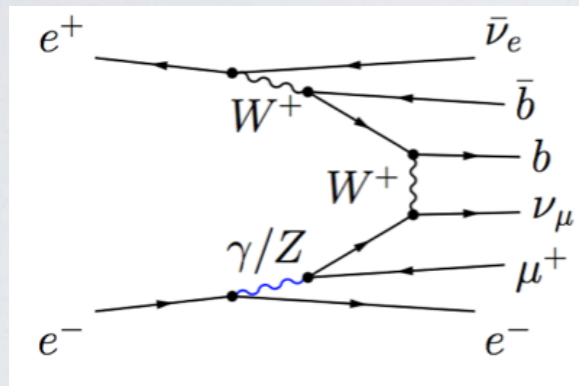


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## INPUT PARAMETERS:

$$m_Z = 91.1876 \text{ GeV},$$
$$m_b = 4.2 \text{ GeV},$$

$$m_W = 80.385 \text{ GeV}$$
$$m_t = 173.2 \text{ GeV}.$$

$$\Gamma_{t \rightarrow Wb}^{\text{LO}} = 1.4986 \text{ GeV},$$
$$\Gamma_{t \rightarrow f\bar{f}b}^{\text{LO}} = 1.4757 \text{ GeV},$$

$$\Gamma_{t \rightarrow Wb}^{\text{NLO}} = 1.3681 \text{ GeV},$$
$$\Gamma_{t \rightarrow f\bar{f}b}^{\text{NLO}} = 1.3475 \text{ GeV}.$$

$$m_H = 125 \text{ GeV}$$

$$\Gamma_H = 0.00431 \text{ GeV}$$

$$\Gamma_Z^{\text{LO}} = 2.4409 \text{ GeV},$$
$$\Gamma_W^{\text{LO}} = 2.0454 \text{ GeV},$$

$$\Gamma_Z^{\text{NLO}} = 2.5060 \text{ GeV},$$
$$\Gamma_W^{\text{NLO}} = 2.0978 \text{ GeV}.$$

## Complex Mass Scheme (CMS):

$$\mu_i^2 = M_i^2 - i\Gamma_i M_i \quad \text{for } i = W, Z, t, H$$

$$s_w^2 = 1 - c_w^2 = 1 - \frac{\mu_W^2}{\mu_Z^2}$$





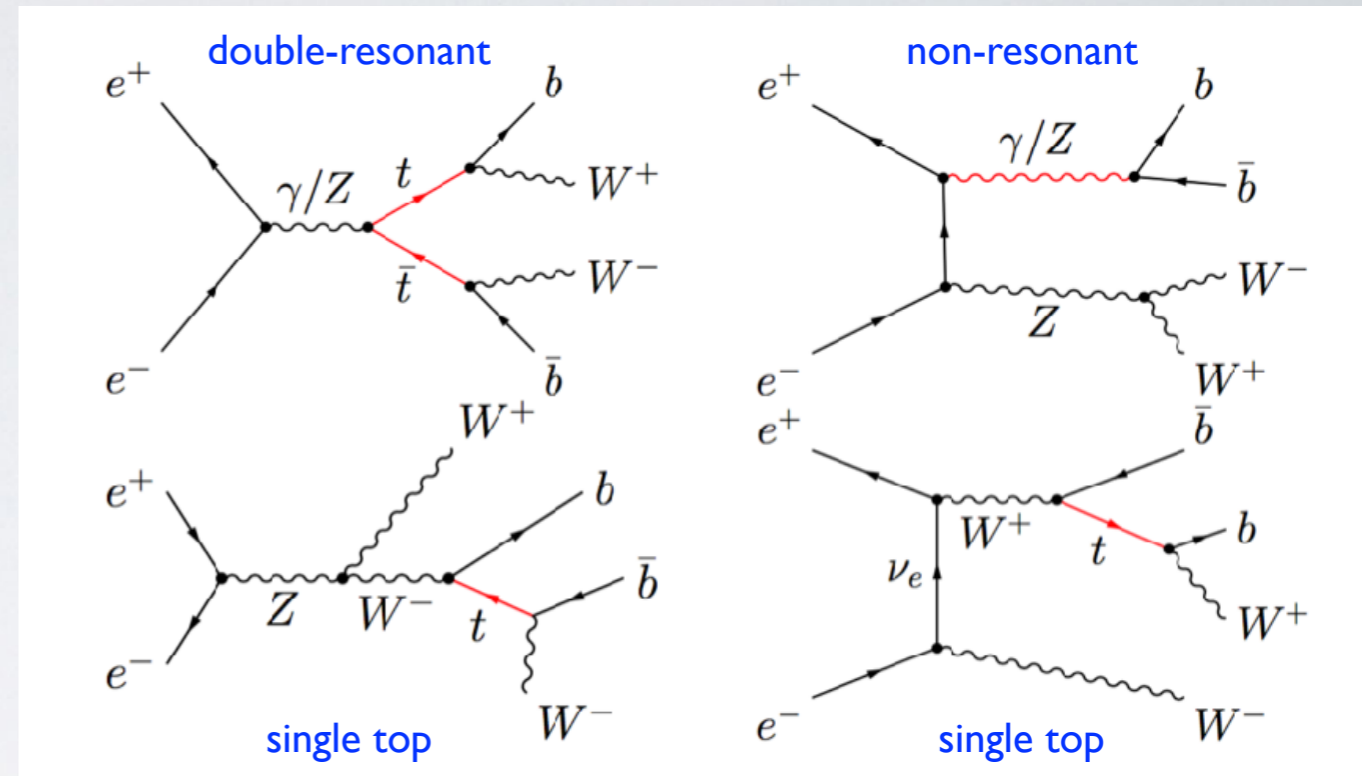
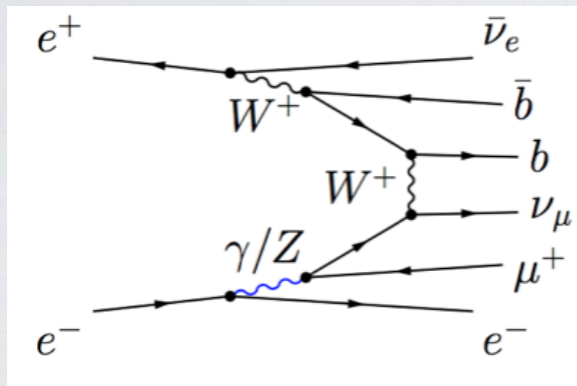


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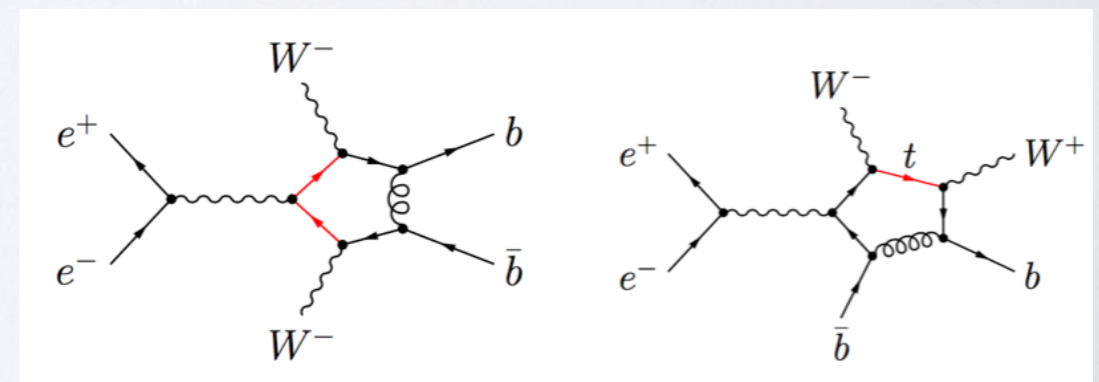
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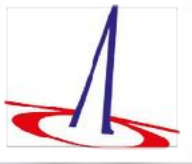
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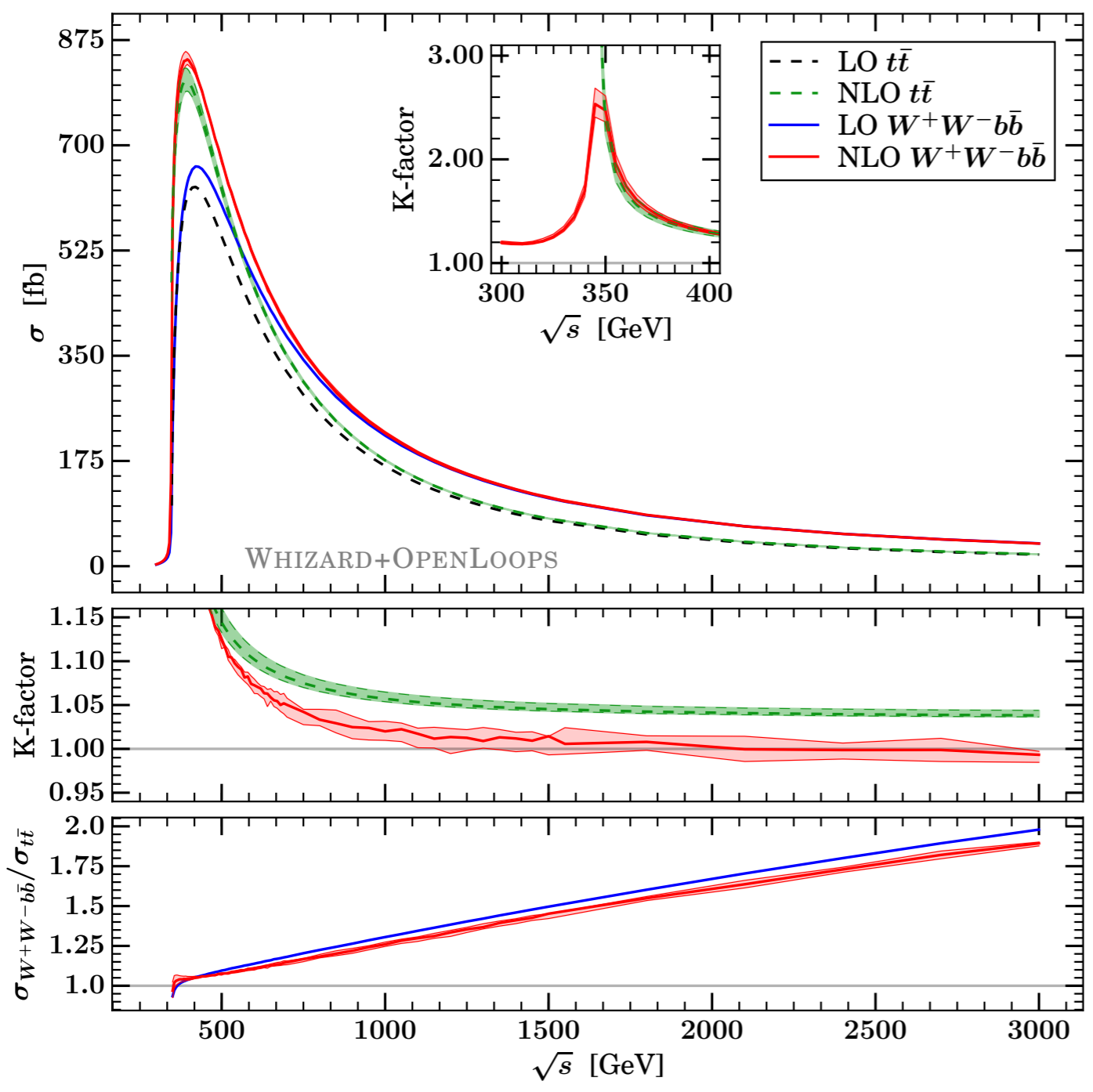
● Typical pentagons:



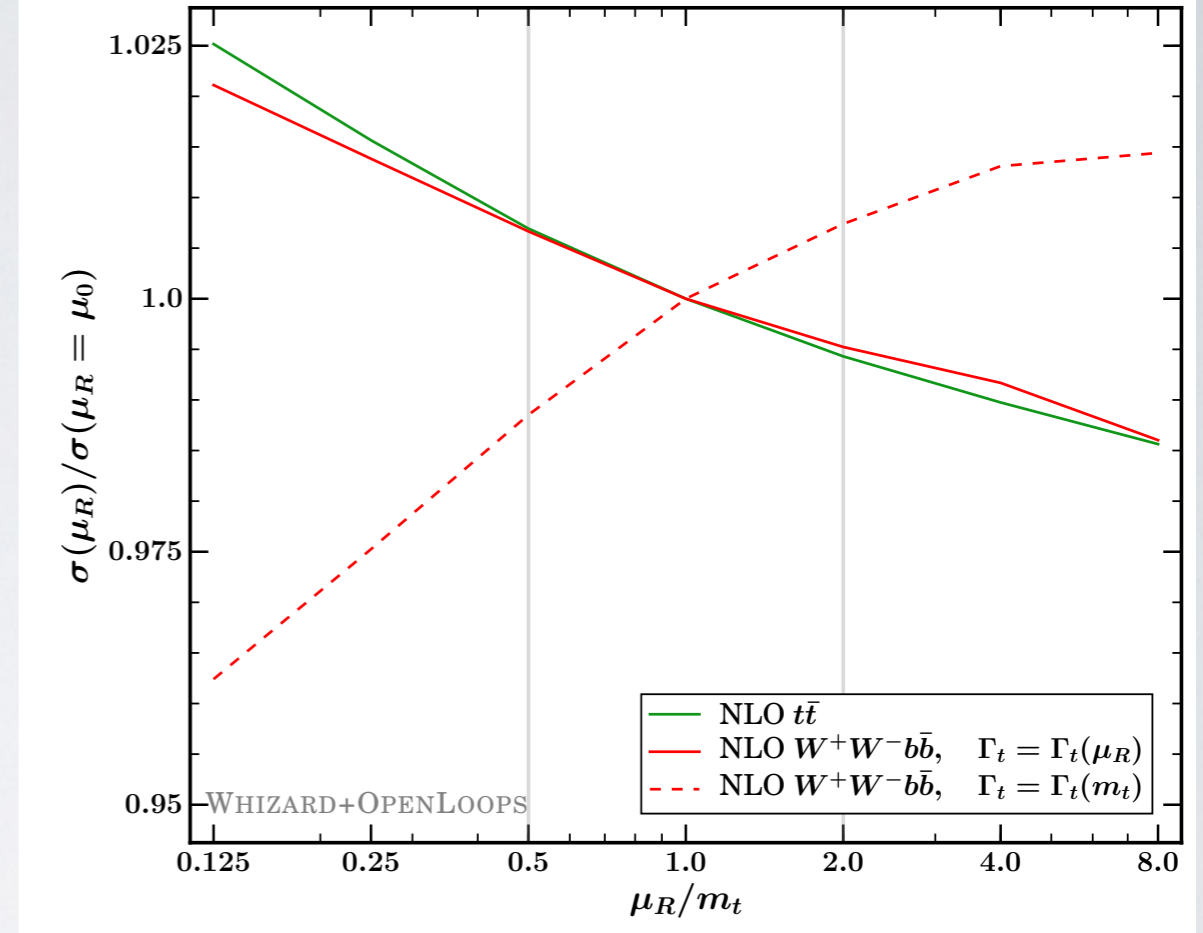


# NLO QCD Results for off-shell $e^+e^- \rightarrow t\bar{t}$

$e^+e^- \rightarrow t\bar{t}$  and  $e^+e^- \rightarrow W^+W^-b\bar{b}$



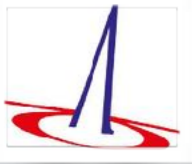
$e^+e^- \rightarrow t\bar{t}$  and  $e^+e^- \rightarrow W^+W^-b\bar{b}$  at  $\sqrt{s} = 800$  GeV



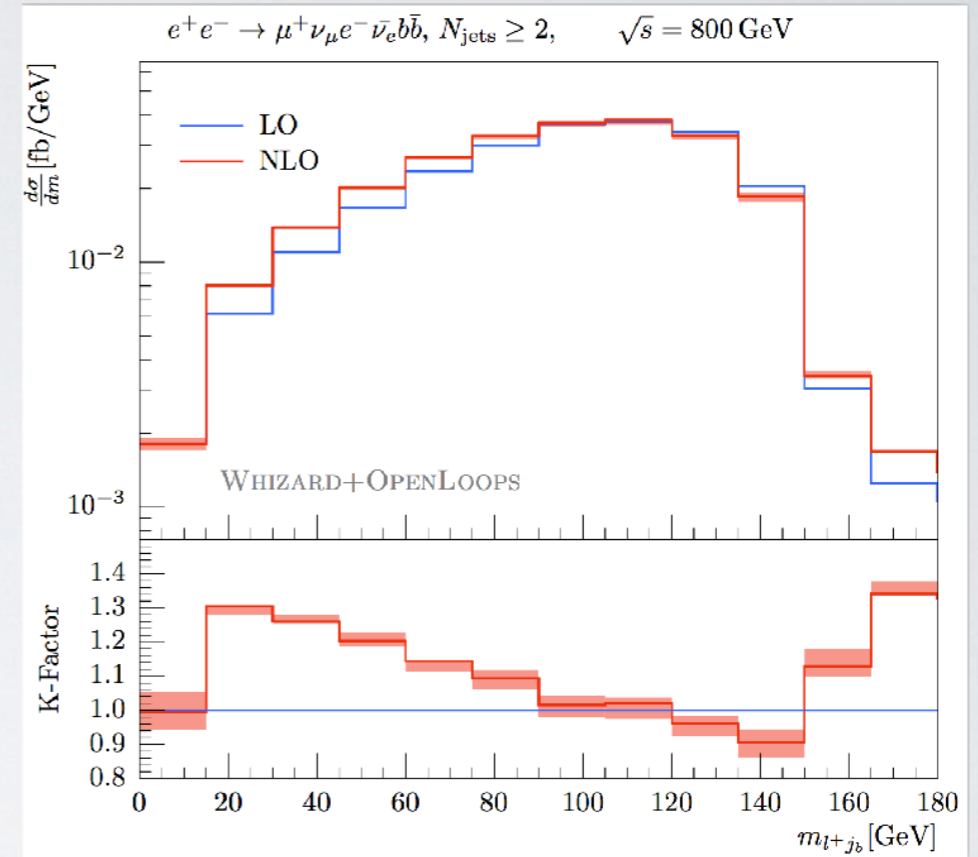
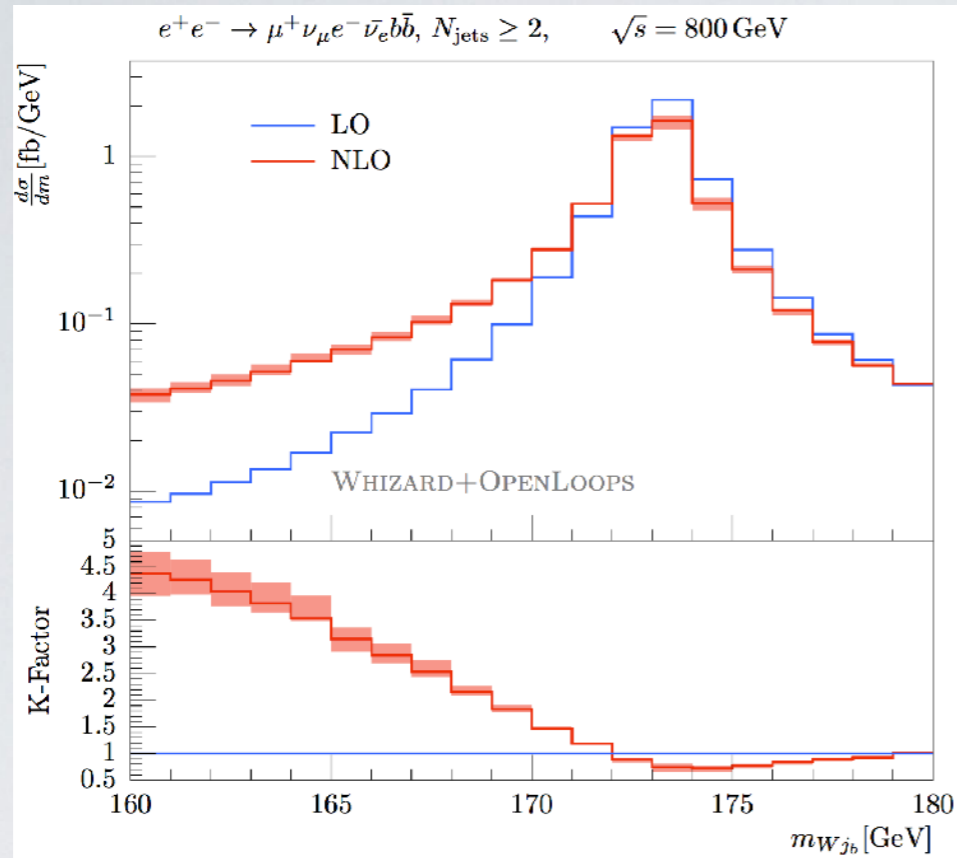
$\sqrt{s}$ [GeV]	$e^+e^- \rightarrow t\bar{t}$			$e^+e^- \rightarrow W^+W^-b\bar{b}$		
	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor	$\sigma^{\text{LO}}$ [fb]	$\sigma^{\text{NLO}}$ [fb]	K-factor
500	548.4	$627.4^{+1.4\%}_{-0.9\%}$	1.14	600.7	$675.1^{+0.4\%}_{-0.8\%}$	1.12
800	253.1	$270.9^{+0.8\%}_{-0.4\%}$	1.07	310.2	$320.7^{+1.1\%}_{-0.7\%}$	1.03
1000	166.4	$175.9^{+0.7\%}_{-0.3\%}$	1.06	217.2	$221.6^{+1.1\%}_{-1.0\%}$	1.02
1400	86.62	$90.66^{+0.6\%}_{-0.2\%}$	1.05	126.4	$127.9^{+0.7\%}_{-1.5\%}$	1.01
3000	19.14	$19.87^{+0.5\%}_{-0.2\%}$	1.04	37.89	$37.63^{+0.4\%}_{-0.9\%}$	0.993

Chokouf/Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

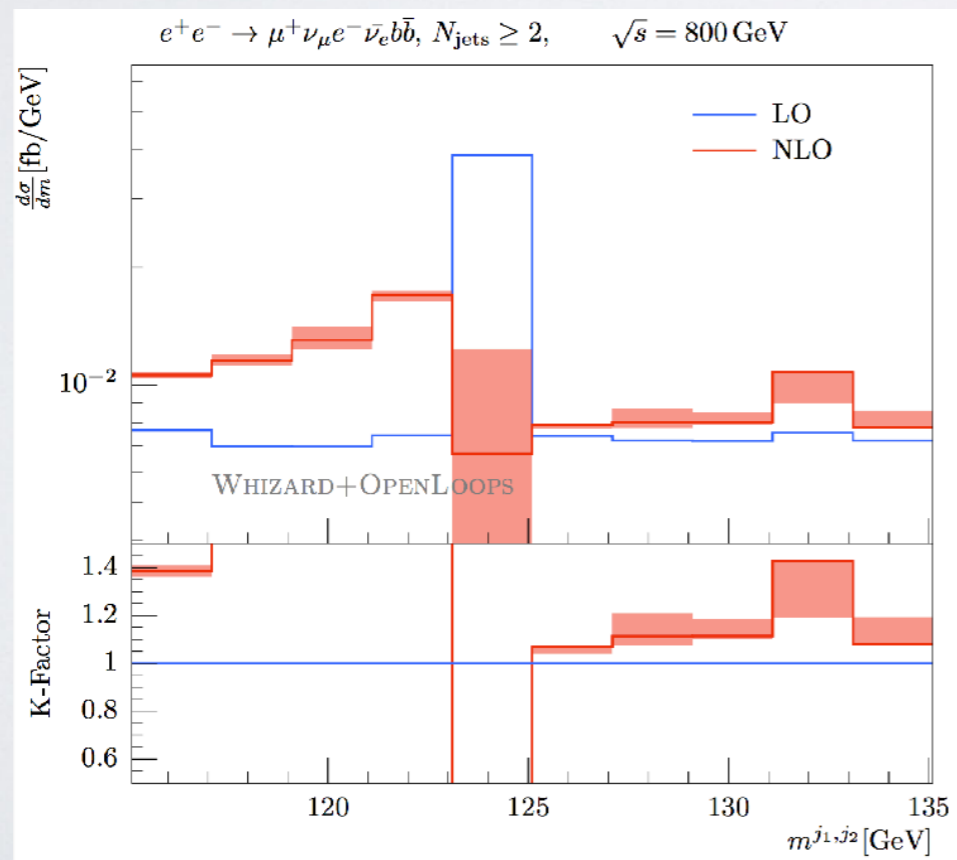




# Differential Results for off-shell $e^+e^- \rightarrow tt$



$$m_t^2 = m_W^2 + \frac{2\langle m_{\ell j_b}^2 \rangle}{1 - \langle \cos \theta_{\ell j_b} \rangle}$$



Chokouf/Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390





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- Integrate out hard quark and gluon d.o.f.
- Resummation of singular terms close to threshold ( $v = 0$ ) Hoang et al. '99-'01; Beneke et al., '13-'14

Phase space of two massive particles

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times$$

$$\times \{1 (\mathbf{LL}); \alpha_s, v (\mathbf{NLL}); \alpha_s^2, \alpha_s v, v^2 (\mathbf{NNLL})\}$$

(p/v)NRQCD EFT w/ RG improvement



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$$R^{\gamma,Z}(s) = \underbrace{F^v(s)R^v(s)}_{\text{s-wave: LL+NLL}} + \underbrace{F^a(s)R^a(s)}_{\text{p-wave} \sim v^2: \text{NNLL}}$$

but contributes at NLL differentially!



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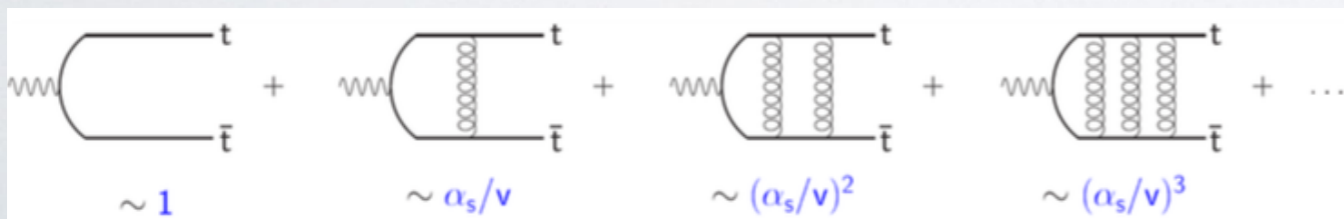
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Coulomb potential gluon ladder resummation





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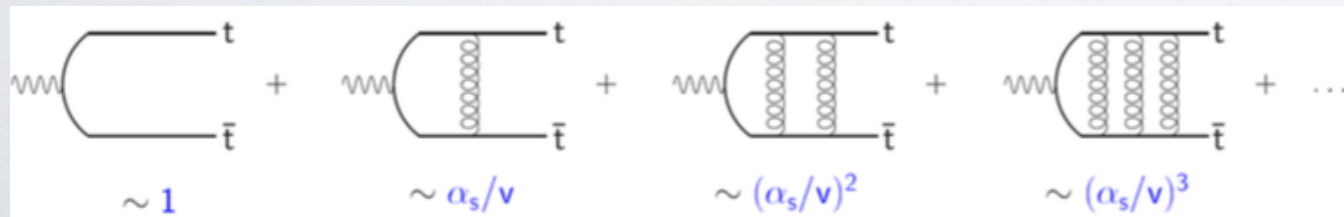
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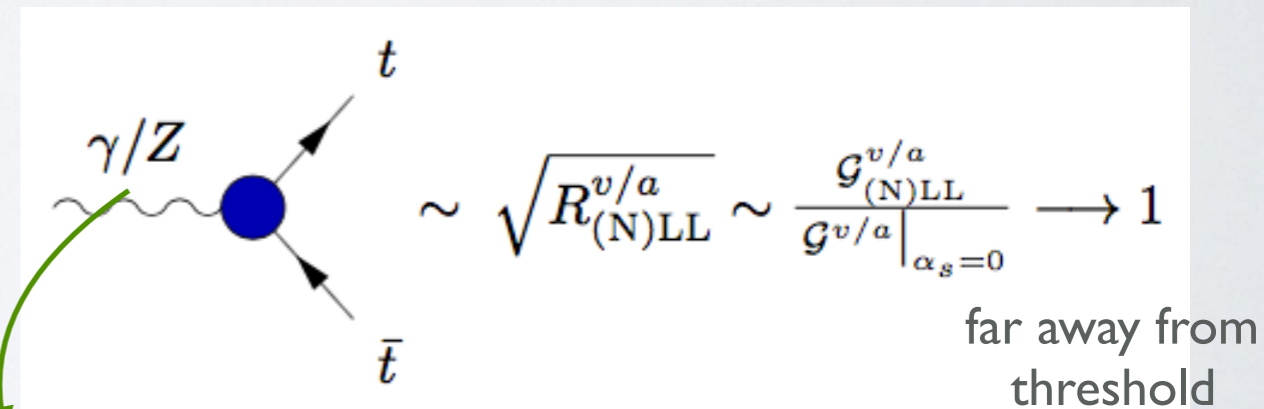
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Coulomb potential gluon ladder resummation



can be mapped onto effective  $t\bar{t}V$  vertex



$$\mathbb{C} \ni \mathcal{G}_{(N)LL}^{v/a} = \mathcal{G}_{(N)LL}^{v/a}(\alpha_s, M_t^{\text{pole}}, \sqrt{s}, |\vec{p}_t|, \Gamma_t)$$

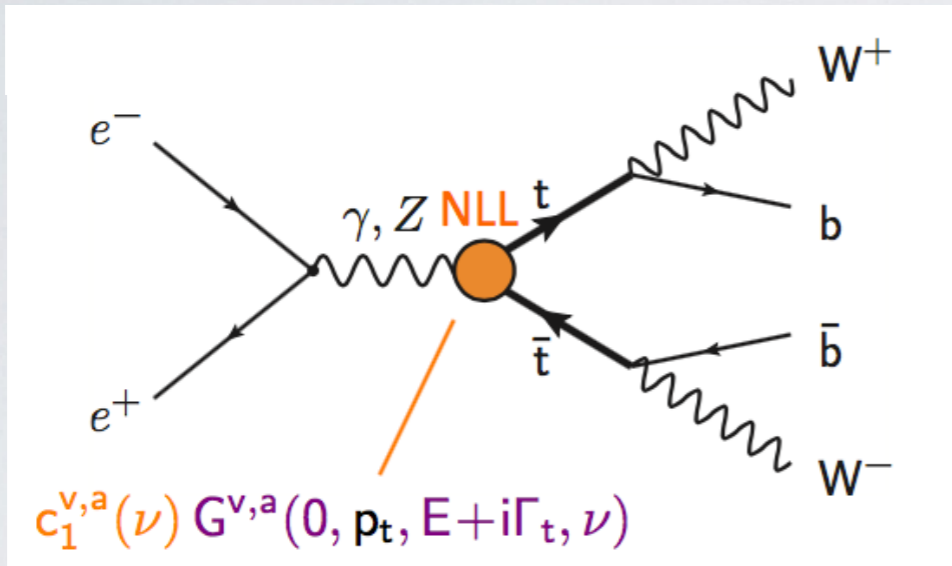
differential in off-shell  $t\bar{t}$  phase space





# Top Threshold in WHIZARD

- Implement resummed threshold effects as effective vertex [form factor] in WHIZARD
- $G^{v,a}(0, p_t, E + i\Gamma_t, \nu)$  from TOPPIK code [Jezabek/Teubner], included in WHIZARD



- Default parameters:

$$M^{1S} = 172 \text{ GeV}, \quad \Gamma_t = 1.54 \text{ GeV},$$

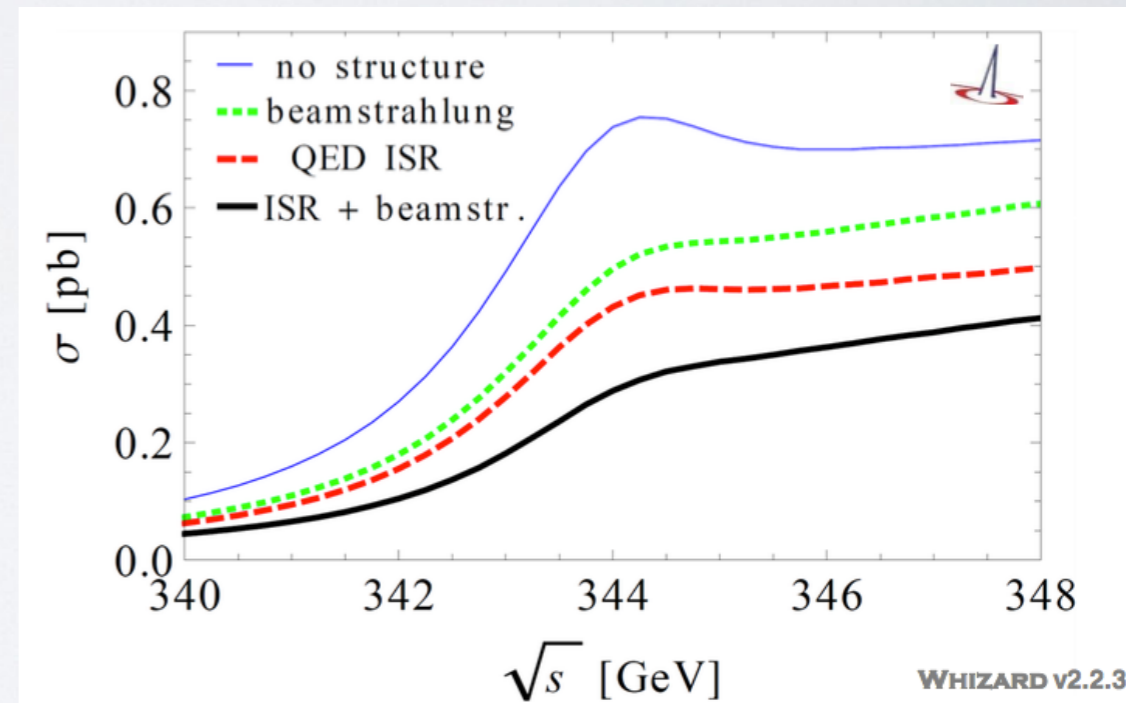
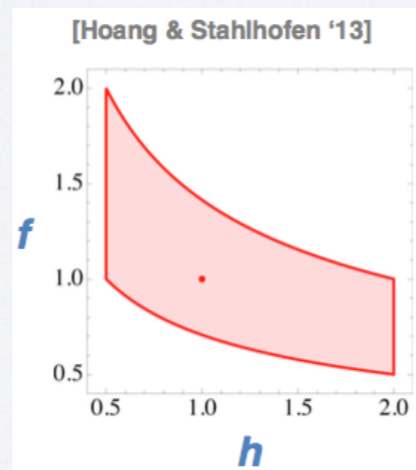
$$\alpha_s(M_Z) = 0.118$$

$$M^{1S} = M_t^{pole} (1 - \Delta_{(Coul.)}^{LL/NLL})$$

- ▶ Important effects: beamstrahlung; ISR; LO EW terms
- ▶ Exclusive observables accessible

Theory uncertainties from scale variations: hard and soft scale

$$\mu_h = h \cdot m_t \quad \mu_s = f \cdot m_t v$$



cf. Frank Simon's talk



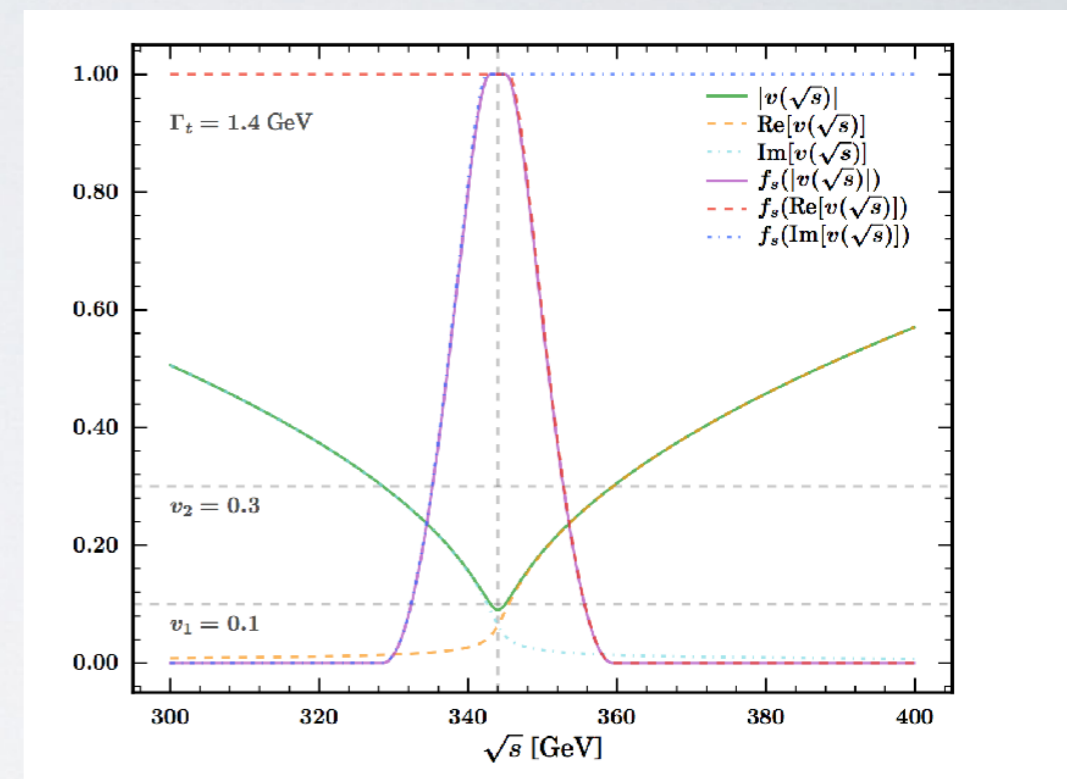




# Top threshold: validation and matching

- Transition region between relativistic and resummation effects

$$\begin{aligned}
 \sigma_{\text{NLO+NLL}} = & \sigma_{\text{NLO}} + \left( \tilde{F}_{\text{NLL}} - \tilde{F}_{\text{NLL}}^{\text{exp}} \right) \left( \begin{array}{c} e^+ \\ \phantom{e^+} \\ \phantom{e^+} \\ e^- \end{array} \rightarrow \begin{array}{c} b \\ W^+ \\ W^- \\ \bar{b} \end{array} \right) \left( \begin{array}{c} b \\ W^+ \\ W^- \\ \bar{b} \end{array} \rightarrow \begin{array}{c} e^+ \\ \phantom{e^+} \\ \phantom{e^+} \\ e^- \end{array} \right) \\
 & + \left| \tilde{F}_{\text{NLL}} \left( \begin{array}{c} e^+ \\ \phantom{e^+} \\ \phantom{e^+} \\ e^- \end{array} \rightarrow \begin{array}{c} b \\ W^+ \\ W^- \\ \bar{b} \end{array} \right) \right|^2 \\
 & + \left\{ \tilde{F}_{\text{NLL}} \left( \begin{array}{c} e^+ \\ \phantom{e^+} \\ \phantom{e^+} \\ e^- \end{array} \rightarrow \begin{array}{c} b \\ W^+ \\ W^- \\ \bar{b} \end{array} \right) \left( \begin{array}{c} e^+ \\ \phantom{e^+} \\ \phantom{e^+} \\ e^- \end{array} \rightarrow \begin{array}{c} b \\ W^+ \\ W^- \\ \bar{b} \end{array} \right) \right. \\
 & \quad \left. + \left( \begin{array}{c} e^+ \\ \phantom{e^+} \\ \phantom{e^+} \\ e^- \end{array} \rightarrow \begin{array}{c} b \\ W^+ \\ W^- \\ \bar{b} \end{array} \right) \tilde{F}_{\text{NLL}} \left( \begin{array}{c} e^+ \\ \phantom{e^+} \\ \phantom{e^+} \\ e^- \end{array} \rightarrow \begin{array}{c} b \\ W^+ \\ W^- \\ \bar{b} \end{array} \right) \right\} \\
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 \end{aligned}$$



Smoothstep matching function:

$$\begin{aligned}
 \sigma_{\text{matched}} = & \sigma_{\text{FO}} [\alpha_H] + \sigma_{\text{NRQCD}}^{\text{full}} [f_s \alpha_H, f_s \alpha_S, f_s \alpha_{US}] \\
 & - \sigma_{\text{NRQCD}}^{\text{expanded}} [f_s \alpha_H, f_s \alpha_H],
 \end{aligned}$$

$$\begin{aligned}
 \alpha_H &= \alpha_s [\mu_H = hM_t^{1S}], \\
 \alpha_S &= \alpha_s [\mu_S = hM_t^{1S} f\nu_*],
 \end{aligned}$$

$$\begin{aligned}
 \alpha_F &= \alpha_s [\mu_F = hM_t^{1S} \sqrt{\nu_*}], \\
 \alpha_U &= \alpha_s [\mu_{US} = hM_t^{1S} (f\nu_*)^2]
 \end{aligned}$$

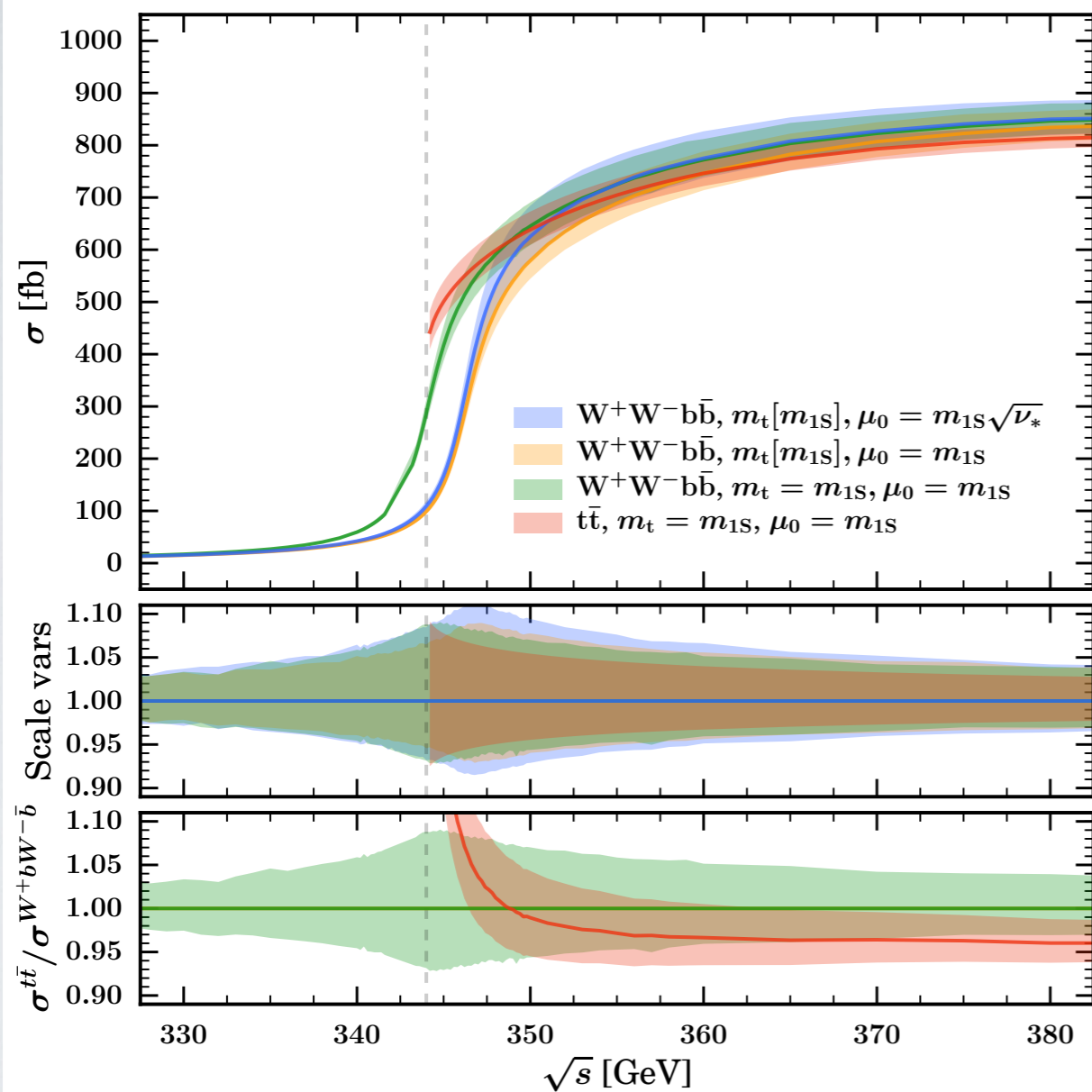
$$f_s(v) = \begin{cases} 1 & v < v_1 \\ 1 - 3 \left( \frac{v-v_1}{v_2-v_1} \right)^2 - 2 \left( \frac{v-v_1}{v_2-v_1} \right)^3 & v_1 \leq v \leq v_2 \\ 0 & v > v_2 \end{cases}$$



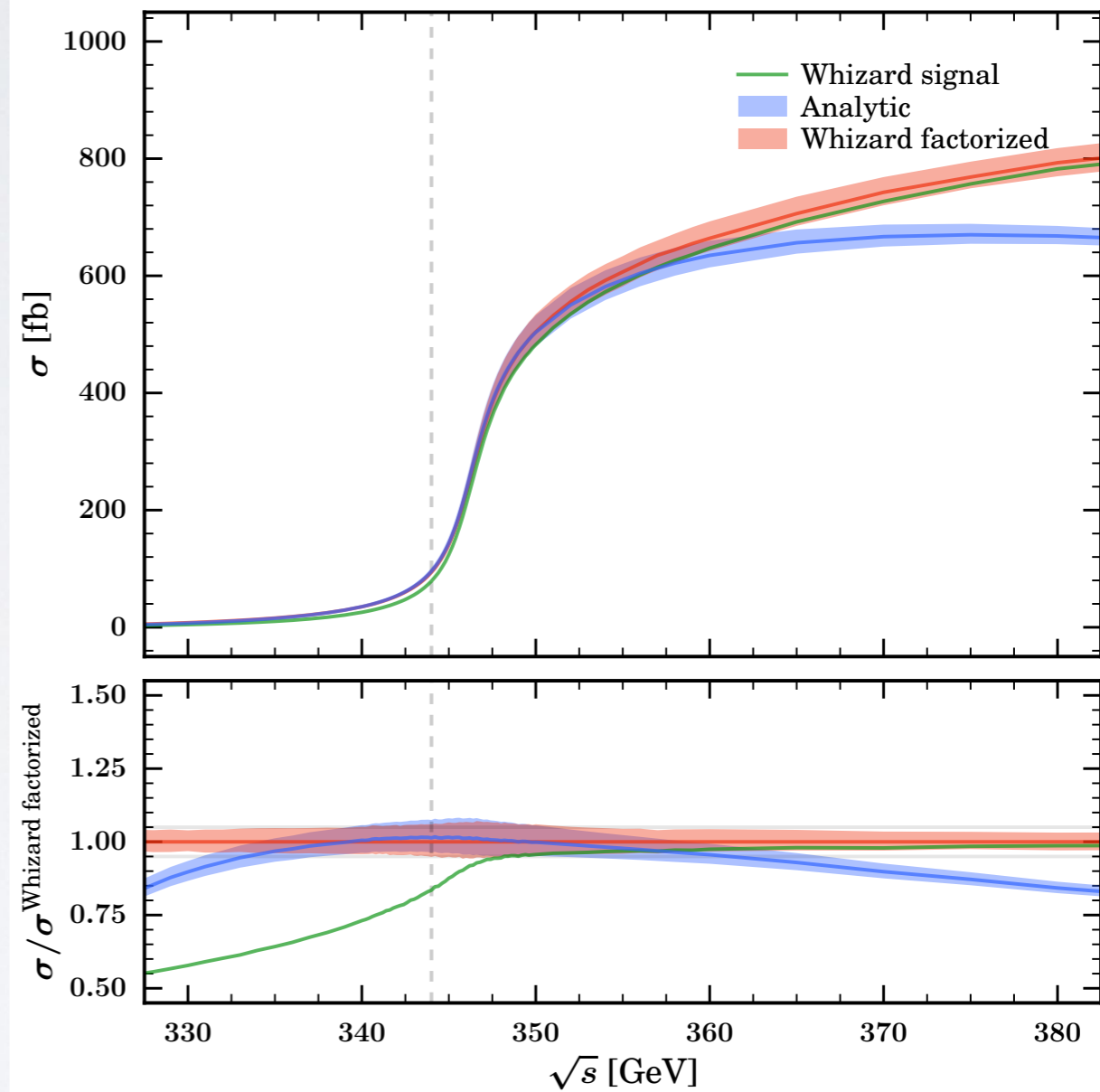


# Top threshold: validation and matching

NLO predictions for on- and off-shell  $t\bar{t}$  production



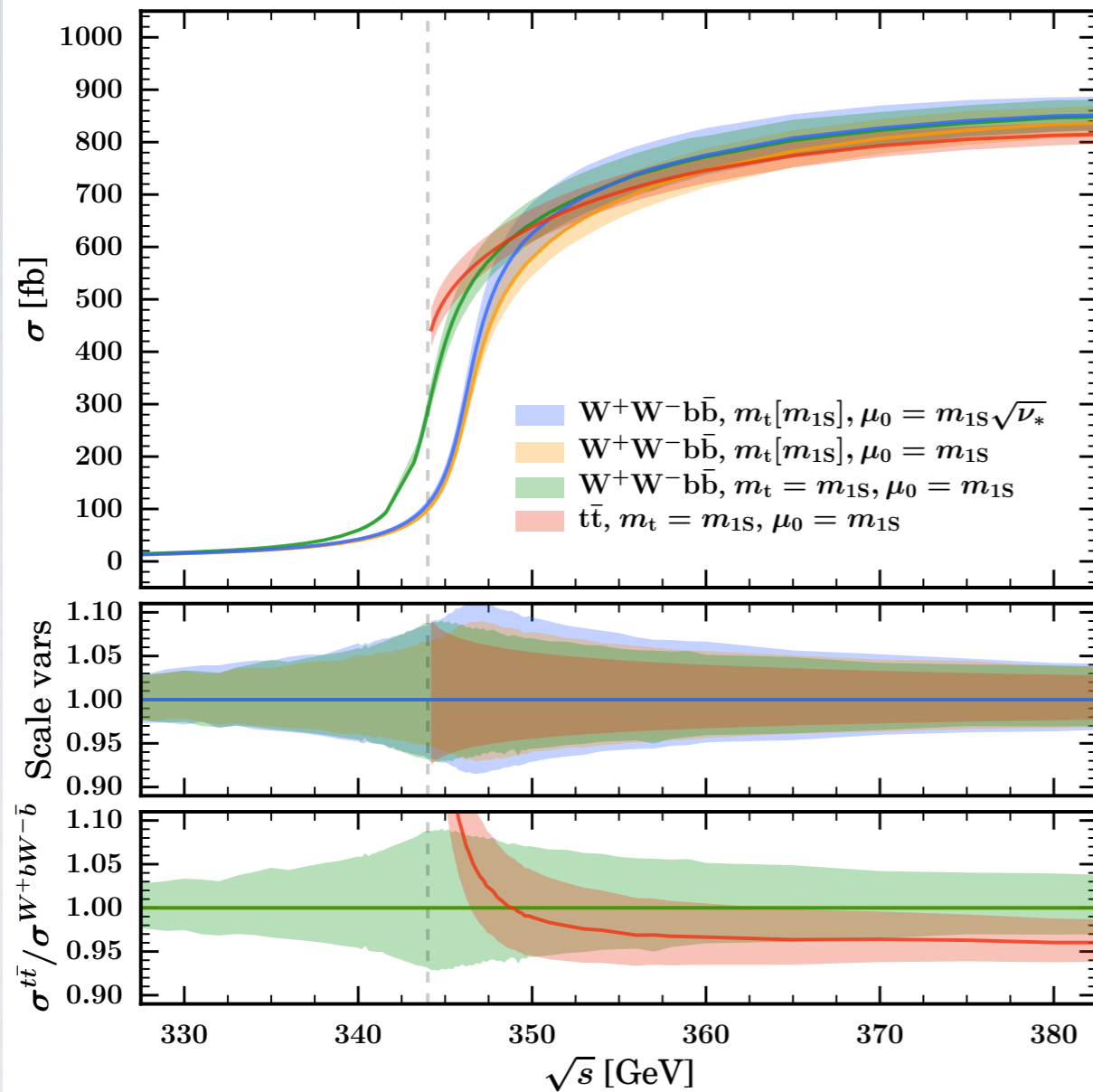
$\Delta_{m_t} = 30$  GeV, expanded, evaluated with  $\alpha_H$ , only s-wave contributions



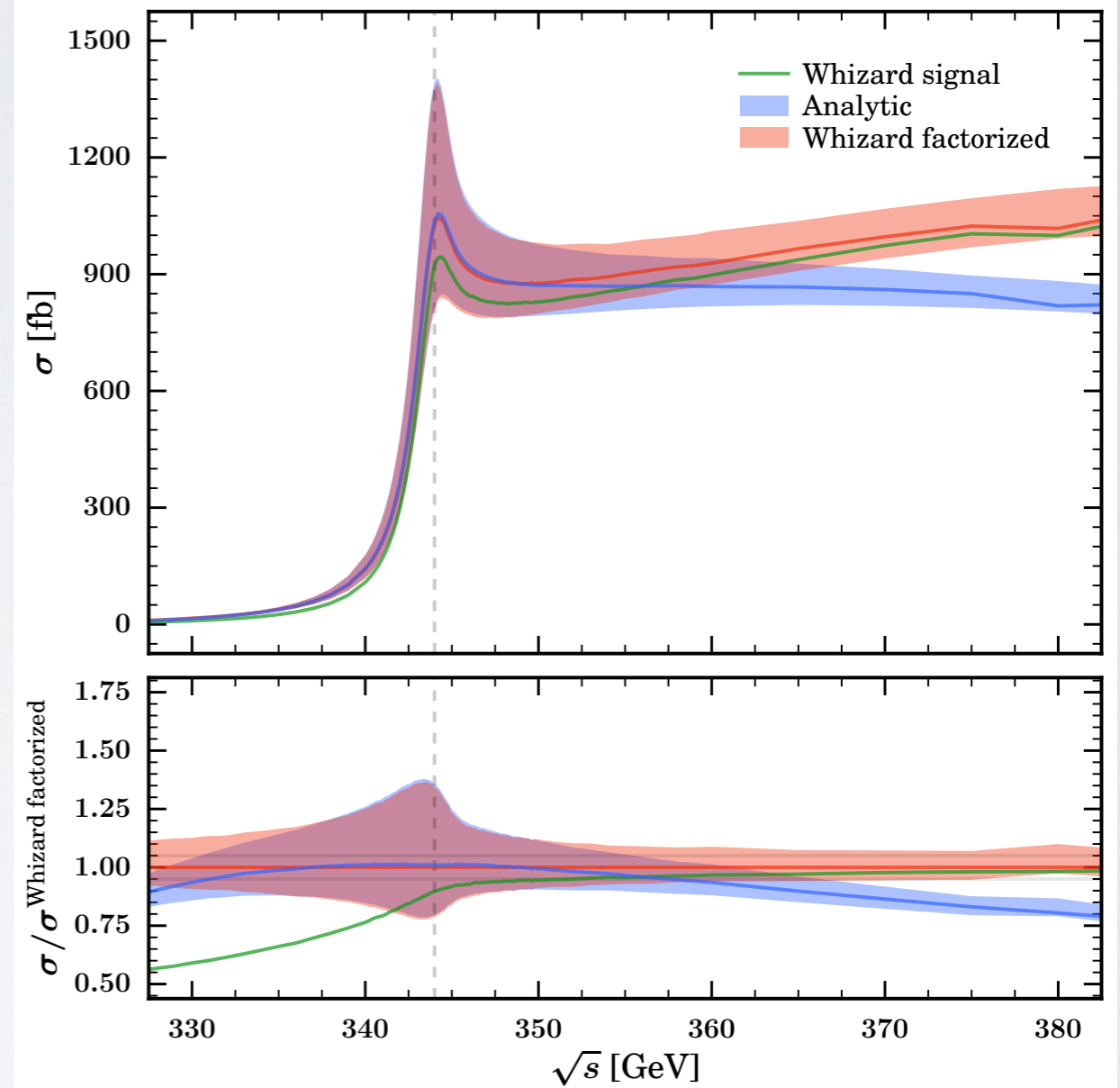


# Top threshold: validation and matching

NLO predictions for on- and off-shell  $t\bar{t}$  production



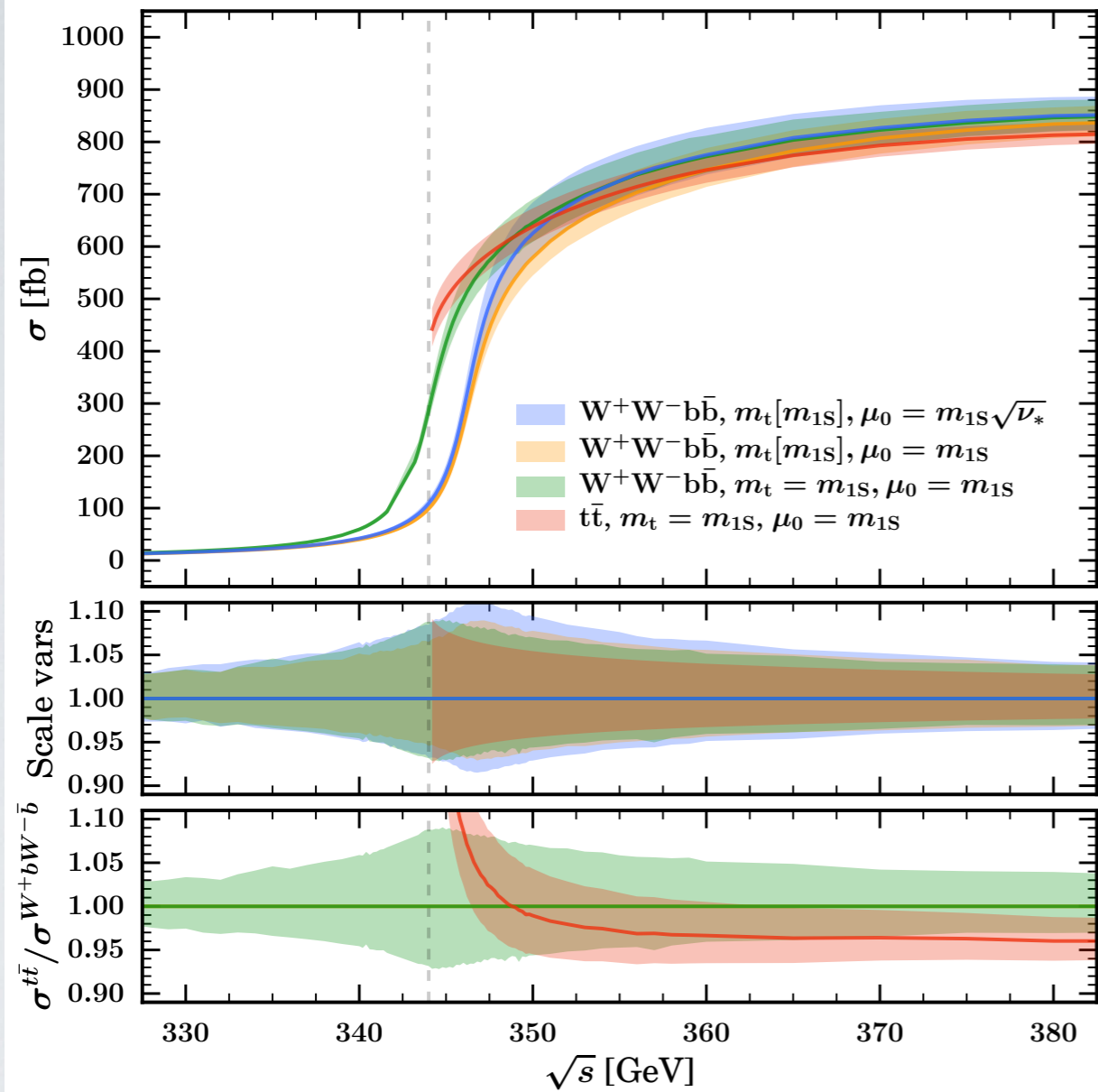
$\Delta_{m_t} = 30$  GeV, LL, only s-wave contributions



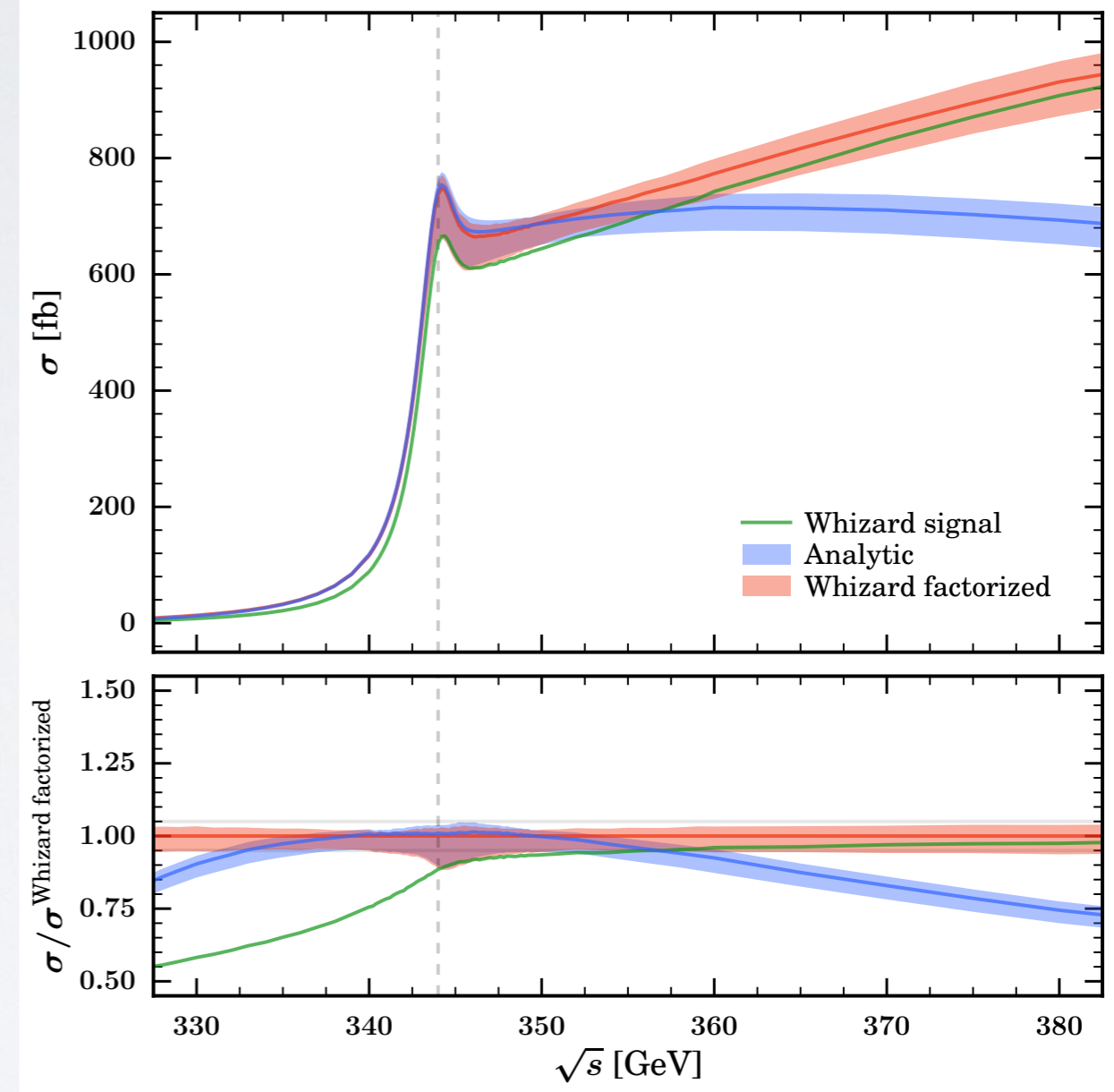


# Top threshold: validation and matching

NLO predictions for on- and off-shell  $t\bar{t}$  production

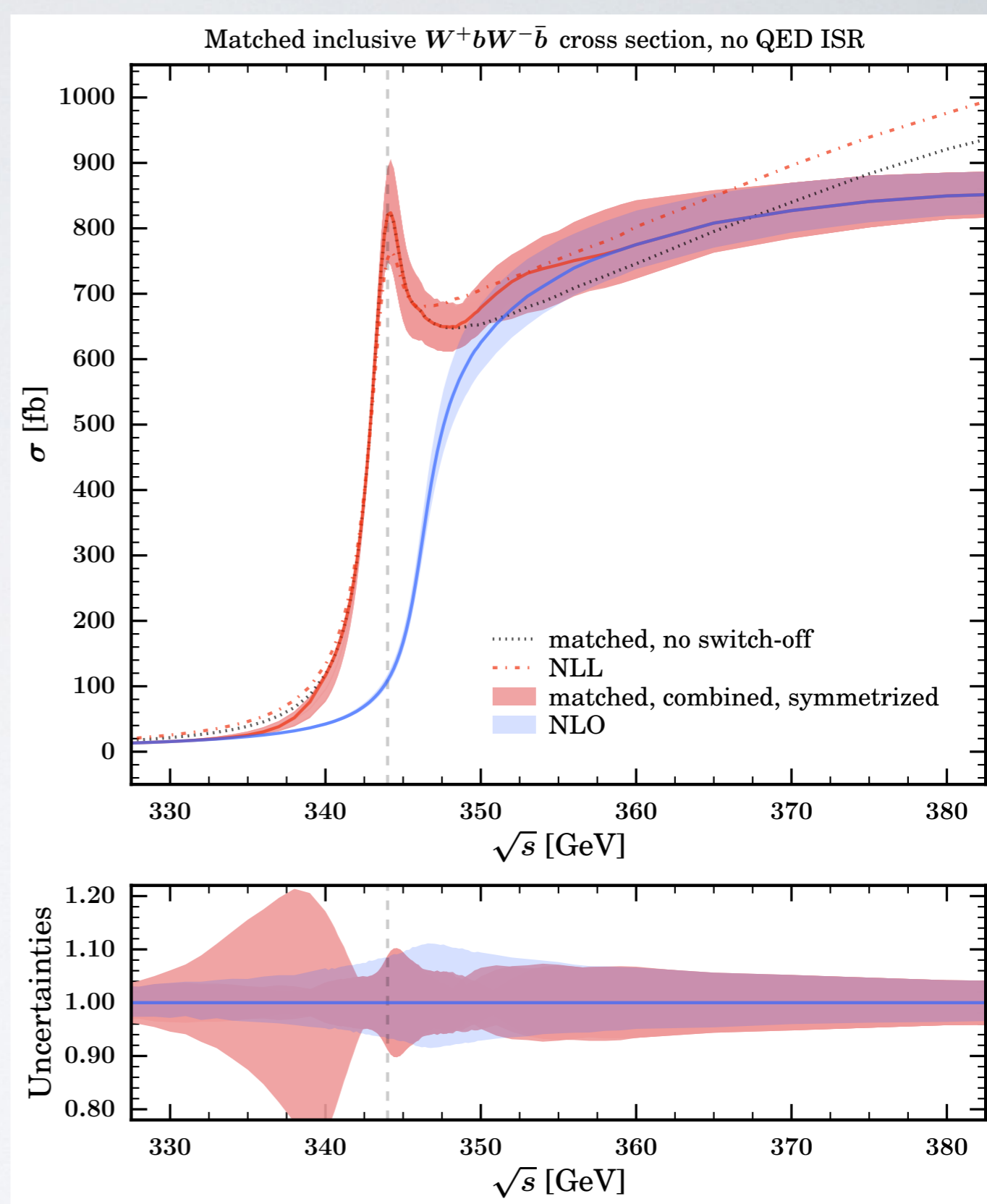
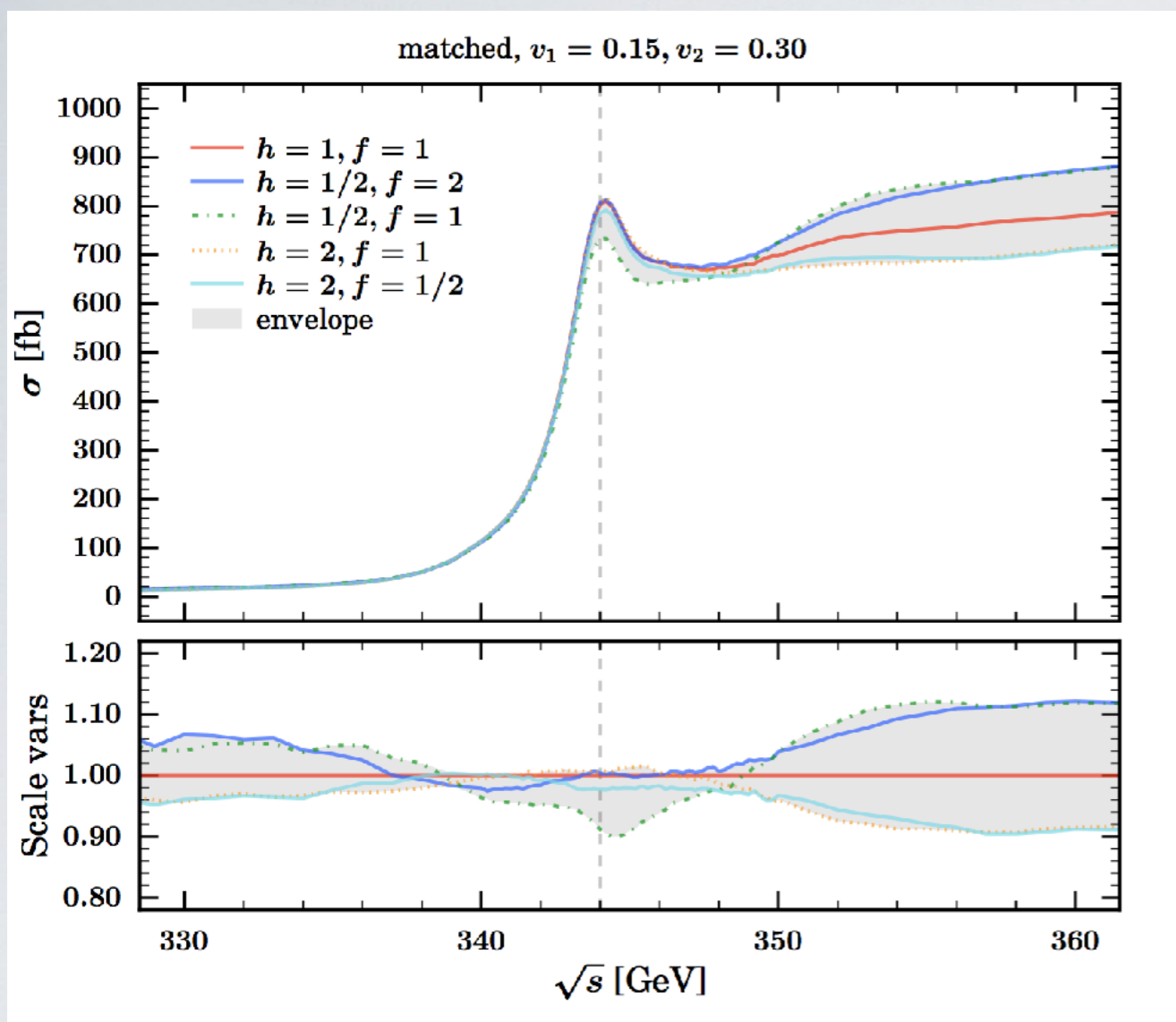


$\Delta_{m_t} = 30$  GeV, NLL, only s-wave contributions





# Matching threshold NLL to continuum NLO



**Total uncertainty:  $h$ - $f$  variation band and matching [switch-off function]**

Symmetrization of error bands:

$$\sigma_{\max} = \max \left[ \max_{i \in \text{HF}} \sigma_i, \sigma_0 + (\sigma_0 - \min_{i \in \text{HF}} \sigma_i) \right]$$

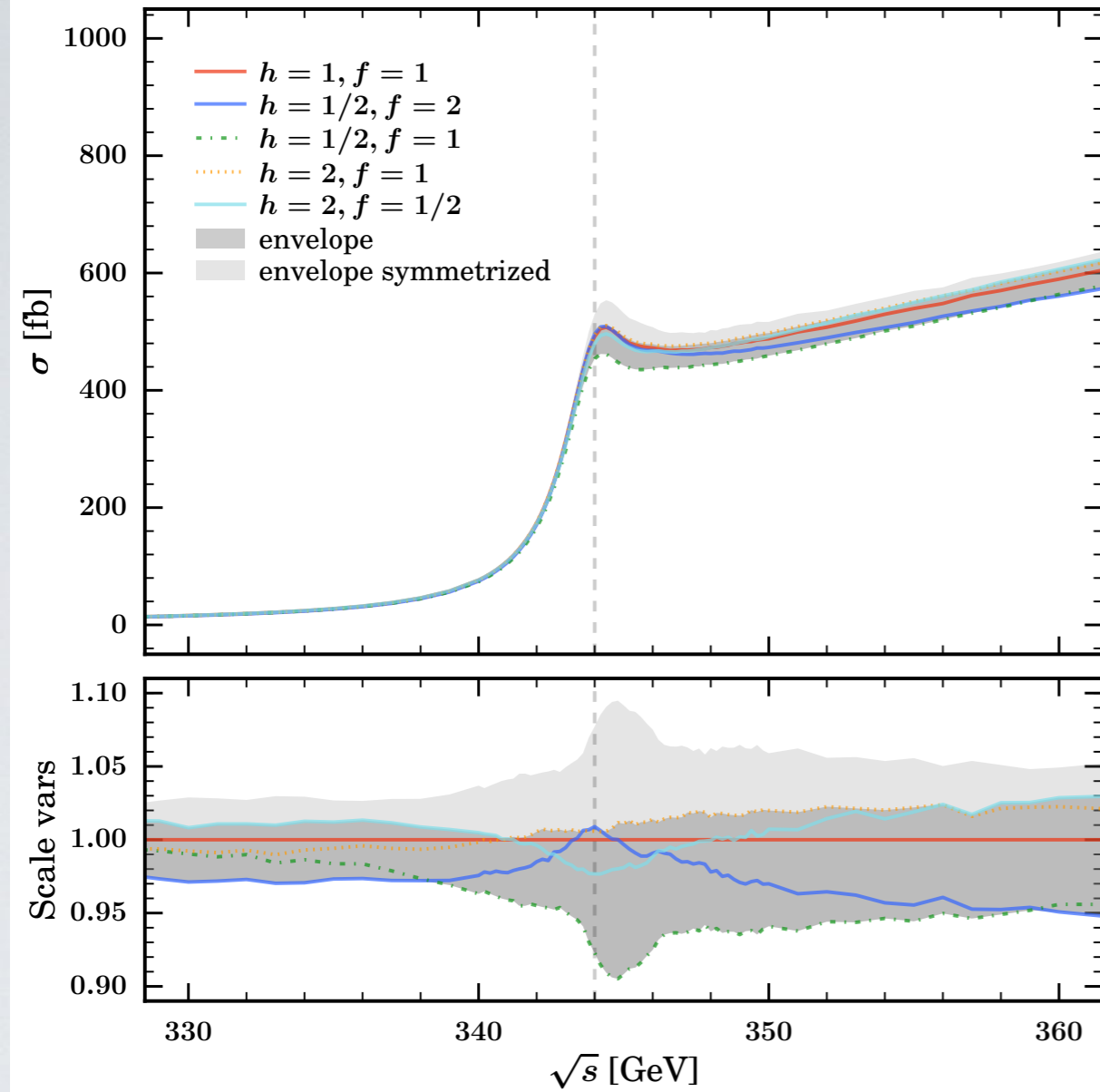
$$\sigma_{\min} = \min \left[ \min_{i \in \text{HF}} \sigma_i, \sigma_0 - (\max_{i \in \text{HF}} \sigma_i - \sigma_0) \right]$$



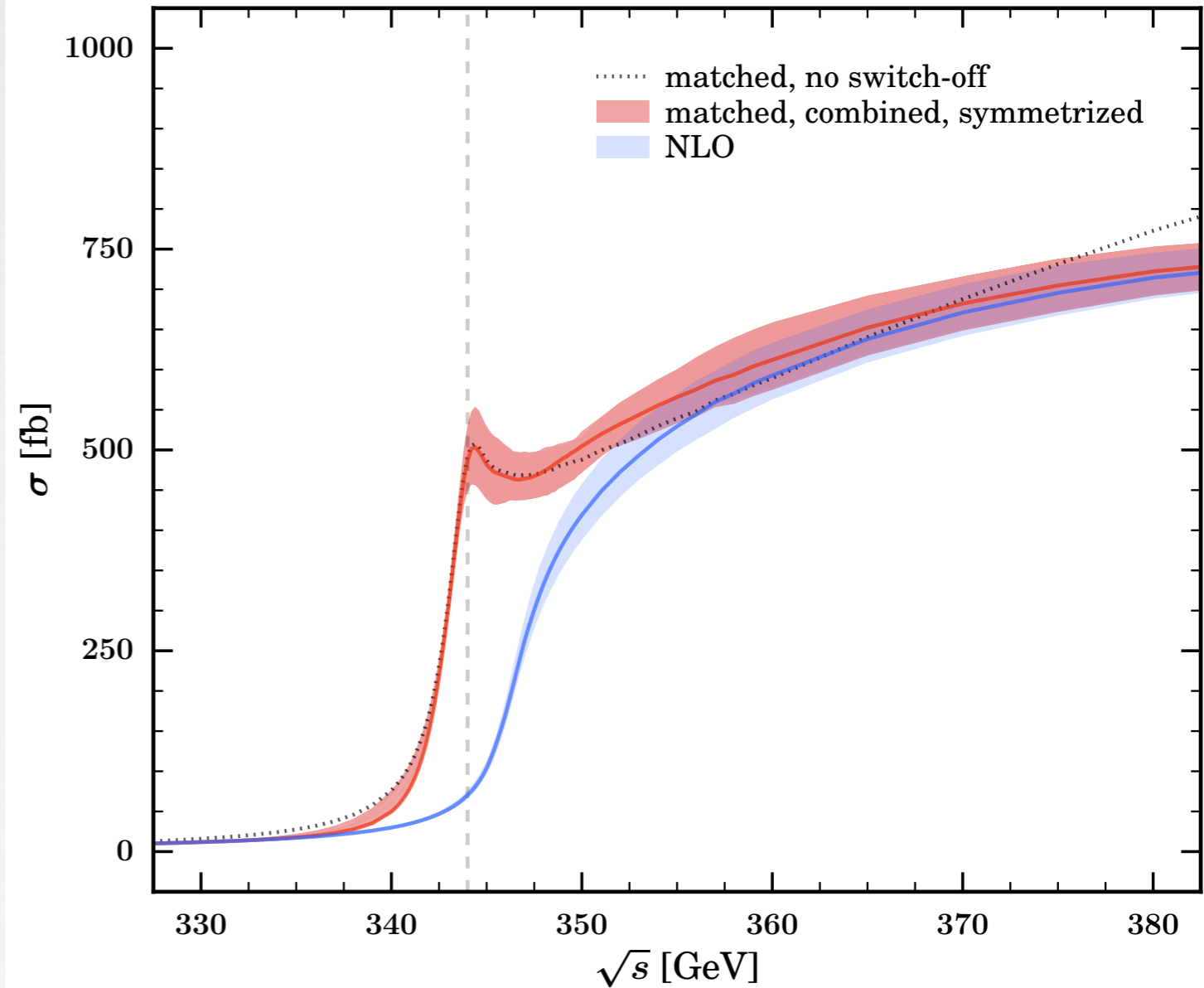


# Threshold matching with QED ISR

matched, no switch-off

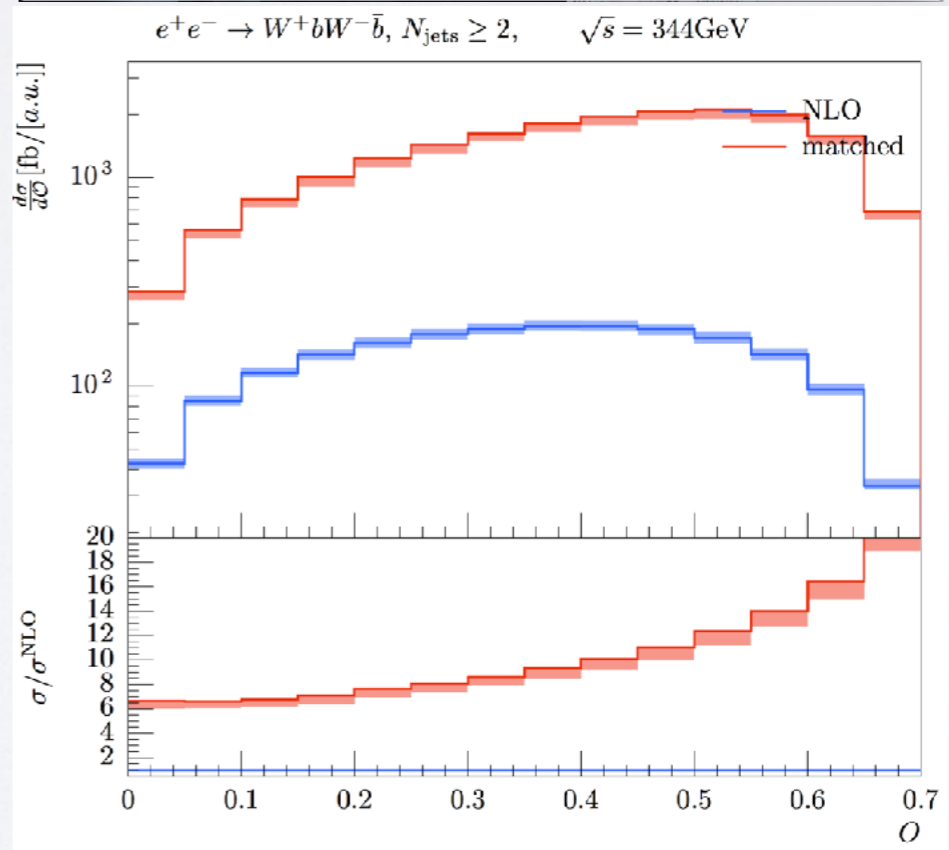
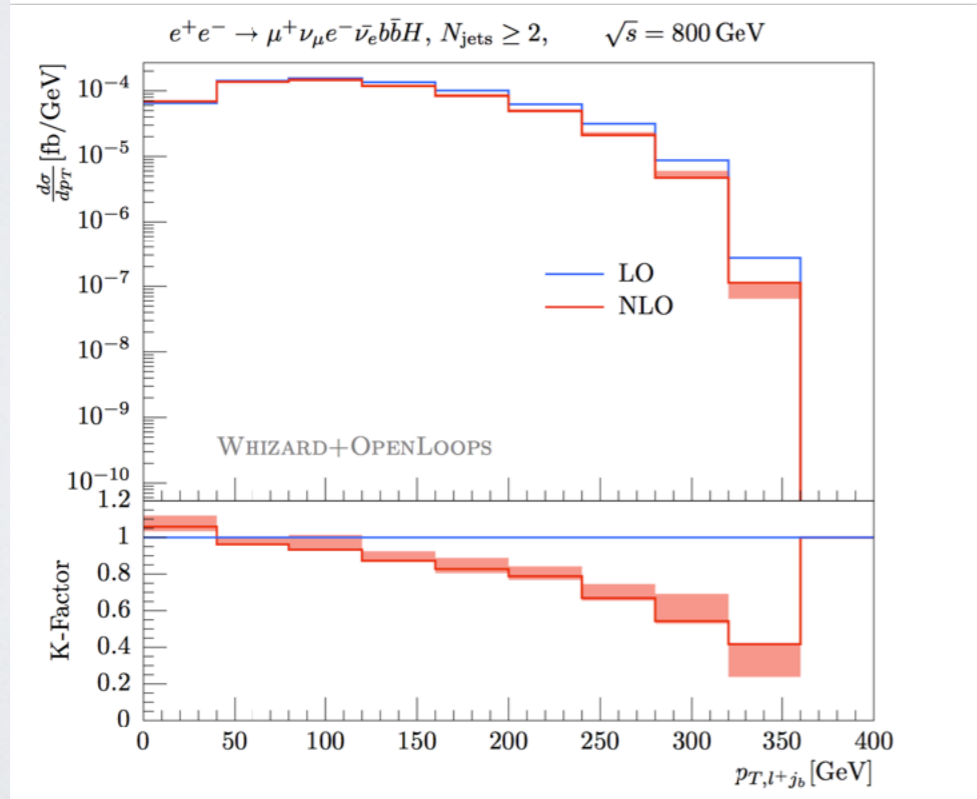
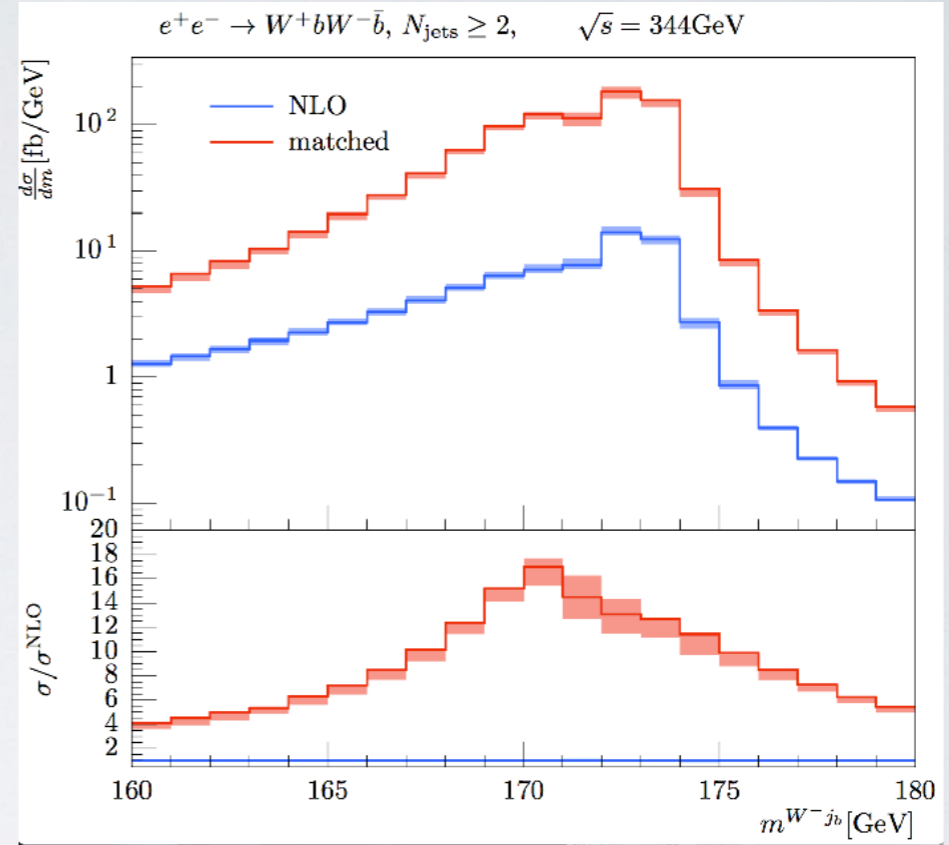
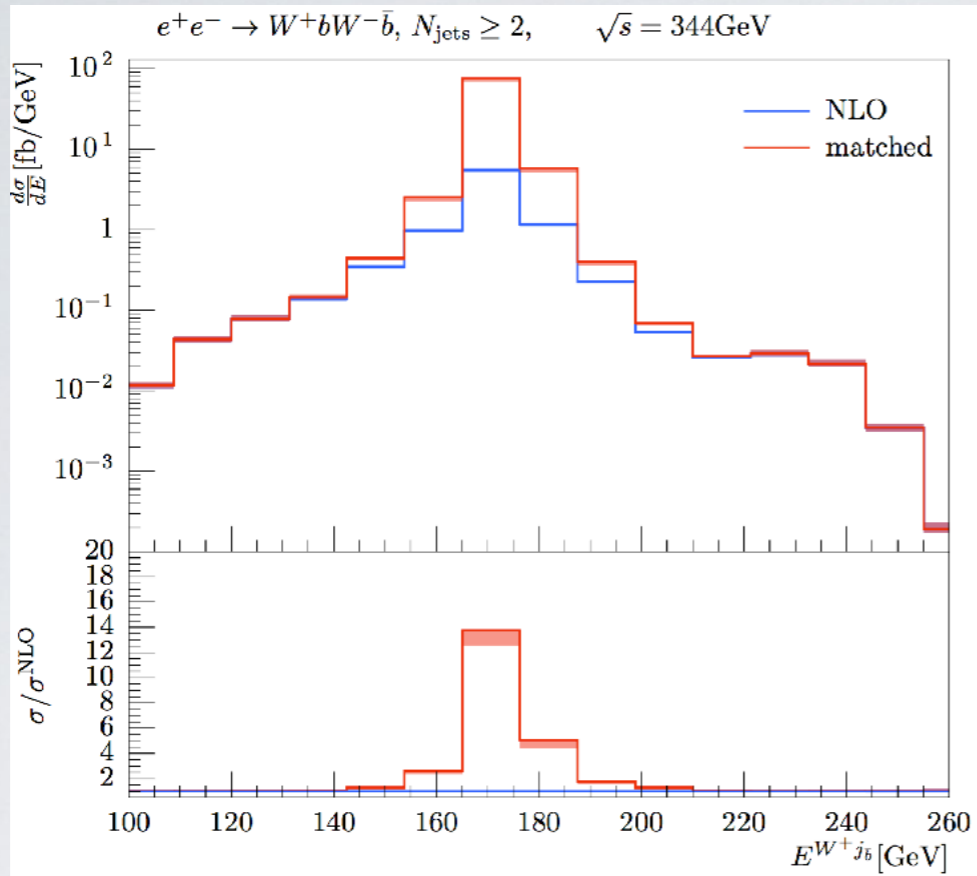


Matched inclusive  $W^+bW^-b$  cross section, with QED ISR





# Matched threshold differential distributions

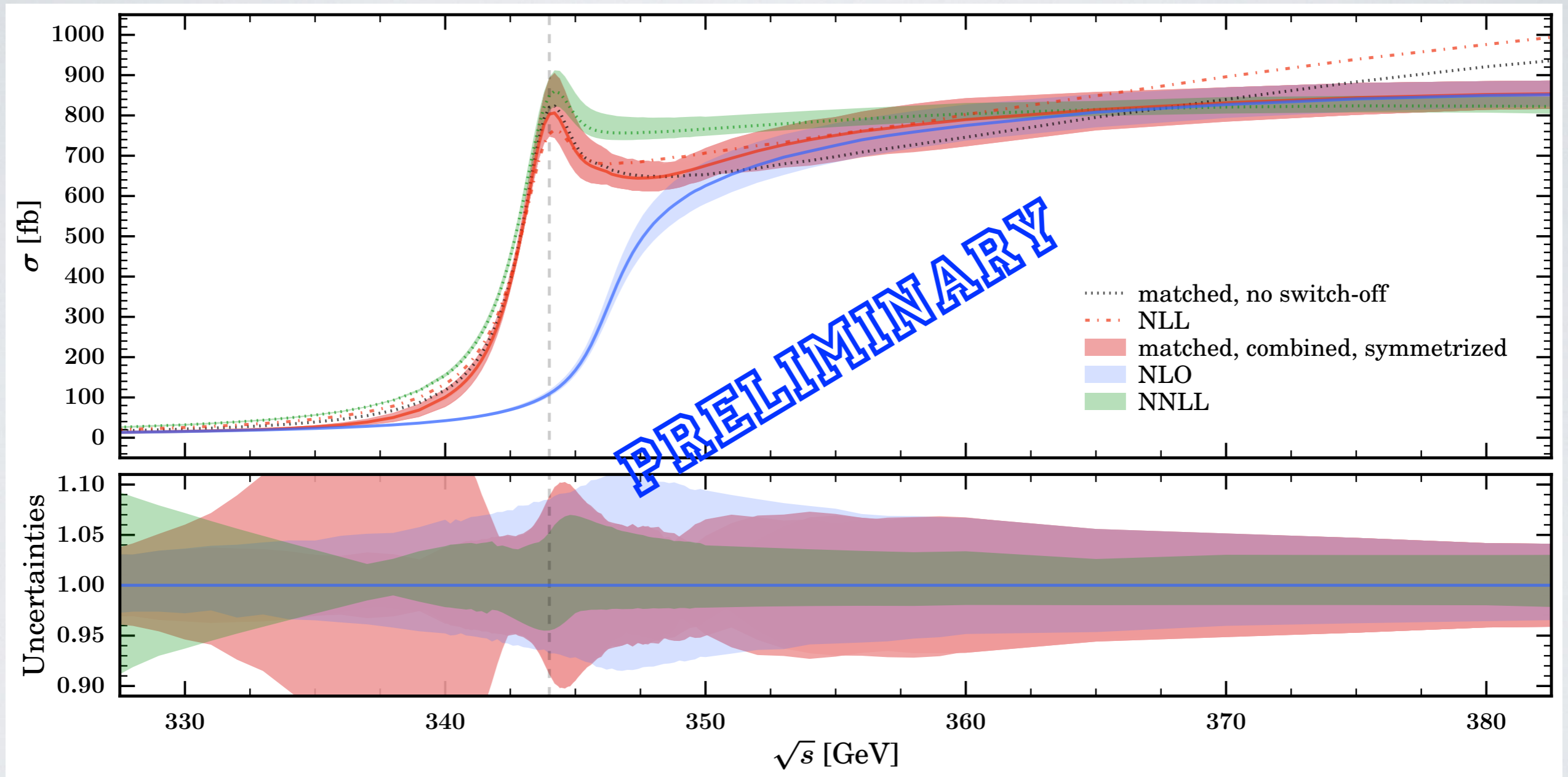




# Summary & Outlook

Next steps: higher QCD order, EW corrections (ISR matching!!), soft gluons ... ..

$$e^+e^- \rightarrow W^+bW^-\bar{b}$$







- Top physics is cornerstone of future lepton collider program
- **Leptonic top fully off-shell at NLO QCD**
- Inclusive processes: off-shell background grows with energy

- **Complete  $v$ NRQCD threshold / QCD-NLO continuum matching**
- **Offers framework for new differential top mass measurements**
- Can in principle be reweighted to NNNLO QCD accuracy at threshold
- Next steps: EW corrections, semi-leptonic/hadronic top decays,  $ttH$  threshold matching, top threshold matched with EW corrections

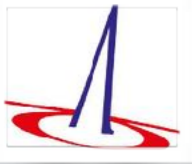
- WHIZARD 2.6 framework for automated (QCD) NLO
- **NLO QCD** (almost) done → WHIZARD 3.0 [EW in validation]



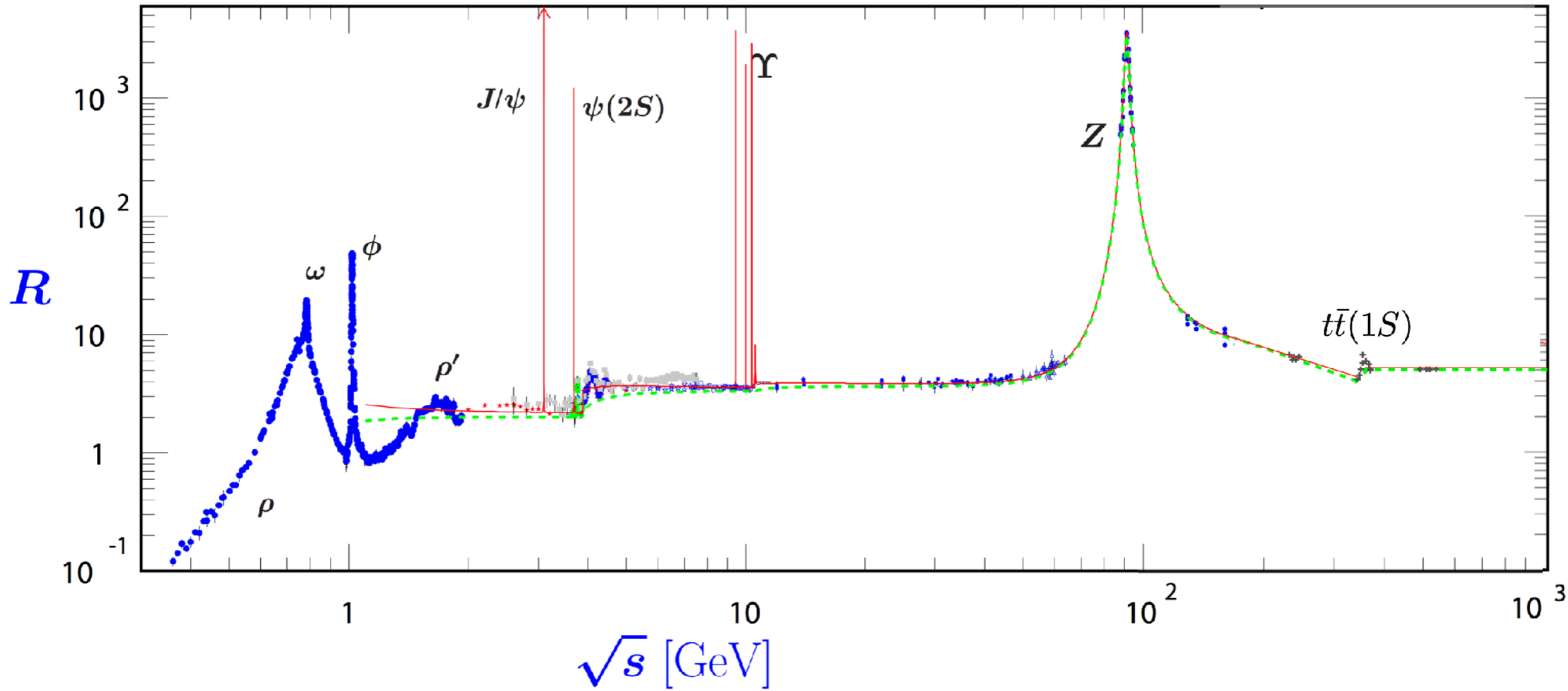


# Outlook to PDG 203x:





# Outlook to PDG 203x:





# BACKUP





# tt continuum production (on- & off-shell)

- Paradigm processes at lepton colliders: **precision determination of top properties**
- Major background for EW measurements (VVV and VBS); any [most] BSM searches

On-Shell process:  $e^+e^- \rightarrow t\bar{t}$

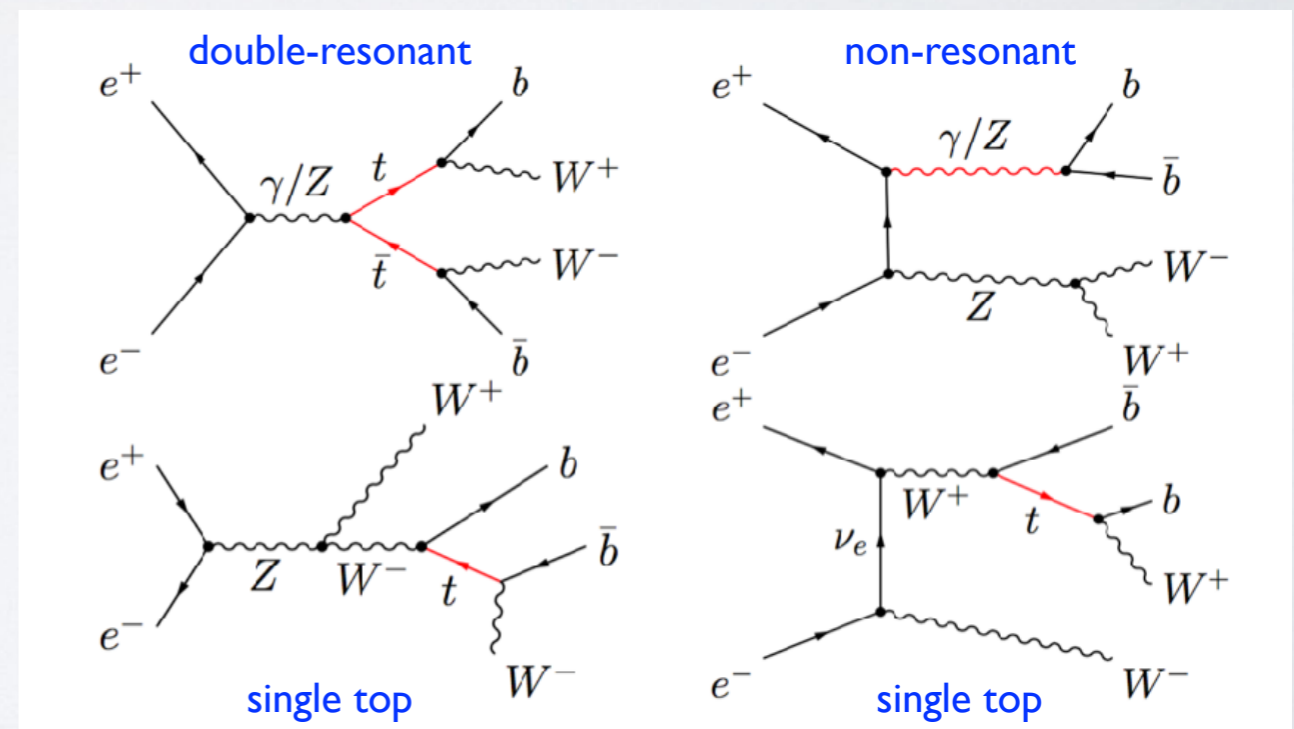
- NLO QCD [Jersak/Laermann/Zerwas, 1982]
- NNLO QCD [Chetyrkin/Kühn/Steinhauser, 1996; Harlander/Steinhauser, 1998]
- NLO EW [Beenakker/von der Marck/Hollik, 1991; Beenakker/Denner/Kraft, 1993]
- Threshold enhancement [Fadin/Khoze, 1987; Strassler/Peskin, 1991; Jezabek/Kühn/Teubner, 1992; Sumino et al., 1992]

Top width:  $t \rightarrow W^+b$

- NLO QCD [Jezabek/Kühn, 1989]
- NNLO QCD [Guo/Li/Zhu, 2012]

Off-Shell process:  $e^+e^- \rightarrow W^+\bar{b}W^-b$

- NLO QCD [Guo/Ma/Wang/Zhang, 2008] ✗
- NLO QCD diff. [Chokoufe/JRR/Weiss, 2015; Liebler/Moortgat-Pick/Papanastasiou, 2015; Chokoufe/Kilian/Lindert/JRR/Pozzorini/Weiss, 2016]





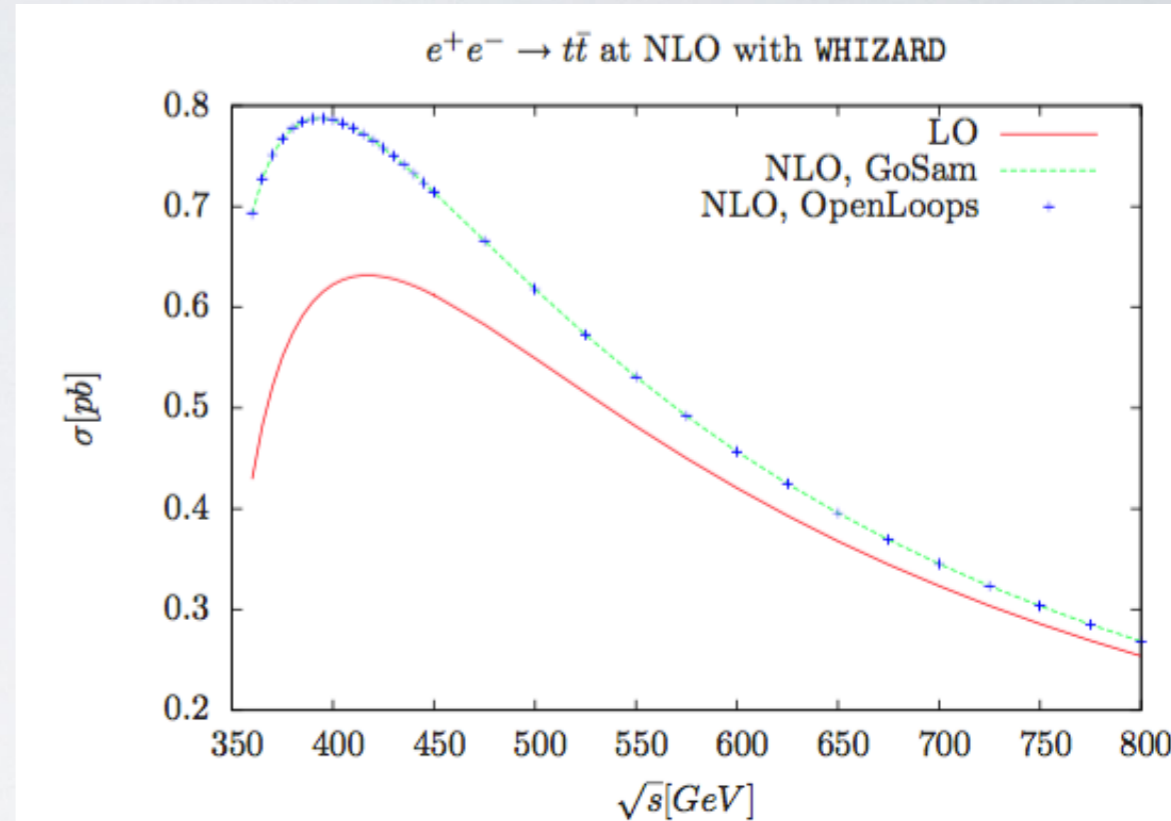
# NLO Automation in WHIZARD

Working NLO interfaces to:

- ★ GoSam [N. Greiner, G. Heinrich, J. v. Soden-Fraunhofen et al.]
- ★ OpenLoops [F. Cascioli, J. Lindert, P. Maierhöfer, S. Pozzorini]
- ★ RecoLa [A. Denner, L. Hofer, J.-N. Lang, S. Uccirati]

```
alpha_power = 2
alphas_power = 0

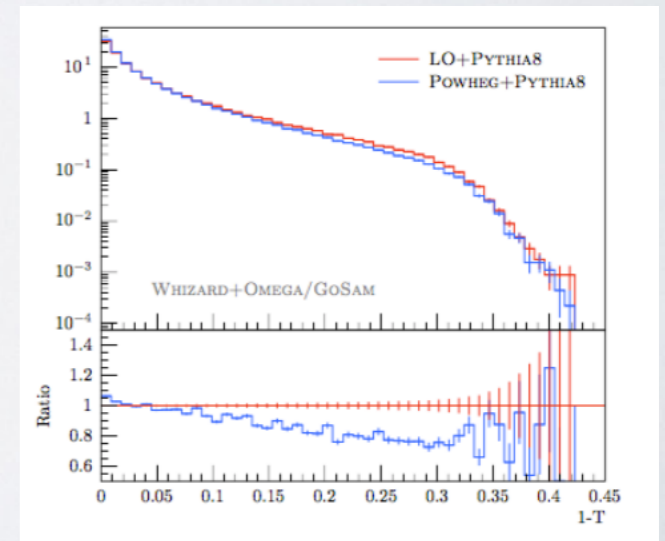
process eett = e1,E1 => t, tbar
{ nlo_calculation = "full" }
```



- FKS subtraction [Frixione/Kunszt/Signer, 1995]
- Resonance-aware treatment [Ježo/Nason, 1509.09071]
- Virtual MEs external
- Real and virtual subtraction terms internal
- NLO decays available for the NLO processes
- Fixed order events for plotting (weighted, either LHEF or HepMC)
- Automated POWHEG damping and matching
- NLO QCD (massless & massive emitters) fully supported

Status of EW corrections: all parts

technically completed, validation phase started [Rothe et al.]



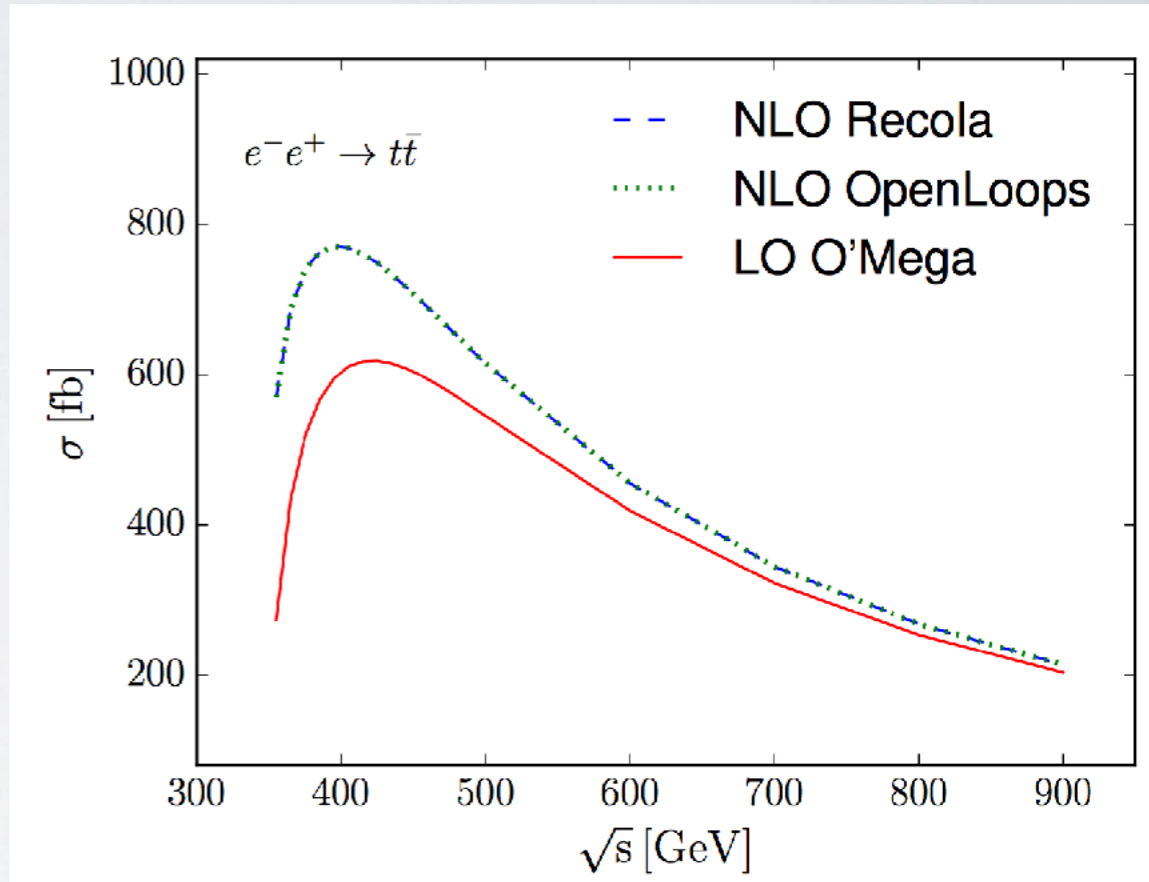


## Working NLO interfaces to:

- ★ GoSam [N. Greiner, G. Heinrich, J. v. Soden-Fraunhofen et al.]
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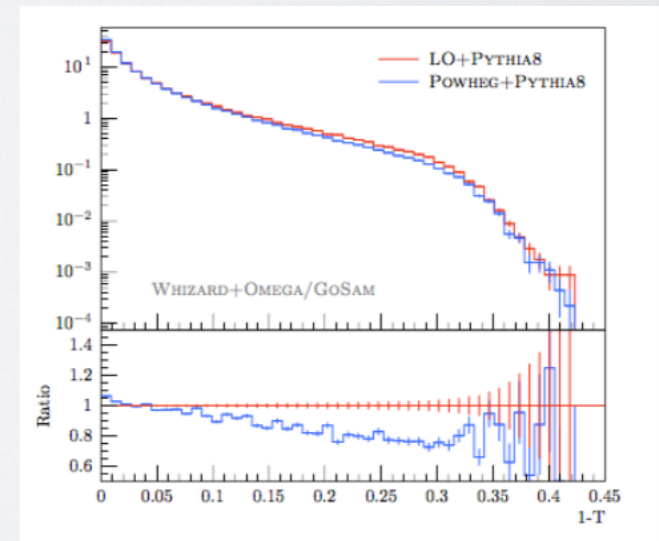
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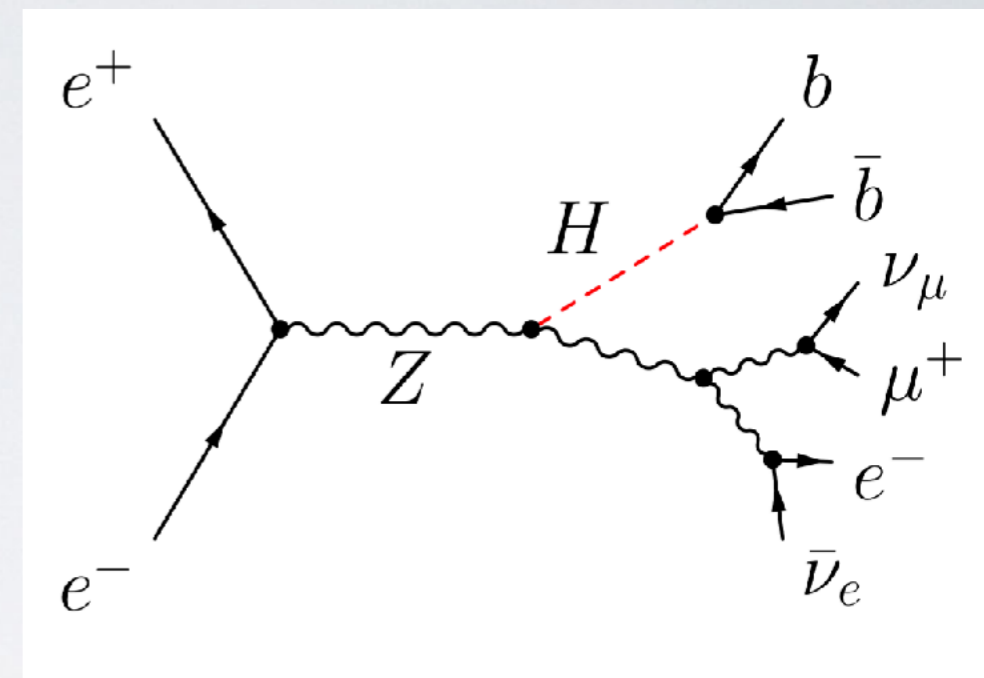


- Amplitudes (except for pure QCD/QED) contain resonances ( $Z, W, H, t$ )
- In general: resonance masses *not* respected by modified kinematics of subtraction terms
- Algorithm to include resonance histories  
[Ježo/Nason, 1509.09071]
- Most important for narrow resonances ( $H \rightarrow bb$ )
- Additional soft mismatch integration component

$$\begin{aligned} \blacktriangleright D_H^{\text{Born}} &= \left[ (\bar{p}_{bb}^2 - m_H^2)^2 + m_H^2 \Gamma_H^2 \right]^{-1}, \\ \blacktriangleright D_H^{\text{Real}} &= \left[ (p_{bbg}^2 - m_H^2)^2 + m_H^2 \Gamma_H^2 \right]^{-1} \end{aligned}$$

$$p_{bbg}^2 = \bar{p}_{bb}^2 + \Delta_{bbg}^2$$

$$\frac{D_H^{\text{Born}}}{D_H^{\text{Real}}} \xrightarrow{\bar{p}_{bb}^2 \rightarrow m_H^2} 1 + \frac{\Delta_{bbg}^4}{m_H^2 \Gamma_H^2}$$





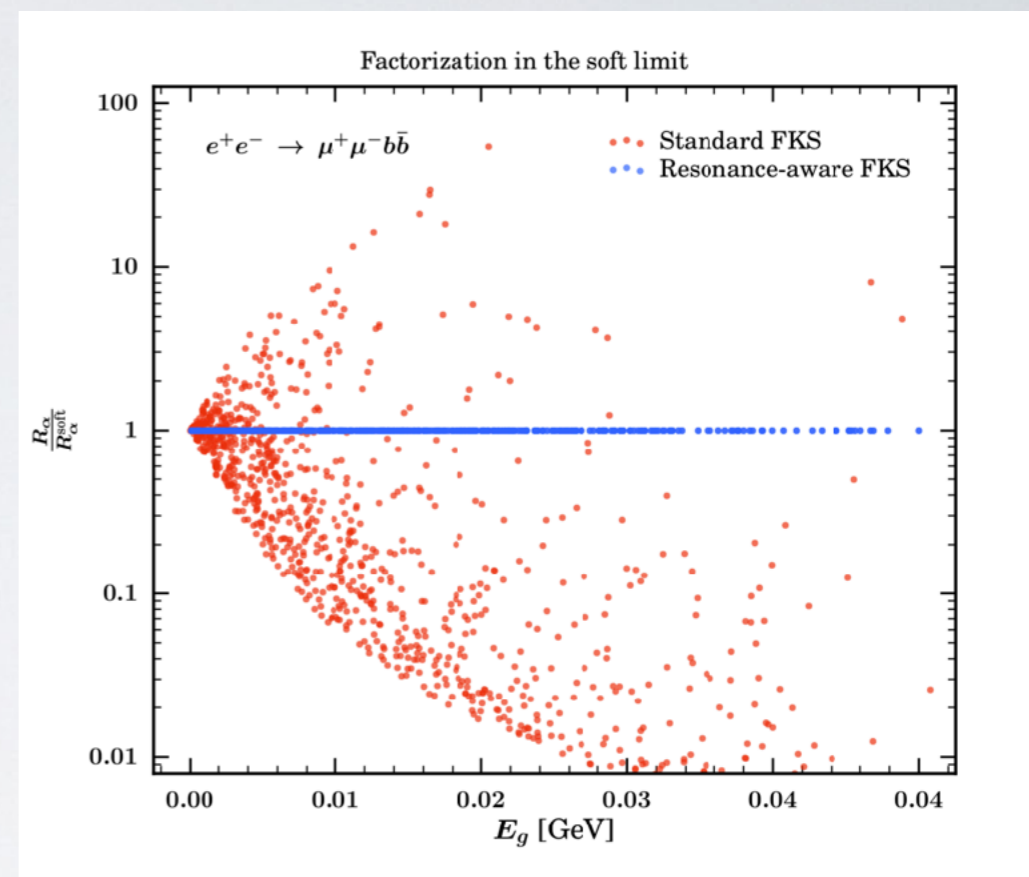


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# Resonance mappings for NLO processes

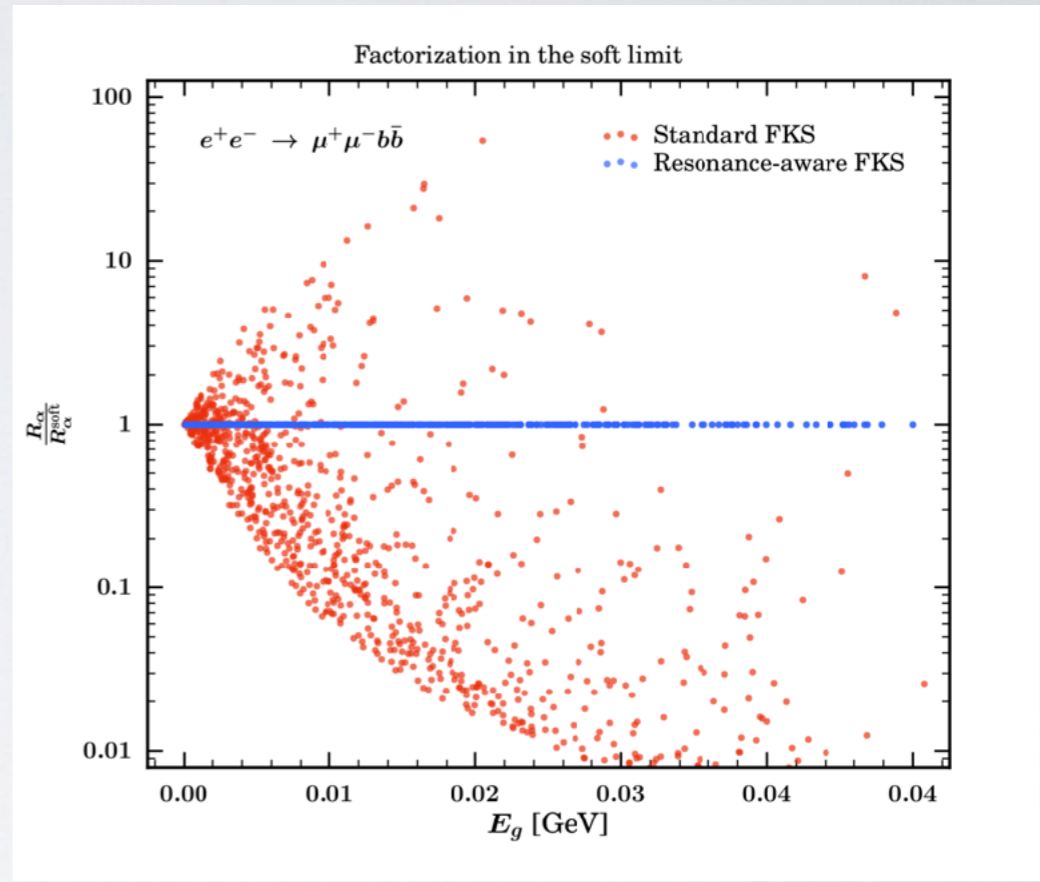
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WHIZARD complete automatic implementation: example  $e^+ e^- \rightarrow \mu\mu bb$  ( $ZZ, ZH$  histories)

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	9.6811847E+00	6.42E+00	66.30	72.60*	0.65		
2	11959	2.8539703E+00	2.35E-01	8.25	9.02*	0.69		
3	11936	2.4907574E+00	6.54E-01	26.25	28.68	0.35		
4	11908	2.7695559E+00	9.67E-01	34.91	38.09	0.30		
5	11874	2.4346151E+00	4.82E-01	19.80	21.57*	0.74		
5	59665	2.7539078E+00	1.97E-01	7.15	17.47	0.74	0.49	5

standard FKS





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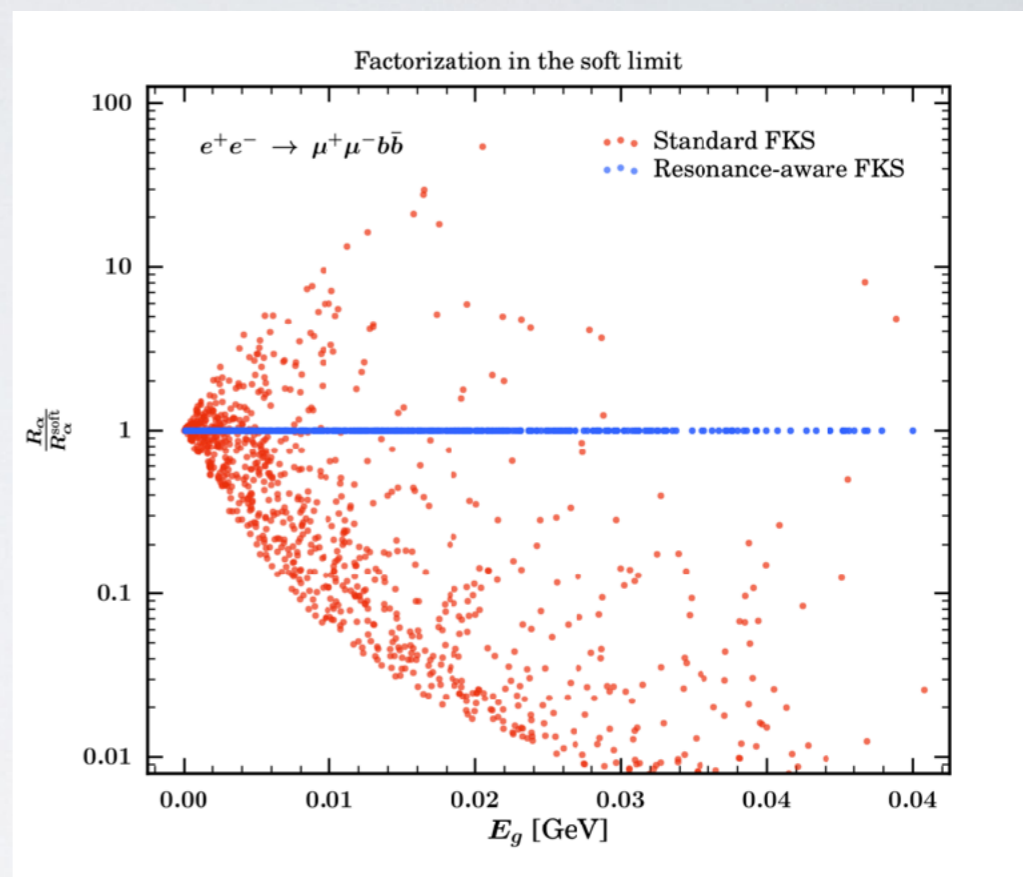
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standard FKS

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	2.9057032E+00	8.35E-02	2.87	3.15*	7.90		
2	11962	2.8591952E+00	5.20E-02	1.82	1.99*	10.91		
3	11936	2.9277880E+00	4.09E-02	1.40	1.52*	14.48		
4	11902	2.8512337E+00	3.98E-02	1.40	1.52*	13.70		
5	11874	2.8855399E+00	3.87E-02	1.34	1.46*	17.15		
5	59662	2.8842006E+00	2.04E-02	0.71	1.72	17.15	0.53	5

FKS with resonance mappings





$$A_{FB} = \frac{\sigma(\cos \theta_t > 0) - \sigma(\cos \theta_t < 0)}{\sigma(\cos \theta_t > 0) + \sigma(\cos \theta_t < 0)}$$

Gluon emission symmetric in  $\theta \Rightarrow$   
NLO QCD corrections small

## $A_{FB}$ of the top quark

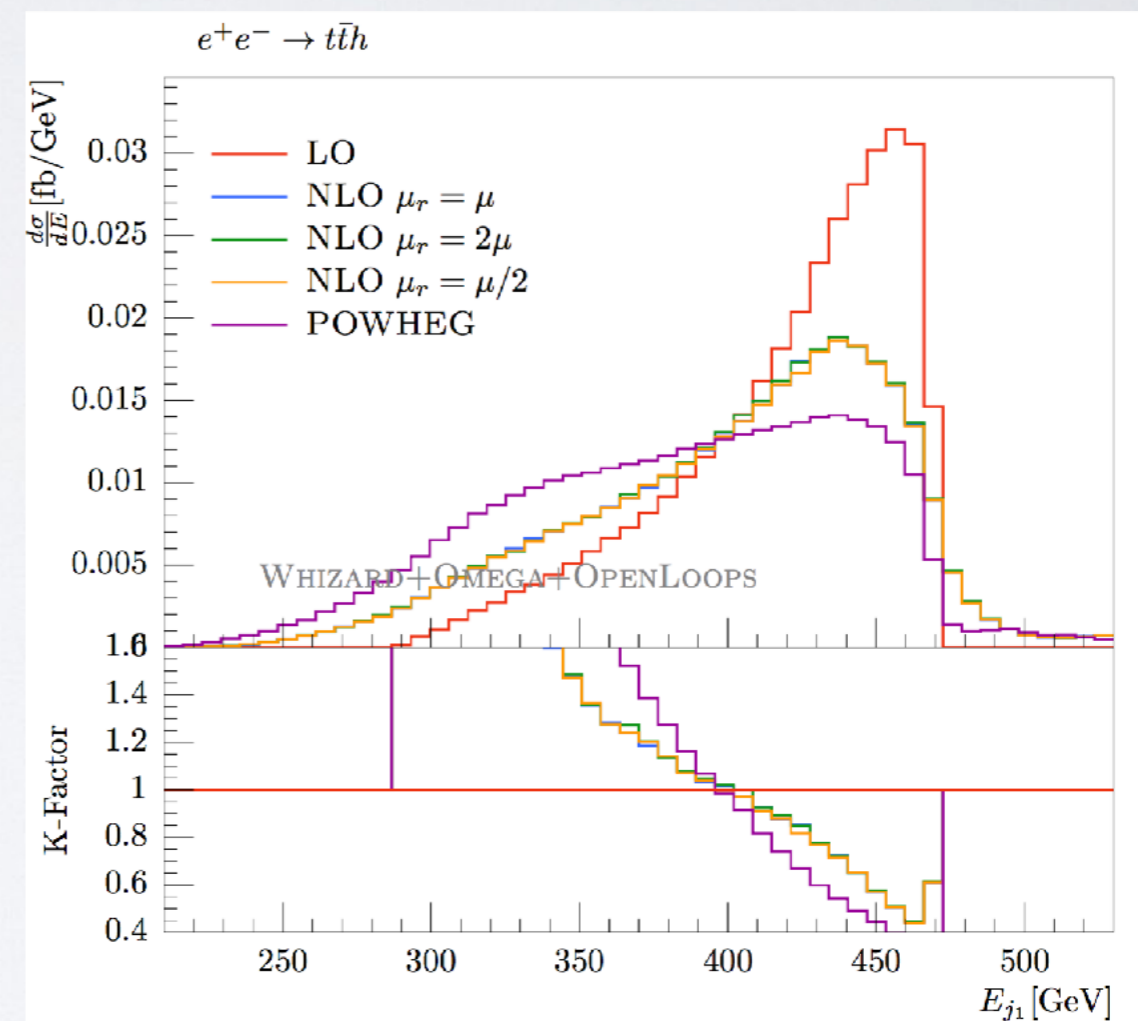
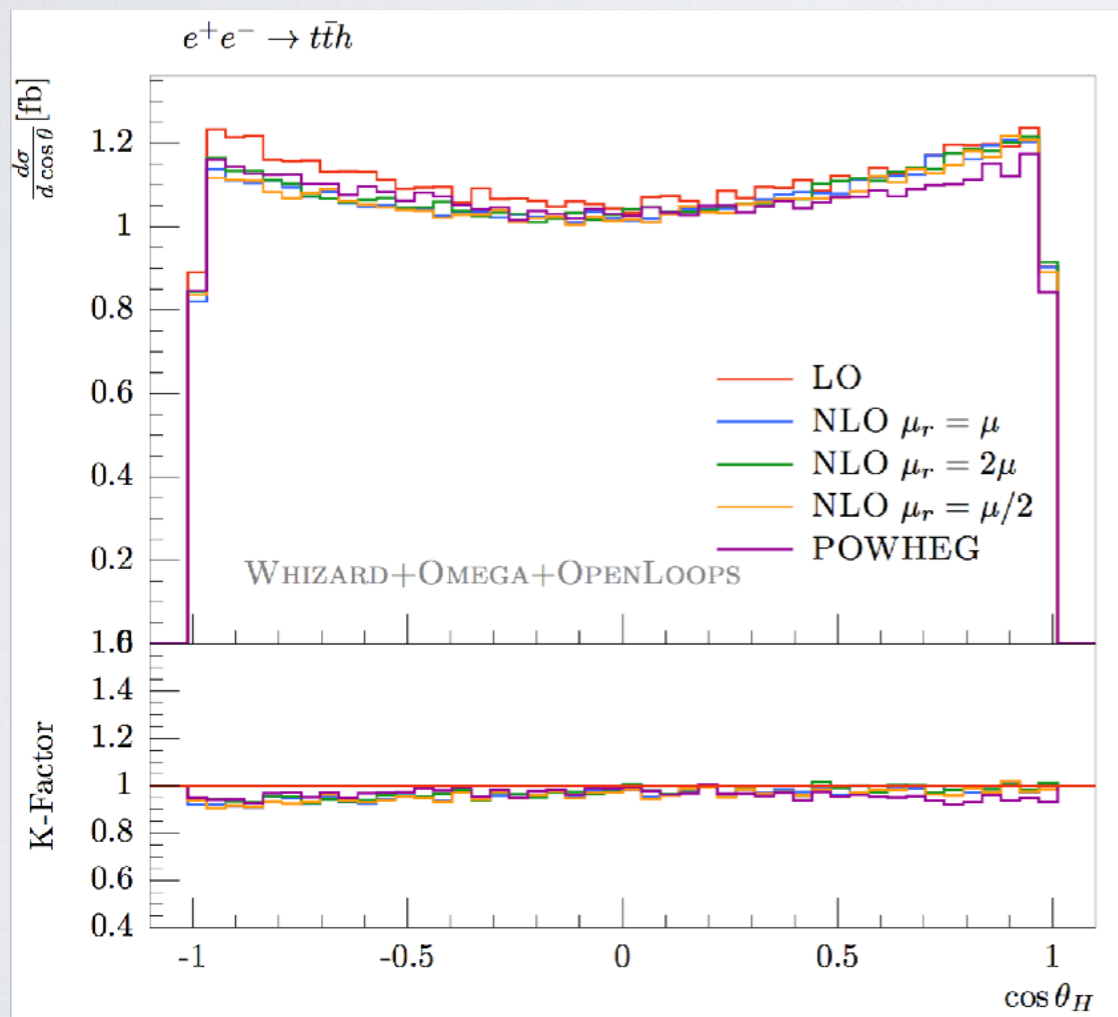
	$e^+e^- \rightarrow$	$A_{FB}^{LO}$	$A_{FB}^{NLO}$	$A_{FB}^{NLO} / A_{FB}^{LO}$
$A_{FB}$	$t\bar{t}$	-0.535	-0.539	1.013
	$W^+W^-b\bar{b}$	-0.428	-0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	-0.415	-0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$ , without neutrinos	-0.402	-0.387	0.964
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	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	0.415	0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$ , without neutrinos	0.377	0.350	0.928



- Precise predictions of multi-parton final states require properly matched samples
- NLO QCD including POWHEG matching already available [WHIZARD+OpenLoops]
- All descriptions at NLO at the moment for the on-shell process
- Even LO simulations are demanding, e.g.:  $e^+e^- \rightarrow b\bar{b}b\bar{b}jjl\nu_e, \quad b\bar{b}jjjjjjl\nu_e$



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[Chokoufe/JRR/Weiss]