EW/Higgs Precision for Future Lepton Colliders

SnowMass2021

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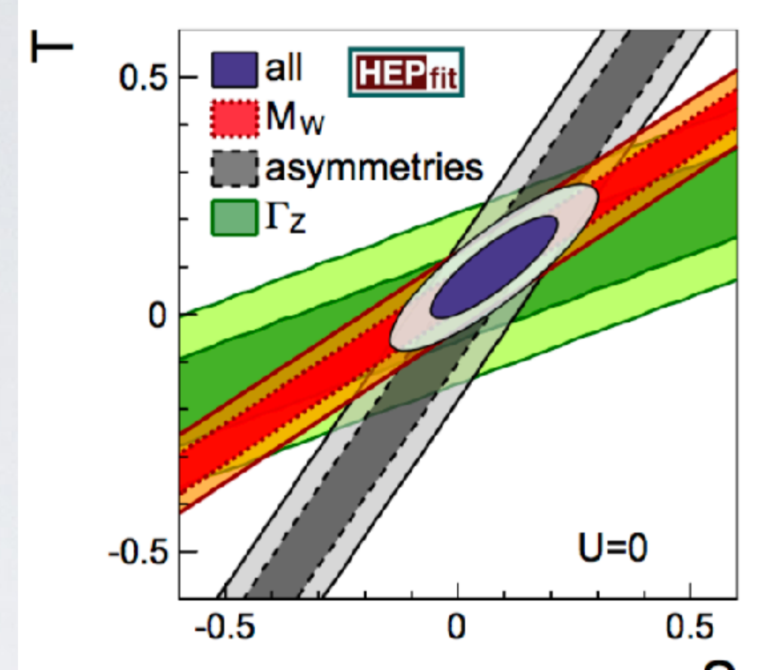
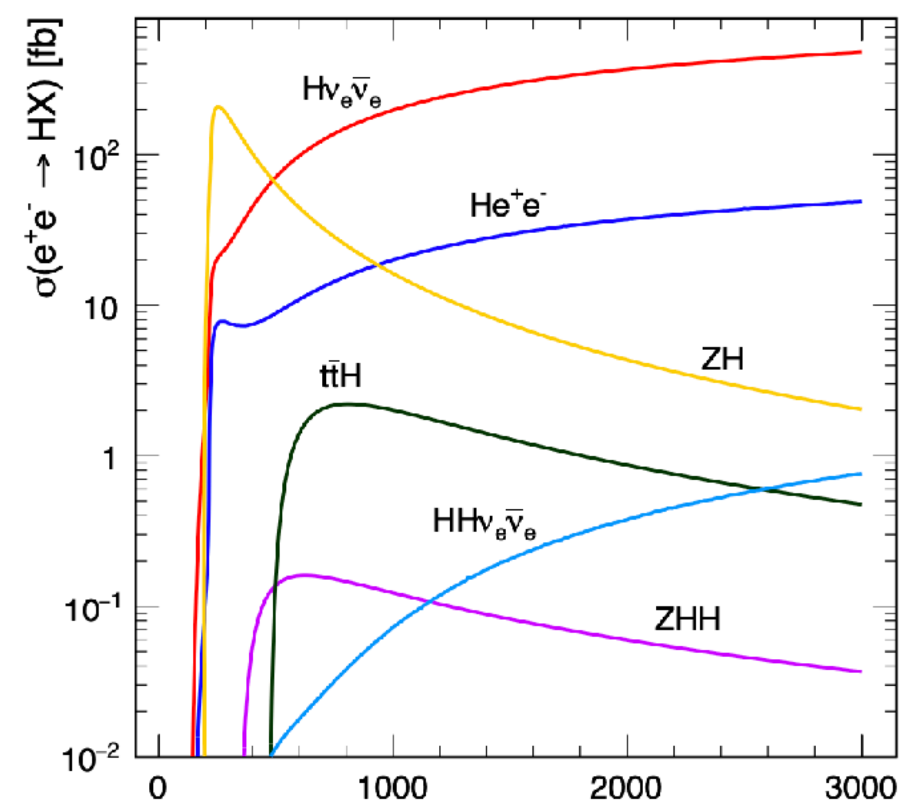
Snowmass 2020 Community Planning Meeting

Motivation — Findings from ESU

1. Higgs (and EW) main driver of particle physics for next decade(s)
2. Broad consensus on lepton (e^+e^-) collider as next step
3. Well-defined initial state: electroweak production
4. High precision, triggerless operation: complete coverage of final states

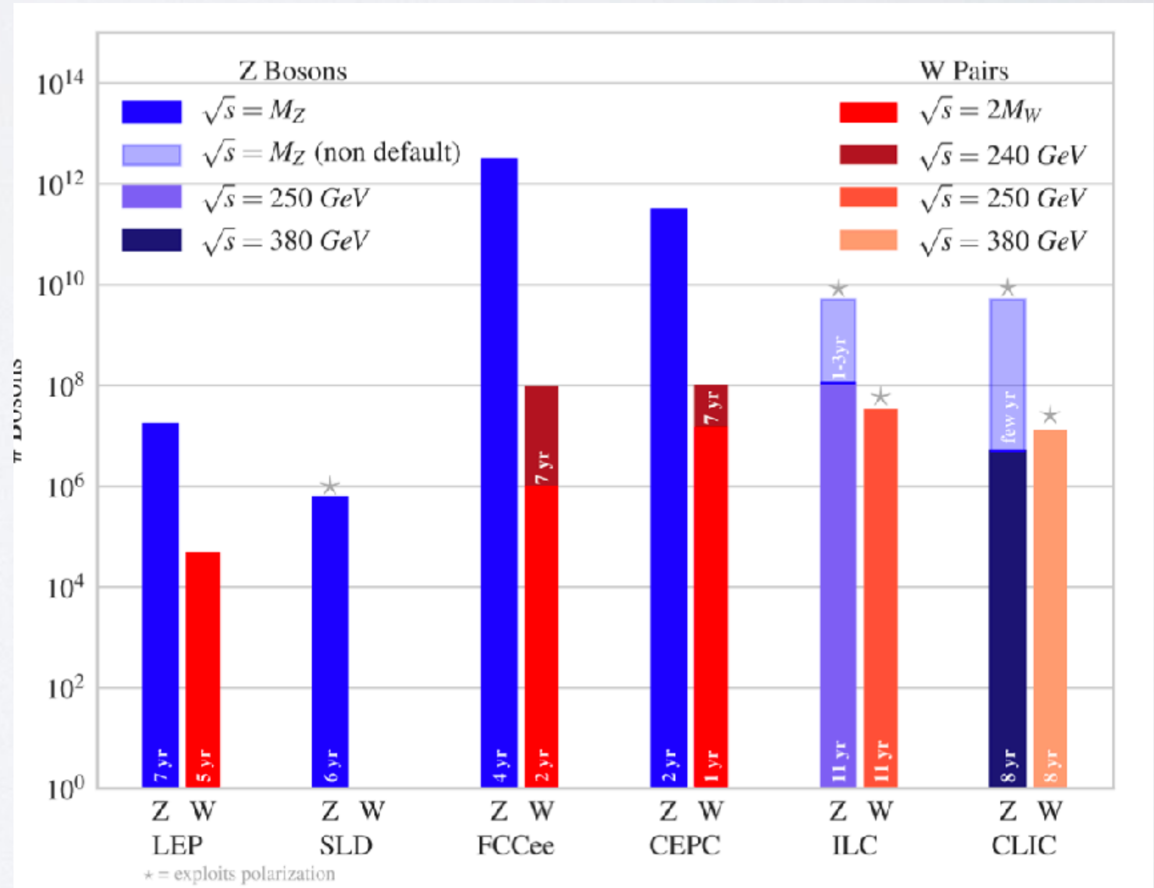


de Blas et al., 1608.01509



Abramowicz et al. , 1608.07538

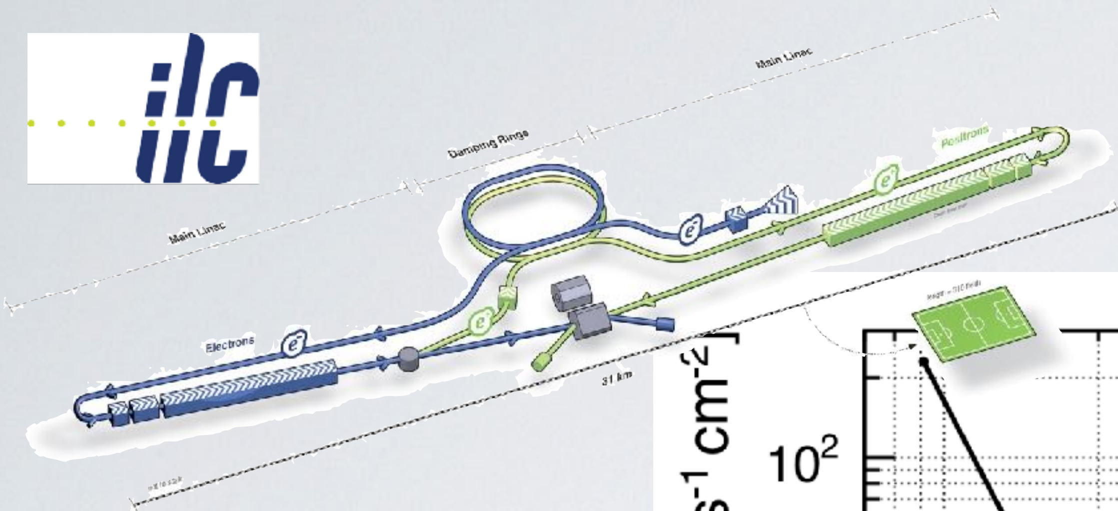
Polarisation	Scaling factor		
$P(e^-):P(e^+)$	$e^+e^- \to ZH$	$e^+e^- \to H\nu_e\bar{\nu}_e$	$e^+e^- \to He^+e^-$
unpolarised	1.00	1.00	1.00
−80% : 0%	1.12	1.80	1.12
−80% : +30%	1.40	2.34	1.17
−80% : −30%	0.83	1.26	1.07
+80% : 0%	0.88	0.20	0.88
+80% : +30%	0.69	0.26	0.92
+80% : −30%	1.08	0.14	0.84

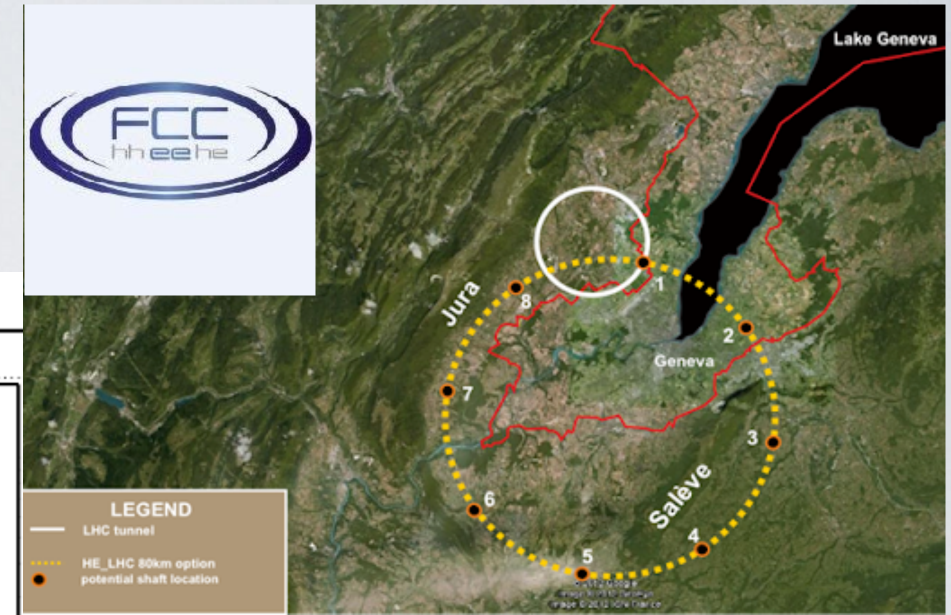


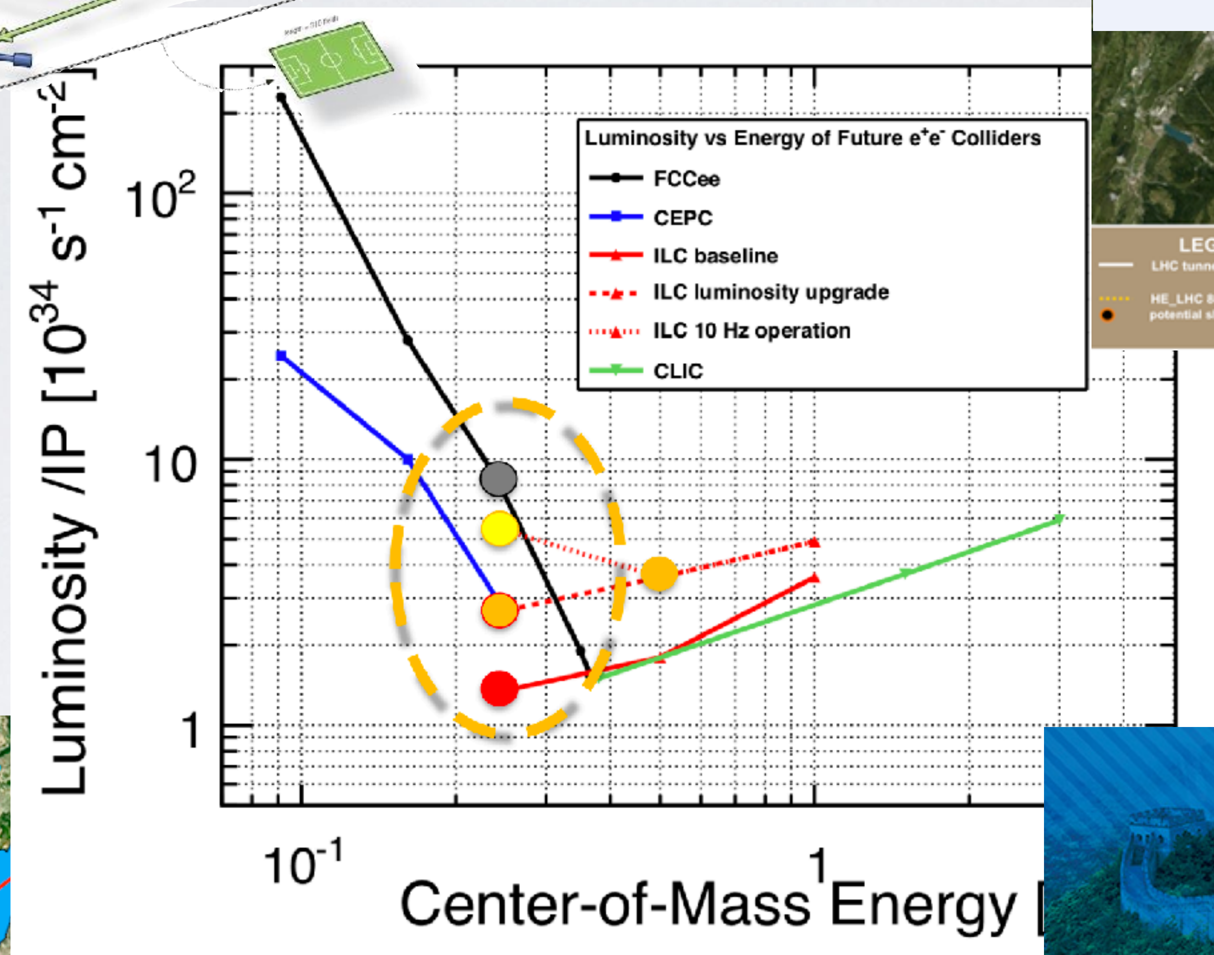
1910.11775

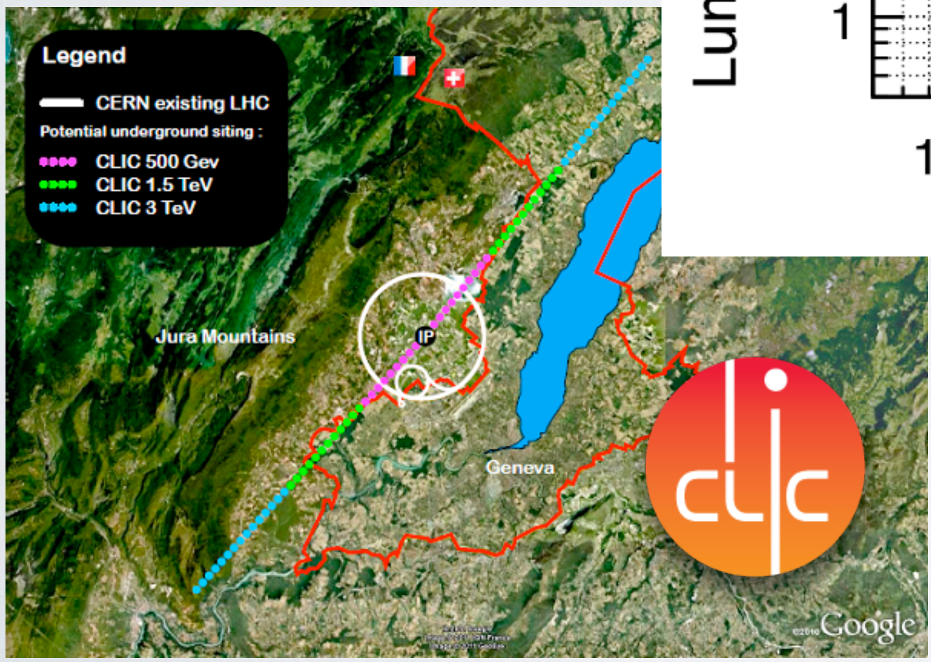
	Current	HL-LHC	ILC_{250}	(& ILC_{91})	CEPC	FCC-ee	$CLIC_{380}$	(& $CLIC_{91}$)
S	0.13	0.053	0.012	0.009	0.0068	0.0038	0.032	0.011
T	0.08	0.041	0.014	0.013	0.0072	0.0022	0.023	0.012

The e⁺e⁻ Precision Landscape



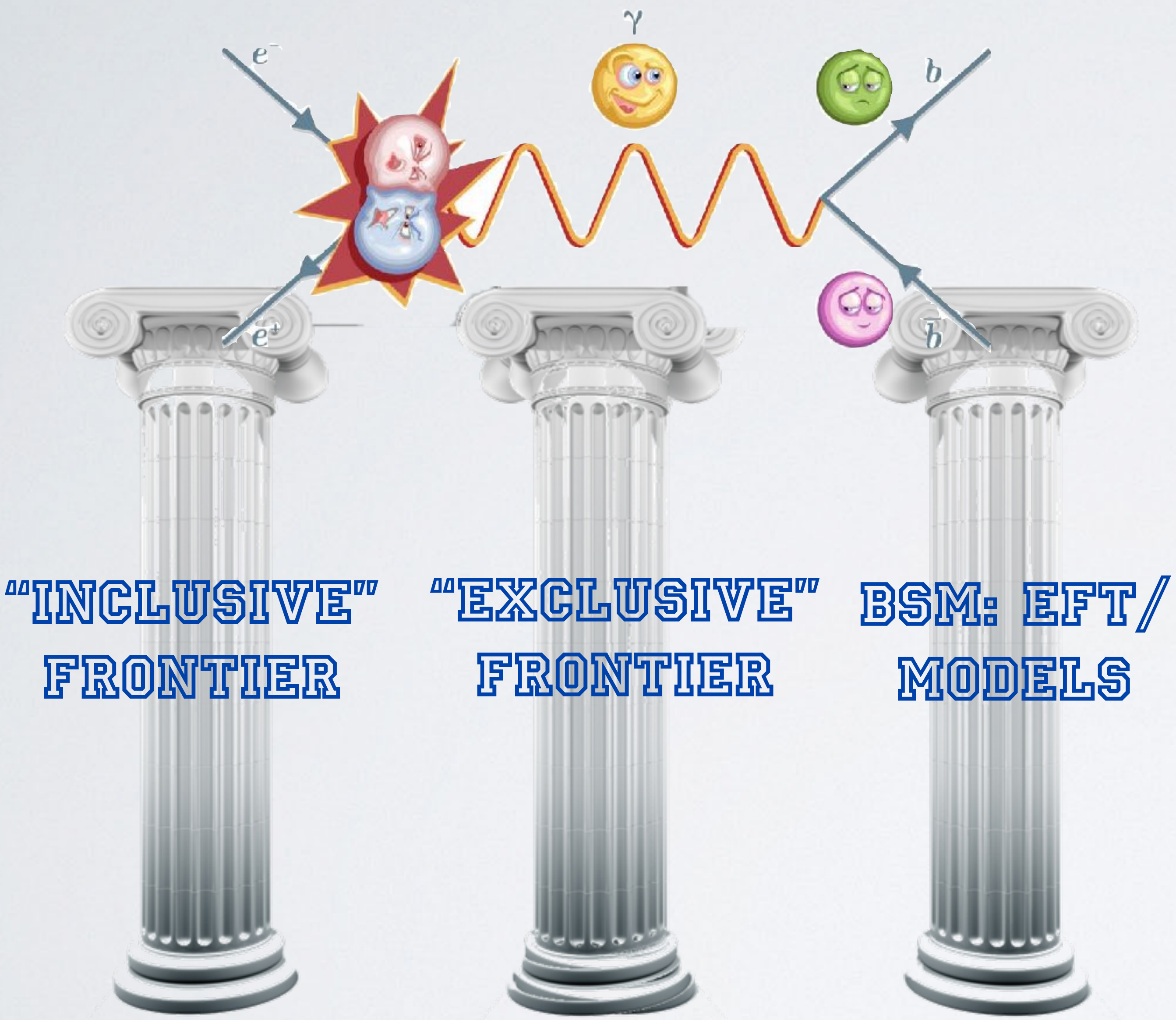


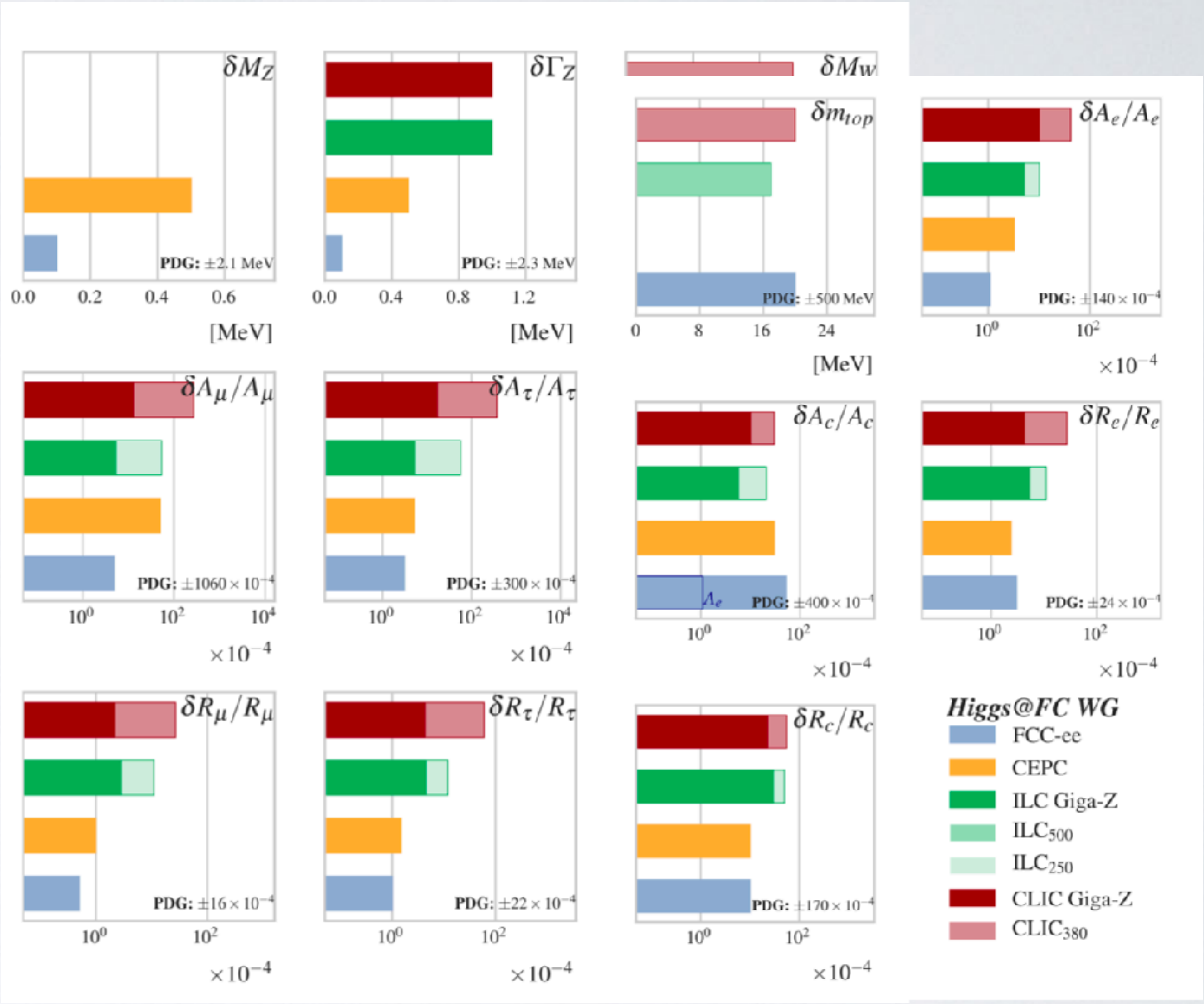




Theoretical Uncertainties

- LHC HXSWG: 1101.0593, 1201.3084, 1307.1347, 1610.07922
- FCC-ee theory effort: CERN workshops '18-'20: 1906.05379
- Data extraction needs theory input (bkgd., acceptance): "exclusive" "pseudo-observables"





- ❑ **Intrinsic uncertainties:** missing higher-order calculations of observable
- ❑ **Parametric uncertainties:** imperfect knowledge or theoretic data extraction of SM input

Theoretical Uncertainties

- Strip loop amps. of group theory / mass ratios / multiplicities / couplings. → $\mathcal{O}(1)$
- Extrapolate to higher orders from geometric series (beware of renormalons)
- Scale dependence for missing higher order corrections (QCD, $\overline{\mathrm{MS}}$, less useful for EW)
- Compare differences in renormalisation schemes (e.g. On-Shell vs. $\overline{\mathrm{MS}}$)

Parametric uncertainties

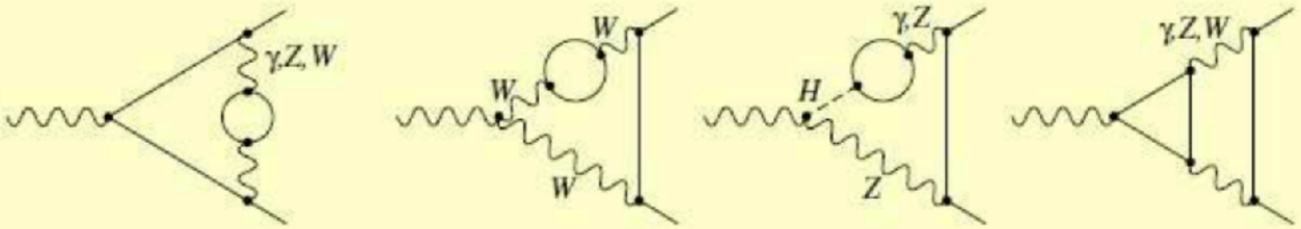
- M_H: Higgsstrahlung at threshold, 10 MeV uncertainty, leptonic recoil, minor th. uncertainties
- M_Z: *Z* lineshape, ~0.1 MeV exp., QED ISR+ISR/FSR, EW box diagrams, Jadach/Skrzypek/Pietrzik, 1999
- $\alpha_s(M_Z)$: global fit of overconstrained EW pseudo-observables at *Z* pole, pert. uncertainties
- $m_t^{MS}(m_t)$: N³LO QCD/NNLO EW, resummed NNLL, 4-loop mass translation., off-shell corr.
  Beneke et al., 1506.06864/1711.10429, Hoang et al. 1309.6323, Marquard et al. 1502.01030, Chokoufé et al. 1609.03390, Bach et al. 1712.02220
- $m_{c/s}^{MS}(m_{c/s})$: lattice QCD, sum rules, NNLO jet ratios. 1404.0319, 1401.7035, 0907.2110, 1411.3132, 1504.07638
- $\Delta\alpha$: extracted from $e^+e^- \to$ *hadrons* , τ *decays* (BESIII, VEPP-2000, Belle II), radiative return
  Proposal for direct measurement below/above *Z* pole: subtract EW from QED corrections
  available @ 1-loop; needed fermionic 2-loop corr., $\mathcal{O}(\alpha\alpha_s^2)$ corr. $\Longrightarrow 10^{-4}$
  2-/3-loop box diagrams: full $\mathcal{O}(\alpha^2, \alpha^2\alpha_s)$, double-fermionic $\mathcal{O}(\alpha^3)$ corr. $\Longrightarrow 10^{-5}$

The "Inclusive" Frontier

Parametric uncertainties

$$\delta m_t = 50\text{ MeV}, \quad \delta m_b = 13\text{ MeV}, \quad \delta M_Z = 0.1\text{ MeV}, \quad \delta\alpha_s = 0.0002\ (0.0001),$$
$$\delta(\Delta\alpha) = 5\times 10^{-5}\ (3\times 10^{-5}).$$

Known corrections to Δr, $\sin^2\theta^f_{\rm eff}$, g_{Vf}, g_{Af}: A. Freitas, 1604.00406



- Complete NNLO corrections (Δr, $\sin^2\theta^\ell_{\rm eff}$) Freitas, Hollik, Walter, Weiglein '00; Awramik, Czakon '02; Onishchenko, Veretin '02; Awramik, Czakon, Freitas, Weiglein '04; Awramik, Czakon, Freitas '06; Hollik, Meier, Uccirati '05,07; Degrassi, Gambino, Giardino '14
- "Fermionic" NNLO corrections (g_{Vf}, g_{Af}) Czarnecki, Kühn '96; Harlander, Seidensticker, Steinhauser '98; Freitas '13,14
- Partial 3/4-loop corrections to ρ/T-parameter $\mathcal{O}(\alpha_t\alpha_s^2)$, $\mathcal{O}(\alpha_t^2\alpha_s)$, $\mathcal{O}(\alpha_t\alpha_s^3)$ Chetyrkin, Kühn, Steinhauser '95; Faisst, Kühn, Seidensticker, Veretin '03; Boughezal, Tausk, v. d. Bij '05; Schröder, Steinhauser '05; Chetyrkin et al. '06; Boughezal, Czakon '06

  $(\alpha_t \equiv \frac{y_t^2}{4\pi})$

EW Precision Physics

$$\sigma^0_{\rm had} = \sum_q \sigma_q(M_Z^2),$$

$$\Gamma_Z = \sum_f \Gamma[Z \to f\bar{f}], \qquad \text{(from a fit to } \sigma_f(s) \text{ at various values of } s\text{)}$$

$$R_\ell = \left[\textstyle\sum_q \sigma_q(M_Z^2)\right]/\sigma_\ell(M_Z^2), \qquad (\ell = e, \mu, \tau)$$

$$R_q = \sigma_q(M_Z^2)/\left[\textstyle\sum_q \sigma_q(M_Z^2)\right], \qquad (q = b, c)$$

$$A^f_{\rm FB} = \frac{\sigma_f(\theta < \frac{\pi}{2}) - \sigma_f(\theta > \frac{\pi}{2})}{\sigma_f(\theta < \frac{\pi}{2}) + \sigma_f(\theta > \frac{\pi}{2})} \equiv \tfrac{3}{4}\mathcal{A}_e\mathcal{A}_f,$$

$$A^f_{\rm LR} = \frac{\sigma_f(P_e < 0) - \sigma_f(P_e > 0)}{\sigma_f(P_e < 0) + \sigma_f(P_e > 0)} \equiv \mathcal{A}_e|P_e|.$$

$$\text{total} = \sqrt{\text{experimental}^2 + \text{parametric}^2} + \text{intrinsic}$$

$\mathcal{O}(\alpha\alpha_s^2)$ complete
$\mathcal{O}(\alpha^2\alpha_s)$ fermionic
$\mathcal{O}(\alpha^3)$ double-fermionic
$\mathcal{O}(\alpha_t\alpha_s^3)$ 4-loop

$$\mathcal{A}_f = \frac{1 - 4|Q_f|\sin^2\theta^f_{\rm eff}}{1 - 4|Q_f|\sin^2\theta^f_{\rm eff} + 8(Q_f\sin^2\theta^f_{\rm eff})^2}.$$

Quantity	FCC-ee	Current intrinsic error		Projected intrinsic error
M_W [MeV]	0.5–1 [‡]	4	$(\alpha^3, \alpha^2\alpha_s)$	1
$\sin^2\theta^\ell_{\rm eff}$ [10^{-5}]	0.6	4.5	$(\alpha^3, \alpha^2\alpha_s)$	1.5
Γ_Z [MeV]	0.1	0.4	$(\alpha^3, \alpha^2\alpha_s, \alpha\alpha_s^2)$	0.15
R_b [10^{-5}]	6	11	$(\alpha^3, \alpha^2\alpha_s)$	5
R_l [10^{-3}]	1	6	$(\alpha^3, \alpha^2\alpha_s)$	1.5

Theoretical uncertainties for *WW* threshold don't match exp. precision: 3 GeV uncertainty

Beneke/Falgari/Schwinn/Signer/Zanderighi, 0707.0773; Actis/Beneke/Falgari/Schwinn, 0807.0102; C. Schwinn, in 1905.05078

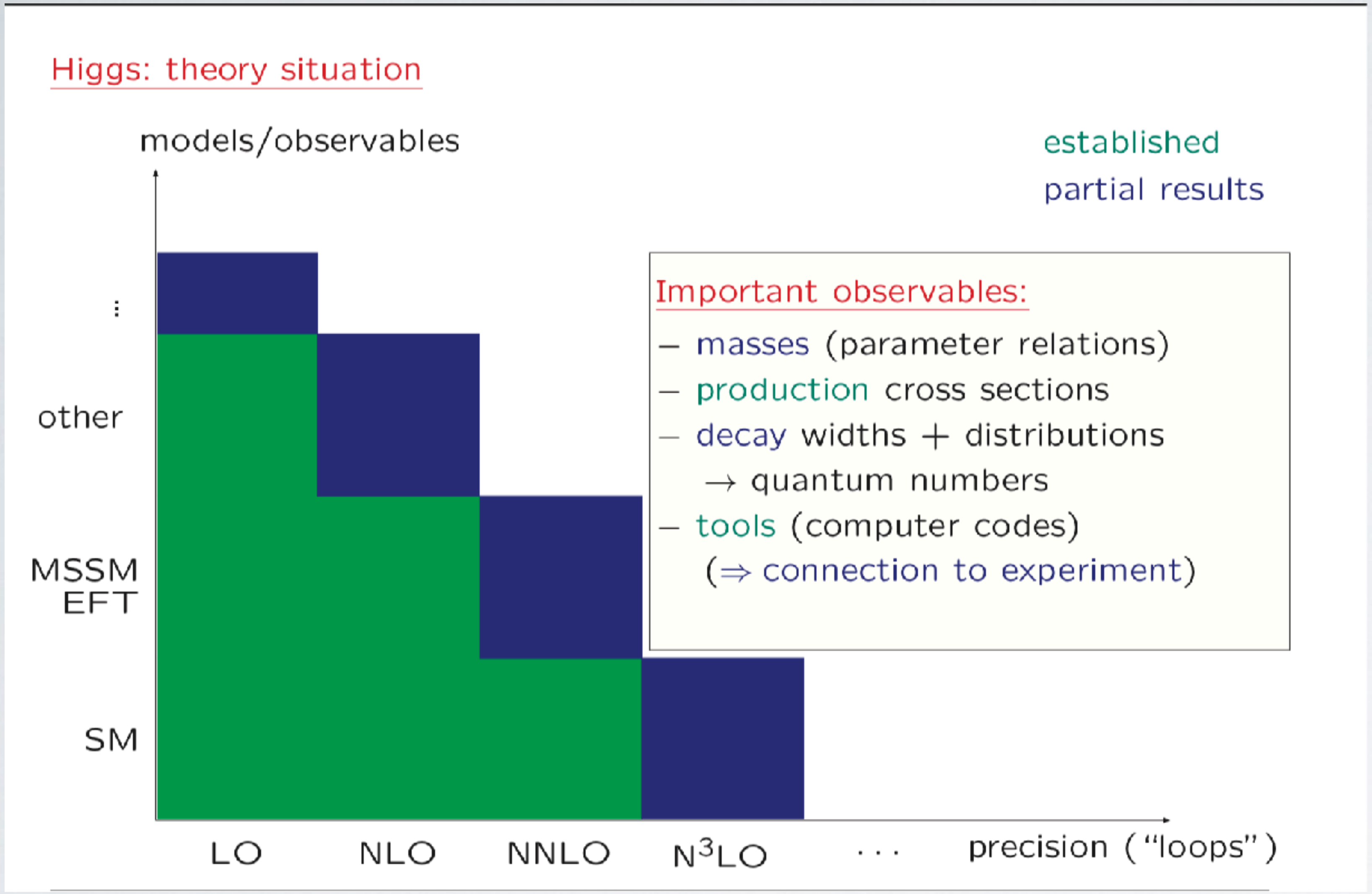
needed: full 2-loop corr. $e^+e^- \to W^+W^-$ and $W \to ff$, ISR & matching (later); 3-loop Coulomb-enh.

New efforts in $e^+e^- \to ff$ (2-loop, logarithmic corr.)

Blümlein/de Freitas/Raab/Schönwald, 1901.08018, 1910.05759, 2003.14283, 2004.04287

Higgs Precision Calculations

Heinemeyer, LCWS 2017



Partial width	QCD	electroweak	total
$H \to b\bar{b}/c\bar{c}$	$\sim 0.2\%$	$< 0.3\%$	$< 0.4\%$
$H \to \tau^+\tau^-/\mu^+\mu^-$	–	$< 0.3\%$	$< 0.3\%$
$H \to gg$	$\sim 3\%$	$\sim 1\%$	$\sim 3.2\%$
$H \to \gamma\gamma$	$< 0.1\%$	$< 1\%$	$<1\%$
$H \to Z\gamma$	$\lesssim 0.1\%$	$\sim 5\%$	$\sim 5\%$
$H \to WW/ZZ \to 4f$	$< 0.5\%$	$< 0.3\%$	$\sim 0.5\%$

Intrinsic Higgs decay uncertainties, LHCHXSWG

decay	para. m_q	para. α_s	para. M_H
$H \to b\bar{b}$	1.4%	0.4%	–
$H \to c\bar{c}$	4.0%	0.4%	–
$H \to \tau^+\tau^-$	–	–	–
$H \to \mu^+\mu^-$	–	–	–
$H \to gg$	$< 0.2\%$	3.7%	–
$H \to \gamma\gamma$	$< 0.2\%$	–	–
$H \to Z\gamma$	–	–	2.1%
$H \to WW$	–	–	2.6%
$H \to ZZ$	–	–	3.0%

Parametric Higgs decay uncertainties, Lepage/McKenzie/Peskin, 1404.0319

- Full 1-loop exists for $ee \to ZH$, Denner/Dittmaier/Roth/Weber, hep-ph/0311089
  $ee \to \nu\nu H$ Belanger/Boudjema/Fujimoto/Ishikawa/Kaneki/Kato/Shimizu, hep-ph/0212261

5-10% NLO corrections

Higgs Precision Calculations

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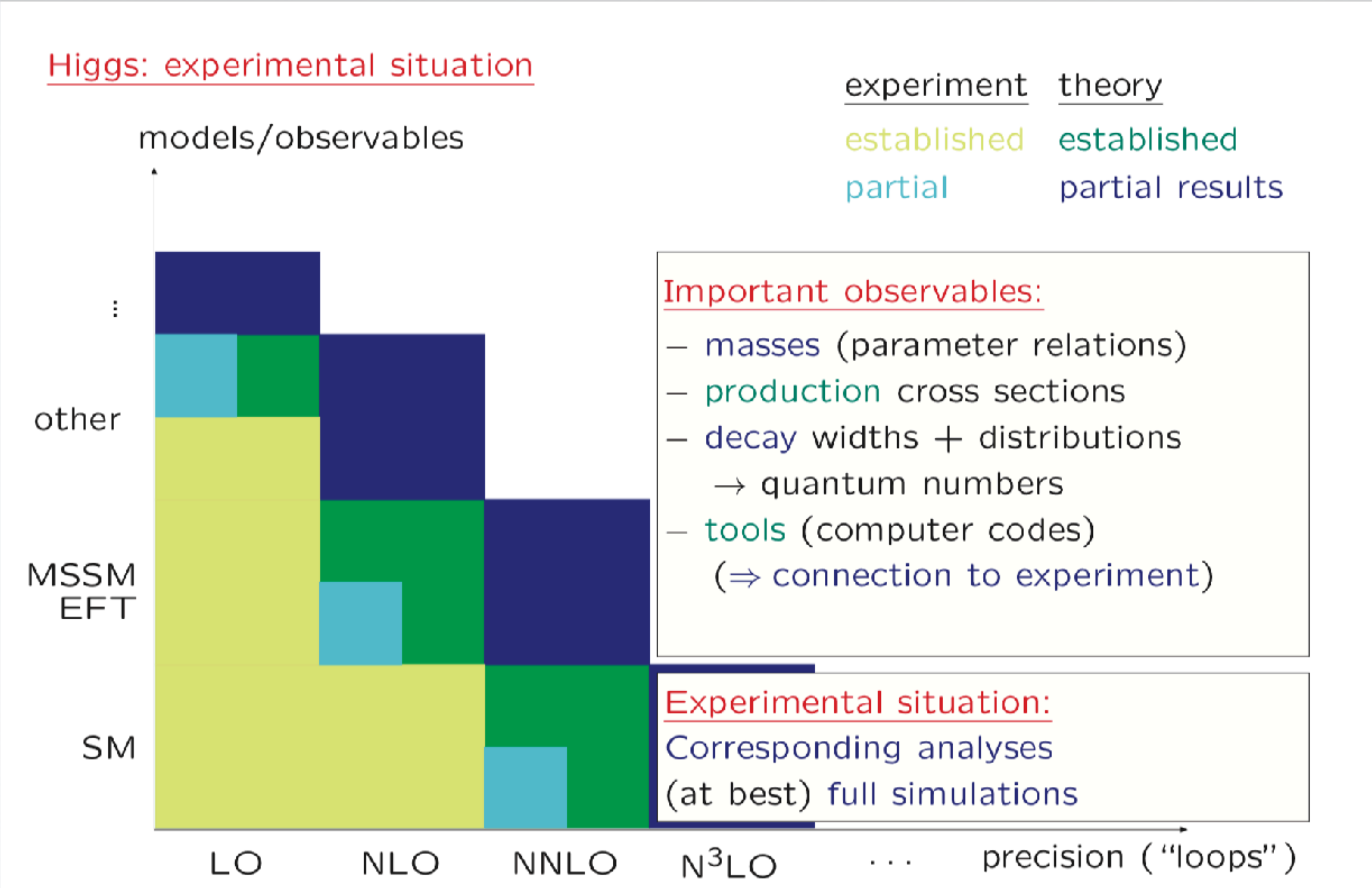

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$H \to \mu^+\mu^-$	–	–	–
$H \to gg$	$< 0.2\%$	3.7%	–
$H \to \gamma\gamma$	$< 0.2\%$	–	–
$H \to Z\gamma$	–	–	2.1%
$H \to WW$	–	–	2.6%
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5-10% NLO corrections

- Missing 2-loop corrections [2→2, 2→3] : intrinsic uncertainty 1%
- Compared to experimental uncertainty of 0.5-1.0%

NNLO hard task for VBF !

Higgs Precision Calculations

ILC/FCC-ee projections

decay	intrinsic	para. m_q	para. α_s	para. M_H	FCC-ee prec. on g^2_{HXX}
$H \to b\bar{b}$	$\sim 0.2\%$	0.6%	$< 0.1\%$	–	$\sim 0.8\%$
$H \to c\bar{c}$	$\sim 0.2\%$	$\sim 1\%$	$< 0.1\%$	–	$\sim 1.4\%$
$H \to \tau^+\tau^-$	$< 0.1\%$	–	–	–	$\sim 1.1\%$
$H \to \mu^+\mu^-$	$< 0.1\%$	–	–	–	$\sim 12\%$
$H \to gg$	$\sim 1\%$		0.5% (0.3%)	–	$\sim 1.6\%$
$H \to \gamma\gamma$	$< 1\%$	–	–	–	$\sim 3.0\%$
$H \to Z\gamma$	$\sim 1\%$	–	–	$\sim 0.1\%$	
$H \to WW$	$\lesssim 0.3\%$	–	–	$\sim 0.1\%$	$\sim 0.4\%$
$H \to ZZ$	$\lesssim 0.3\%^{\dagger}$	–	–	$\sim 0.1\%$	$\sim 0.3\%$
Γ_{tot}	$\sim 0.3\%$	$\sim 0.4\%$	$< 0.1\%$	$< 0.1\%$	$\sim 1\%$

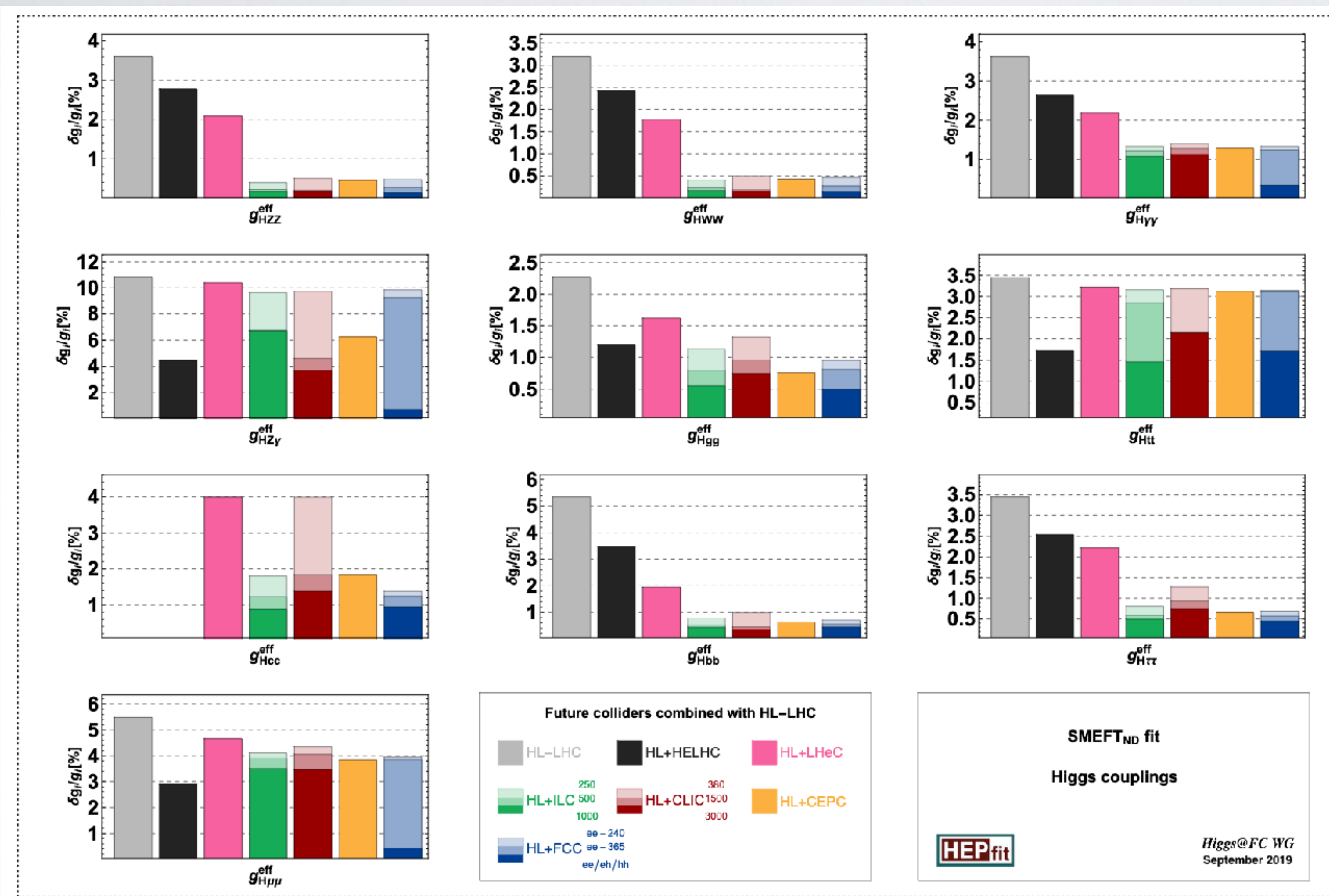
† From $e^+e^- \to HZ$ production

Needed theory effort

- $H \to qq$ $\sim$ N^{4}LO QCD, $\lesssim \mathcal{O}(\alpha^2, \alpha\alpha_s)$ ✅
- $H \to gg$ $\sim$ N^{3}LO QCD scale, $\lesssim \mathcal{O}(\alpha^2)$ ❌ [N^{4}LO QCD: massless 4-loop]
- $H \to \gamma\gamma$ $\lesssim \mathcal{O}(\alpha^2)$ light-fermion dominate ✅
- $H \to Z\gamma$ $\lesssim \mathcal{O}(\alpha)$ *NLO EW* smaller than exp. ✅
- $H \to WW, ZZ$ NLO QCD corr., ❌ [non-factorizable NNLO QCD]

BSM Frontier — SMEFT — Global Fits

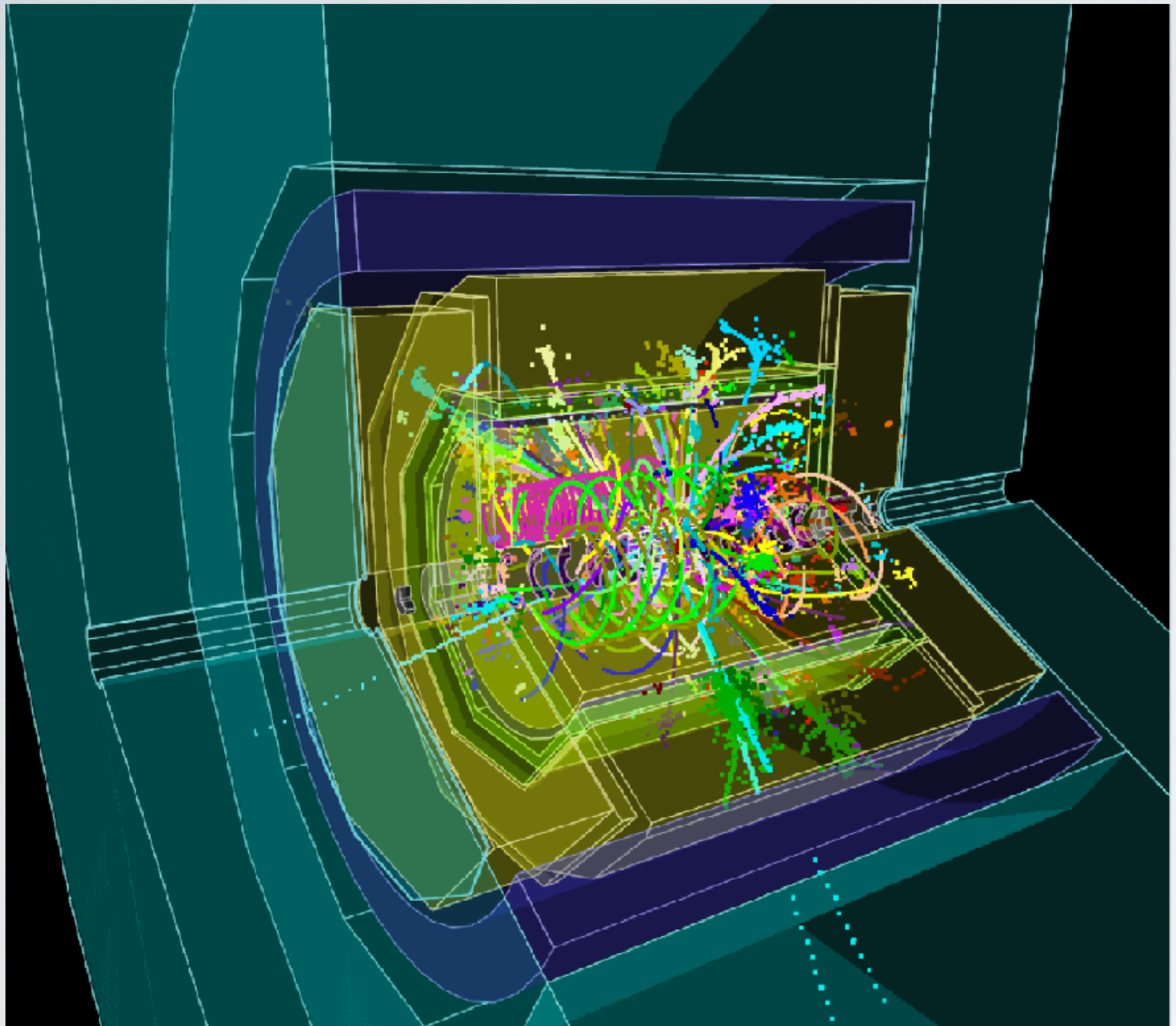
- Higgs (and EW) precision calculations needed for BSM models, e.g. SUSY, RS, composite Higgs …
- SMEFT calculations at NLO QCD ⊕ EW Degrande/Durieux/Maltoni/Mimasu/Vryonidou, 2008.11743
- Global fits: simultaneous extraction / limit setting of *all* relevant parameters



de Blas et al., 1905.03764

- Technical handling of fits
- What can we learn from correlations?
- Extraction of Wilson coeff. @ NLO (interpretation?)
- SMEFT ⟺ BSM models: needs better mapping

The “Exclusive” Frontier



$e^+ e^- \to tth$ @ 1 TeV in 8 jets

- Mapping observables — pseudo-observables
- Determination of systematic uncertainties
- Need $e^+e^- \to 2f,\ 4f,\ 6f,\ [8f]$ @ NLO QCD ⊕ EW

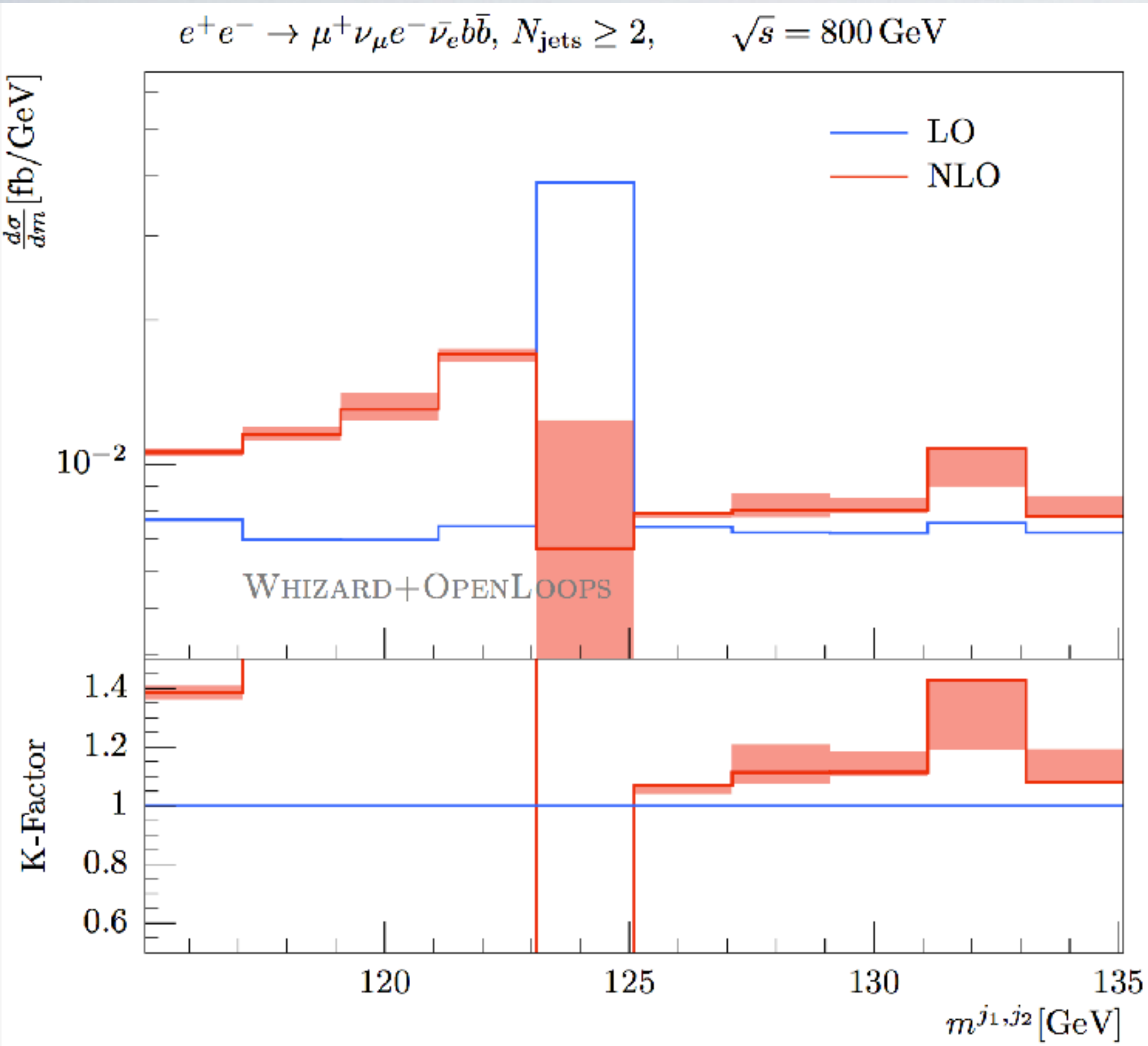
  (arbitrary cuts, fully differentially)

Process	MG5_aMC $\sigma^{\rm LO}$[fb]	MG5_aMC $\sigma^{\rm NLO}$[fb]	MG5_aMC K	WHIZARD $\sigma^{\rm LO}$[fb]	WHIZARD $\sigma^{\rm NLO}$[fb]	WHIZARD K
$e^+e^- \to jj$	622.3(5)	639.3(1)	1.02733	622.73(4)	639.41(9)	1.02678
$e^+e^- \to jjj$	340.1(2)	317.3(8)	0.93297	342.4(5)	318.6(7)	0.9305
$e^+e^- \to jjjj$	104.7(1)	103.7(3)	0.99045	105.1(4)	103.0(6)	0.98003
$e^+e^- \to jjjjj$	22.11(6)	24.65(4)	1.11488	22.80(2)	24.35(15)	1.06798
$e^+e^- \to jjjjjj$	N/A	N/A	N/A	3.62(2)	0.0(0)	0.0
$e^+e^- \to b\bar{b}$	92.37(6)	94.89(1)	1.02728	92.32(1)	94.78(7)	1.02664
$e^+e^- \to b\bar{b}b\bar{b}$	$1.644(3) \cdot 10^{-1}$	$3.60(1) \cdot 10^{-1}$	2.1897	$1.64(2) \cdot 10^{-1}$	$3.67(4) \cdot 10^{-1}$	2.2378
$e^+e^- \to t\bar{t}$	166.2(2)	174.5(3)	1.04994	166.4(1)	174.53(6)	1.04886
$e^+e^- \to t\bar{t}j$	48.13(5)	53.36(1)	1.10867	48.3(2)	53.25(6)	1.10248
$e^+e^- \to t\bar{t}jj$	8.614(9)	10.49(3)	1.21777	8.612(8)	10.46(6)	1.21458
$e^+e^- \to t\bar{t}jjj$	1.044(2)	1.420(4)	1.3601	1.040(1)	1.414(10)	1.3595
$e^+e^- \to t\bar{t}t\bar{t}$	$6.45(1) \cdot 10^{-4}$	$11.94(2) \cdot 10^{-4}$	1.85117	$6.463(2) \cdot 10^{-4}$	$11.91(2) \cdot 10^{-4}$	1.8428
$e^+e^- \to t\bar{t}t\bar{t}j$	$2.719(5) \cdot 10^{-5}$	$5.264(8) \cdot 10^{-5}$	1.93602	$2.722(1) \cdot 10^{-5}$	$5.250(14) \cdot 10^{-5}$	1.92873
$e^+e^- \to t\bar{t}b\bar{b}$	0.1819(3)	0.292(1)	1.60533	0.186(1)	0.293(2)	1.57527
$e^+e^- \to t\bar{t}H$	2.018(3)	1.909(3)	0.94601	2.022(3)	1.912(3)	0.9456
$e^+e^- \to t\bar{t}Hj$	$0.2533(3) \cdot 10^{-0}$	$0.2665(6) \cdot 10^{-0}$	1.05212	0.2540(9)	0.2664(5)	1.04889
$e^+e^- \to t\bar{t}Hjj$	$2.663(4) \cdot 10^{-2}$	$3.141(9) \cdot 10^{-2}$	1.1795	$2.666(4) \cdot 10^{-2}$	$3.144(9) \cdot 10^{-2}$	1.17928
$e^+e^- \to t\bar{t}\gamma$	12.7(2)	13.3(4)	1.04726	12.71(4)	13.78(4)	1.08418
$e^+e^- \to t\bar{t}Z$	4.642(6)	4.95(1)	1.06636	4.64(1)	4.94(1)	1.06467
$e^+e^- \to t\bar{t}Zj$	0.6059(6)	0.6917(24)	1.14168	0.610(4)	0.6927(14)	1.13565
$e^+e^- \to t\bar{t}Zjj$	$6.251(28) \cdot 10^{-2}$	$8.181(21) \cdot 10^{-2}$	1.30875	$6.233(8) \cdot 10^{-2}$	$8.201(14) \cdot 10^{-2}$	1.31573
$e^+e^- \to t\bar{t}W^\pm jj$	$2.400(4) \cdot 10^{-4}$	$3.714(8) \cdot 10^{-4}$	1.54747	$2.41(1) \cdot 10^{-4}$	$3.695(9) \cdot 10^{-4}$	1.5332
$e^+e^- \to t\bar{t}\gamma\gamma$	0.383(5)	0.416(2)	1.08618	0.382(3)	0.420(3)	1.09952
$e^+e^- \to t\bar{t}\gamma Z$	0.2212(3)	0.2364(6)	1.06873	0.220(1)	0.240(2)	1.09094
$e^+e^- \to t\bar{t}\gamma H$	$9.75(1) \cdot 10^{-2}$	$9.42(3) \cdot 10^{-2}$	0.96614	$9.748(6) \cdot 10^{-2}$	$9.58(7) \cdot 10^{-2}$	0.98277
$e^+e^- \to t\bar{t}ZZ$	$3.788(4) \cdot 10^{-2}$	$4.00(1) \cdot 10^{-2}$	1.05597	$3.756(4) \cdot 10^{-2}$	$4.005(2) \cdot 10^{-2}$	1.0663
$e^+e^- \to t\bar{t}W^+W^-$	0.1372(3)	0.1540(6)	1.1225	0.1370(4)	0.1538(4)	1.12257
$e^+e^- \to t\bar{t}HH$	$1.358(1) \cdot 10^{-2}$	$1.206(3) \cdot 10^{-2}$	0.888	$1.367(1) \cdot 10^{-2}$	$1.218(1) \cdot 10^{-2}$	0.8909
$e^+e^- \to t\bar{t}HZ$	$3.600(6) \cdot 10^{-2}$	$3.58(1) \cdot 10^{-2}$	0.99445	$3.596(1) \cdot 10^{-2}$	$3.581(2) \cdot 10^{-2}$	0.9958

- ☑ [Beam spectra and overlays: “non-perturbative” part of event simulation]
- ☑ QED ISR: correct normalization [inclusive part], ISR photons [exclusive part]
- ☑ QED FSR: interference w/ ISR, correct normalization of matrix elements
- ☑ QED showers: proper matching of exclusive and resummed prescriptions
- ☑ High-energy colliders (CLIC, Plasma, Muon): EW PDFs, EW showers, event selection/definition!?

↪ Talk by S. Prestel

The "Exclusive" Frontier



- Mapping observables — pseudo-observables
- Determination of systematic uncertainties
- Need $e^+e^- \to$ *2f, 4f, 6f, [8f] @ NLO QCD ⊕ EW*

  (arbitrary cuts, fully differentially)

	MG5_aMC			WHIZARD		
Process	σ^{LO}[fb]	σ^{NLO}[fb]	K	σ^{LO}[fb]	σ^{NLO}[fb]	K
$e^+e^- \to jj$	622.3(5)	639.3(1)	1.02733	622.73(4)	639.41(9)	1.02678
$e^+e^- \to jjj$	340.1(2)	317.3(8)	0.93297	342.4(5)	318.6(7)	0.9305
$e^+e^- \to jjjj$	104.7(1)	103.7(3)	0.99045	105.1(4)	103.0(6)	0.98003
$e^+e^- \to jjjjj$	22.11(6)	24.65(4)	1.11488	22.80(2)	24.35(15)	1.06798
$e^+e^- \to jjjjjj$	N/A	N/A	N/A	3.62(2)	0.0(0)	0.0
$e^+e^- \to b\bar{b}$	92.37(6)	94.89(1)	1.02728	92.32(1)	94.78(7)	1.02664
$e^+e^- \to b\bar{b}b\bar{b}$	$1.644(3) \cdot 10^{-1}$	$3.60(1) \cdot 10^{-1}$	2.1897	$1.64(2) \cdot 10^{-1}$	$3.67(4) \cdot 10^{-1}$	2.2378
$e^+e^- \to t\bar{t}$	166.2(2)	174.5(3)	1.04994	166.4(1)	174.53(6)	1.04886
$e^+e^- \to t\bar{t}j$	48.13(5)	53.36(1)	1.10867	48.3(2)	53.25(6)	1.10248
$e^+e^- \to t\bar{t}jj$	8.614(9)	10.49(3)	1.21777	8.612(8)	10.46(6)	1.21458
$e^+e^- \to t\bar{t}jjj$	1.044(2)	1.420(4)	1.3601	1.040(1)	1.414(10)	1.3595
$e^+e^- \to t\bar{t}t\bar{t}$	$6.45(1) \cdot 10^{-4}$	$11.94(2) \cdot 10^{-4}$	1.85117	$6.463(2) \cdot 10^{-4}$	$11.91(2) \cdot 10^{-4}$	1.8428
$e^+e^- \to t\bar{t}t\bar{t}j$	$2.719(5) \cdot 10^{-5}$	$5.264(8) \cdot 10^{-5}$	1.93602	$2.722(1) \cdot 10^{-5}$	$5.250(14) \cdot 10^{-5}$	1.92873
$e^+e^- \to t\bar{t}b\bar{b}$	0.1819(3)	0.292(1)	1.60533	0.186(1)	0.293(2)	1.57527
$e^+e^- \to t\bar{t}H$	2.018(3)	1.909(3)	0.94601	2.022(3)	1.912(3)	0.9456
$e^+e^- \to t\bar{t}Hj$	$0.2533(3) \cdot 10^{-0}$	$0.2665(6) \cdot 10^{-0}$	1.05212	0.2540(9)	0.2664(5)	1.04889
$e^+e^- \to t\bar{t}Hjj$	$2.663(4) \cdot 10^{-2}$	$3.141(9) \cdot 10^{-2}$	1.1795	$2.666(4) \cdot 10^{-2}$	$3.144(9) \cdot 10^{-2}$	1.17928
$e^+e^- \to t\bar{t}\gamma$	12.7(2)	13.3(4)	1.04726	12.71(4)	13.78(4)	1.08418
$e^+e^- \to t\bar{t}Z$	4.642(6)	4.95(1)	1.06636	4.64(1)	4.94(1)	1.06467
$e^+e^- \to t\bar{t}Zj$	0.6059(6)	0.6917(24)	1.14168	0.610(4)	0.6927(14)	1.13565
$e^+e^- \to t\bar{t}Zjj$	$6.251(28) \cdot 10^{-2}$	$8.181(21) \cdot 10^{-2}$	1.30875	$6.233(8) \cdot 10^{-2}$	$8.201(14) \cdot 10^{-2}$	1.31573
$e^+e^- \to t\bar{t}W^\pm jj$	$2.400(4) \cdot 10^{-4}$	$3.714(8) \cdot 10^{-4}$	1.54747	$2.41(1) \cdot 10^{-4}$	$3.695(9) \cdot 10^{-4}$	1.5332
$e^+e^- \to t\bar{t}\gamma\gamma$	0.383(5)	0.416(2)	1.08618	0.382(3)	0.420(3)	1.09952
$e^+e^- \to t\bar{t}\gamma Z$	0.2212(3)	0.2364(6)	1.06873	0.220(1)	0.240(2)	1.09094
$e^+e^- \to t\bar{t}\gamma H$	$9.75(1) \cdot 10^{-2}$	$9.42(3) \cdot 10^{-2}$	0.96614	$9.748(6) \cdot 10^{-2}$	$9.58(7) \cdot 10^{-2}$	0.98277
$e^+e^- \to t\bar{t}ZZ$	$3.788(4) \cdot 10^{-2}$	$4.00(1) \cdot 10^{-2}$	1.05597	$3.756(4) \cdot 10^{-2}$	$4.005(2) \cdot 10^{-2}$	1.0663
$e^+e^- \to t\bar{t}W^+W^-$	0.1372(3)	0.1540(6)	1.1225	0.1370(4)	0.1538(4)	1.12257
$e^+e^- \to t\bar{t}HH$	$1.358(1) \cdot 10^{-2}$	$1.206(3) \cdot 10^{-2}$	0.888	$1.367(1) \cdot 10^{-2}$	$1.218(1) \cdot 10^{-2}$	0.8909
$e^+e^- \to t\bar{t}HZ$	$3.600(6) \cdot 10^{-2}$	$3.58(1) \cdot 10^{-2}$	0.99445	$3.596(1) \cdot 10^{-2}$	$3.581(2) \cdot 10^{-2}$	0.9958

- ☑ [Beam spectra and overlays: "non-perturbative" part of event simulation]
- ☑ QED ISR: correct normalization [inclusive part], ISR photons [exclusive part]
- ☑ QED FSR: interference w/ ISR, correct normalization of matrix elements
- ☑ QED showers: proper matching of exclusive and resummed prescriptions
- ☑ High-energy colliders (CLIC, Plasma, Muon): EW PDFs, EW showers, event selection/definition!?

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The "Exclusive" Frontier

QED ISR, inclusive part

- Soft resummation to all orders Gribov/Lipatov, 1972; Kuraev/Fadin, 1985
- Hard collinear radiation $\mathcal{O}(\alpha^2)$ Kuraev/Fadin, 1985 , $\mathcal{O}(\alpha^3)$ Skrzypek/Jadach, 1992
- LO boundary conditions, collinear evolution @ LL Skrzypek/Jadach, 1992; Cacciari/Deandrea/Montagna/Nicrosini, 1992
- NLO boundary conditions for QED PDFs Frixione, 1909.03886
- NLO QED PDFs, collinear evolution @ NLL Bertone/Cacciari/Frixione/Stagnitto, 1911.12040
- Crucial: numerical stability at kinematically peaked limit $z \to 1$

$$\mathbb{P}_{\rm S} = \begin{pmatrix} P_{\Sigma\Sigma} & P_{\Sigma\gamma} \\ P_{\gamma\Sigma} & P_{\gamma\gamma} \end{pmatrix},$$
$$P_{\rm NS} = P_{e^\pm e^\pm} - P_{e^\pm e^\mp} \equiv P^{\rm V}_{ee} - P^{\rm V}_{e\bar{e}}$$

$$\Gamma_i^{[0]}(z,\mu_0^2) = \delta_{ie^-}\,\delta(1-z)\,,$$
$$\Gamma_{e^-}^{[1]}(z,\mu_0^2) = \left[\frac{1+z^2}{1-z}\left(\log\frac{\mu_0^2}{m^2} - 2\log(1-z) - 1\right)\right]_+ + K_{ee}(z)\,,$$
$$\Gamma_{\gamma}^{[1]}(z,\mu_0^2) = \frac{1+(1-z)^2}{z}\left(\log\frac{\mu_0^2}{m^2} - 2\log z - 1\right) + K_{\gamma e}(z)\,,$$
$$\Gamma_{e^+}^{[1]}(z,\mu_0^2) = 0\,,$$

$$\frac{\partial \mathbb{E}_N(t)}{\partial t} = \frac{b_0\alpha^2(\mu)}{\beta(\alpha(\mu))}\sum_{k=0}^{\infty}\left(\frac{\alpha(\mu)}{2\pi}\right)^k \mathbb{P}_N^{[k]}\,\mathbb{E}_N(t)$$
$$= \left[\mathbb{P}_N^{[0]} + \frac{\alpha(\mu)}{2\pi}\left(\mathbb{P}_N^{[1]} - \frac{2\pi b_1}{b_0}\mathbb{P}_N^{[0]}\right)\right]\mathbb{E}_N(t) + \mathcal{O}(\alpha^2)$$

QED ISR [+FSR], exclusive part

- Exclusive ("coherent") resummation Yennie/Frautschi/Suura, 1961
- Explicitly matches ME photons Jadach/Ward/Yost, hep-ph/0103163+0104049+0211132+0602197
- Coherent exponentiated EW corrections (CEEX) Jadach/Ward/Wąs, hep-ph/0006359; 1409.4173

QED Full Factorization

- Fully factorized QED amplitudes for small/vanishing m_e Laenen et al. 2008.01736

Conclusions & Outlook

- Amazing experimental Higgs + EW precision program in e^+e^- collisions
- Most measurements allow per-cent down to (sub-) permil level precision
- Hard theoretical work needed !
- *Z* / *WW* threshold: up to massive 2- and 3-loop 4-point functions needed
- Massive 3-loop diagrams: PDE, sector decomposition, Mellin methods etc.
- Higgs precision program: production processes NNLO, decays @ 3-loop
- "Exclusive frontier" (I): $2 \rightarrow 4, 6, (8)$ NLO SM corrections, NLO $e^\pm$ PDFs
- "Exclusive frontier" (II): Exclusive exponentiation, QED showers & matching
- Tools, tools, tools: community must value and support codes (loops, MC, fits)
- BSM models / SMEFT: precision needs to catch up; global fits …

- Ultimate challenges for 10-50 TeV e^+e^- or $\mu^+\mu^-$ (EW PDFs, EW showers etc.)