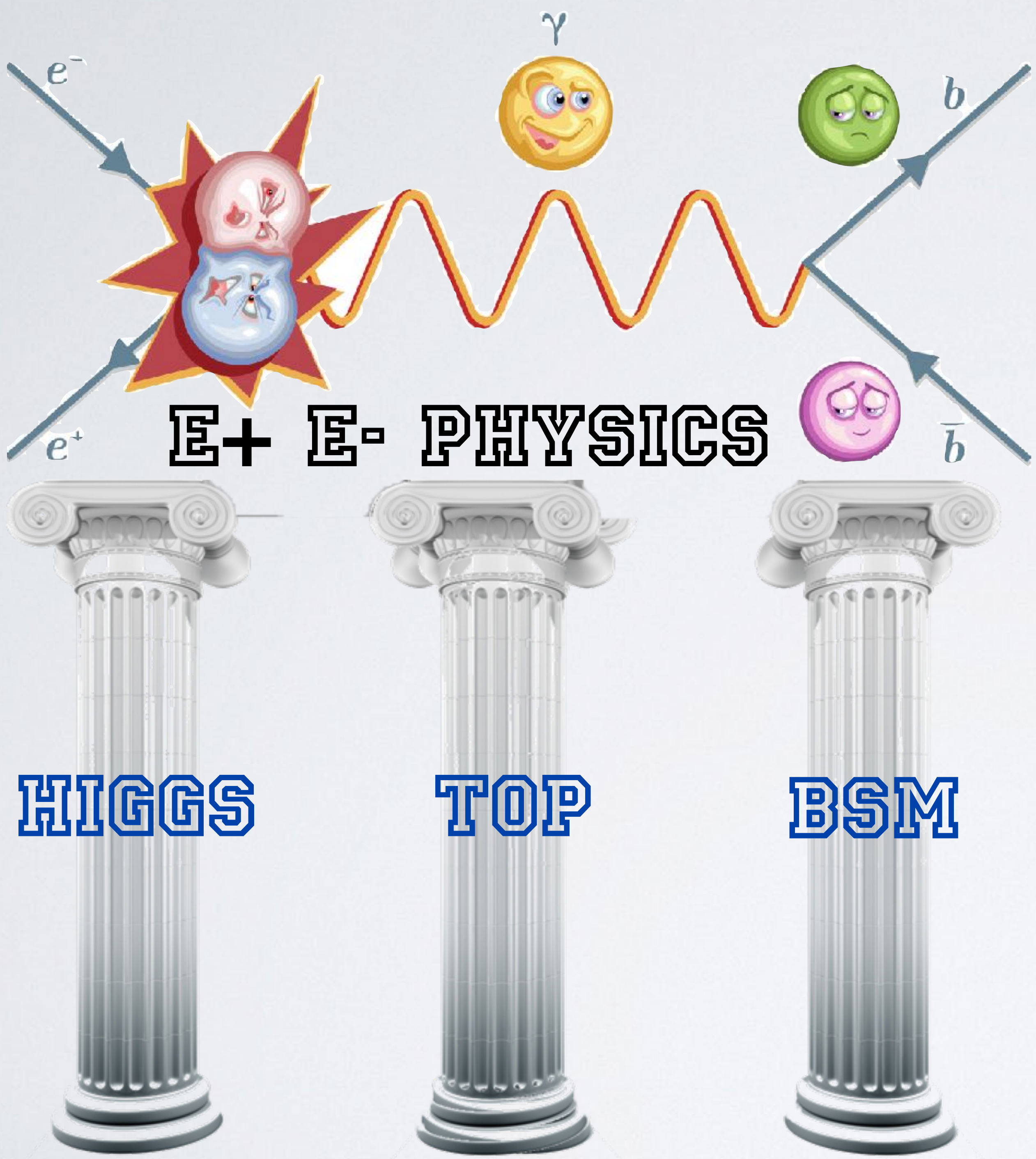
Precision Predictions for Top Physics — Threshold & Continuum

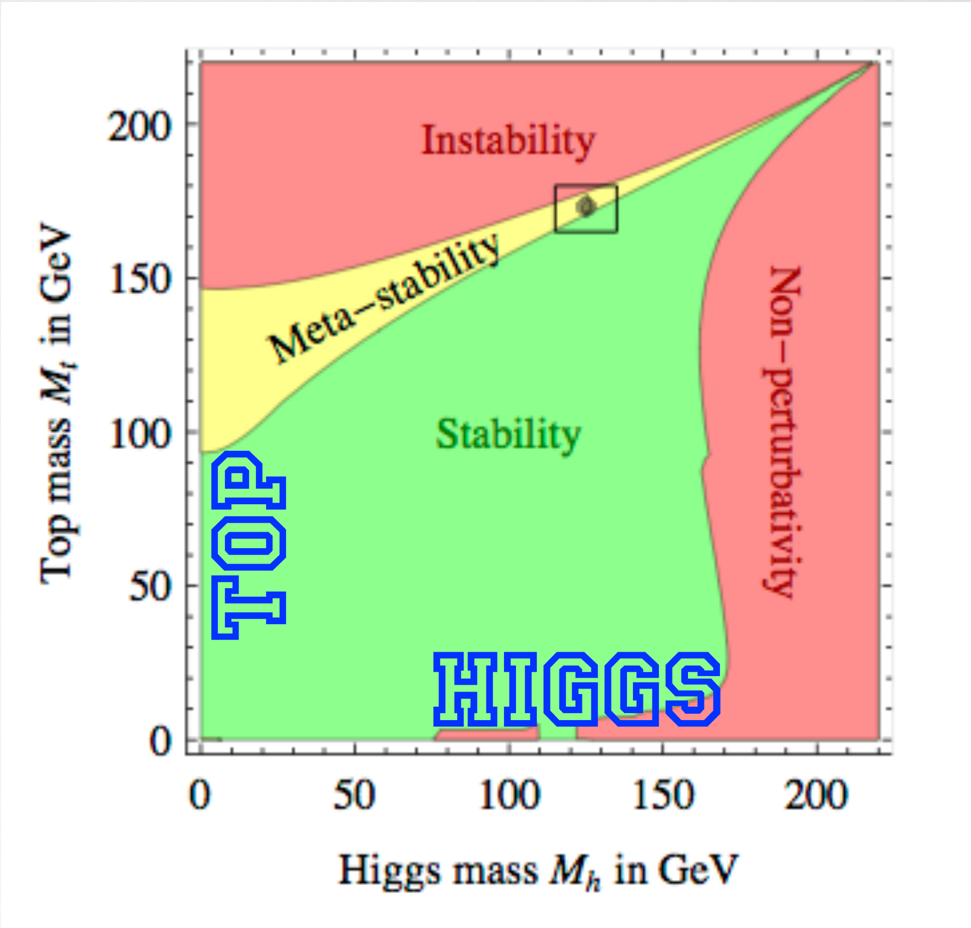
Jürgen R. Reuter, DESY

HELMHOLTZ GEMEINSCHAFT

The importance of top / Higgs



Top mass and top Yukawa coupling



from [Degrassi *et al.*, 2012]

(note: plot under assumptions of NO additional **BSM**)

tt continuum production (on- & off-shell)

- Paradigm processes at lepton colliders: precision determination of top properties
- Major background for EW measurements (*VVV* and VBS); any [most] BSM searches

On-Shell process: $e^+e^- \to t\bar{t}$

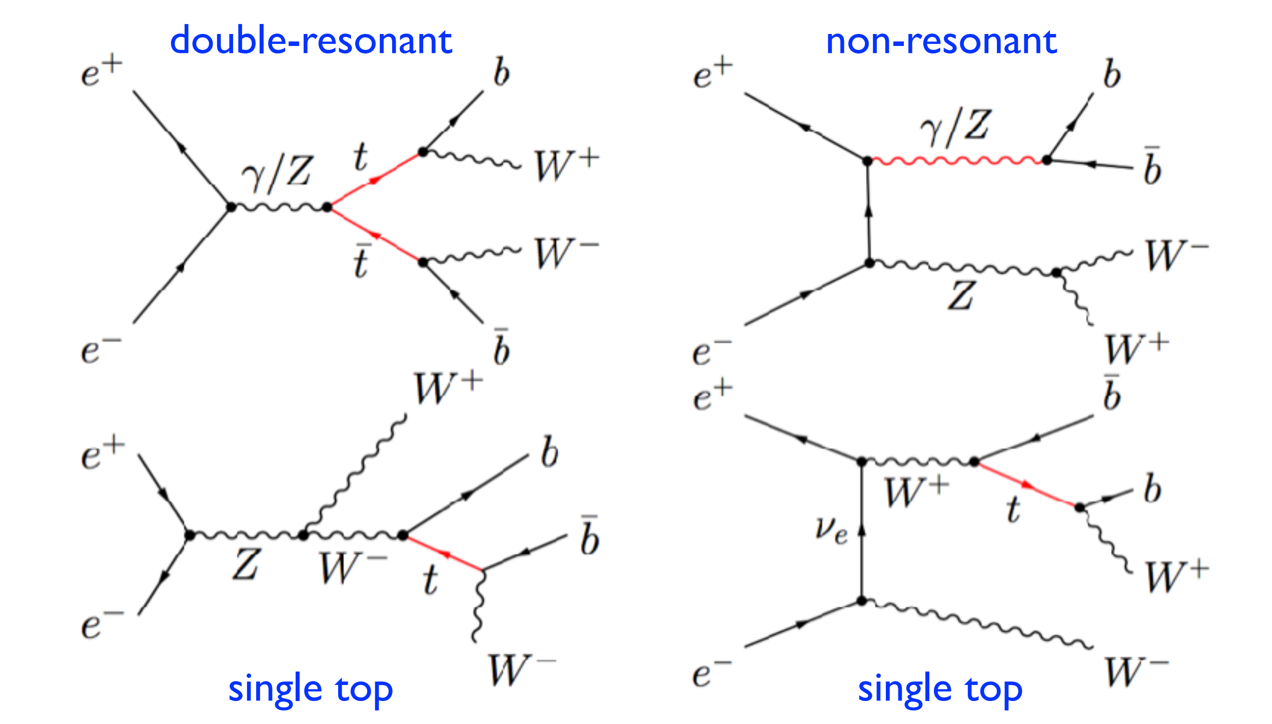
- NLO QCD [Jersak/Laermann/Zerwas, 1982]
- NNLO QCD [Chetyrkin/Kühn/Steinhauser, 1996; Harlander/Steinhauser, 1998]
- NLO EW [Beenakker/von der Marck/Hollik, 1991; Beenakker/Denner/Kraft, 1993]
- Threshold enhancement [Fadin/Khoze, 1987; Strassler/Peskin, 1991; Jezabek/Kühn/Teubner, 1992; Sumino et al., 1992

Top width: $t \to W^+ b$

- NLO QCD [Jezabek/Kühn, 1989]
- NNLO QCD [Guo/Li/Zhu, 2012]

Off-Shell process: $e^+e^- \to W^+\bar{b}W^-b$

- NLO QCD [Guo/Ma/Wang/Zhang, 2008] ✘
- NLO QCD diff. [Chokoufe/JRR/Weiss, 2015; Liebler/Moortgat-Pick/Papanasthasiou, 2015; Chokoufe/Kilian/Lindert/JRR/Pozzorini/Weiss, 2016]



WHIZARD: our MC framework

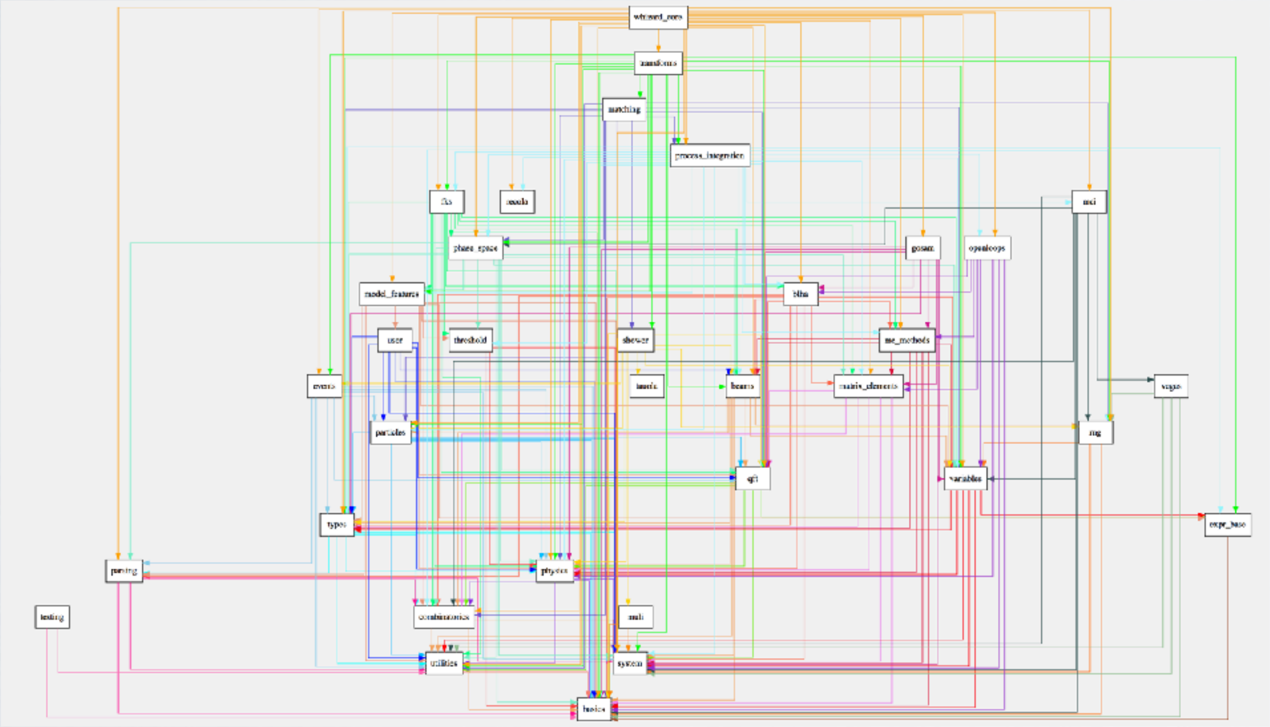
WHIZARD v2.6.1 (03.11.2017) http://whizard.hepforge.org

<whizard@desy.de>

Ancient acronym: ***W**, **Hi**ggs, **Z**, and **R**espective **D**ecays*

WHIZARD Team: *Wolfgang Kilian, Thorsten Ohl, JRR*

Simon Braß/Vincent Rothe/Christian Schwinn/Marco Sekulla/So Young Shim/Florian Staub/Pascal Stienemeier/ Zhijie Zhao + 2 Master



PUBLICATIONS

General WHIZARD reference: EPJ C71 (2011) 1742, arXiv:0708.4241

O'Mega (ME generator): LC-TOOL (2001) 040; arXiv:hep-ph/0102195

VAMP (MC integrator): CPC 120 (1999) 13; arXiv:hep-ph/9806432

CIRCE (beamstrahlung): CPC 101 (1997) 269; arXiv:hep-ph/9607454

Parton shower: JHEP 1204 (2012) 013; arXiv:1112.1039

Color flow formalism: JHEP 1210 (2012) 022; arXiv:1206.3700

NLO capabilities: JHEP 1612 (2016) 075; arXiv: 1609.03390

Parallelization of MEs: CPC 196 (2015) 58; arXiv:1411.3834

POWHEG matching: EPS-HEP (2015) 317; arXiv: 1510.02739

↪ cf. Wolfgang Kilian's talk

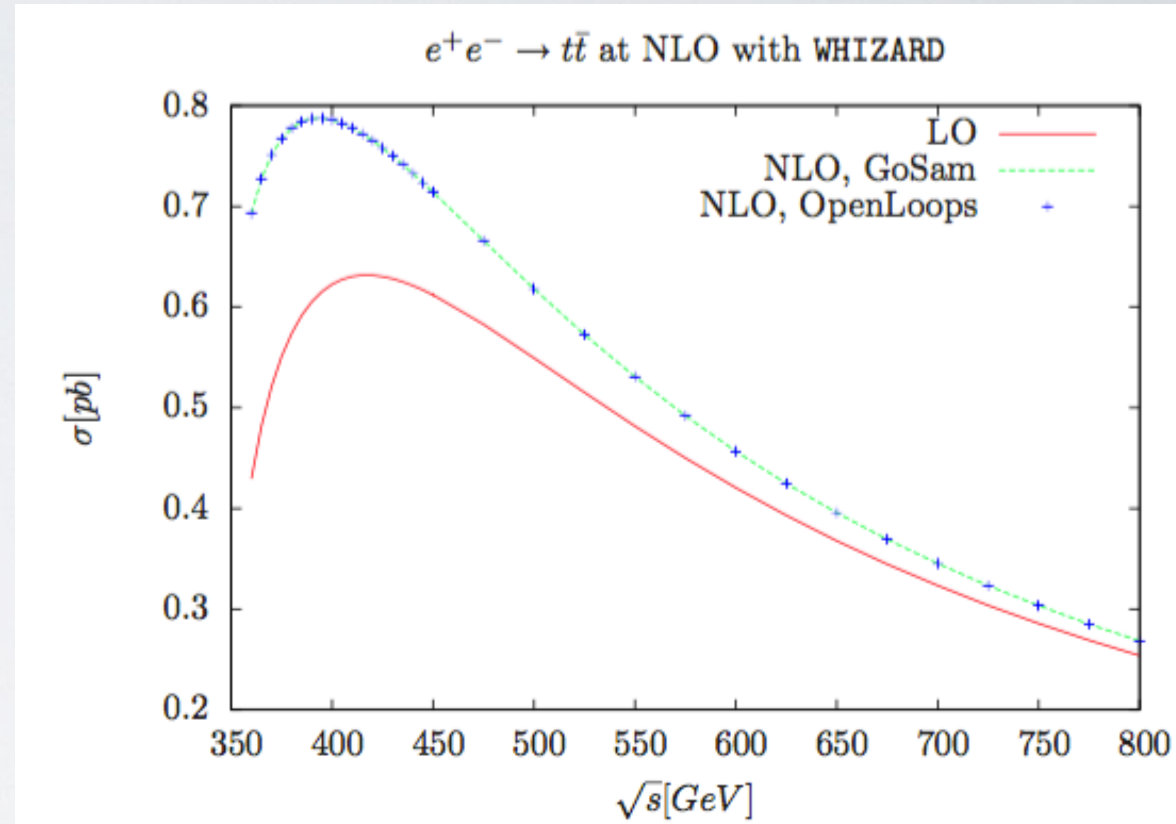
NLO Automation in WHIZARD

Working NLO interfaces to:

- ★ GoSam [N. Greiner, G. Heinrich, J. v. Soden-Fraunhofen et al.]
- ★ OpenLoops [F. Cascioli, J. Lindert, P. Maierhöfer, S. Pozzorini]
- ★ Recola [A. Denner, L. Hofer, J.-N. Lang, S. Uccirati]

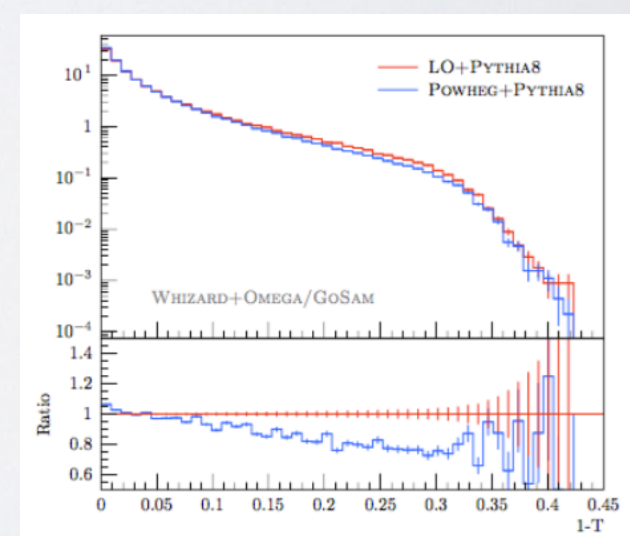
```
alpha_power = 2
alphas_power = 0

process eett = e1,E1 => t, tbar
     { nlo_calculation = “full” }
```



- FKS subtraction [Frixione/Kunszt/Signer, 1995]
- Resonance-aware treatment [Ježo/Nason, 1509.09071]
- Virtual MEs external
- Real and virtual subtraction terms internal
- NLO decays available for the NLO processes
- Fixed order events for plotting (weighted, either LHEF or HepMC)
- Automated POWHEG damping and matching
- NLO QCD (massless & massive emitters) fully supported
- Status of EW corrections: all parts

  technically completed, validation phase started [Rothe et al.]



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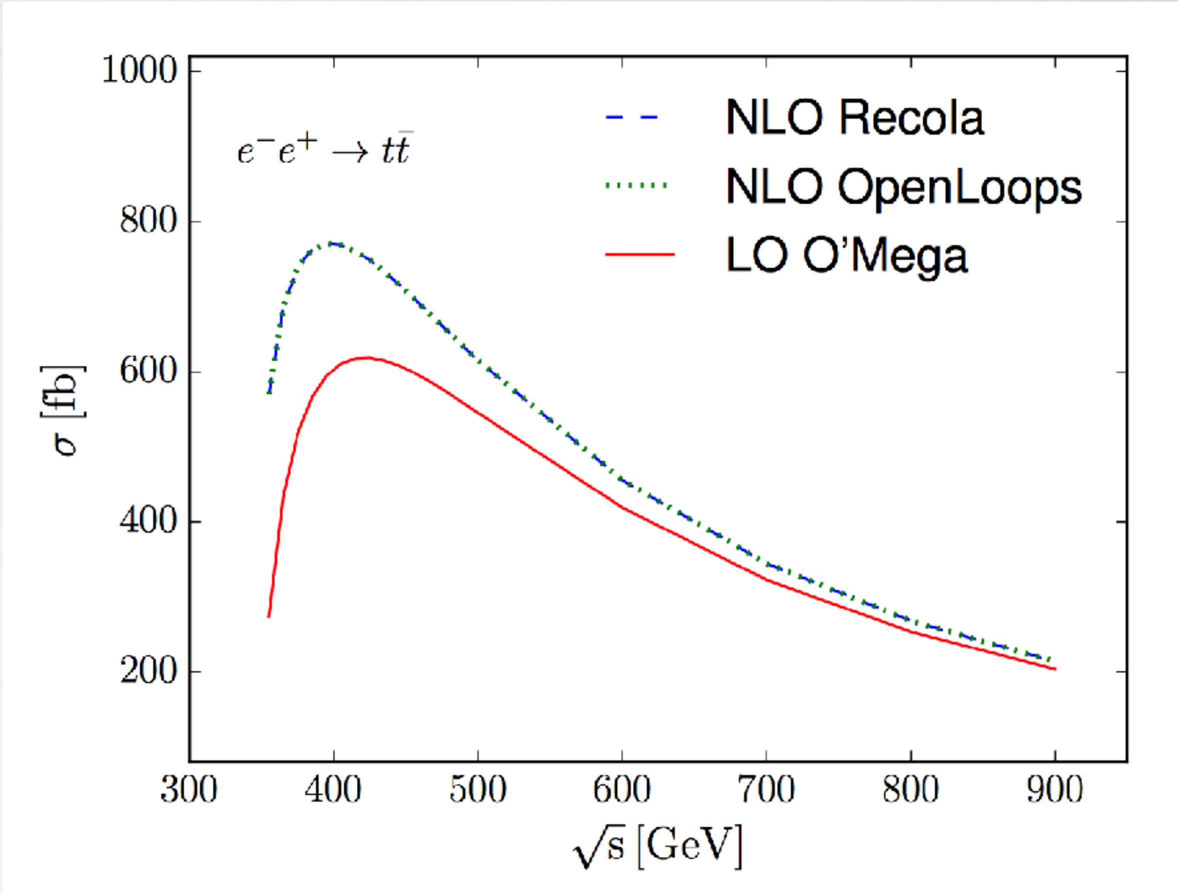

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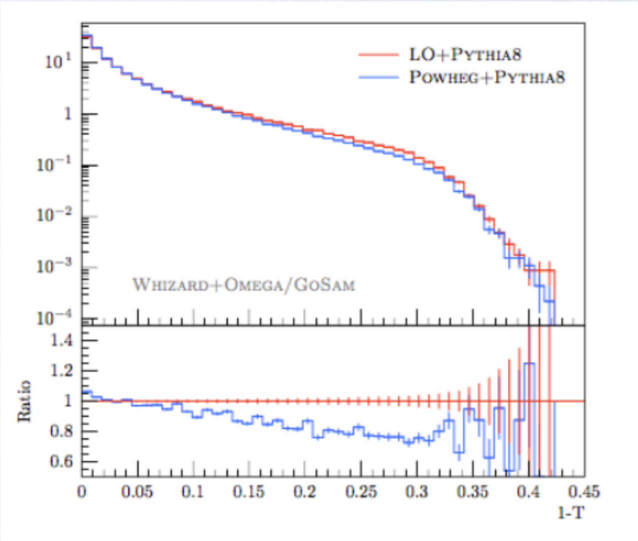

Resonance mappings for NLO processes

- Amplitudes (except for pure QCD/QED) contain resonances (Z, W, H, t)
- In general: resonance masses *not* respected by modified kinematics of subtraction terms
- Algorithm to include resonance histories

[Ježo/Nason, 1509.09071]

- Most important for narrow resonances ($H \to bb$)
- Additional soft mismatch integration component

$$D_H^{\text{Born}} = \left[\left(\bar{p}_{bb}^2 - m_H^2\right)^2 + m_H^2\Gamma_H^2\right]^{-1},$$

$$D_H^{\text{Real}} = \left[\left(p_{bbg}^2 - m_H^2\right)^2 + m_H^2\Gamma_H^2\right]^{-1}$$

$$p_{bbg}^2 = \bar{p}_{bb}^2 + \Delta_{bbg}^2$$

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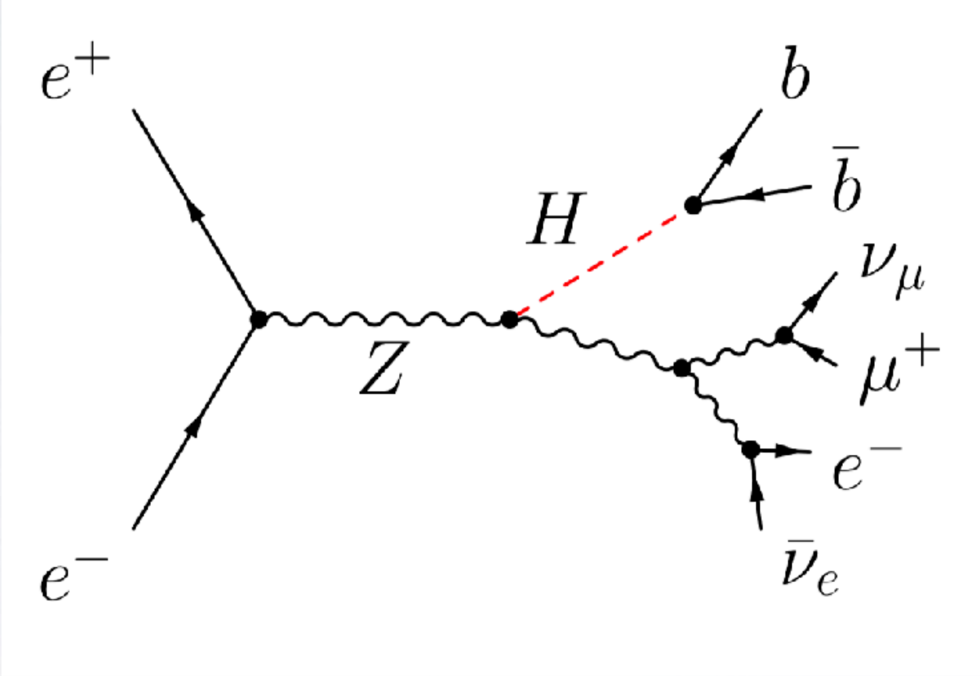

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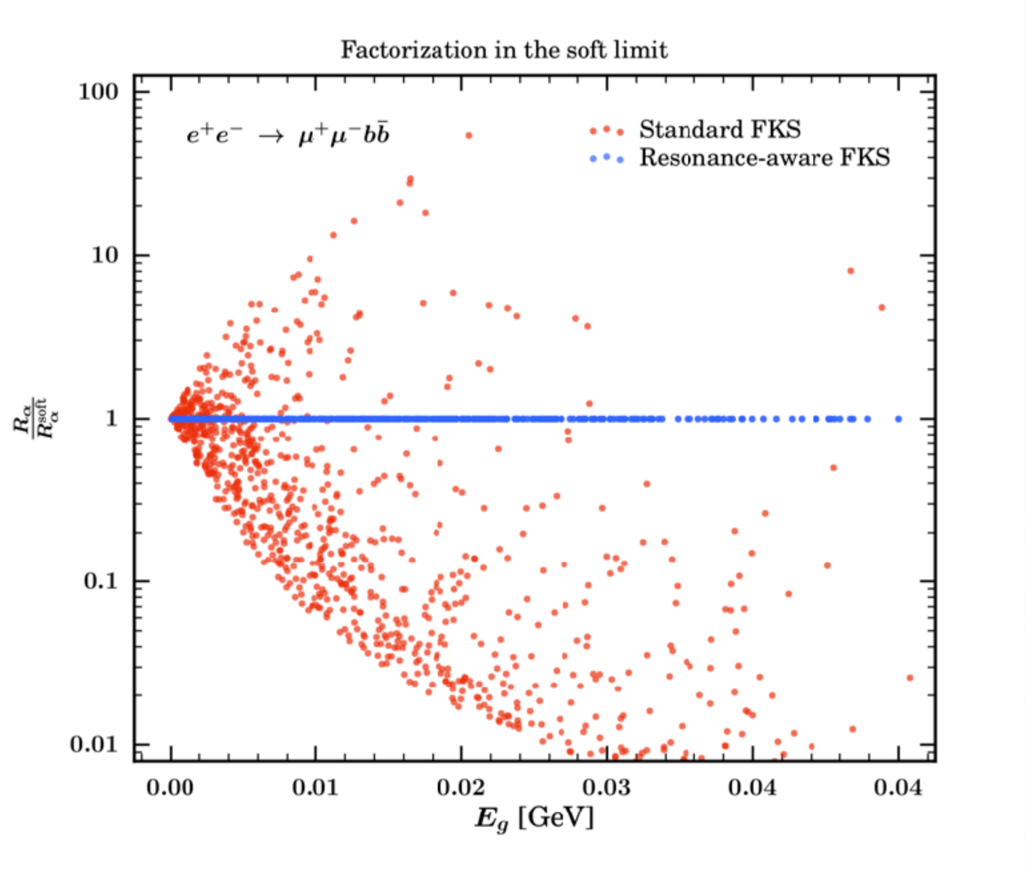

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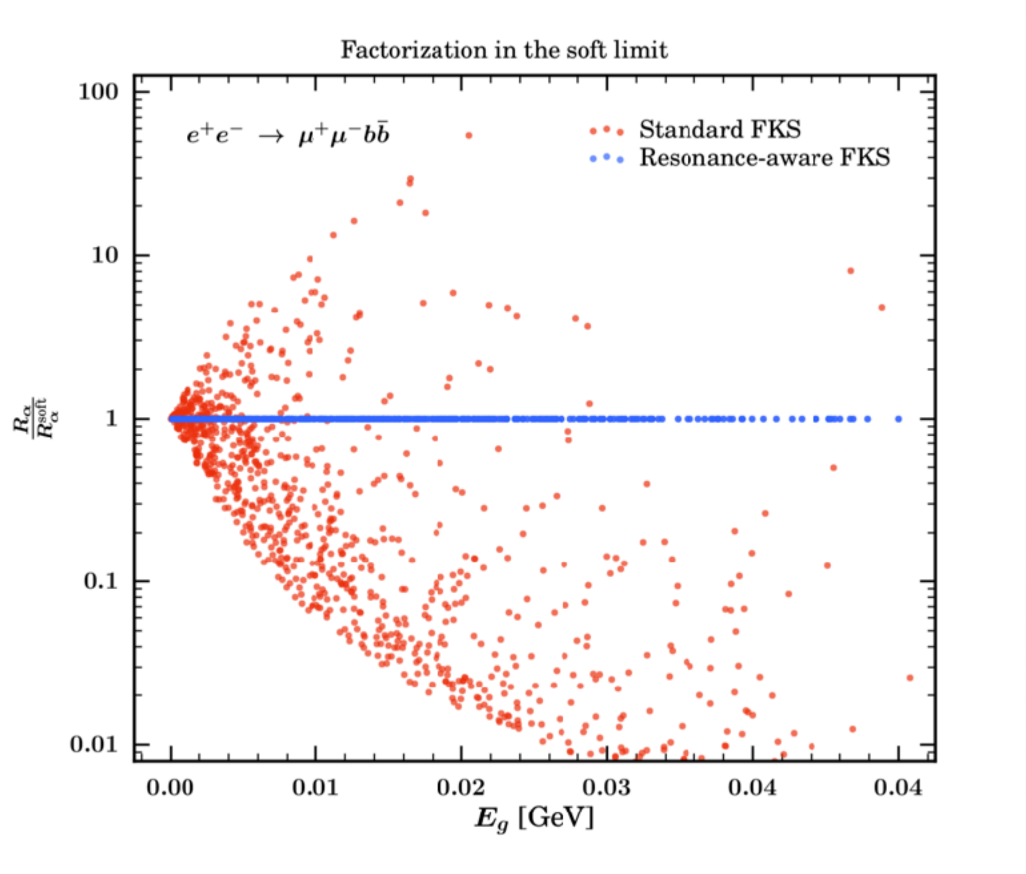

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- `WHIZARD` complete automatic implementation: example $e^+ e^- \to \mu\mu bb$ (ZZ, ZH histories)

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	9.6811847E+00	6.42E+00	66.30	72.60*	0.65		
2	11959	2.8539703E+00	2.35E-01	8.25	9.02*	0.69		
3	11936	2.4907574E+00	6.54E-01	26.25	28.68	0.35		
4	11908	2.7695559E+00	9.67E-01	34.91	38.09	0.30		
5	11874	2.4346151E+00	4.82E-01	19.80	21.57*	0.74		
5	59665	2.7539078E+00	1.97E-01	7.15	17.47	0.74	0.49	5

standard FKS

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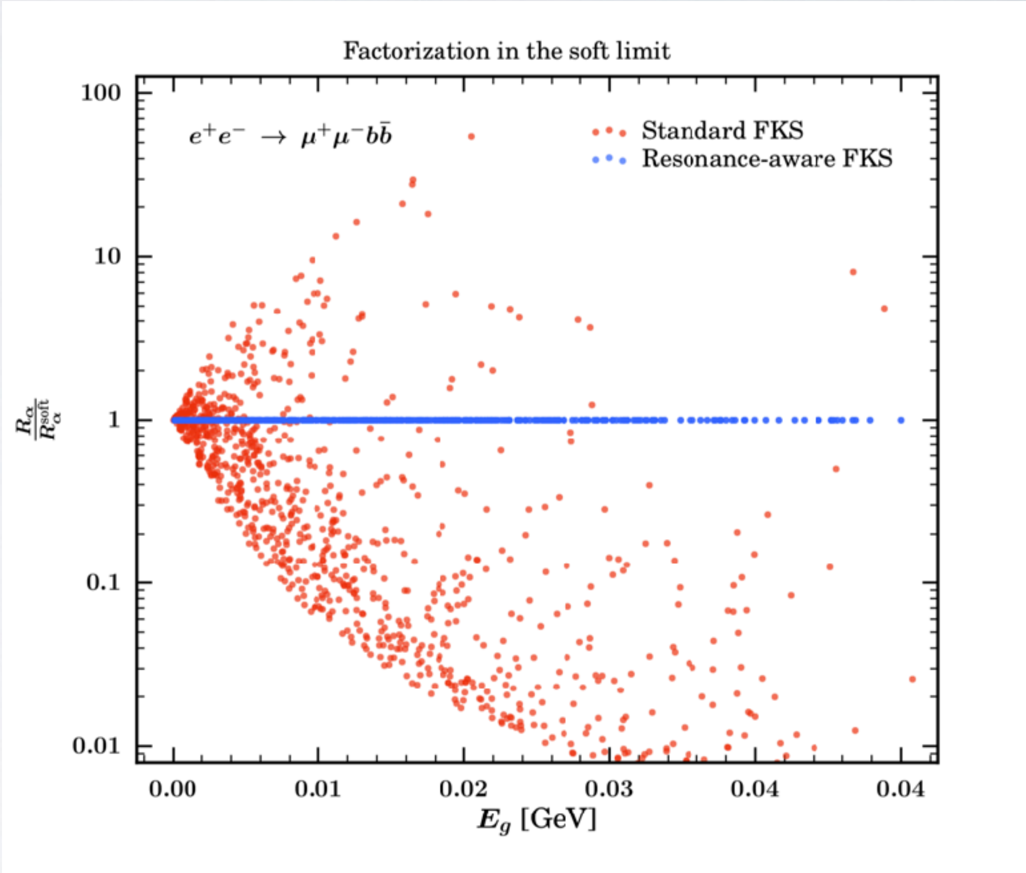

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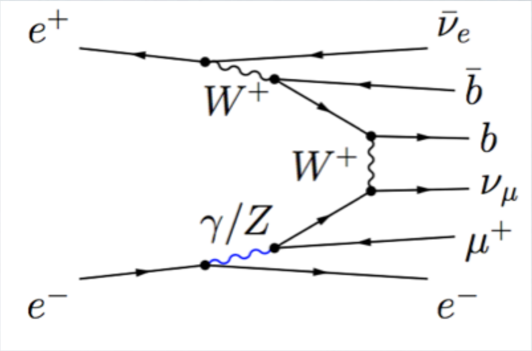
2	11962	2.8591952E+00	5.20E-02	1.82	1.99*	10.91		
3	11936	2.9277880E+00	4.09E-02	1.40	1.52*	14.48		
4	11902	2.8512337E+00	3.98E-02	1.40	1.52*	13.70		
5	11874	2.8855399E+00	3.87E-02	1.34	1.46*	17.15		
5	59662	2.8842006E+00	2.04E-02	0.71	1.72	17.15	0.53	5

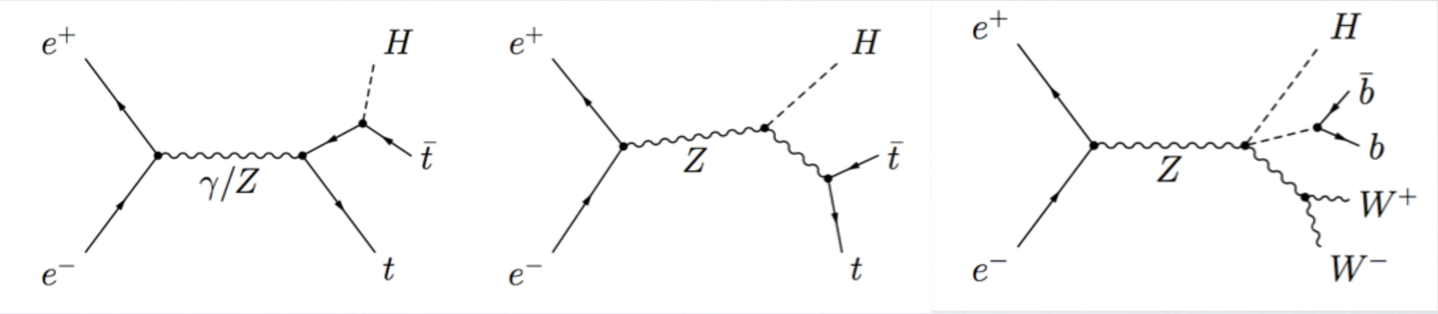
FKS with resonance mappings

Lepton colliders: *tt* and *ttH* (on- & off-shell)

- Cross checks for 2 → 2 and 2 → 4 processes with `Sherpa` and `Munich`
- Using massive *b* quarks: no cuts necessary for $e^+e^- \to W^+W^-bb$
- Full process $e^+e^- \to \mu^+\nu_\mu e^-\nu_e bb$ exhibits Coulomb singularity:



- *ttH* production: 8% contamination from Higgsstrahlung
- Contribution from quartic SM vertices



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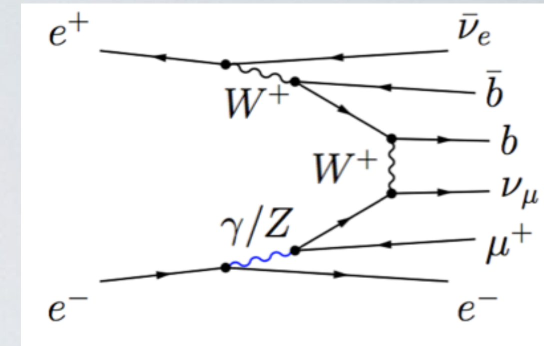

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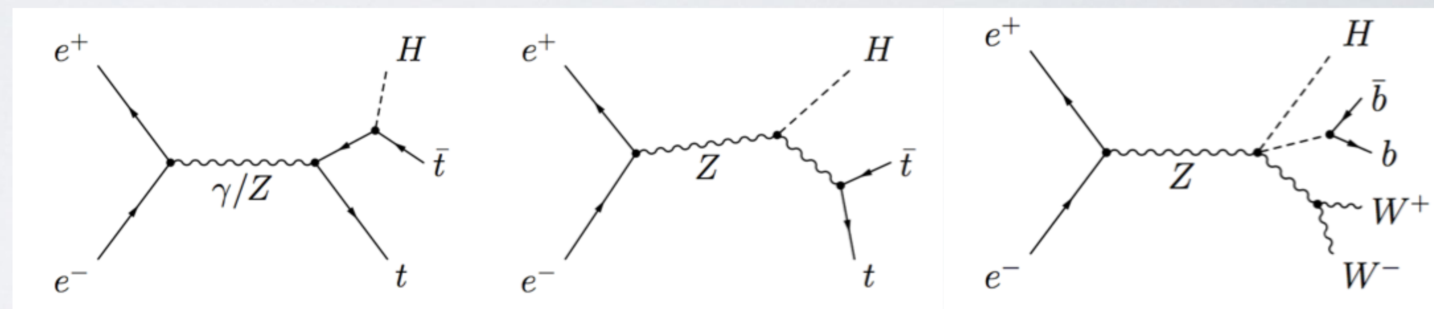

INPUT PARAMETERS:

$$m_Z = 91.1876\,\text{GeV}, \qquad m_W = 80.385\,\text{GeV}$$
$$m_b = 4.2\,\text{GeV}, \qquad m_t = 173.2\,\text{GeV}.$$

$$\Gamma_Z^{\text{LO}} = 2.4409\,\text{GeV}, \qquad \Gamma_Z^{\text{NLO}} = 2.5060\,\text{GeV},$$
$$\Gamma_W^{\text{LO}} = 2.0454\,\text{GeV}, \qquad \Gamma_W^{\text{NLO}} = 2.0978\,\text{GeV}.$$

$$\Gamma^{\text{LO}}_{t\to Wb} = 1.4986\,\text{GeV}, \qquad \Gamma^{\text{NLO}}_{t\to Wb} = 1.3681\,\text{GeV},$$
$$\Gamma^{\text{LO}}_{t\to f\bar{f}b} = 1.4757\,\text{GeV}, \qquad \Gamma^{\text{NLO}}_{t\to f\bar{f}b} = 1.3475\,\text{GeV}.$$

complex mass scheme:

$$m_H = 125\ \text{GeV} \qquad \Gamma_H = 0.000431\ \text{GeV}$$

$$\mu_i^2 = M_i^2 - \mathrm{i}\Gamma_i M_i \qquad \text{for } i = \mathrm{W}, \mathrm{Z}, \mathrm{t}, \mathrm{H}$$

$$s_w^2 = 1 - c_w^2 = 1 - \frac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$$

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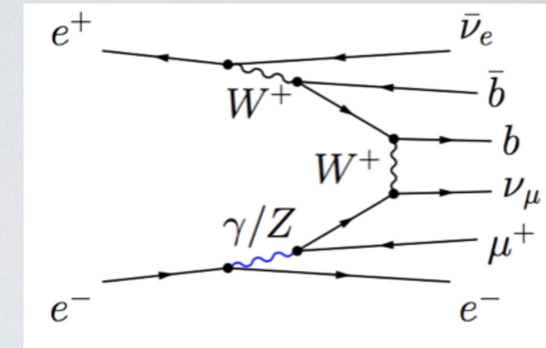

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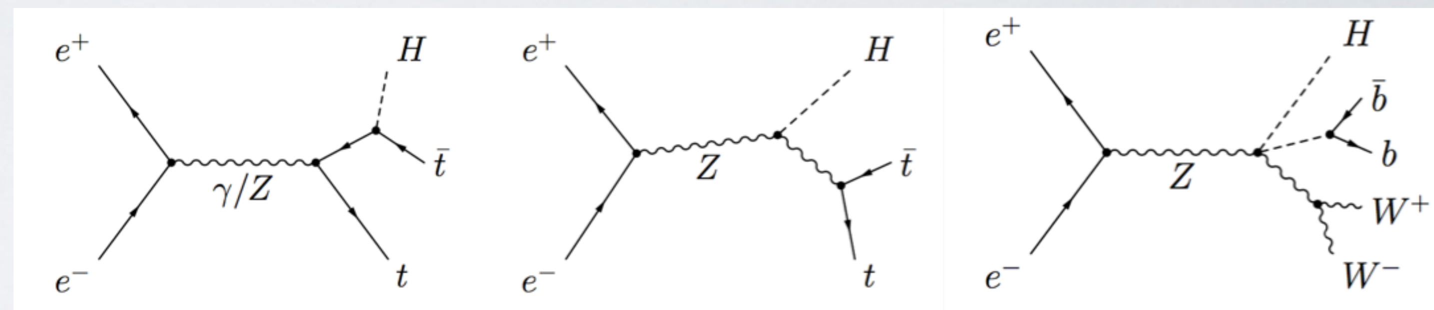

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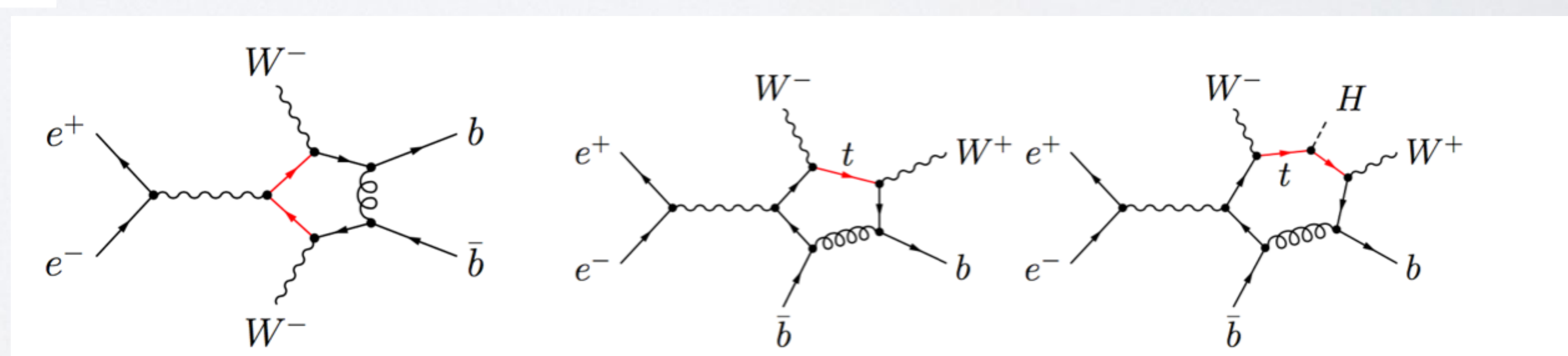
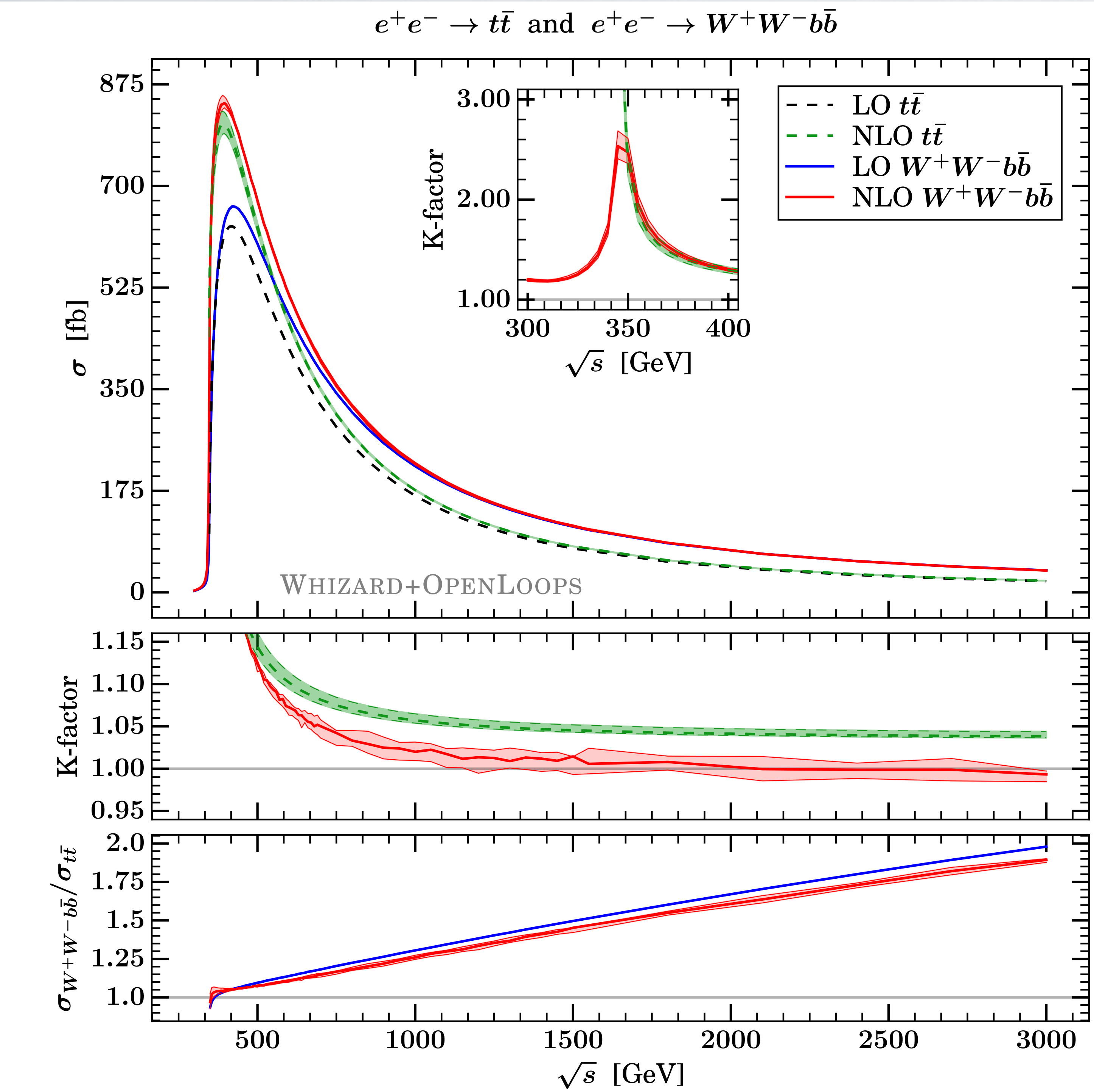
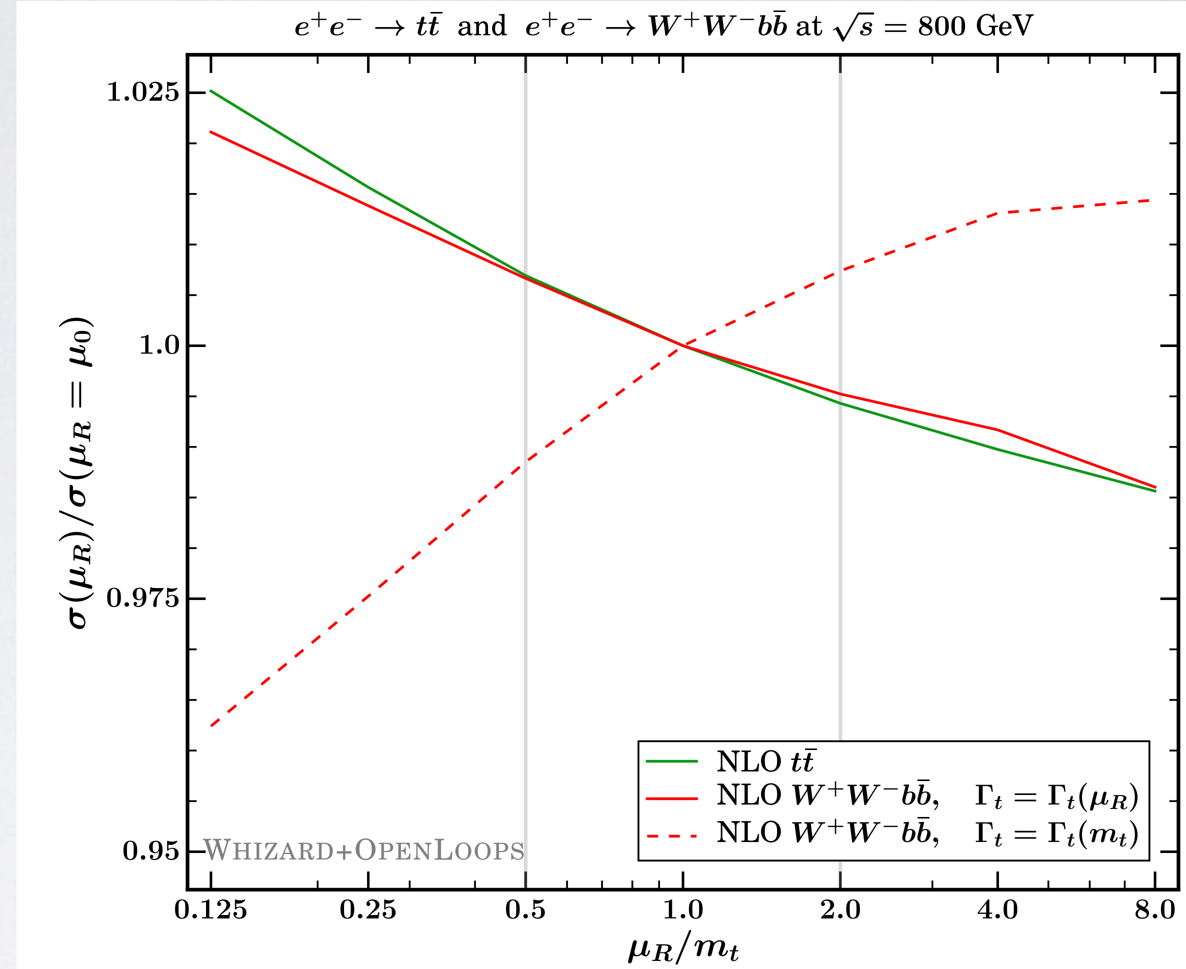
- Typical pentagons/hexagons:



NLO QCD Results for off-shell $e^+e^- \to tt$





	$e^+e^- \to t\bar{t}$			$e^+e^- \to W^+W^-b\bar{b}$		
$\sqrt{s}$ [GeV]	$\sigma^{\rm LO}$[fb]	$\sigma^{\rm NLO}$[fb]	K-factor	$\sigma^{\rm LO}$[fb]	$\sigma^{\rm NLO}$[fb]	K-factor
500	548.4	$627.4^{+1.4\%}_{-0.9\%}$	1.14	600.7	$675.1^{+0.4\%}_{-0.8\%}$	1.12
800	253.1	$270.9^{+0.8\%}_{-0.4\%}$	1.07	310.2	$320.7^{+1.1\%}_{-0.7\%}$	1.03
1000	166.4	$175.9^{+0.7\%}_{-0.3\%}$	1.06	217.2	$221.6^{+1.1\%}_{-1.0\%}$	1.02
1400	86.62	$90.66^{+0.6\%}_{-0.2\%}$	1.05	126.4	$127.9^{+0.7\%}_{-1.5\%}$	1.01
3000	19.14	$19.87^{+0.5\%}_{-0.2\%}$	1.04	37.89	$37.63^{+0.4\%}_{-0.9\%}$	0.993

Chokoufé/Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

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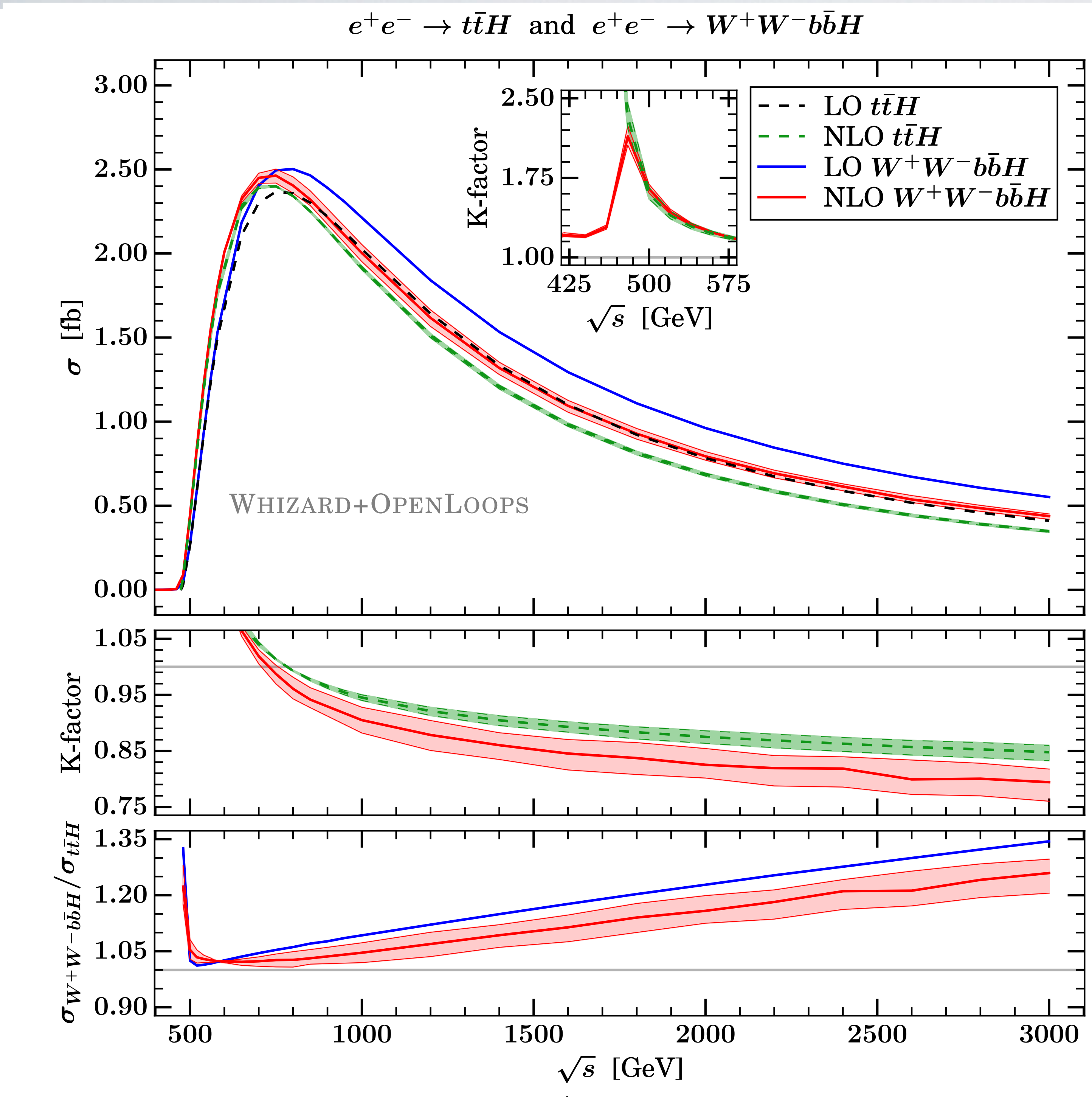
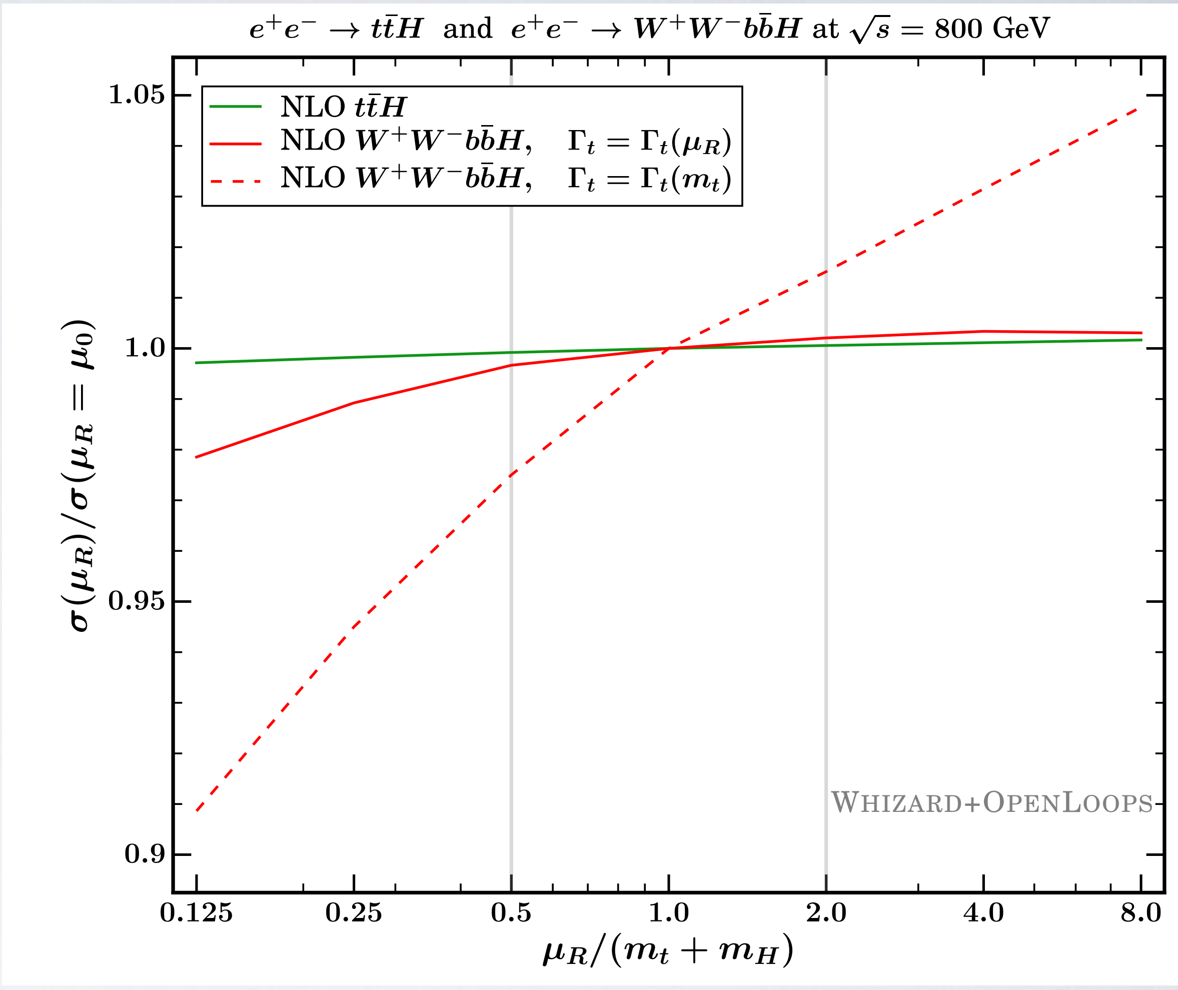
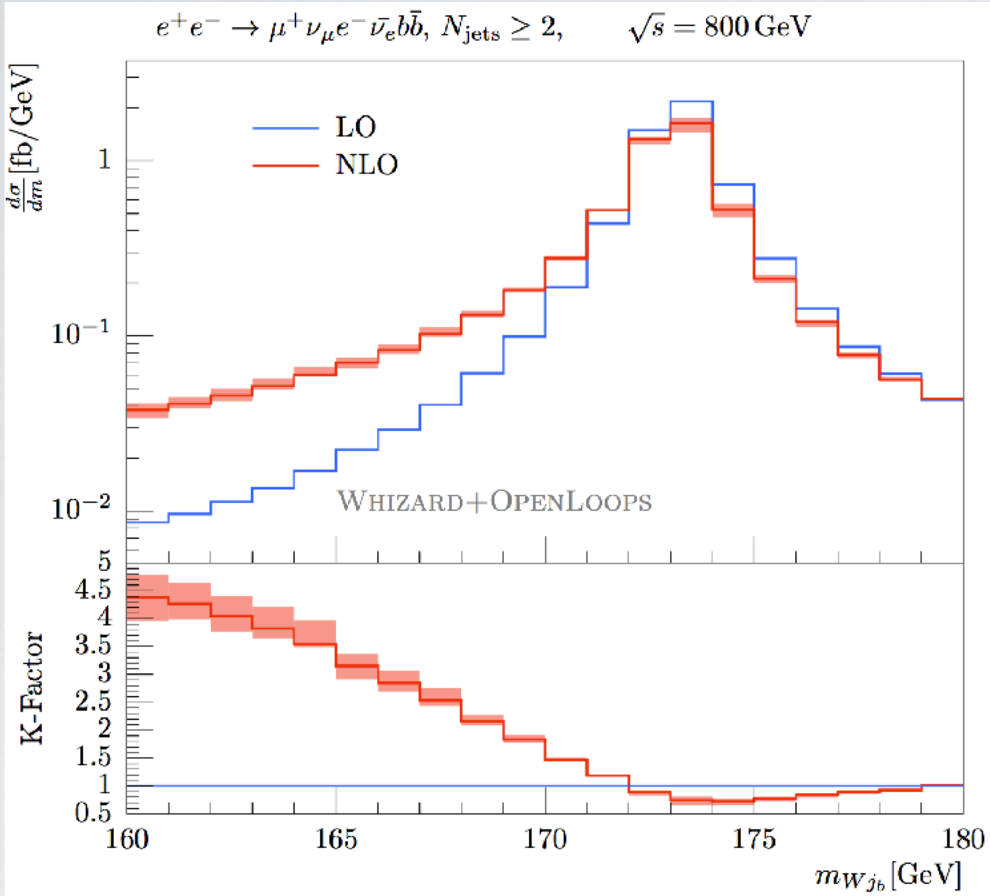


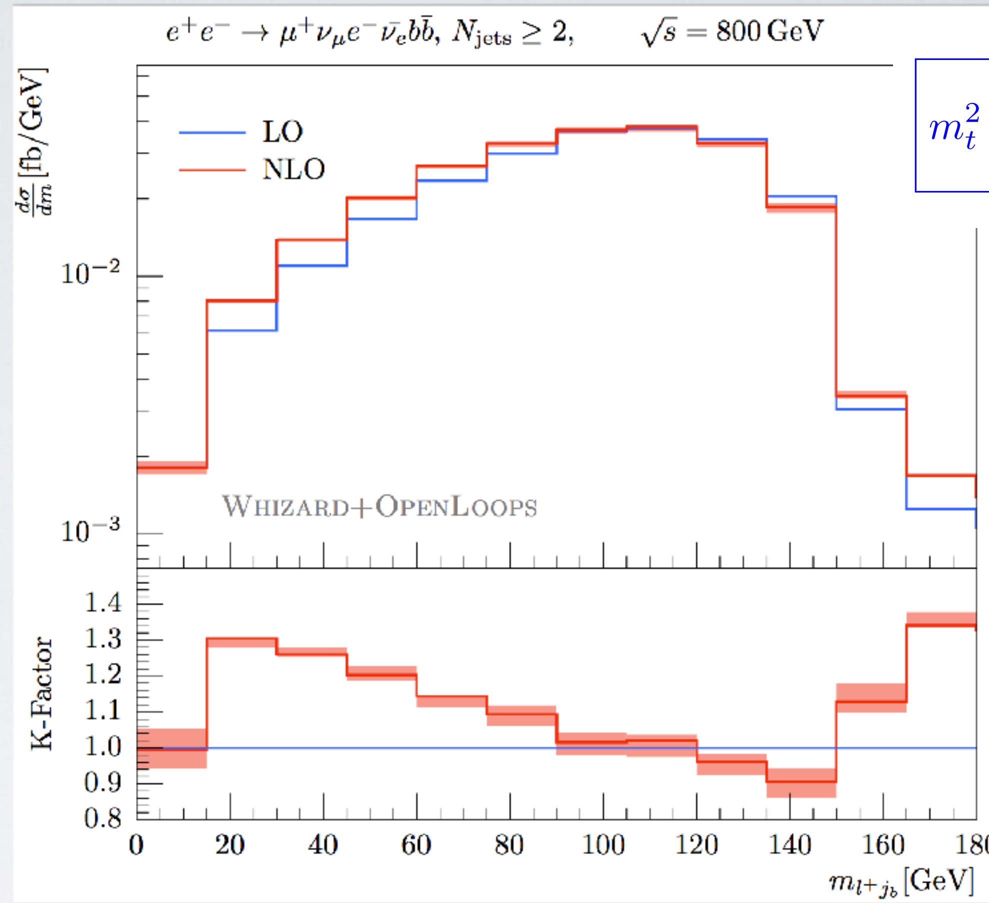


	$e^+e^- \to t\bar{t}H$			$e^+e^- \to W^+W^-b\bar{b}H$		
$\sqrt{s}$ [GeV]	$\sigma^{\rm LO}$[fb]	$\sigma^{\rm NLO}$[fb]	K-factor	$\sigma^{\rm LO}$[fb]	$\sigma^{\rm NLO}$[fb]	K-factor
500	0.26	$0.42^{+3.6\%}_{-3.1\%}$	1.60	0.27	$0.44^{+2.6\%}_{-2.4\%}$	1.63
800	2.36	$2.34^{+0.1\%}_{-0.1\%}$	0.99	2.50	$2.40^{+2.1\%}_{-1.9\%}$	0.96
1000	2.02	$1.91^{+0.5\%}_{-0.5\%}$	0.95	2.21	$2.00^{+2.5\%}_{-2.5\%}$	0.90
1400	1.33	$1.21^{+0.9\%}_{-1.0\%}$	0.90	1.53	$1.32^{+2.6\%}_{-3.0\%}$	0.86
3000	0.41	$0.35^{+1.4\%}_{-1.8\%}$	0.84	0.55	$0.44^{+2.9\%}_{-4.3\%}$	0.79

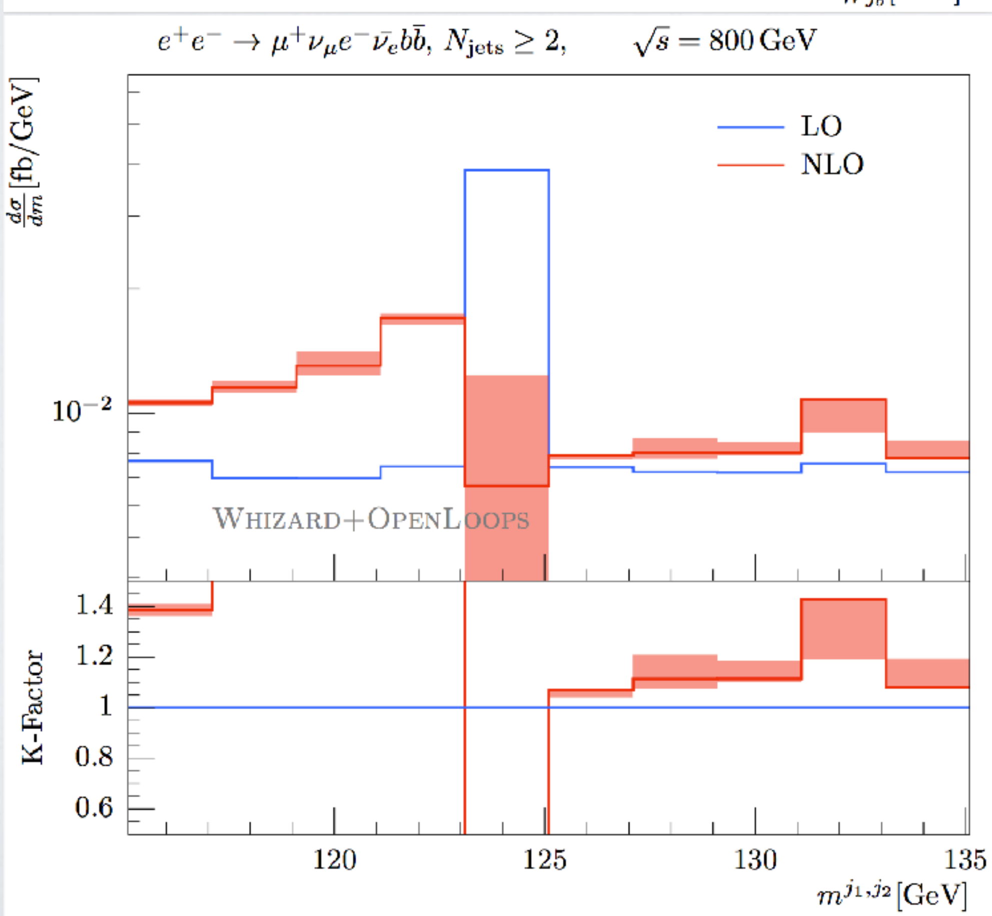
Chokoufé/Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

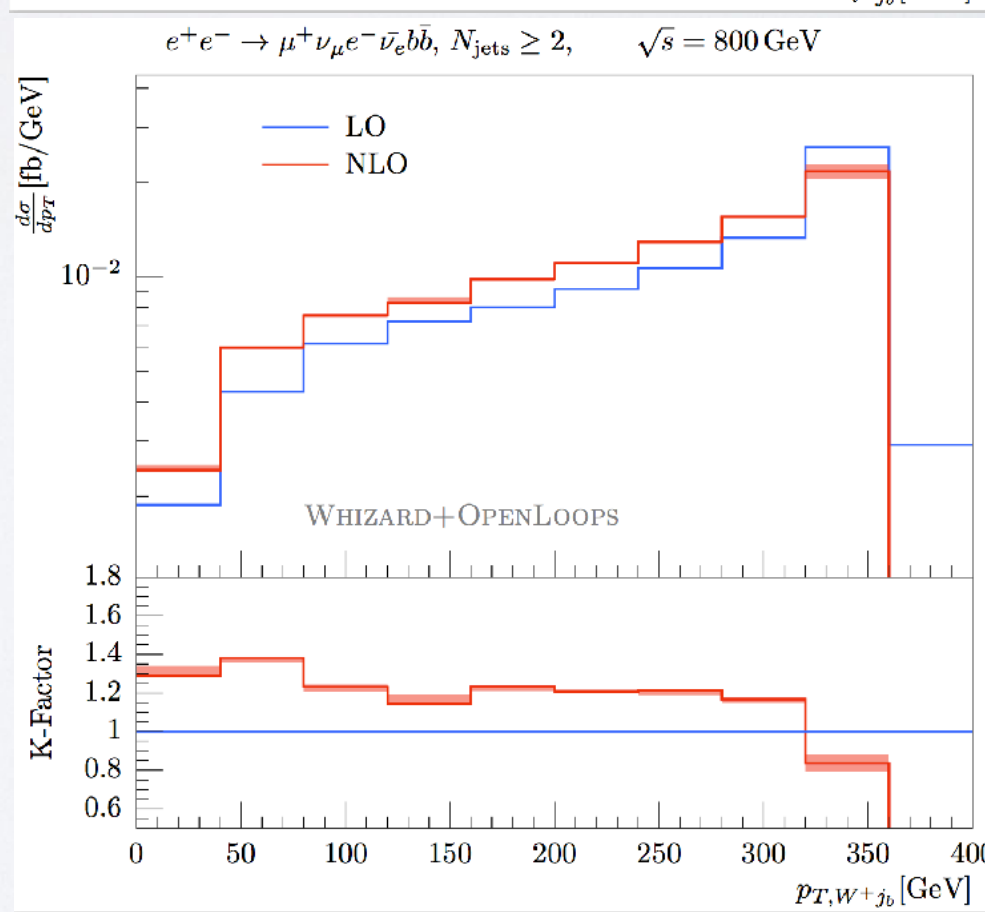
Differential Results for off-shell $e^+e^- \to tt$



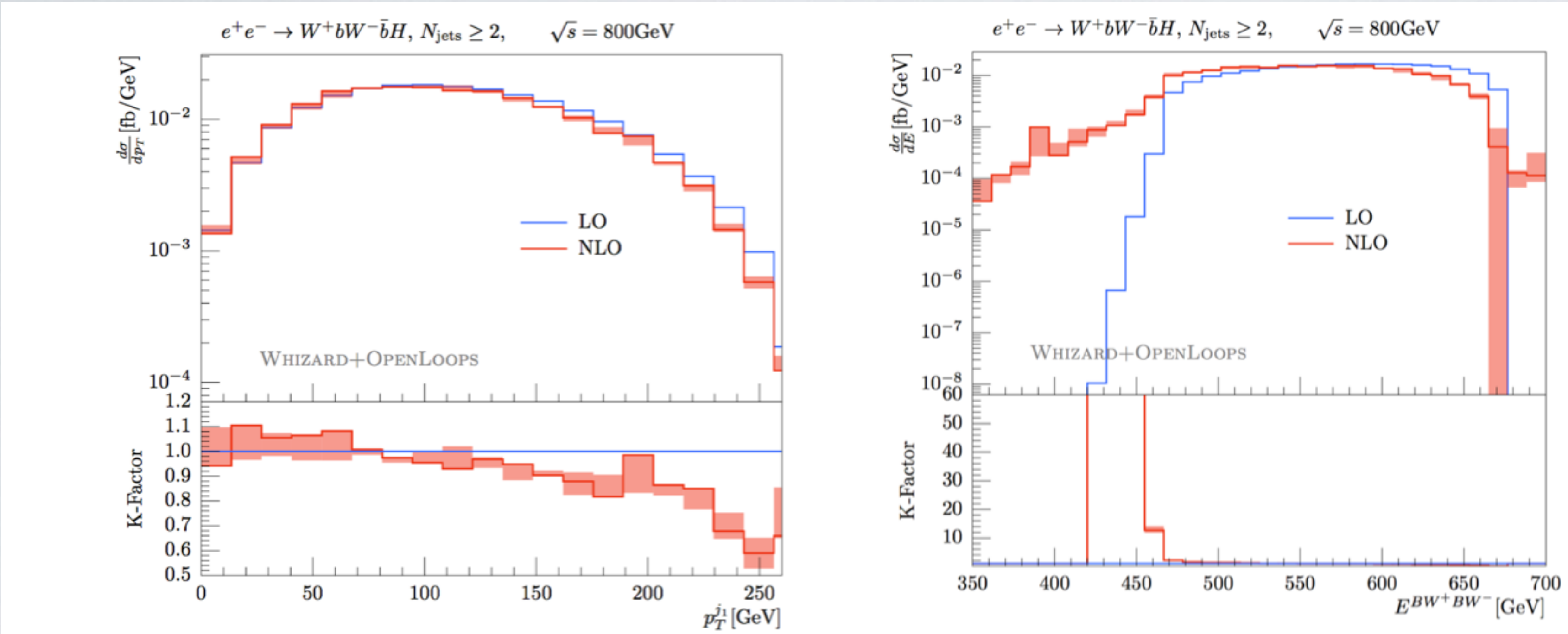


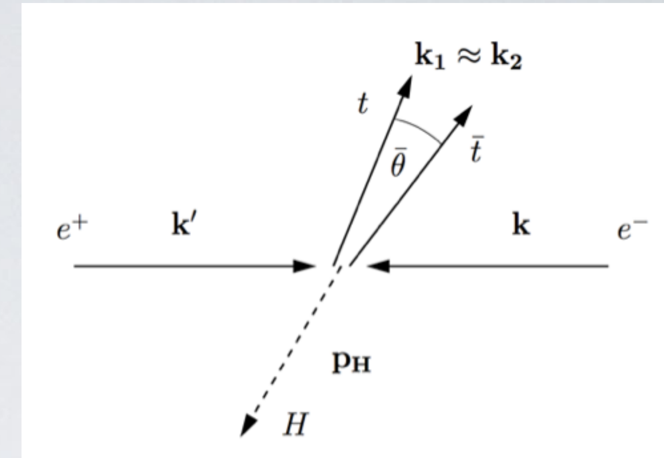
$$m_t^2 = m_W^2 + \frac{2\langle m_{\ell j_b}^2\rangle}{1 - \langle\cos\theta_{\ell j_b}\rangle}$$





Differential Results for off-shell *ttH*

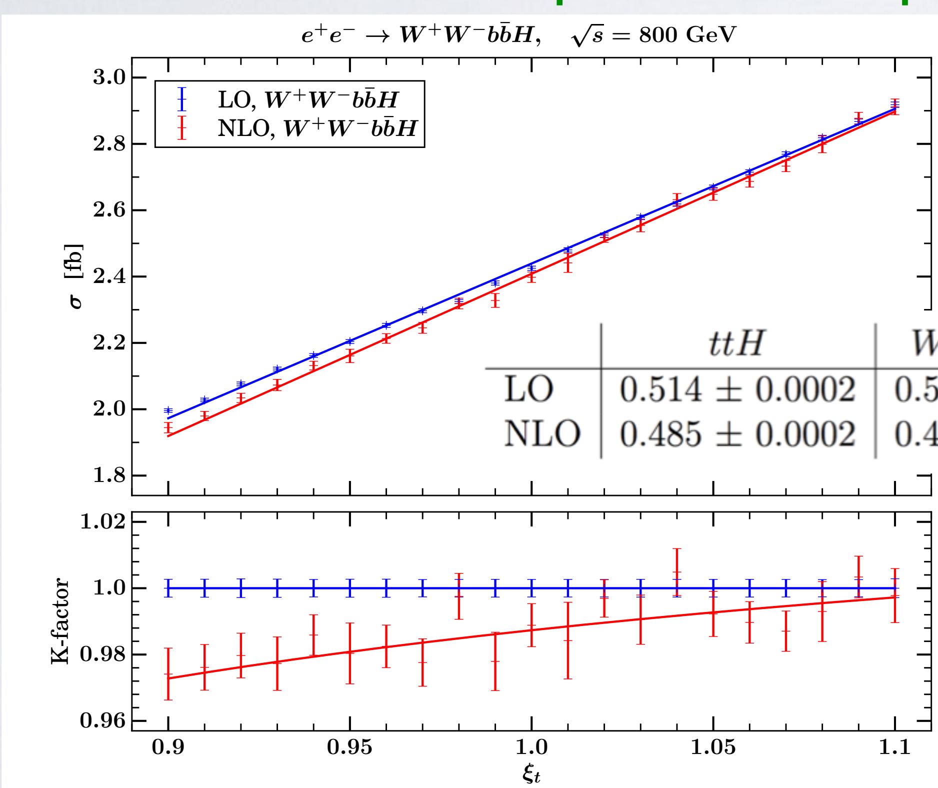




$$E_h = \frac{1}{2\sqrt{s}}\left[s + M_h^2 - (k_1 + k_2)^2\right]$$

Determination of top Yukawa coupling (*ttH*)

Chokoufé/Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390



	ttH	$W^+W^-b\bar{b}H$
LO	0.514 ± 0.0002	0.520 ± 0.001
NLO	0.485 ± 0.0002	0.497 ± 0.002

$$\lim_{\xi_t \to 1} \sigma(\xi_t)\left[\frac{d\sigma(\xi_t)}{d\xi_t}\right]^{-1} = \frac{S+I+B}{2S+I} = \frac{1}{2} + \frac{I/2+B}{2S+I}$$

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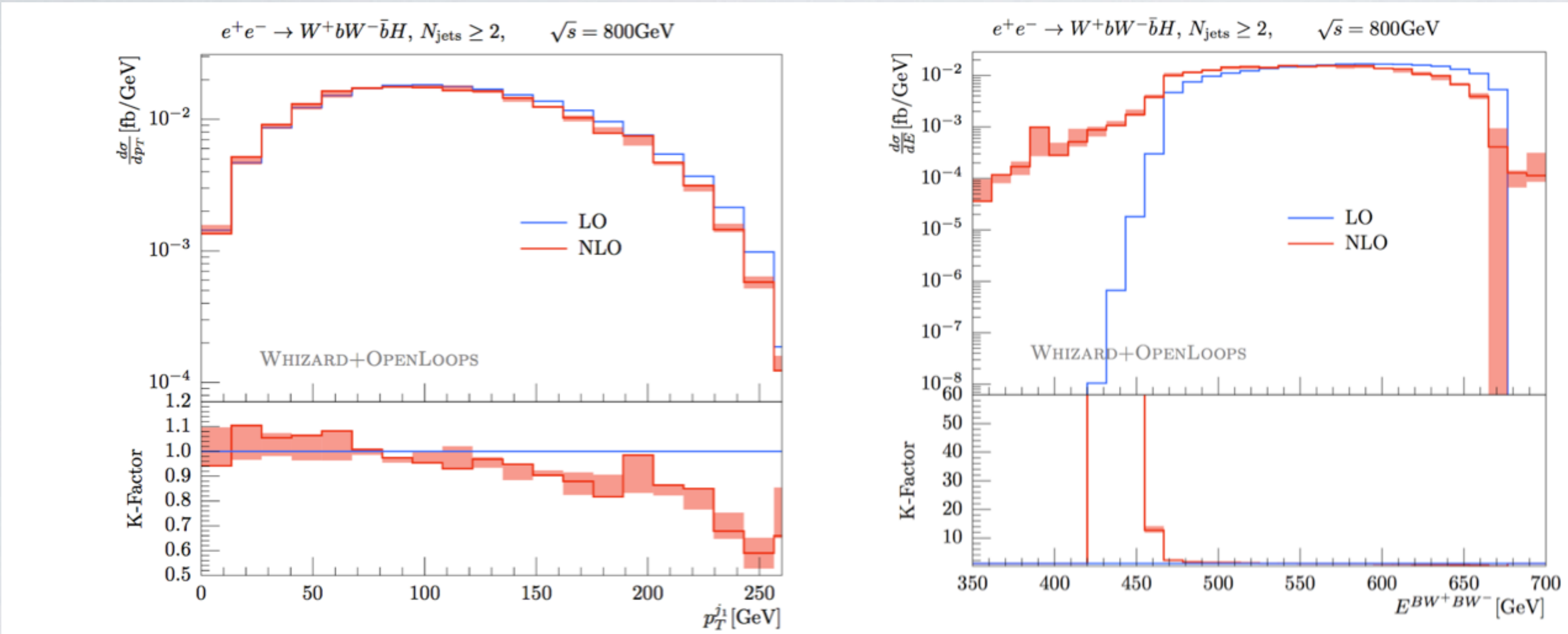

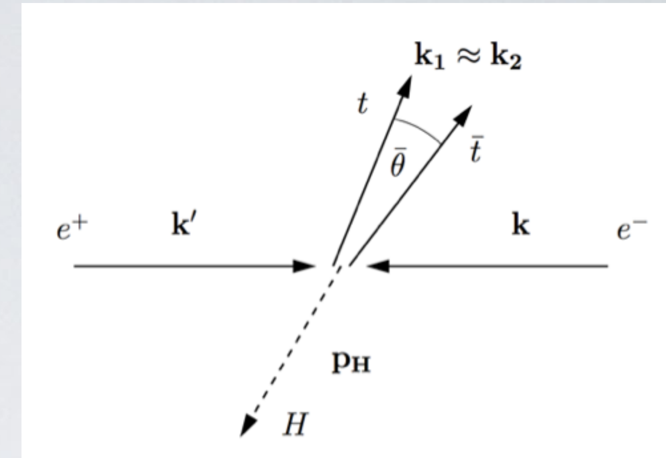

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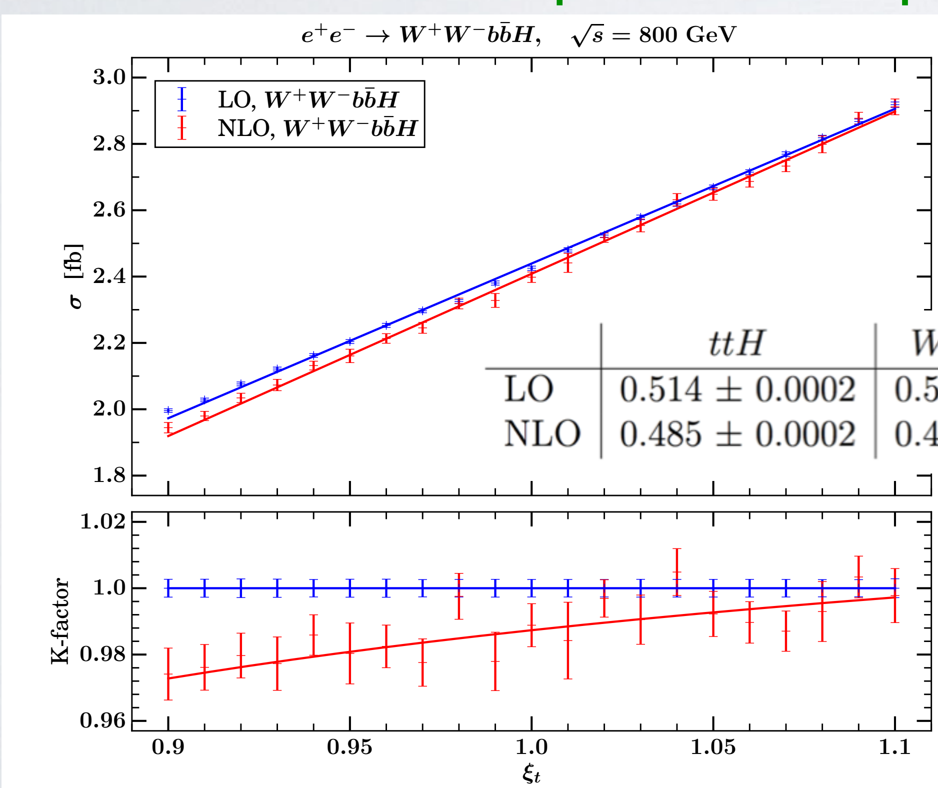

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Polarized Results (*tt*)

- ILC will always run polarized
- Polarized 1-loop amplitudes beyond BLHA

		$\sqrt{s} = 800$ GeV			$\sqrt{s} = 1500$ GeV		
$P(e^-)$	$P(e^+)$	$\sigma^{\rm LO}$[fb]	$\sigma^{\rm NLO}$[fb]	K-factor	$\sigma^{\rm LO}$[fb]	$\sigma^{\rm NLO}$[fb]	K-factor
0%	0%	253.7	272.8	1.075	75.8	79.4	1.049
-80%	0%	176.5	190.0	1.077	98.3	103.1	1.049
+80%	0%	176.5	190.0	1.077	53.2	55.9	1.049
-80%	30%	420.8	452.2	1.074	124.9	131.0	1.048
-80%	60%	510.7	548.7	1.074	151.6	158.9	1.048
80%	-30%	208.4	224.5	1.077	63.0	66.1	1.049
80%	-60%	240.3	258.9	1.077	72.7	76.3	1.049

Top-Forward Backward Asymmetry

$$A_{FB} = \frac{\sigma(\cos\theta_t > 0) - \sigma(\cos\theta_t < 0)}{\sigma(\cos\theta_t > 0) + \sigma(\cos\theta_t < 0)}.$$

Gluon emission symmetric in θ ⇒

NLO QCD corrections small

A_{FB} of the top quark

	$e^+e^- \to$	$A_{FB}^{\rm LO}$	$A_{FB}^{\rm NLO}$	$A_{FB}^{\rm NLO}/A_{FB}^{\rm LO}$
A_{FB}	$t\bar{t}$	-0.535	-0.539	1.013
	$W^+W^-b\bar{b}$	-0.428	-0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	-0.415	-0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$, without neutrinos	-0.402	-0.387	0.964
$\bar{A}_{FB}$	$t\bar{t}$	0.535	0.539	1.013
	$W^+W^-b\bar{b}$	0.428	0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	0.415	0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$, without neutrinos	0.377	0.350	0.928

Matched NLO QCD results

- Precise predictions of multi-parton final states require properly matched samples
- NLO QCD including POWHEG matching already available   [WHIZARD+OpenLoops]
- All descriptions at NLO at the moment for the on-shell process
- Even LO simulations are demanding, e.g.: $e^+e^- \to b\bar{b}b\bar{b}jj\ell\nu_\ell, \quad b\bar{b}jjjjjj\ell\nu_\ell$

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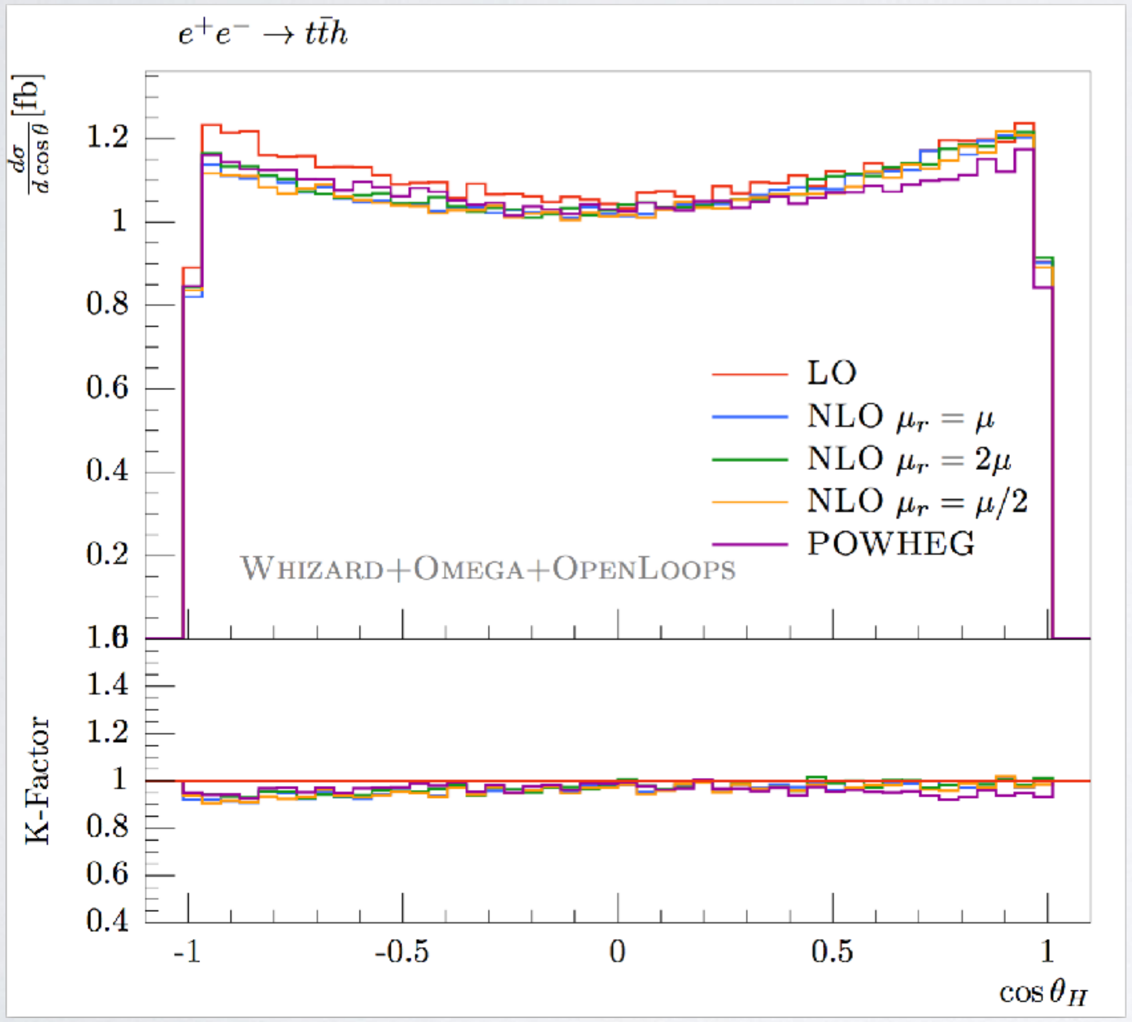
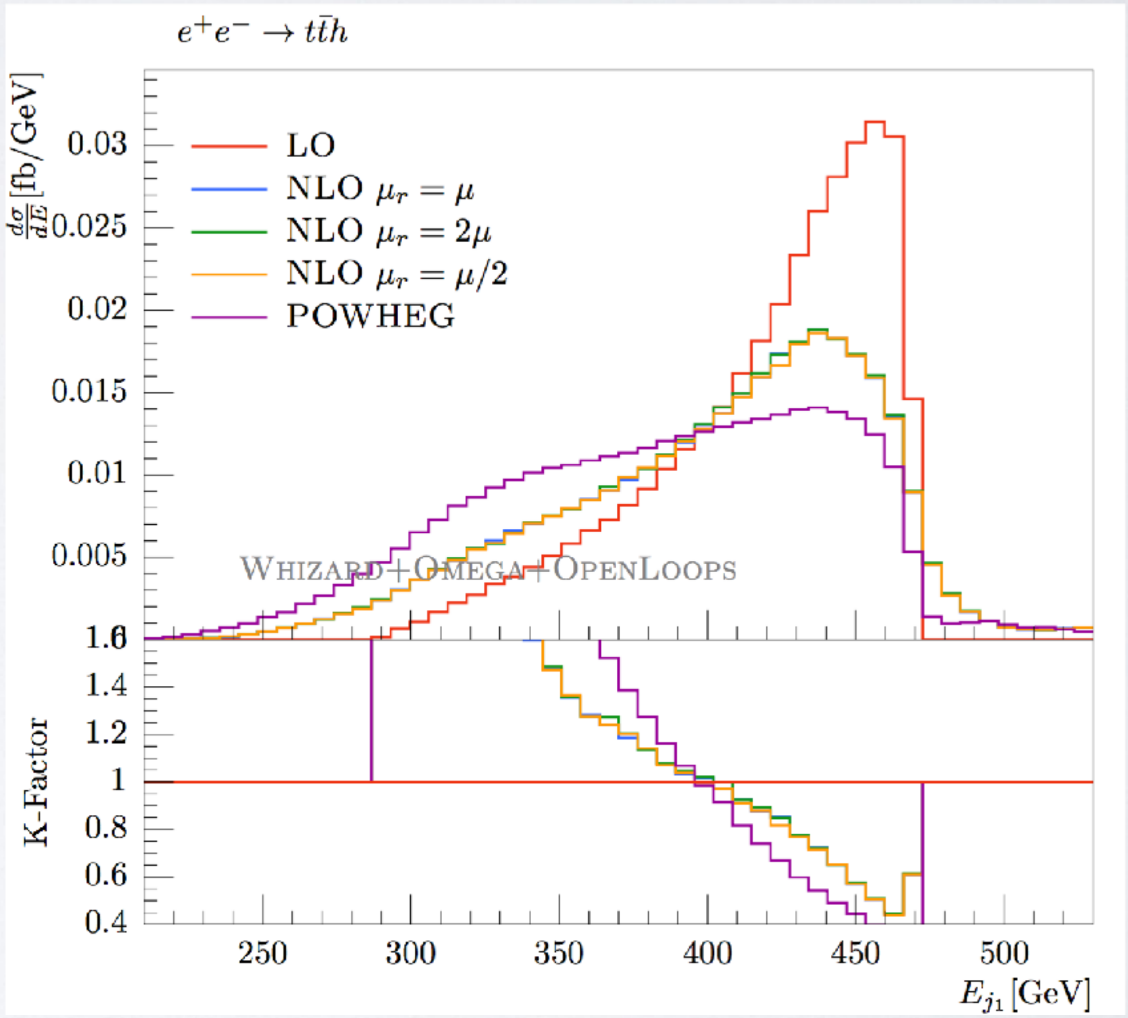


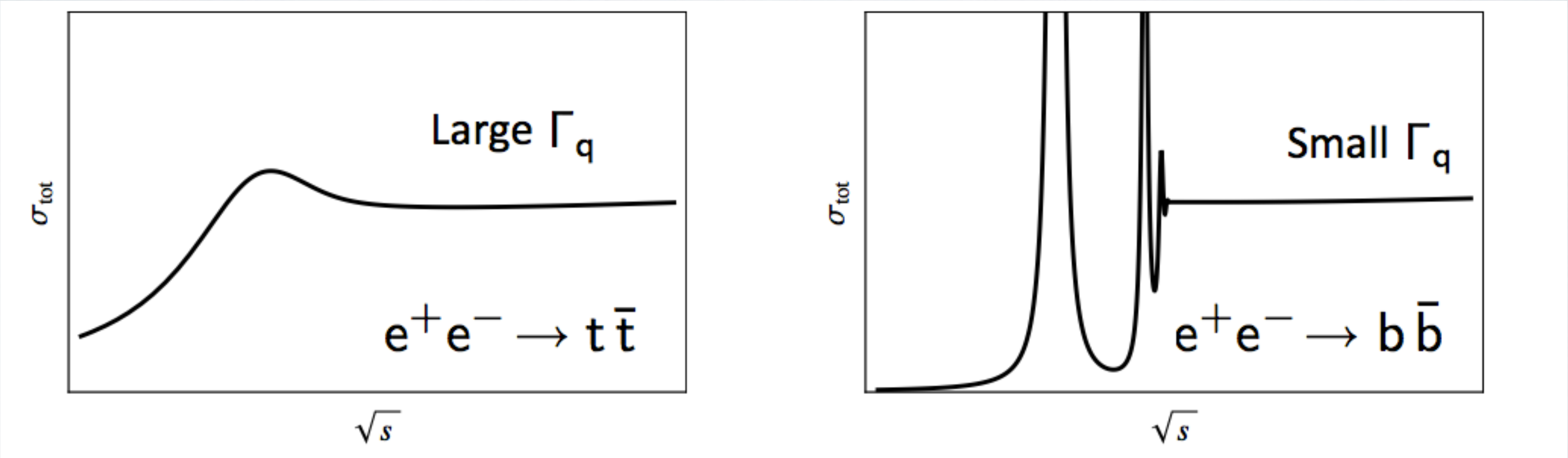


[Chokoufe/JRR/Weiss]

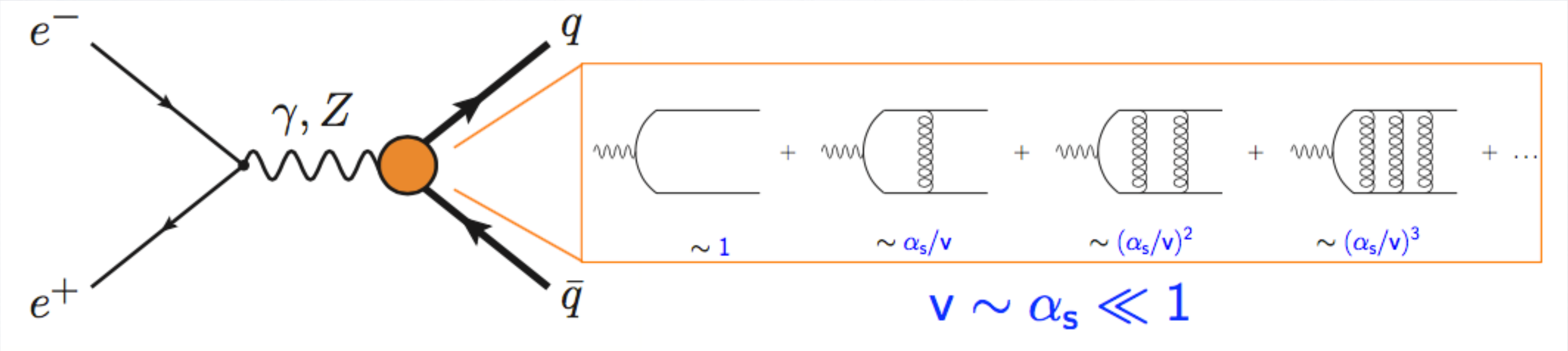
Top Threshold at lepton colliders

ILC top threshold scan best-known method to measure top quark mass, ΔM ~ 30-70 MeV

Heavy quark production at lepton colliders, qualitatively:



Threshold region: top velocity $v \sim \alpha_s \ll 1$



Top Threshold Resummation in (p)NRQCD

- NRQCD is EFT for non-relativistic quark-antiquark systems: separate $M \cdot v$ and $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f.
- Resummation of singular terms close to threshold (v = 0) Hoang et al. '99-'01; Beneke et al., '13-'14

Phase space of two massive particles

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times$$
$$\times \left\{1\,(\mathrm{LL});\ \alpha_s, v\,(\mathrm{NLL});\ \alpha_s^2, \alpha_s v, v^2\,(\mathrm{NNLL})\right\}$$

(p/v)NRQCD EFT w/ RG improvement

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$$R^{\gamma,Z}(s) = \underbrace{F^v(s) R^v(s)}_{\text{s-wave: LL+NLL}} + \underbrace{F^a(s) R^a(s)}_{\text{p-wave}\sim v^2\text{:NNLL}}$$

but contributes
at NLL differentially!

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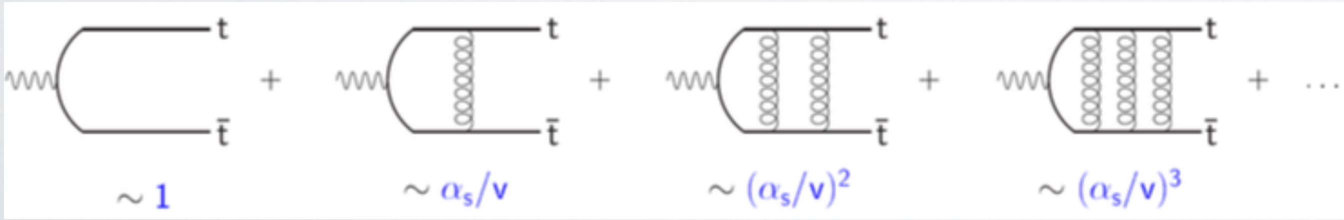
Coulomb potential gluon ladder resummption



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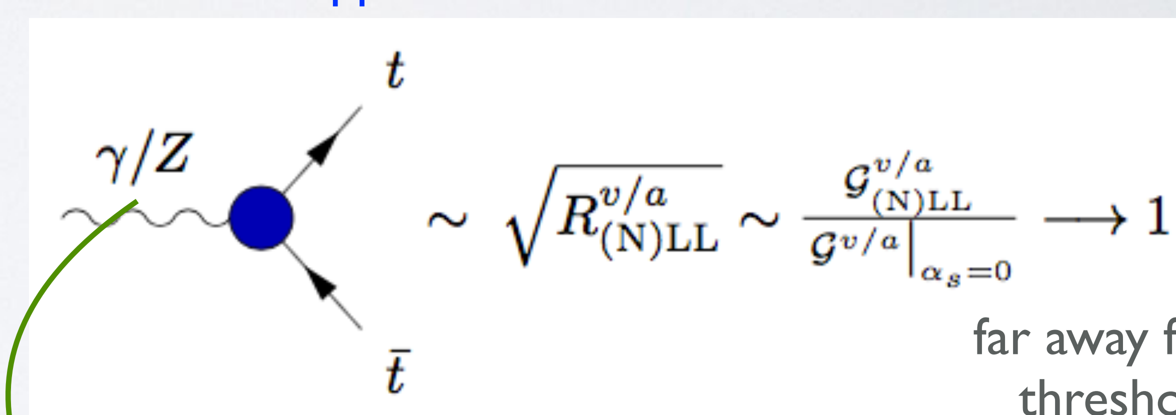
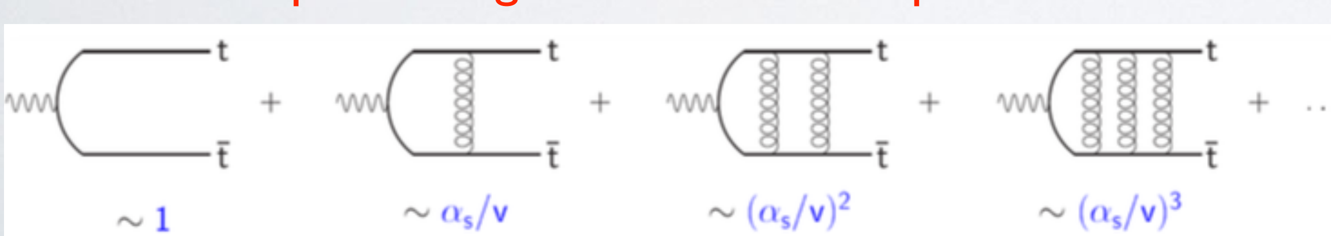
but contributes at NLL differentially!

can be mapped onto effective *ttV* vertex



far away from threshold

Coulomb potential gluon ladder resummation

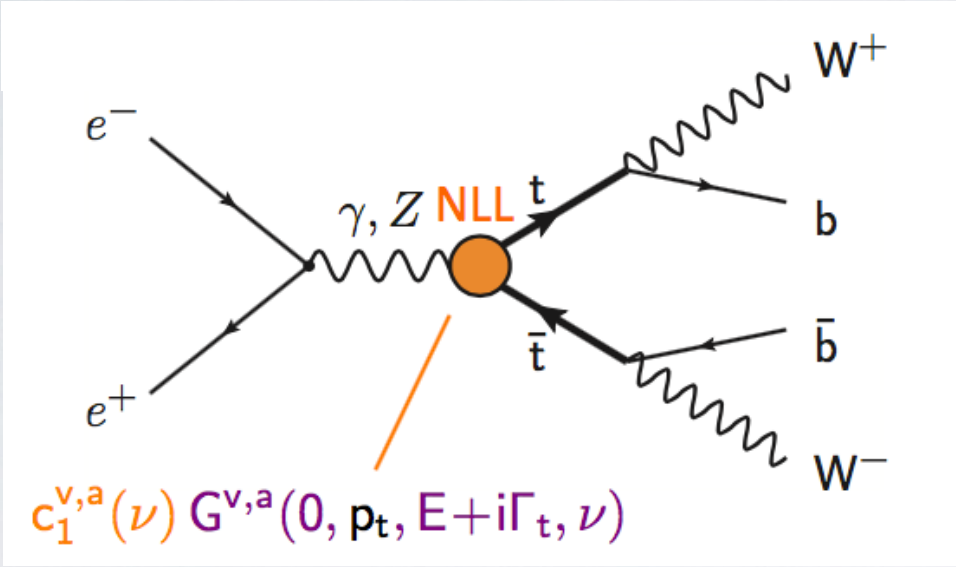


$$\mathbb{C} \ni \mathcal{G}^{v/a}_{\mathrm{(N)LL}} = \mathcal{G}^{v/a}_{\mathrm{(N)LL}}(\alpha_s, M_t^{\mathrm{pole}}, \sqrt{s}, |\vec{p}_t|, \Gamma_t)$$

differential in off-shell *tt* phase space

Top Threshold in WHIZARD

- Implement resummed threshold effects as effective vertex [form factor] in WHIZARD
- $G^{v,a}(0, p_t, E + i\Gamma_t, \nu)$ from TOPPIK code [Jezabek/Teubner], included in WHIZARD

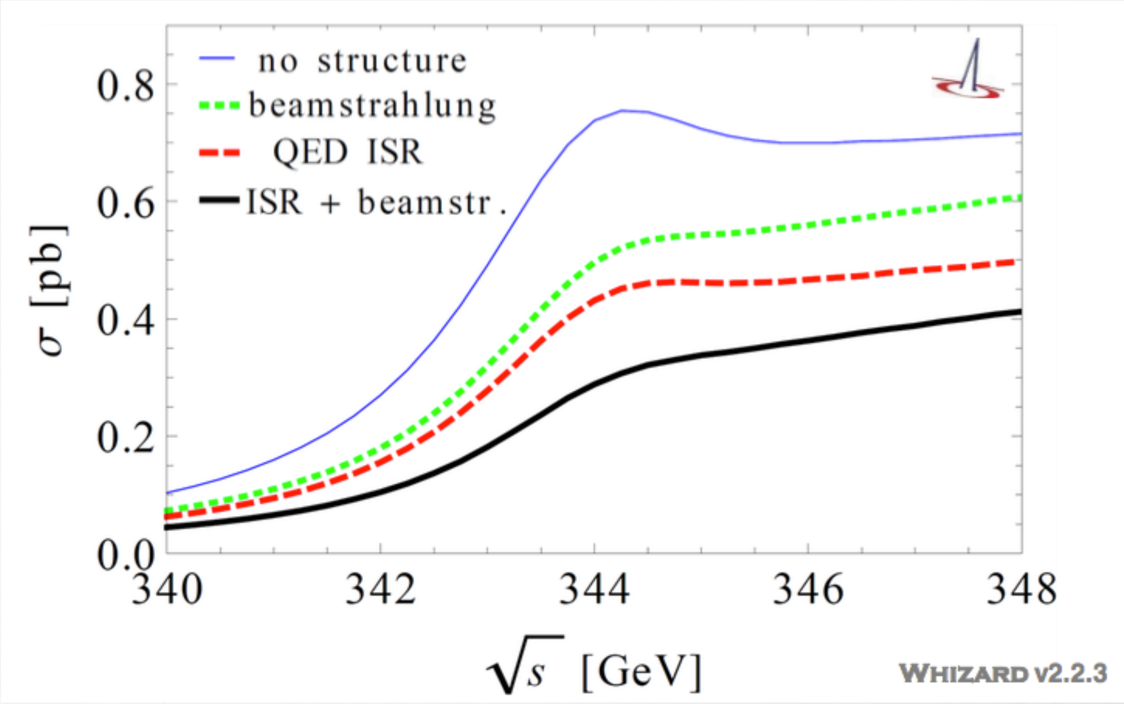


- Default parameters:

$$\mathbf{M^{1S} = 172\,GeV},\ \Gamma_t = 1.54\,\mathrm{GeV},\quad \alpha_s(M_Z) = 0.118$$

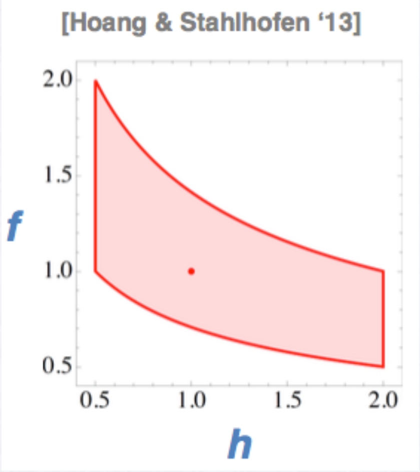
$$M^{1S} = M_t^{pole}(1 - \Delta^{LL/NLL}_{(Coul.)})$$

- Important effects: beamstrahlung; ISR; LO EW terms
- Exclusive observables accessible



Theory uncertainties from scale variations: hard and soft scale

$\mu_h = h \cdot m_t \qquad \mu_s = f \cdot m_t v$



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error source	$\Delta m_t^{\rm PS}$ [MeV]
stat. error (200 fb^{-1})	13
theory (NNLO scale variations, PS scheme)	40
parametric (α_s, current WA)	35
non-resonant contributions (such as single top)	< 40
residual background / selection efficiency	10 – 20
luminosity spectrum uncertainty	< 10
beam energy uncertainty	< 17
combined theory & parametric	30 – 50
combined experimental & backgrounds	25 - 50
total (stat. + syst.)	40 – 75

from 1702.05333

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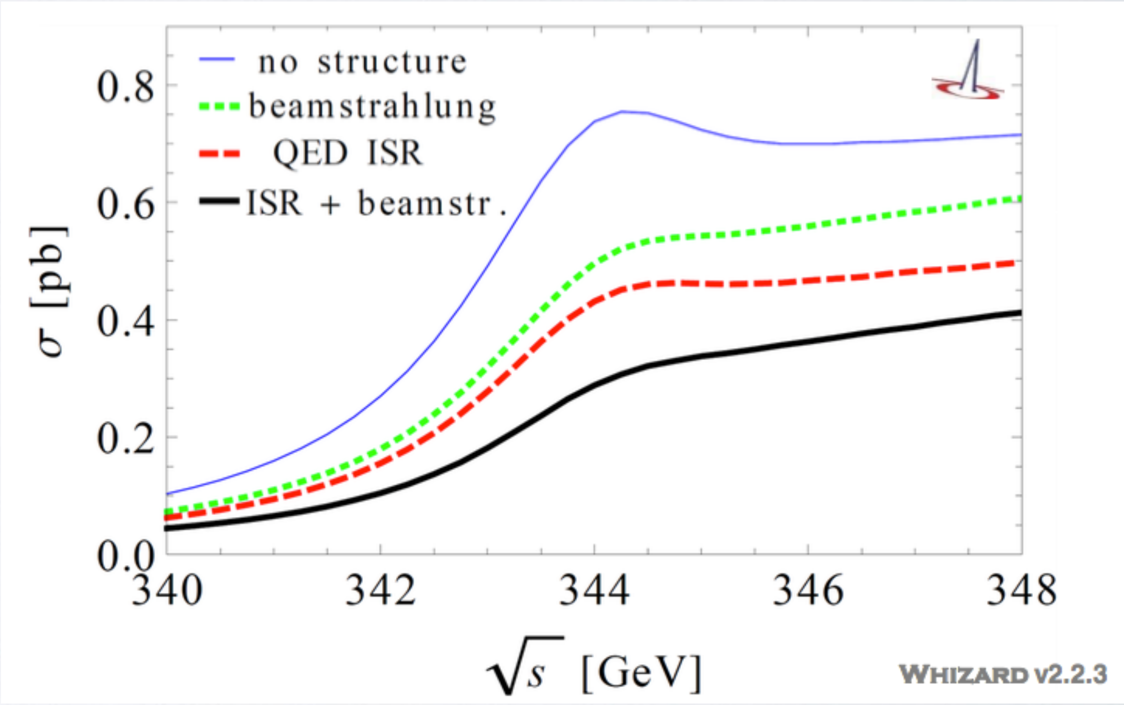

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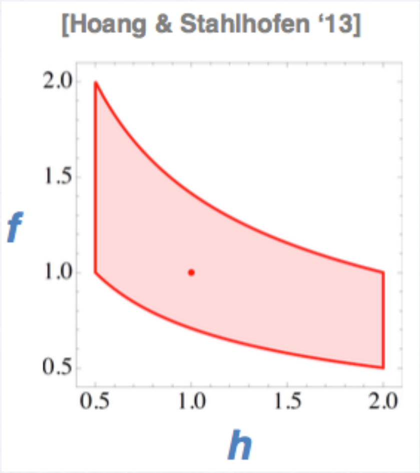
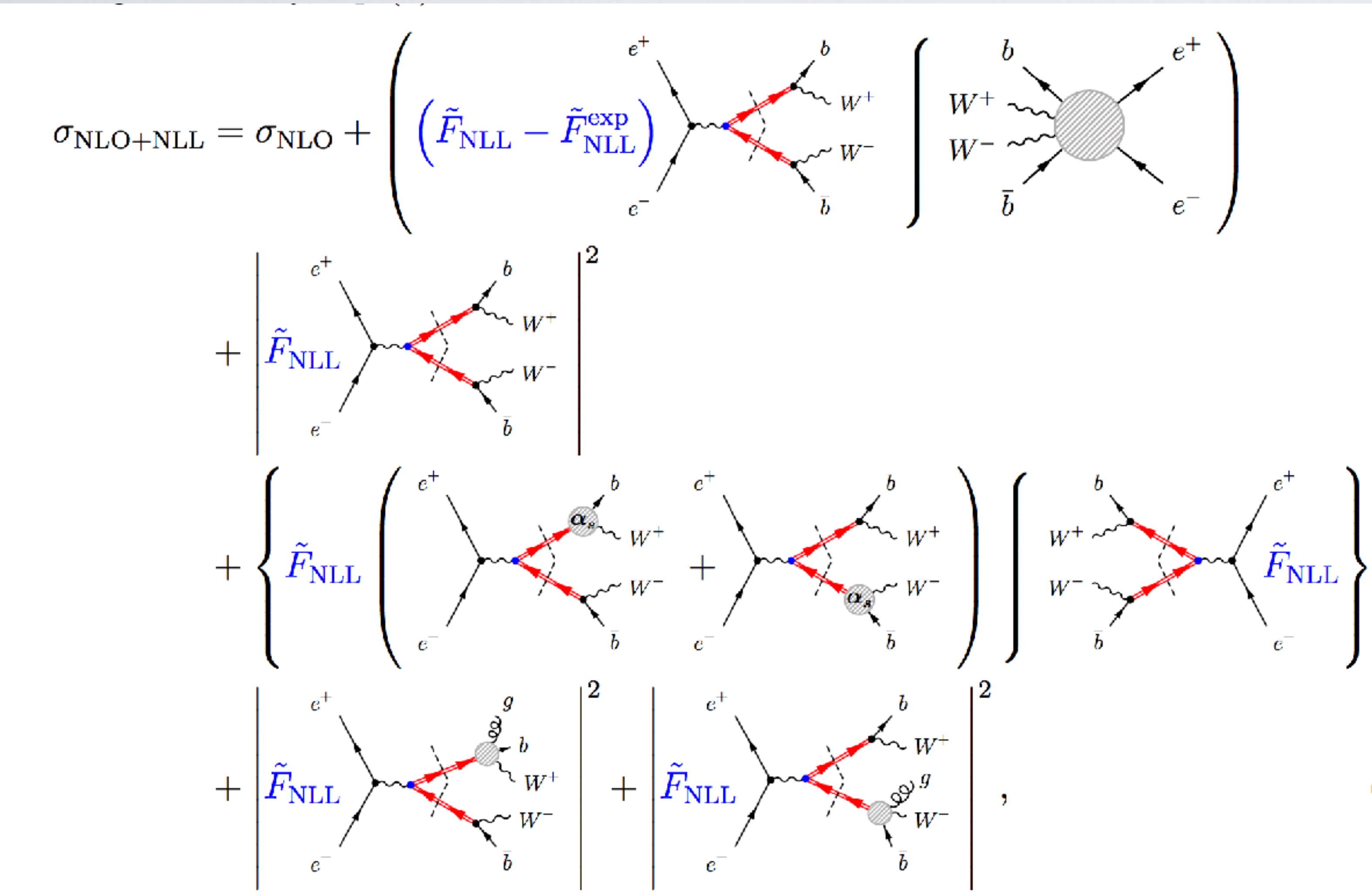
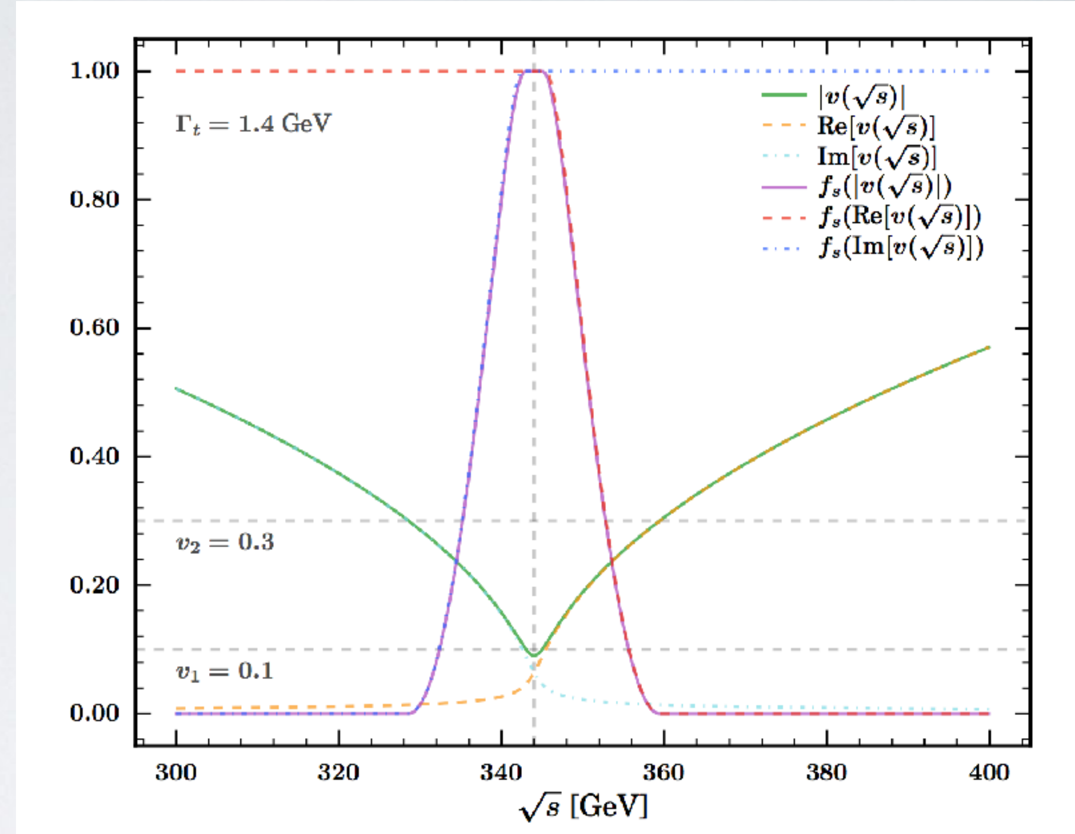


Top threshold: validation and matching

- Transition region between relativistic and resummation effects





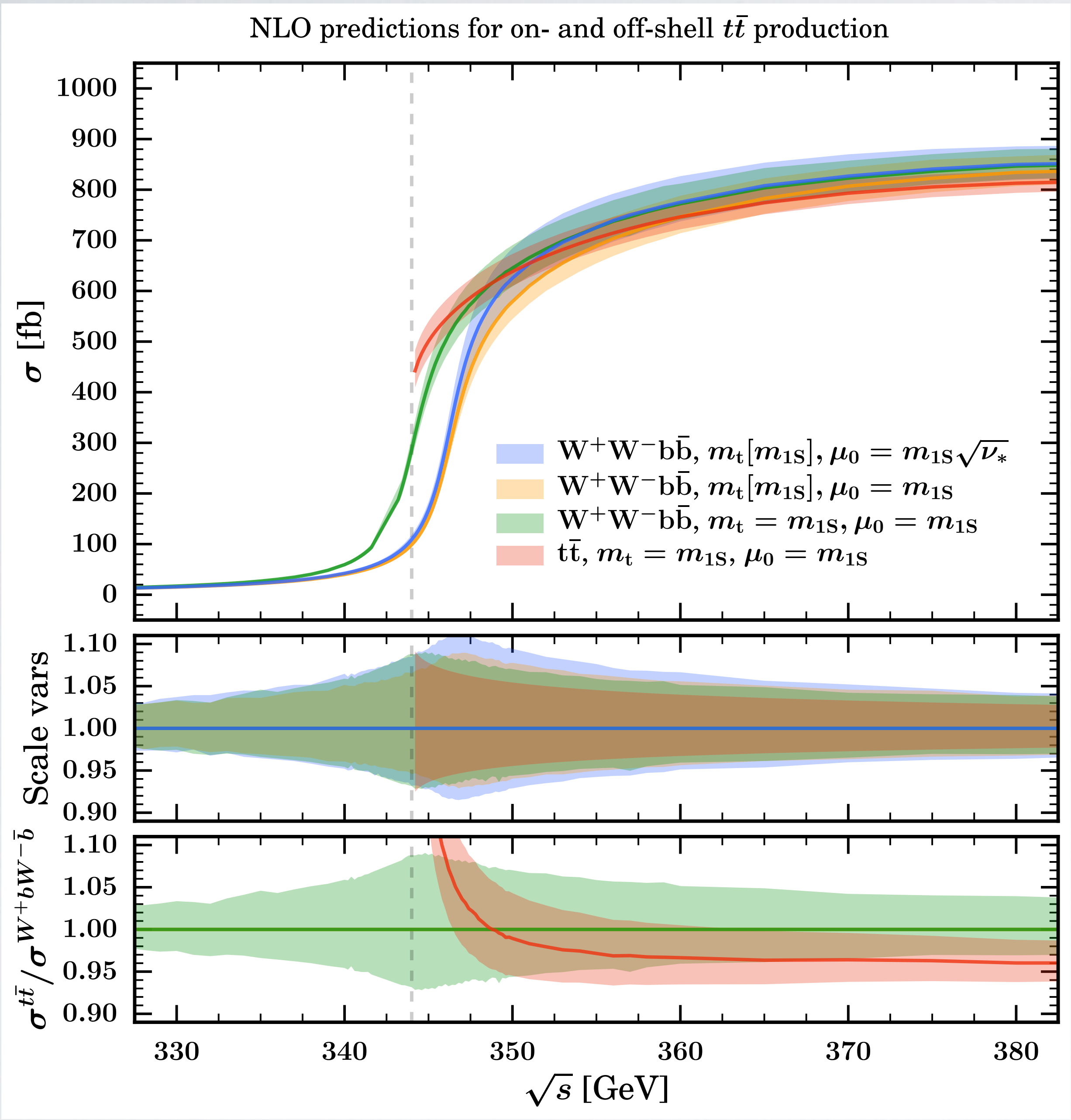
$$\sigma_{\rm matched} = \sigma_{\rm FO}\,[\alpha_{\rm H}] + \sigma^{\rm full}_{\rm NRQCD}\,[f_s\,\alpha_{\rm H},\ f_s\,\alpha_{\rm S},\ f_s\,\alpha_{\rm US}] - \sigma^{\rm expanded}_{\rm NRQCD}\,[f_s\,\alpha_{\rm H},\ f_s\,\alpha_{\rm H}]\ ,$$

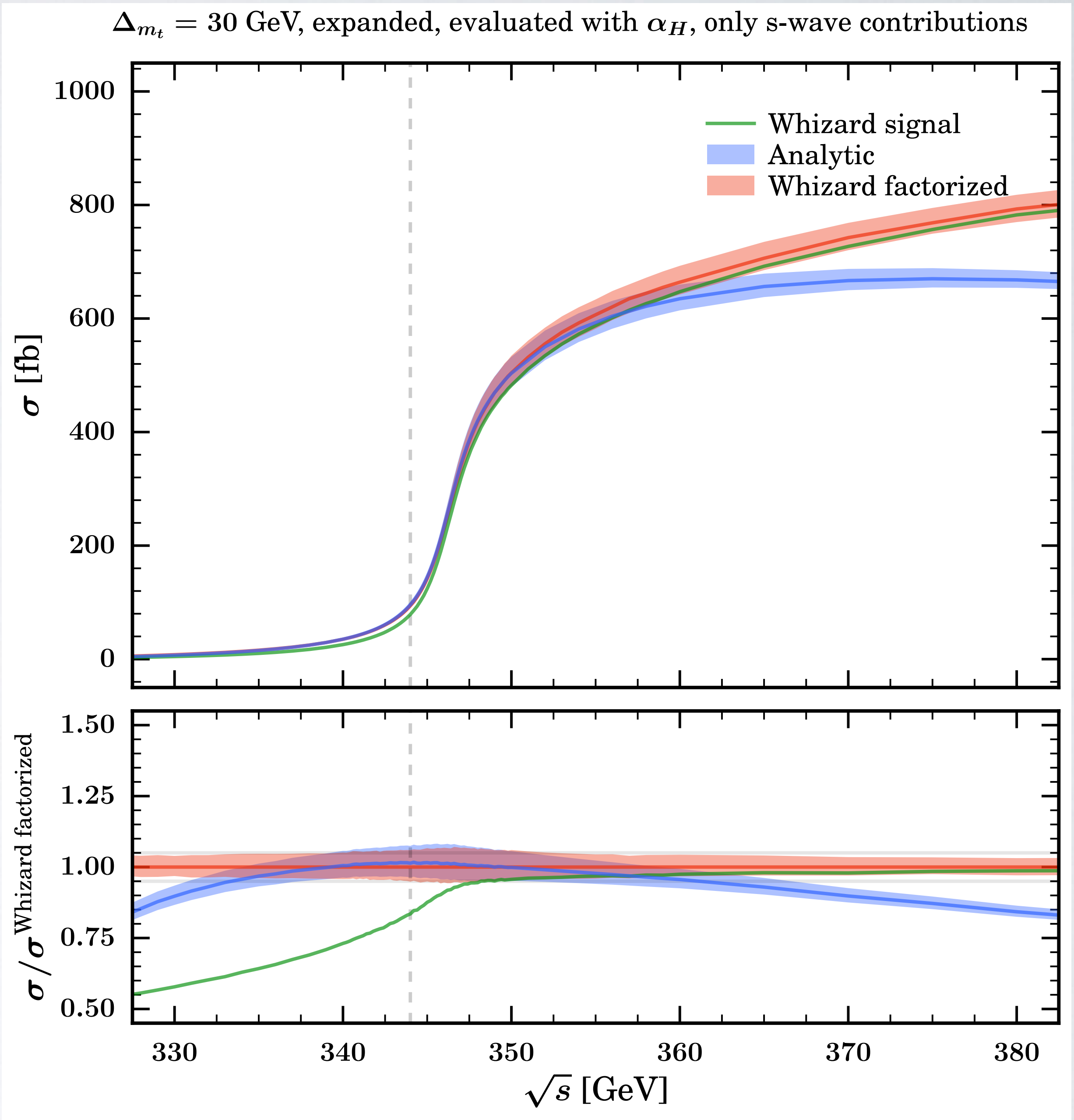
Smoothstep matching function:

$$f_s(v) = \begin{cases} 1 & v < v_1 \\ 1 - 3\left(\frac{v-v_1}{v_2-v_1}\right)^2 - 2\left(\frac{v-v_1}{v_2-v_1}\right)^3 & v_1 \le v \le v_2 \\ 0 & v > v_2 \end{cases}$$

Chokoufé/Hoang/Kilian/JRR/ StahlhofenTeubner/Weiss, *to appear very soon*

Top threshold: validation and matching





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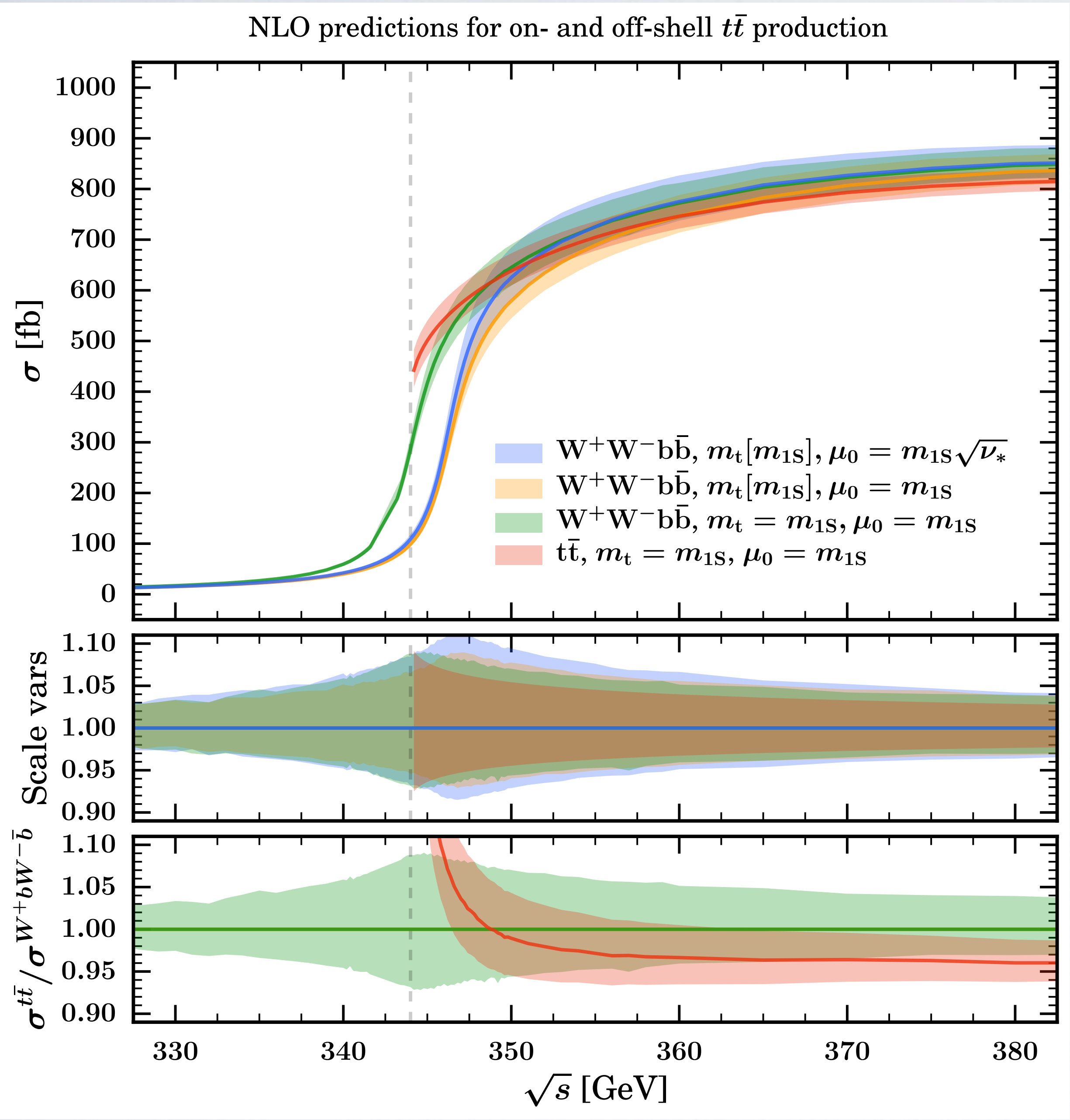

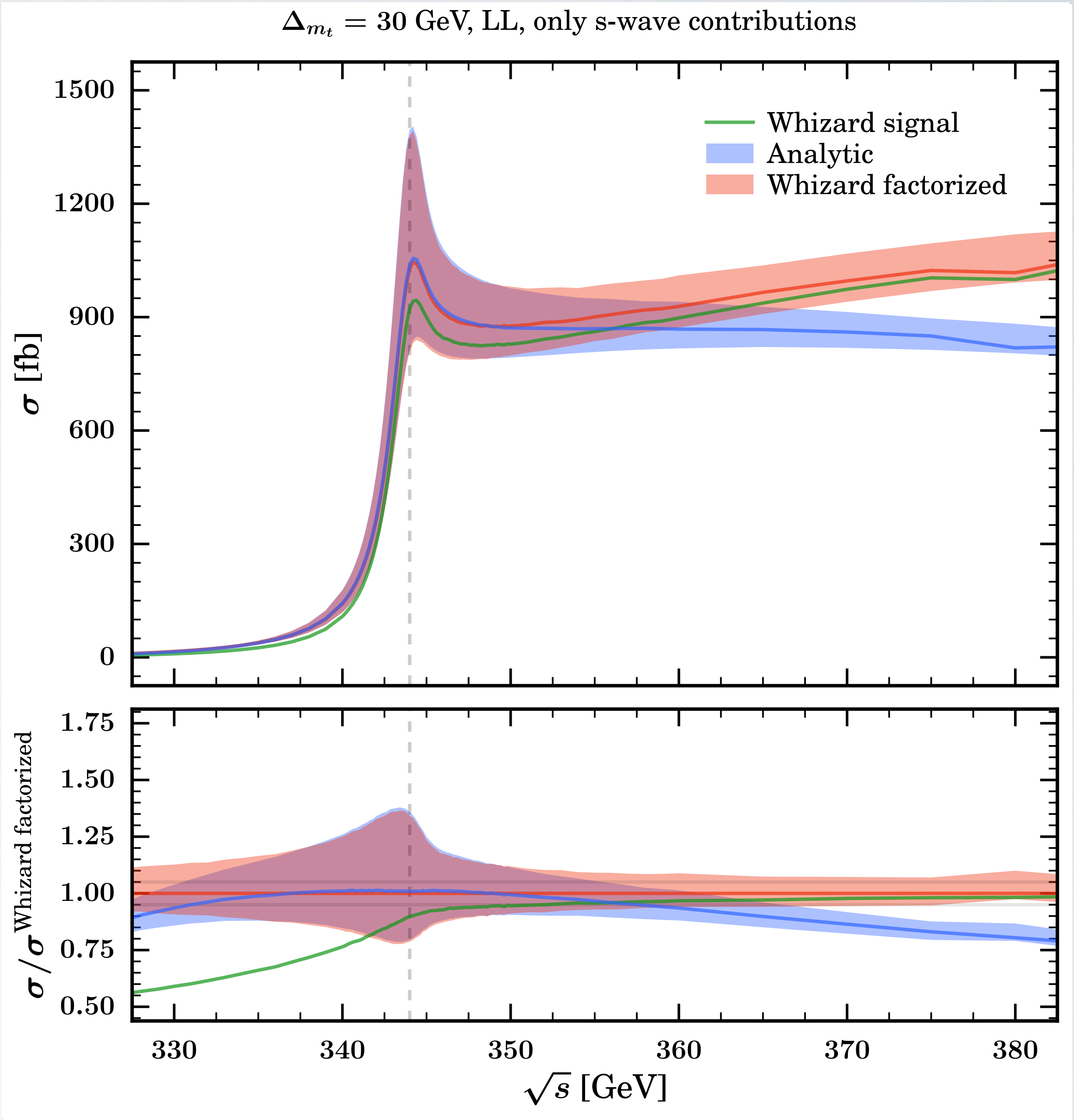

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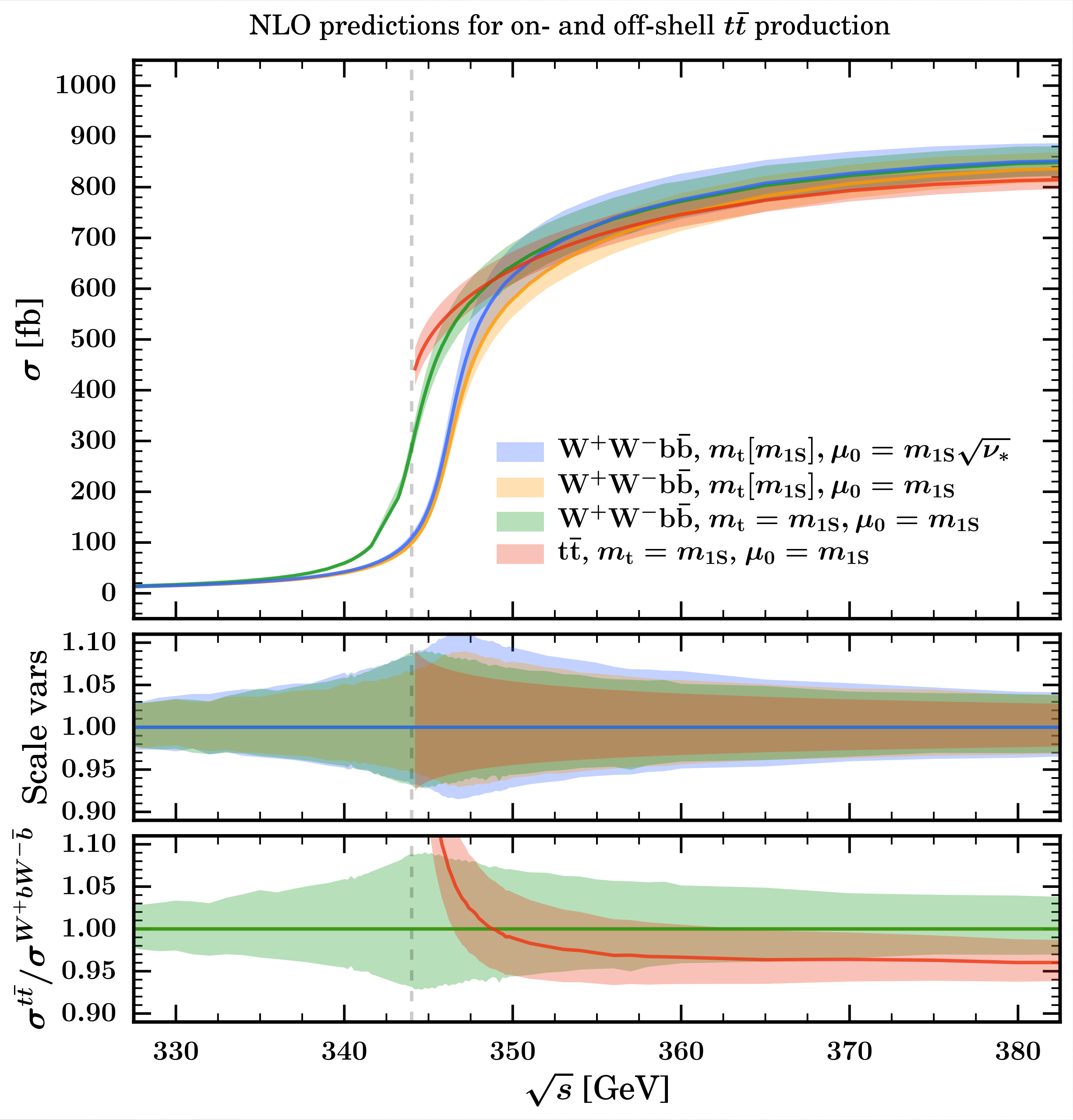
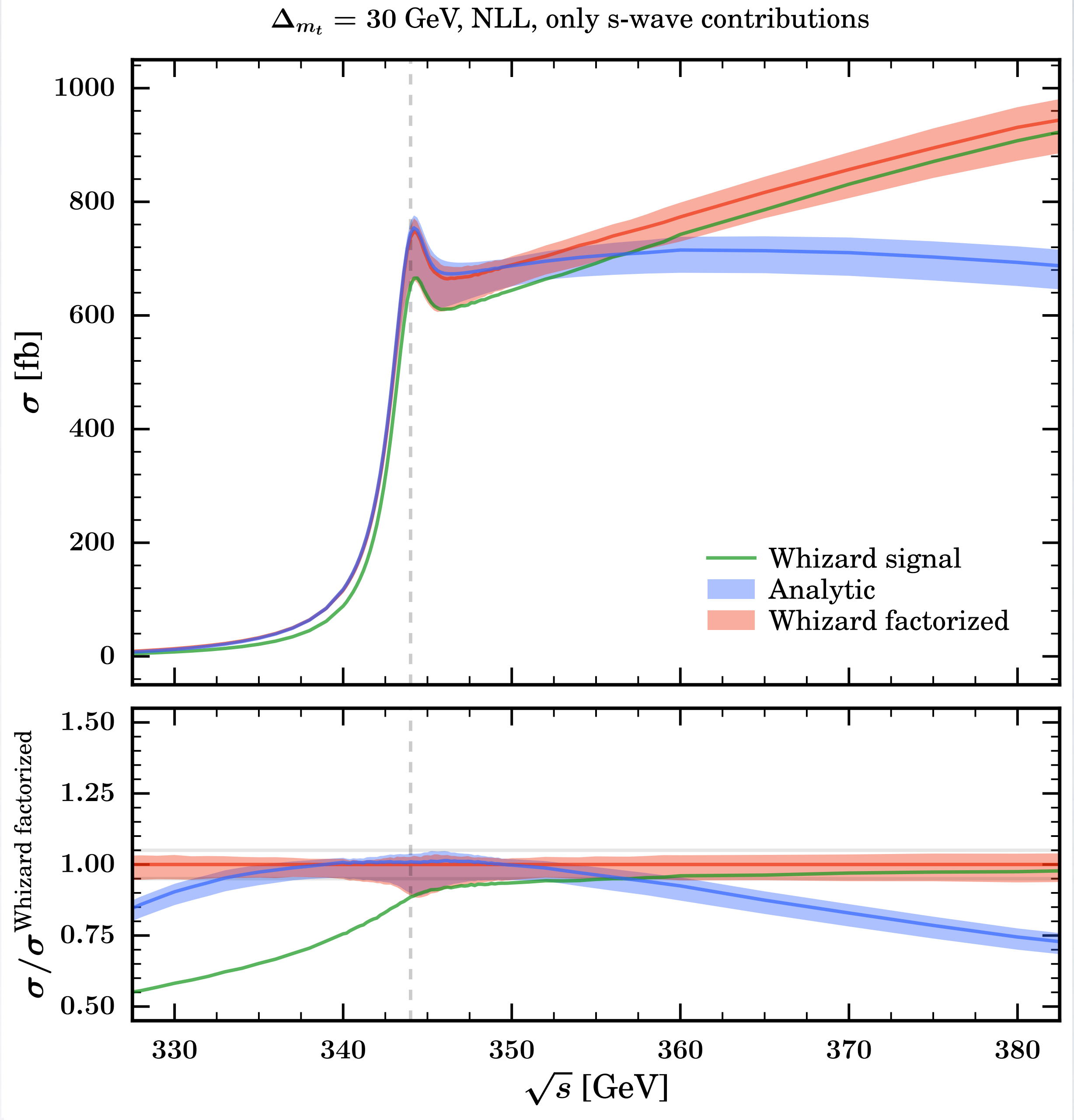
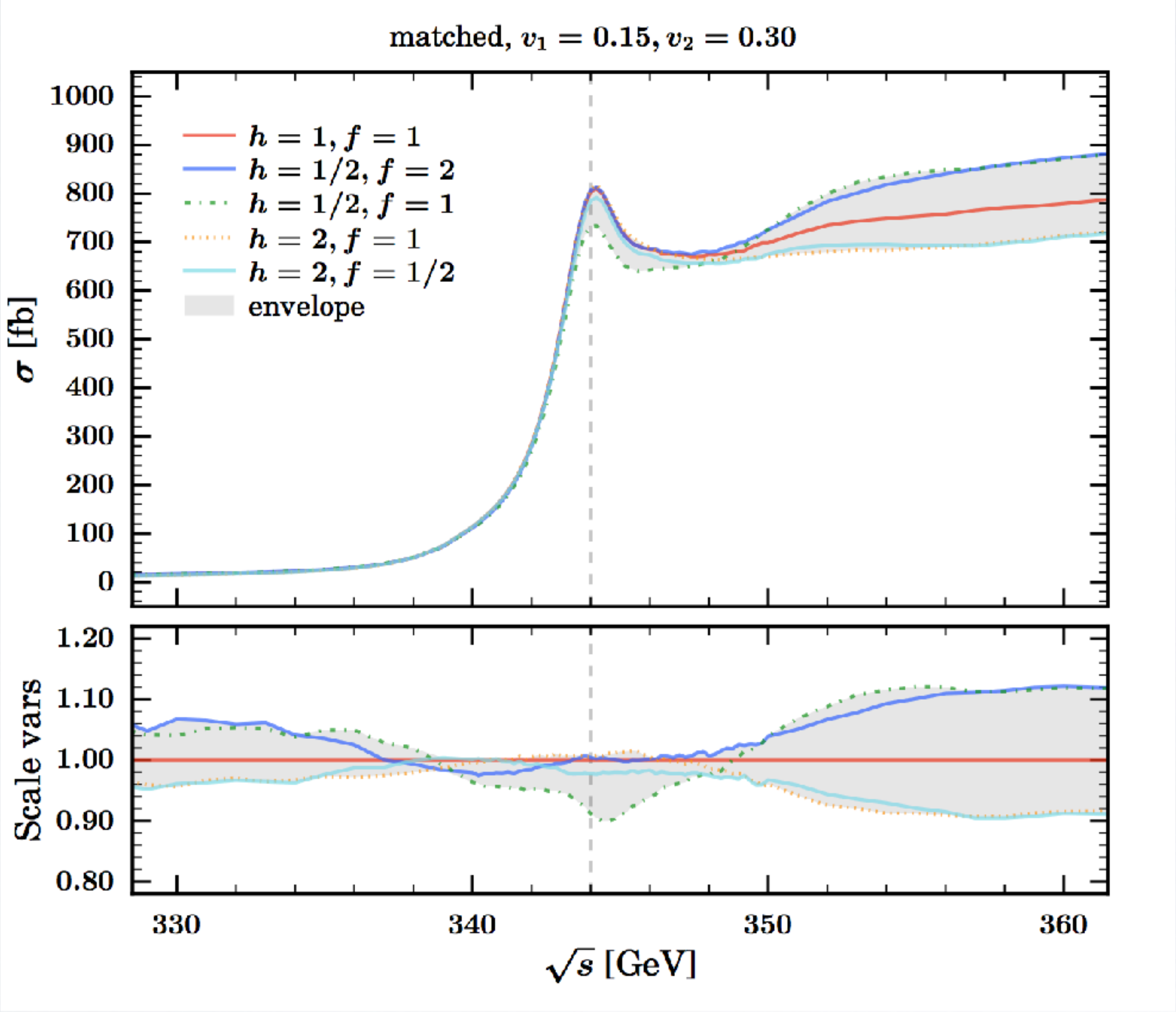
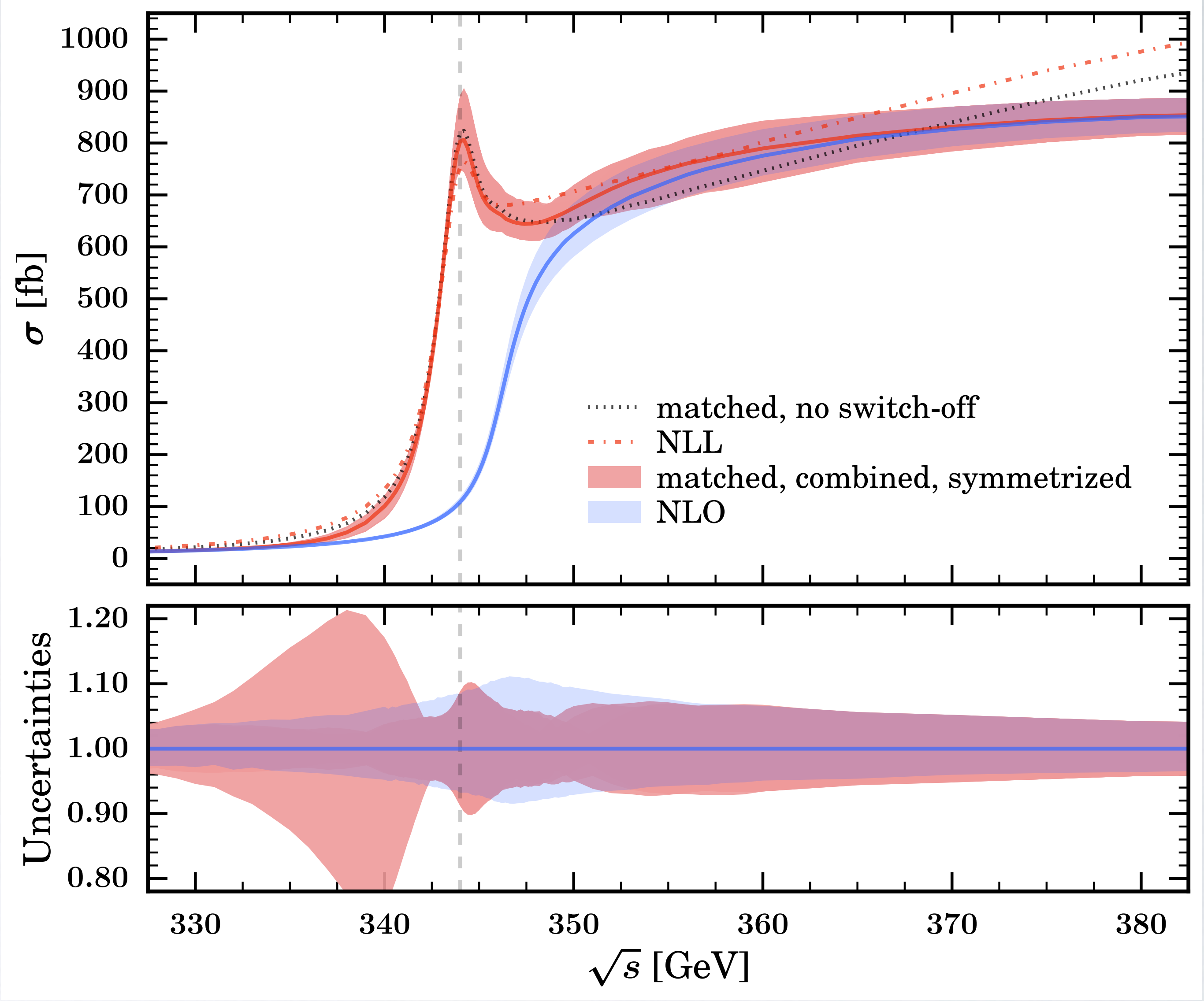




Bach/Chokoufé/Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, *to appear very soon*

Matching threshold NLL to continuum NLO

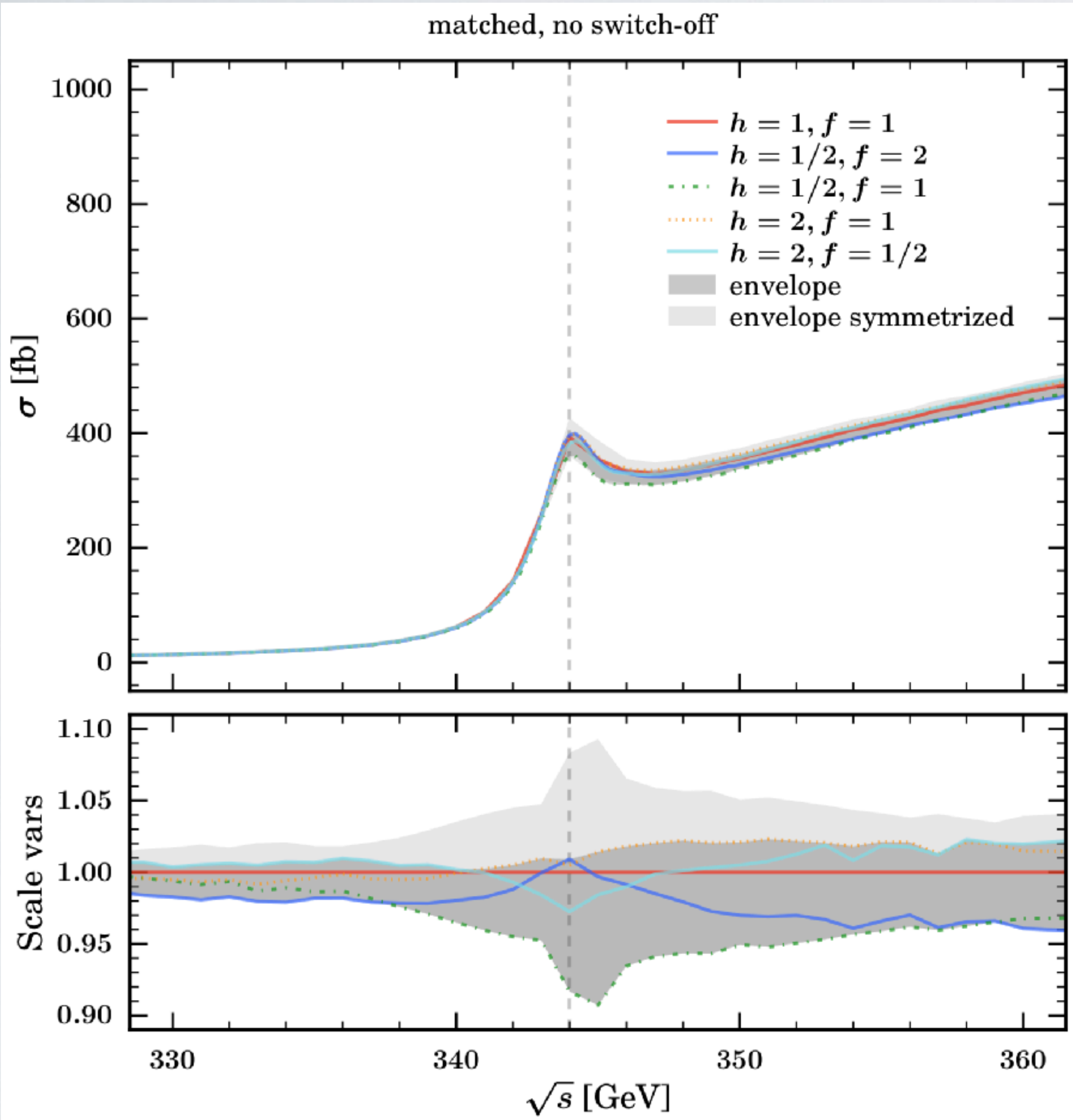


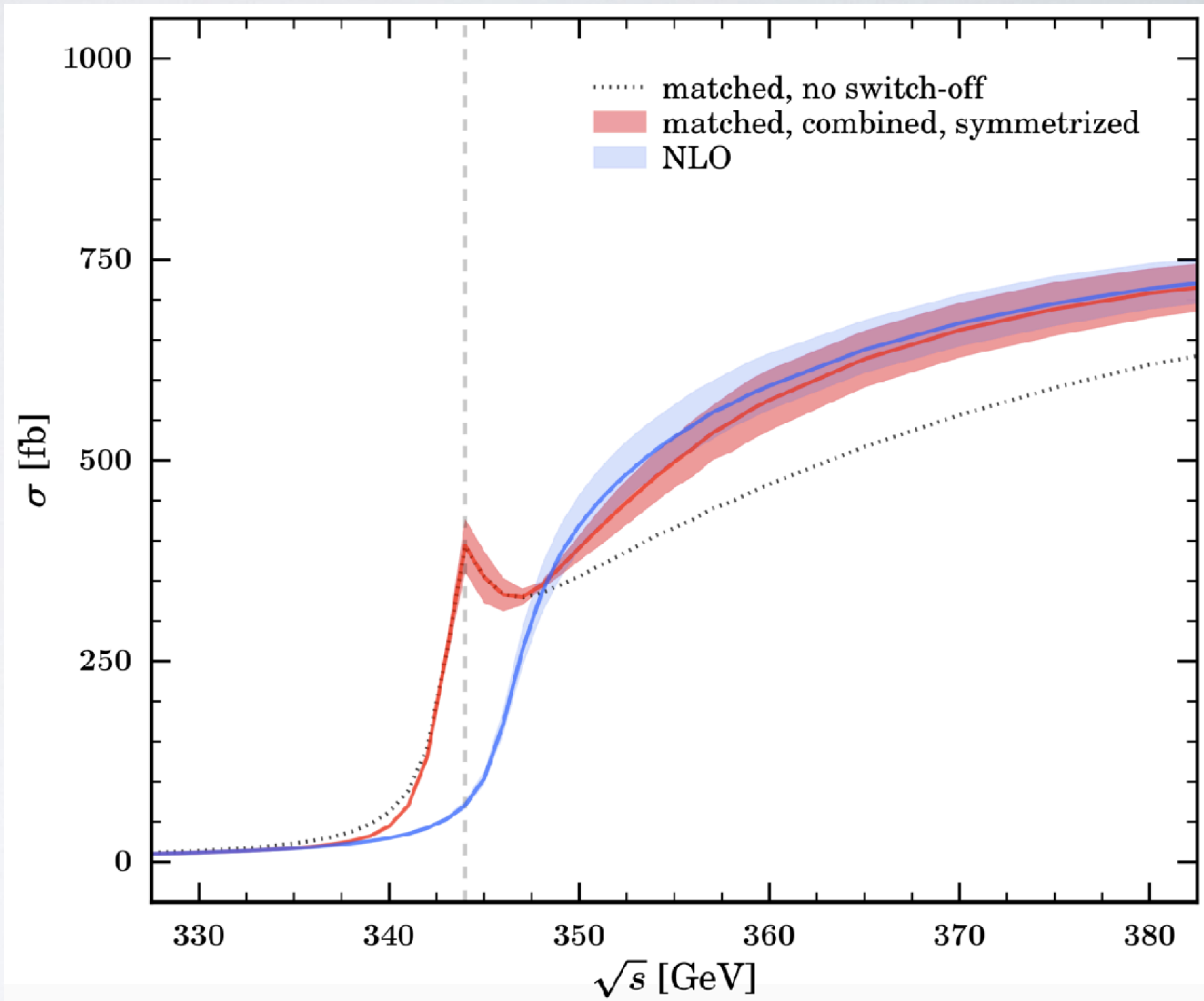


Total uncertainty: **matching and *h-f* variation band**

Bach/Chokoufé/Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, *to appear very soon*

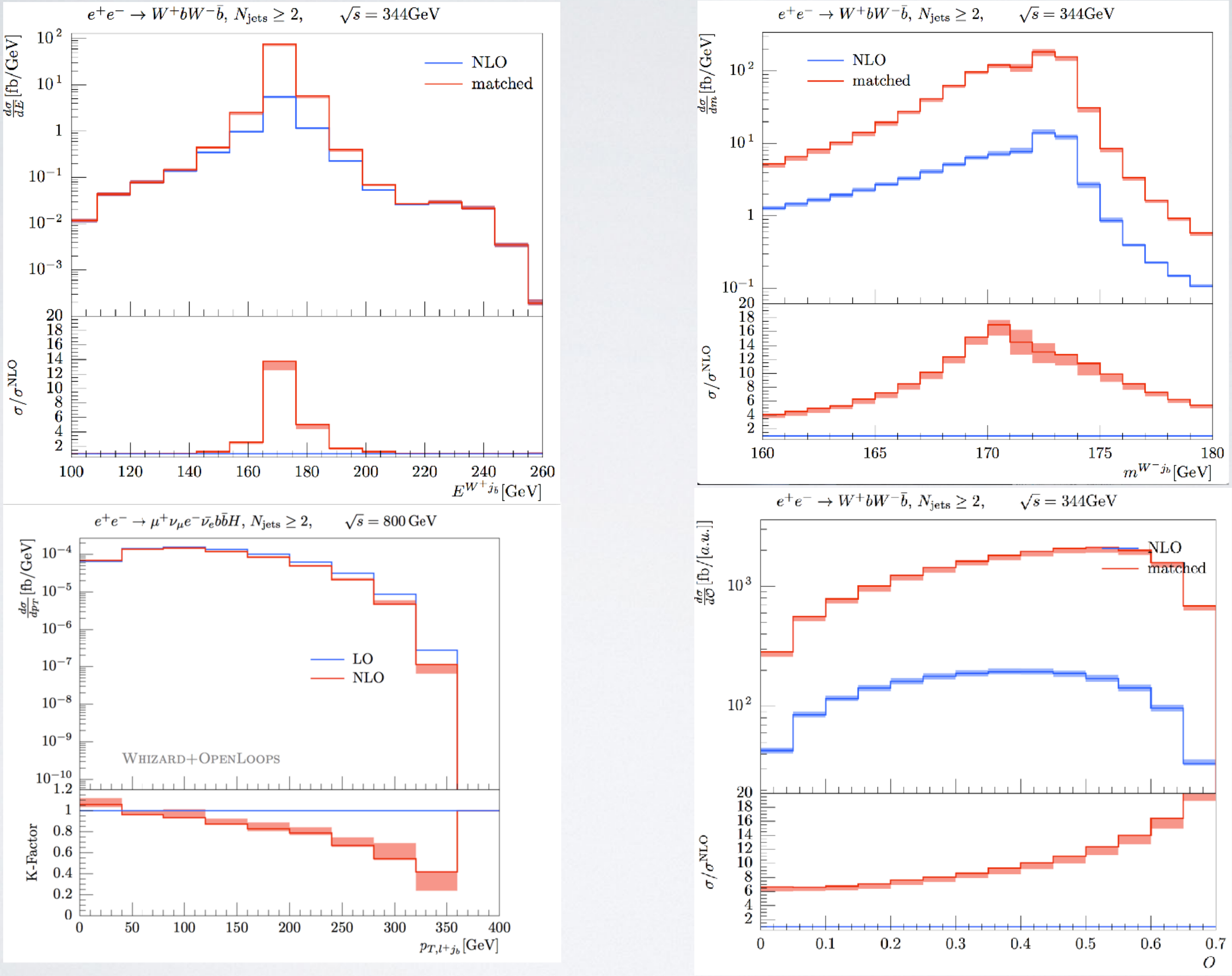
Threshold matching with QED ISR





Bach/Chokoufé/Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, *to appear very soon*

Matched threshold differential distributions



Conclusions & Outlook

- Top physics is cornerstone of future lepton collider program
- Leptonic top and associated Higgs fully off-shell at NLO QCD
- Inclusive processes: off-shell background grows with energy
- Top Yukawa extraction @NLO QCD: stronger interference effects

- Complete NRQCD threshold / NLO continuum matching
- Offers framework for new differential top mass measurements
- Next projects: EW corrections, semi-leptonic/hadronic top decays, ttH threshold matching, top threshold matched with EW corrections

- `WHIZARD 2.6` framework for automated (QCD) NLO
- NLO QCD (almost) done → `WHIZARD 3.0` [EW in validation]
- Automated POWHEG matching

Outlook to PDG 203X:

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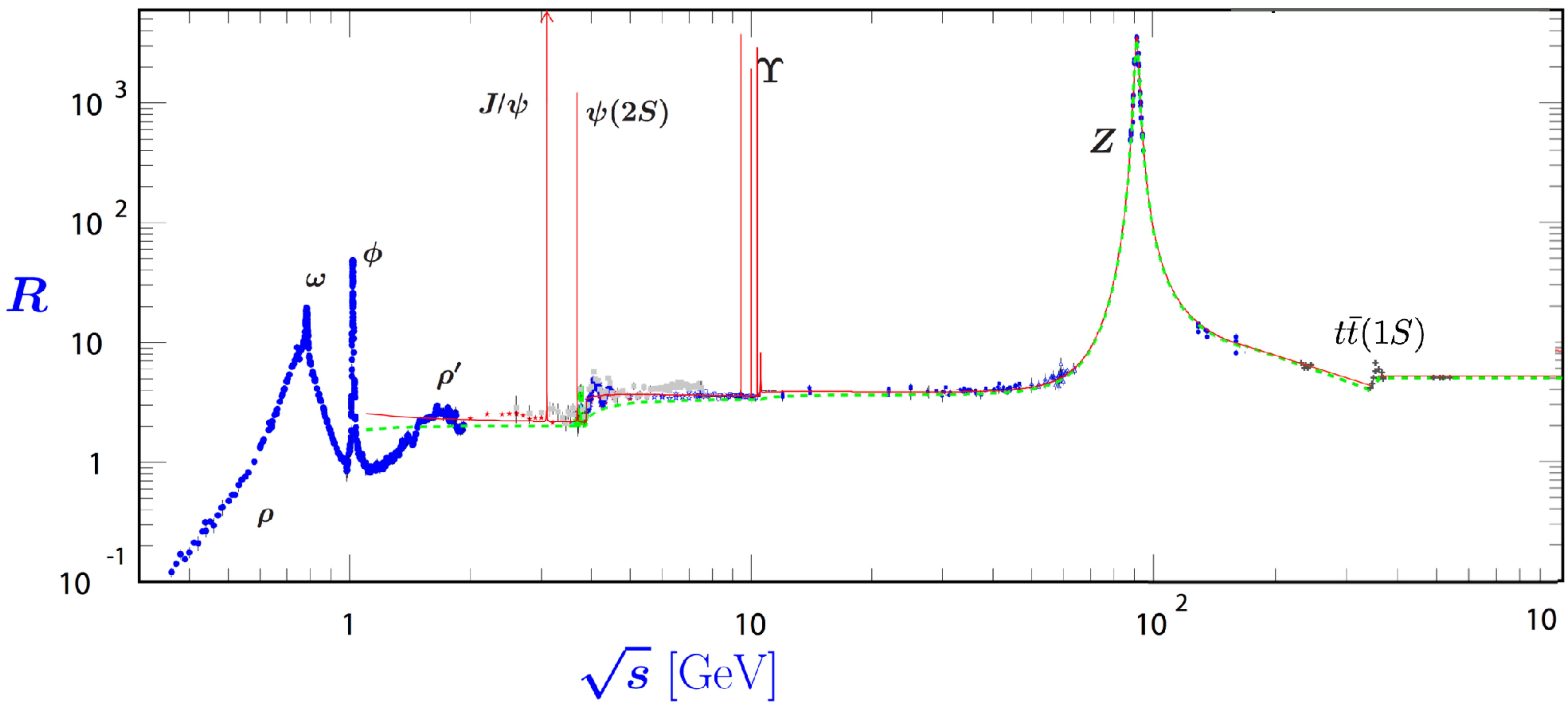


BACKUP

Examples and Validation

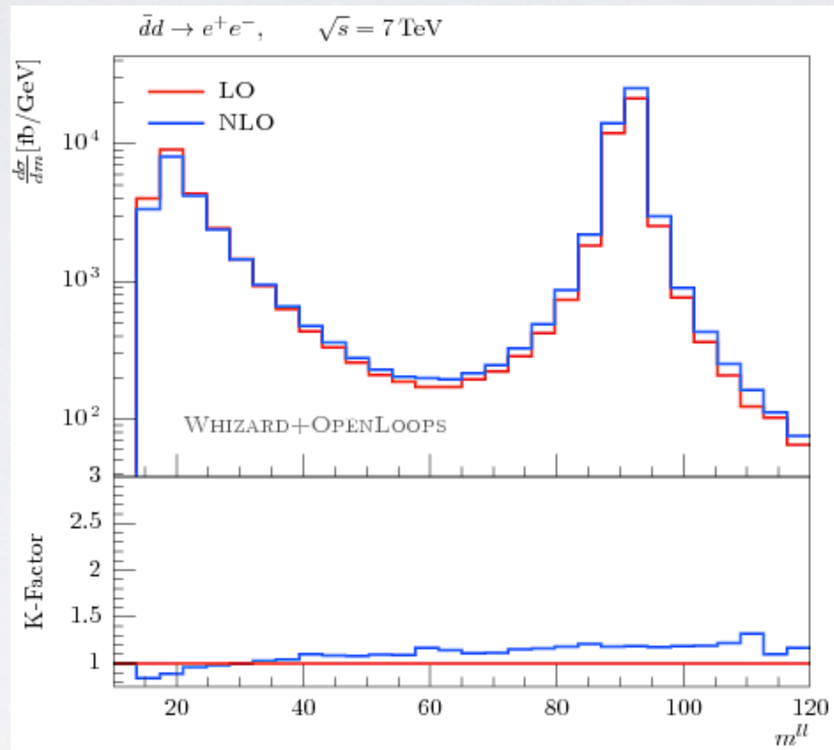
List of validated NLO QCD processes

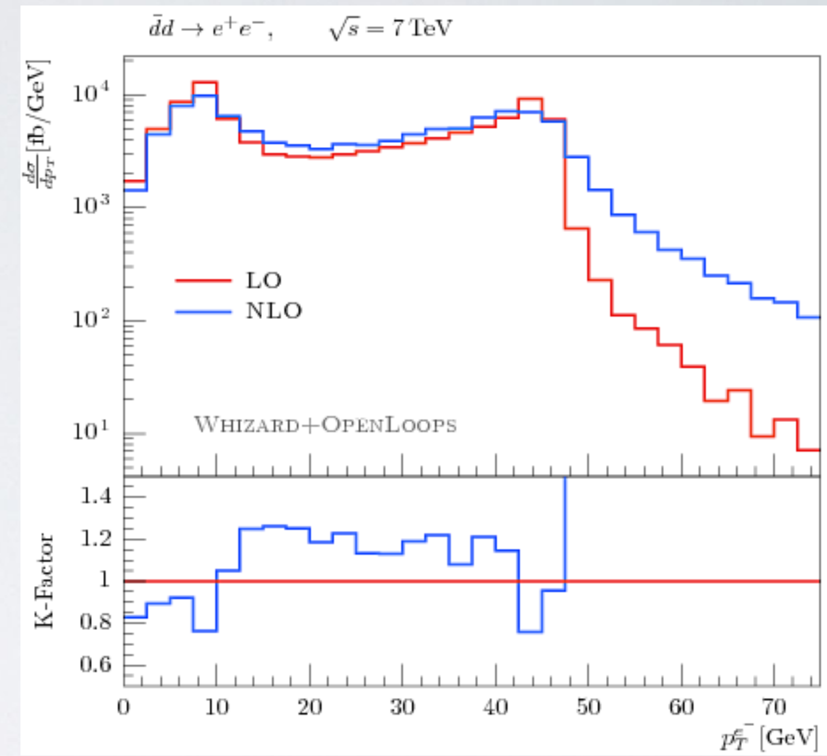
- $e^+e^- \to jj$
- $e^+e^- \to jjj$
- $e^+e^- \to \ell^+\ell^- jj$
- $e^+e^- \to \ell^+\nu_\ell jj$
- $e^+e^- \to t\bar{t}$
- $e^+e^- \to t\bar{t}t\bar{t}$
- $e^+e^- \to t\bar{t}W^+jj$
- $e^+e^- \to tW^-b$
- $e^+e^- \to W^+W^-b\bar{b}, \quad \ell^+\ell^-\nu_\ell\bar{\nu}_\ell b\bar{b}$
- $e^+e^- \to b\bar{b}\ell^+\ell^-$
- $e^+e^- \to t\bar{t}H$
- $e^+e^- \to W^+W^-b\bar{b}H, \quad \ell^+\ell^-\nu_\ell\bar{\nu}_\ell b\bar{b}H$
- $pp \to \ell^+\ell^-$
- $pp \to \ell\nu$
- $pp \to ZZ$

NEW

- Simplest hadron collider processes validated:

$pp \to (Z \to ll) + X, \quad pp \to (W \to l\nu) + X, \quad pp \to ZZ + X$





- ✦ QCD NLO infrastructure in pp close to complete
- ✦ After complete NLO QCD validation: `WHIZARD` v3.0.0
- ✦ Status of EW corrections: all parts technically completed, validation phase started [Rothe et al.]

Validation of NLO QCD for Lepton Collisions

Final state	MG5_aMC $\sigma^{\rm LO}$[fb]	MG5_aMC $\sigma^{\rm NLO}$[fb]	K	WHIZARD $\sigma^{\rm LO}$[fb]	WHIZARD $\sigma^{\rm NLO}$[fb]	K
jj	622.3(5)	639(1)	1.02684	622.73(4)	639.7(2)	1.0272
$b\bar{b}$	92.37(6)	94.89(1)	1.02728	92.32(1)	94.78(7)	1.0266
$t\bar{t}$	166.2(2)	174.5(6)	1.04994	166.4(1)	175.1(1)	1.0522
$t\bar{t}t\bar{t}$	$6.45(1)\cdot 10^{-4}$	$12.21(5)\cdot 10^{-4}$	1.89302	$6.463(2)\cdot 10^{-4}$	$12.16(2)\cdot 10^{-4}$	1.8814
$b\bar{b}b\bar{b}$	$1.644(3)\cdot 10^{-1}$	$3.60(1)\cdot 10^{-1}$	2.1897	$1.64(2)\cdot 10^{-1}$	$3.67(4)\cdot 10^{-1}$	2.2378
$t\bar{t}b\bar{b}$	$1.819(3)\cdot 10^{-1}$	$2.92(1)\cdot 10^{-1}$	1.6052	$1.86(1)\cdot 10^{-1}$	$2.93(2)\cdot 10^{-1}$	1.5752
$t\bar{t}j$	48.13(5)	53.43(1)	1.11012	48.3(2)	53.66(9)	1.1109
$t\bar{t}H$	2.018(3)	1.911(6)	0.947	2.022(3)	1.913(3)	0.9461
$t\bar{t}\gamma$	12.7(2)	13.3(4)	1.04726	12.71(4)	13.78(4)	1.0841
$t\bar{t}Z$	4.642(6)	4.95(1)	1.06636	4.64(1)	4.94(1)	1.0646
$t\bar{t}HZ$	$3.600(6)\cdot 10^{-2}$	$3.58(1)\cdot 10^{-2}$	0.99445	$3.596(1)\cdot 10^{-2}$	$3.581(2)\cdot 10^{-2}$	0.9958
$t\bar{t}\gamma Z$	0.2212(3)	0.2364(6)	1.06873	0.220(1)	0.240(2)	1.0909
$t\bar{t}\gamma H$	$9.75(1)\cdot 10^{-2}$	$9.42(3)\cdot 10^{-2}$	0.96614	$9.748(6)\cdot 10^{-2}$	$9.58(7)\cdot 10^{-2}$	0.9827
$t\bar{t}\gamma\gamma$	0.383(5)	0.416(2)	1.08618	0.382(3)	0.420(3)	1.0995
$t\bar{t}ZZ$	$3.788(4)\cdot 10^{-2}$	$4.00(1)\cdot 10^{-2}$	1.05597	$3.756(4)\cdot 10^{-2}$	$4.005(2)\cdot 10^{-2}$	1.0663
$t\bar{t}HH$	$1.358(1)\cdot 10^{-2}$	$1.206(3)\cdot 10^{-2}$	0.888	$1.367(1)\cdot 10^{-2}$	$1.218(1)\cdot 10^{-2}$	0.8909
$t\bar{t}W^+W^-$	0.1372(3)	0.1540(6)	1.1225	0.1370(4)	0.1538(4)	1.1225
$t\bar{t}W^\pm jj$	$2.400(4)\cdot 10^{-4}$	$3.72(1)\cdot 10^{-4}$	1.54999	$2.41(1)\cdot 10^{-4}$	$3.74(2)\cdot 10^{-4}$	1.5518
jjj	340.1(2)	316(2)	0.92914	342.4(5)	319(1)	0.9316
$jjjj$	104.7(1)	109.0(6)	1.04106	105.1(4)	118(1)	1.1227
$t\bar{t}t\bar{t}j$	$2.719(5)\cdot 10^{-5}$	$5.34(3)\cdot 10^{-5}$	1.96394	$2.722(1)\cdot 10^{-5}$	$4.471(5)\cdot 10^{-5}$	1.6425
$t\bar{t}Hj$	0.2533(3)	0.2658(9)	1.04935	0.254(1)	0.307(1)	1.2087
$t\bar{t}\gamma j$	2.355(2)	2.62(1)	1.11253	2.47(1)	3.14(2)	1.2712
$t\bar{t}Zj$	0.6059(6)	0.694(3)	1.14548	0.610(4)	0.666(5)	1.0918