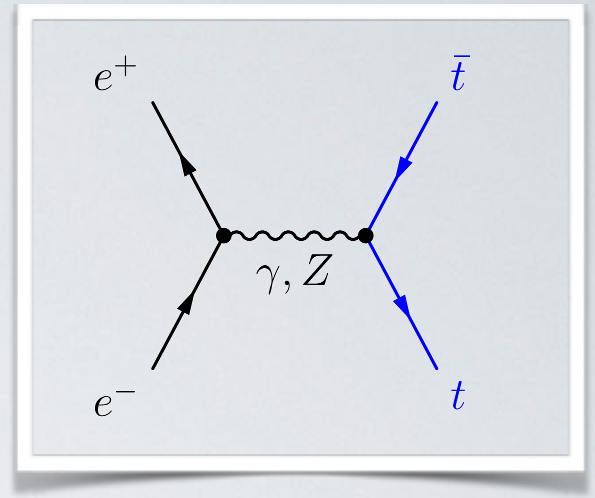
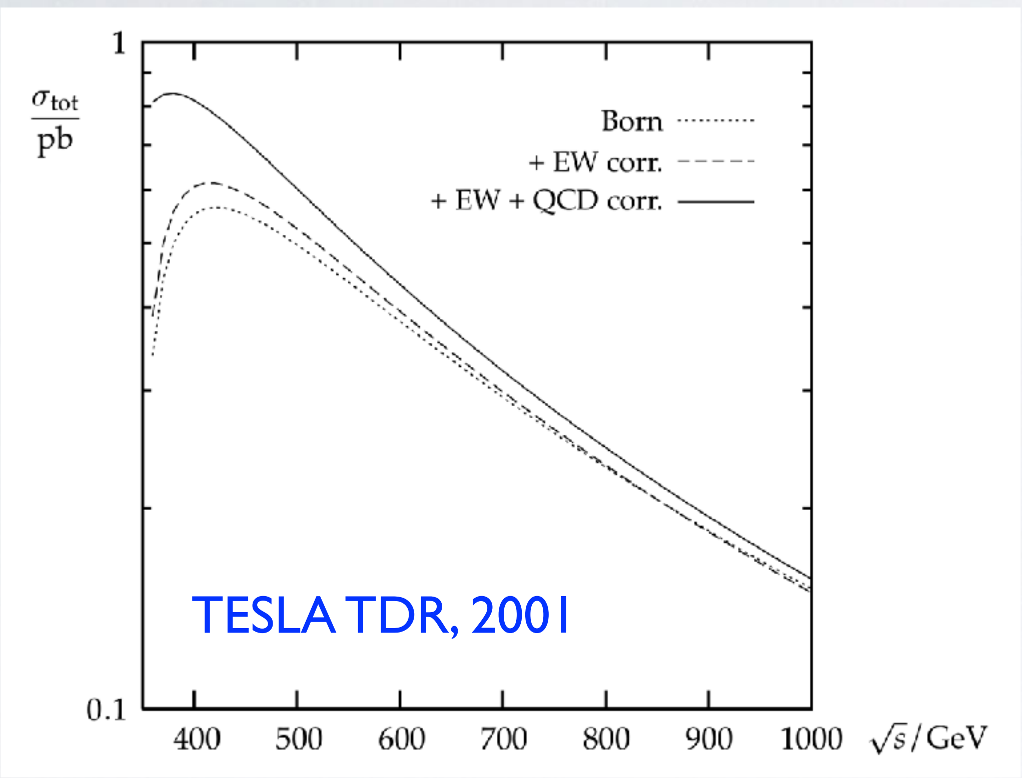
Precision predictions for lepton collider top physics

Jürgen R. Reuter, DESY

Top Quark Production

- *s*-channel production of top-quark pairs
- Top decay happens before hadronization
- Electroweak couplings / polarization studies possible
- Access to vector- and axial-vector *ttZ* couplings





- Production cross section: 0.1 - 1 pb (sub-TeV)
- 100 fb^{-1} : 100,000 top pairs @ threshold
- 2 ab^{-1} : 1,400,000 top pairs @ 500 GeV
- Top helicities available from lepton / jet distributions

- Paradigm processes : precision determination of top properties
- Major bkgd for EW measurements (*VVV* and VBS); any [most] BSM searches

Top Production & Decay: Precision

On-Shell process: $e^+e^- \to t\bar{t}$

- NLO QCD [Jersak/Laermann/Zerwas, 1982]
- NNLO QCD [Chetyrkin/Kühn/Steinhauser, 1996; Harlander/Steinhauser, 1998]
- NLO EW [Beenakker/von der Marck/Hollik, 1991; Beenakker/Denner/Kraft, 1993; Akhundov/Bardin/Leike, 1991]
- Threshold enhancement [Fadin/Khoze, 1987; Strassler/Peskin, 1991; Jezabek/Kühn/Teubner, 1992; Sumino et al., 1992

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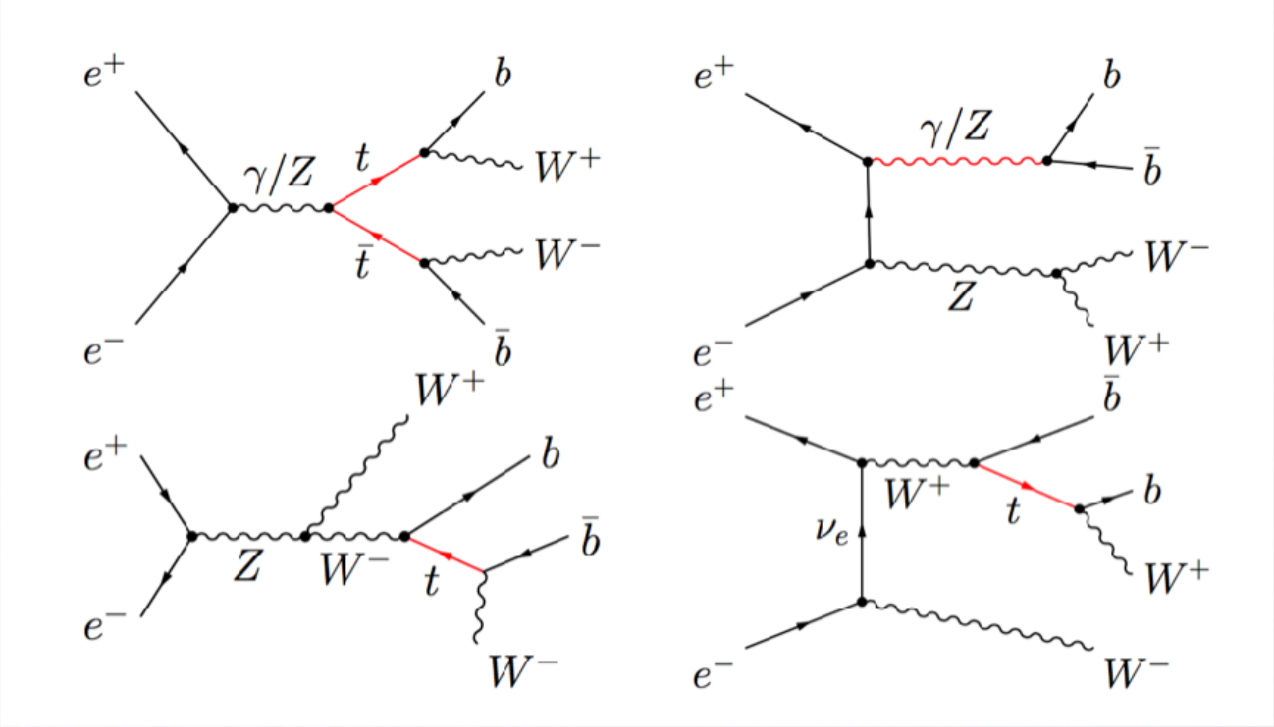
Off-Shell process: $e^+e^- \to W^+\bar{b}W^-b$

- NLO QCD [Guo/Ma/Wang/Zhang, 2008]
- NLO QCD diff. [Chokoufe/JRR/Weiss, 2015; Liebler/Moortgat-Pick/Papanasthasiou, 2015; Chokoufe/Kilian/Lindert/JRR/Pozzorini/Weiss, 2016]
  - Properly model the threshold
  - Assess selection efficiencies
  - Assess systematic uncertainties



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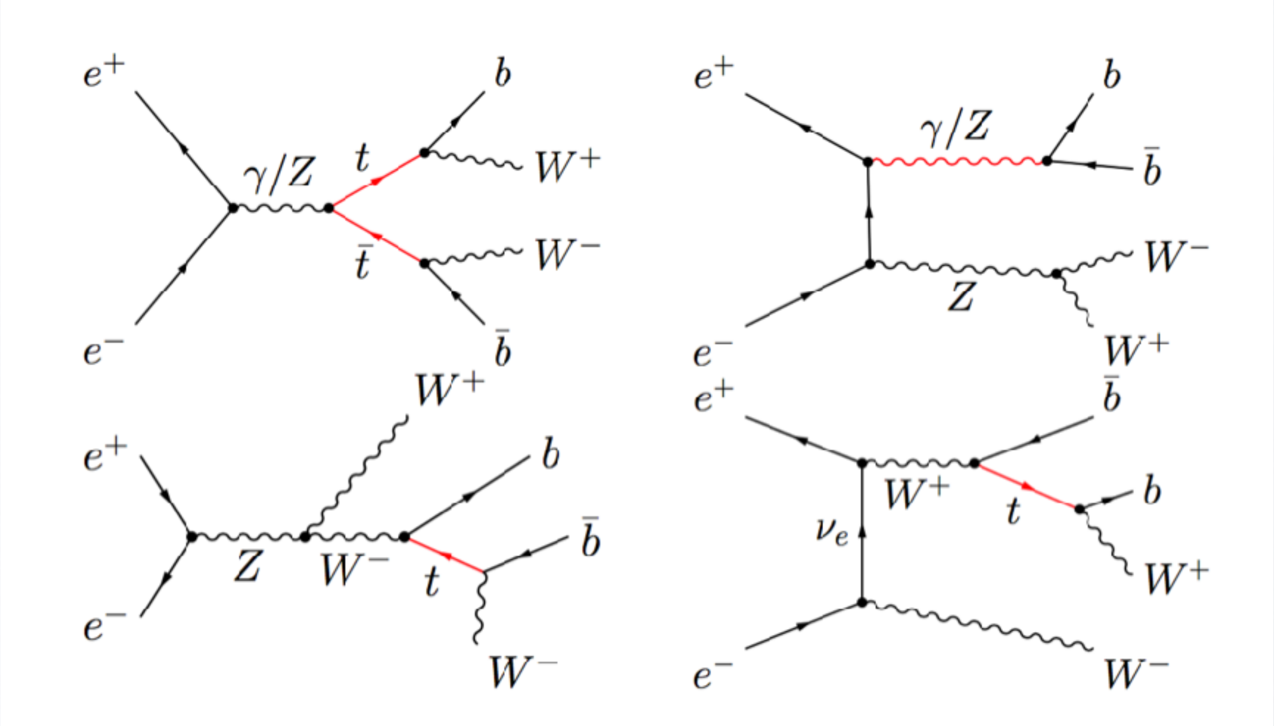
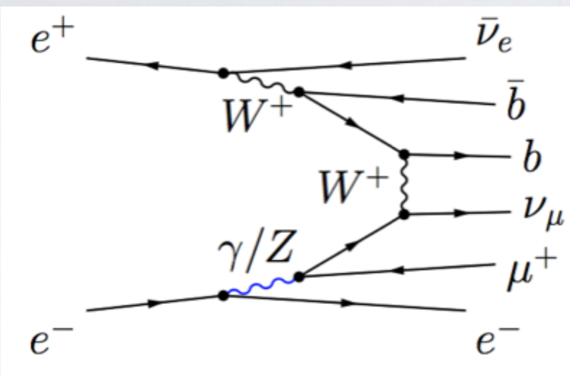


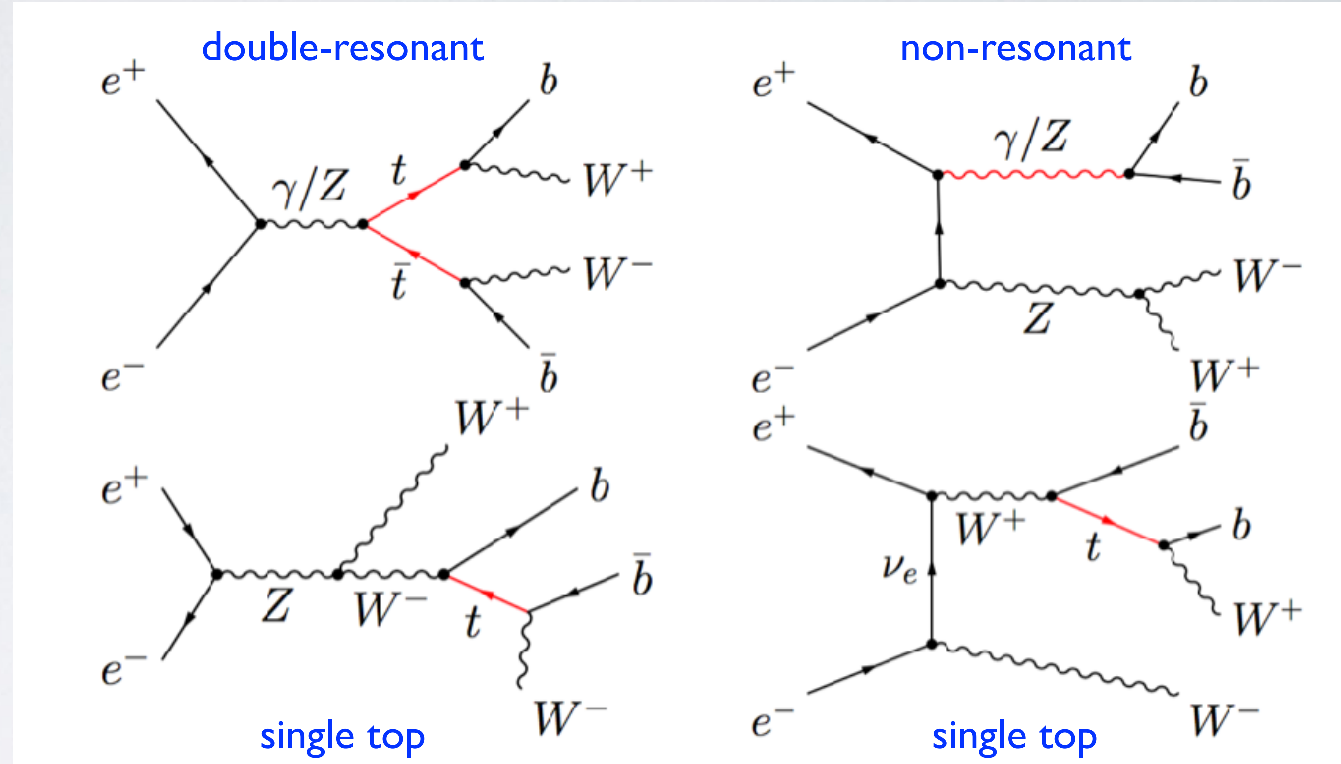
Top width: $t \to W^+b$

- NLO QCD [Jezabek/Kühn, 1989]
- NNLO QCD [Guo/Li/Zhu, 2012]

$e^+e^- \to tt$ off-shell in the continuum

- NLO QCD $2 \to 2$ and $2 \to 4$ calculated with `WHIZARD`, `Sherpa` & `Munich`
- Using massive b quarks:
  no cuts necessary for $e^+e^- \to W^+W^-bb$
- Full process $e^+e^- \to \mu^+\nu_\mu e^-\nu_e bb$ exhibits
  Coulomb singularity:





INPUT PARAMETERS:

$m_Z = 91.1876\,\mathrm{GeV}$,	$m_W = 80.385\,\mathrm{GeV}$
$m_b = 4.2\,\mathrm{GeV}$,	$m_t = 173.2\,\mathrm{GeV}$.

$\Gamma^{\mathrm{LO}}_{t\to Wb} = 1.4986\,\mathrm{GeV}$,	$\Gamma^{\mathrm{NLO}}_{t\to Wb} = 1.3681\,\mathrm{GeV}$,
$\Gamma^{\mathrm{LO}}_{t\to f\bar{f}b} = 1.4757\,\mathrm{GeV}$,	$\Gamma^{\mathrm{NLO}}_{t\to f\bar{f}b} = 1.3475\,\mathrm{GeV}$.

$m_H = 125\ \mathrm{GeV}$	$\Gamma_H = 0.00431\ \mathrm{GeV}$

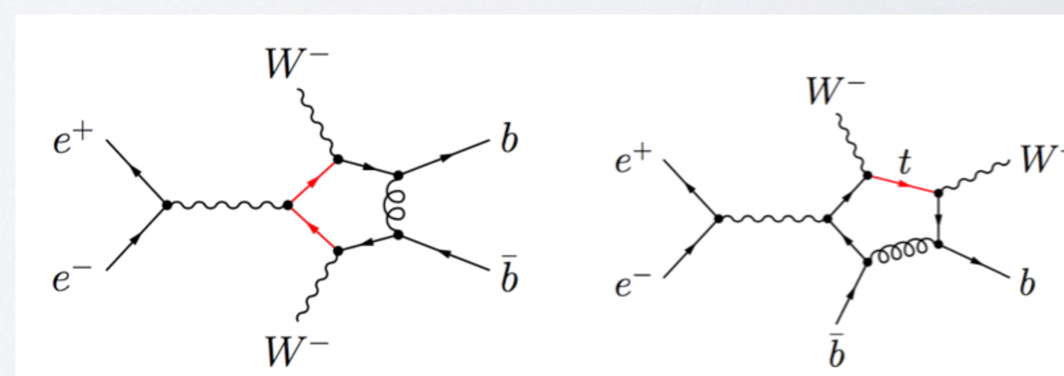
$\Gamma^{\mathrm{LO}}_Z = 2.4409\,\mathrm{GeV}$,	$\Gamma^{\mathrm{NLO}}_Z = 2.5060\,\mathrm{GeV}$,
$\Gamma^{\mathrm{LO}}_W = 2.0454\,\mathrm{GeV}$,	$\Gamma^{\mathrm{NLO}}_W = 2.0978\,\mathrm{GeV}$.

Complex Mass Scheme (CMS):

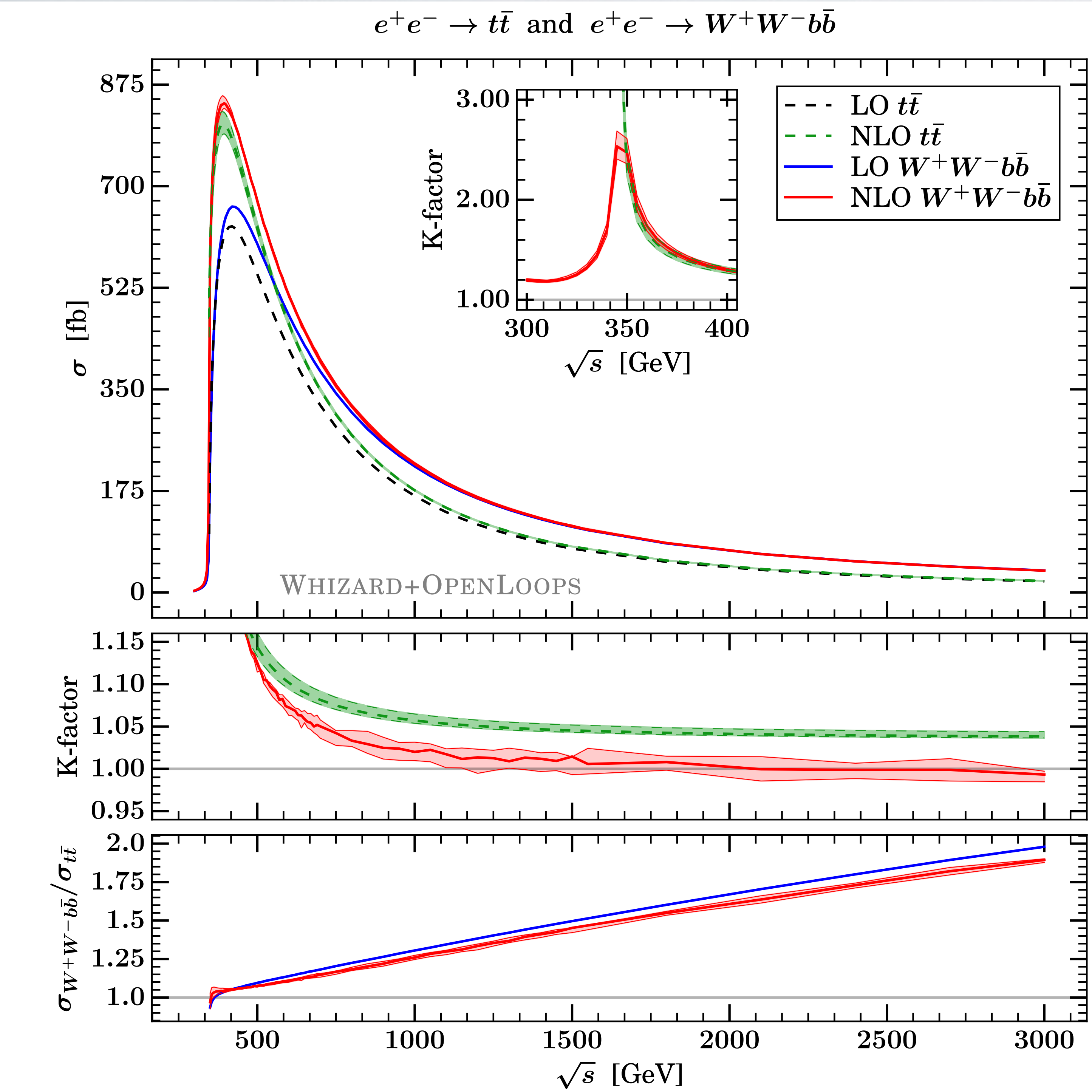
$$\mu_i^2 = M_i^2 - \mathrm{i}\Gamma_i M_i \qquad \text{for } i = \mathrm{W, Z, t, H}$$

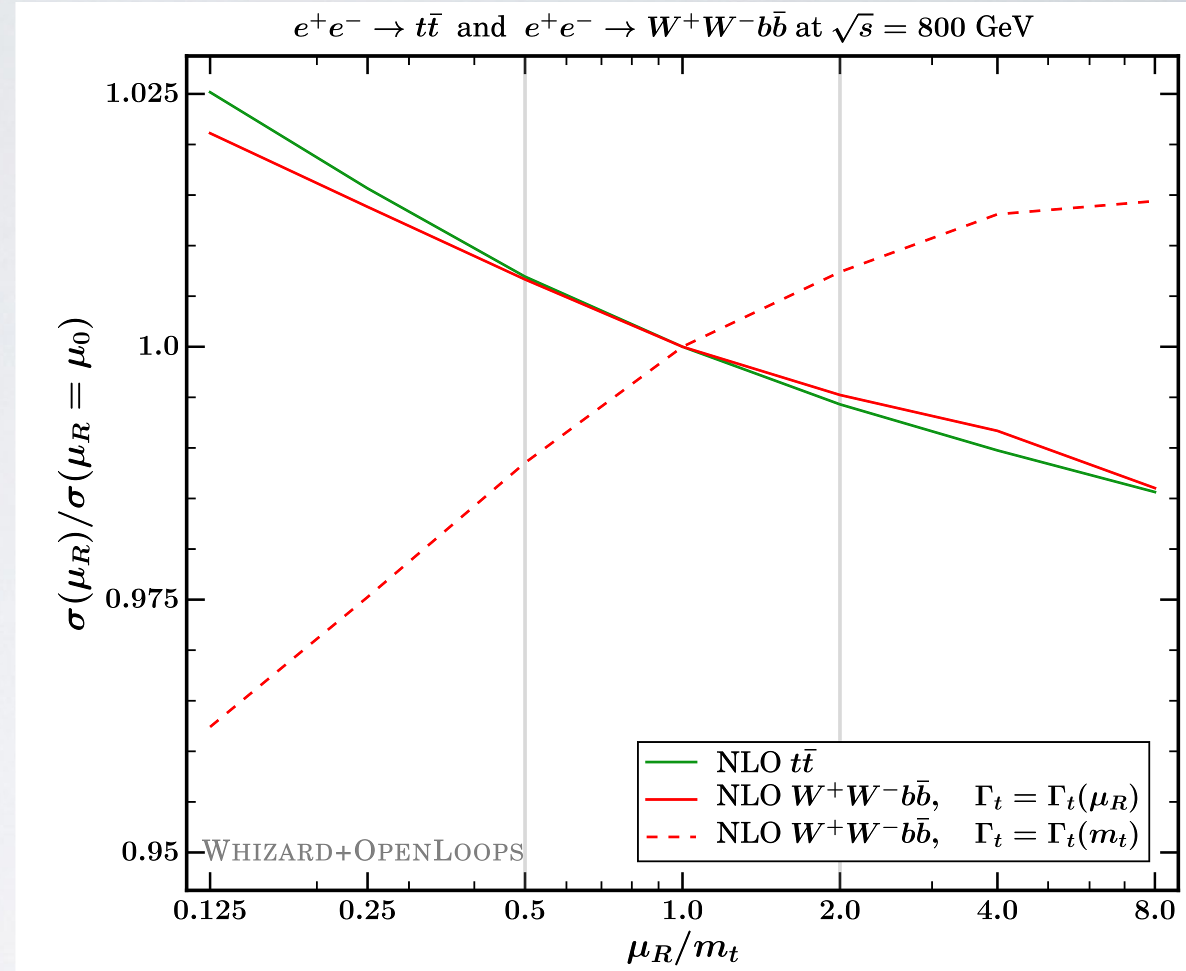
$$s_w^2 = 1 - c_w^2 = 1 - \frac{\mu_\mathrm{W}^2}{\mu_\mathrm{Z}^2}$$

- Typical pentagons:



NLO QCD Results for off-shell $e^+e^- \to tt$

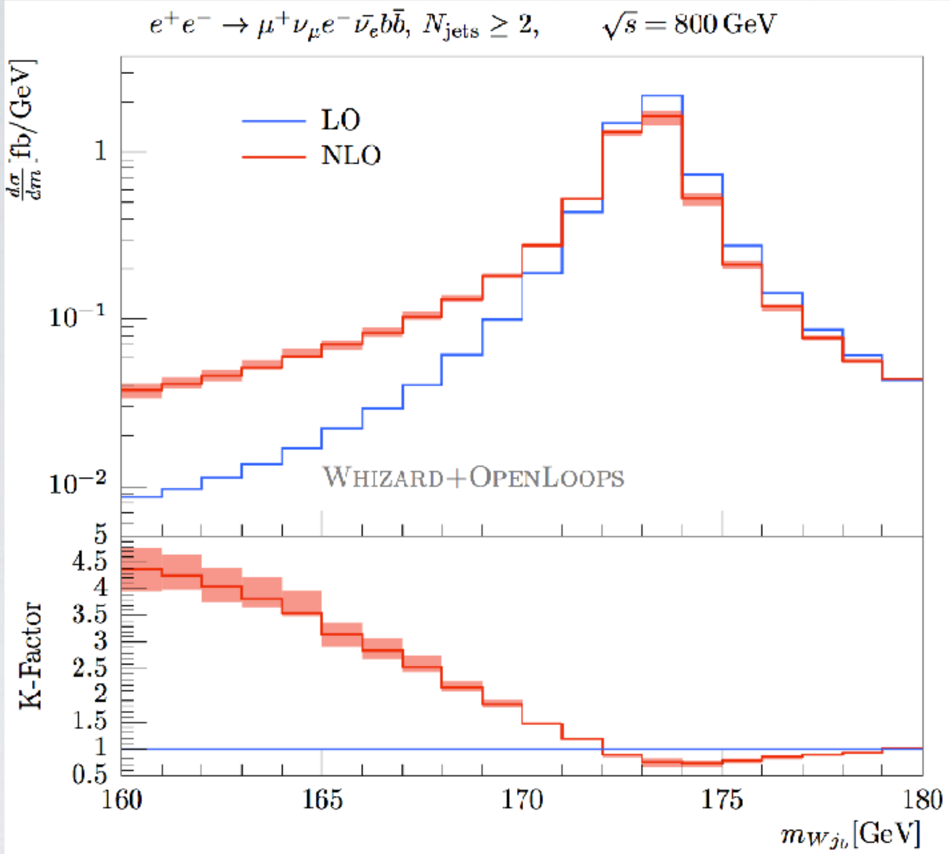


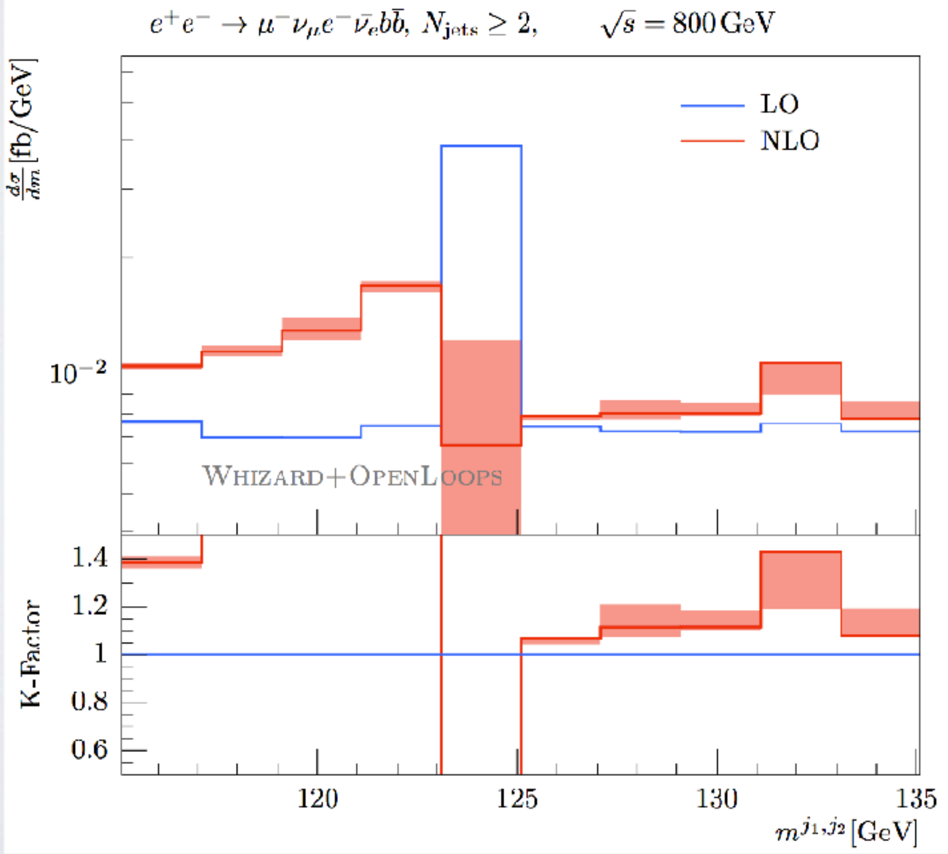


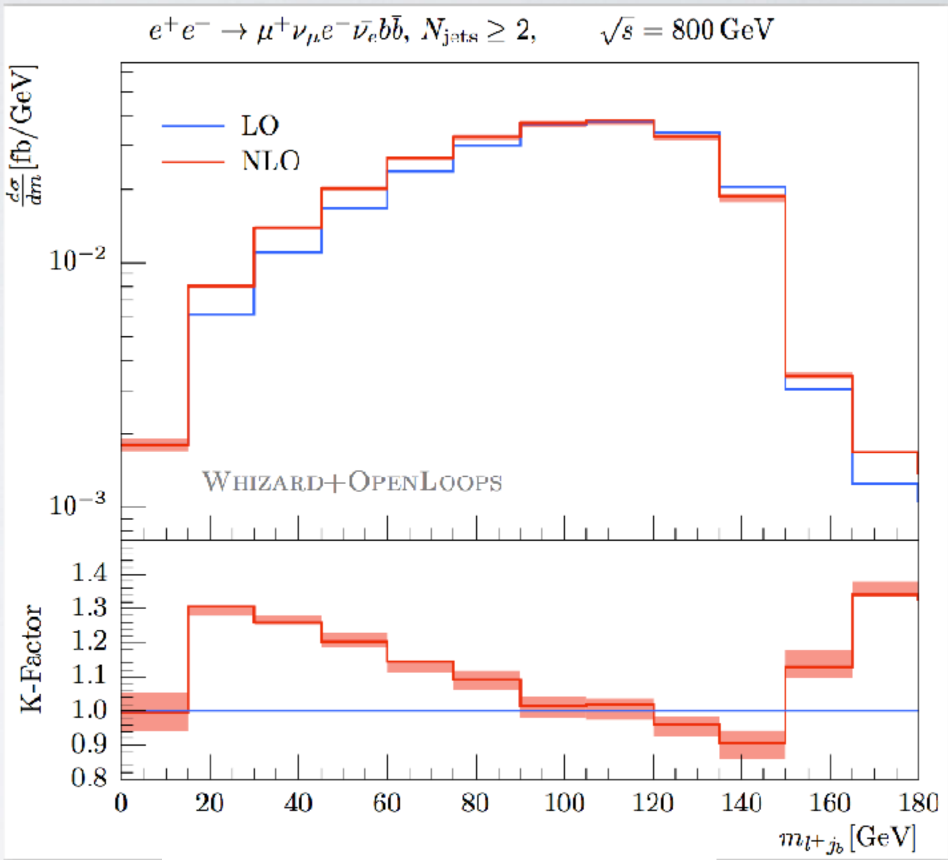
	$e^+e^- \to t\bar{t}$			$e^+e^- \to W^+W^-b\bar{b}$		
$\sqrt{s}$ [GeV]	σ^{LO}[fb]	σ^{NLO}[fb]	K-factor	σ^{LO}[fb]	σ^{NLO}[fb]	K-factor
500	548.4	$627.4^{+1.4\%}_{-0.9\%}$	1.14	600.7	$675.1^{+0.4\%}_{-0.8\%}$	1.12
800	253.1	$270.9^{+0.8\%}_{-0.4\%}$	1.07	310.2	$320.7^{+1.1\%}_{-0.7\%}$	1.03
1000	166.4	$175.9^{+0.7\%}_{-0.3\%}$	1.06	217.2	$221.6^{+1.1\%}_{-1.0\%}$	1.02
1400	86.62	$90.66^{+0.6\%}_{-0.2\%}$	1.05	126.4	$127.9^{+0.7\%}_{-1.5\%}$	1.01
3000	19.14	$19.87^{+0.5\%}_{-0.2\%}$	1.04	37.89	$37.63^{+0.4\%}_{-0.9\%}$	0.993

Chokoufé/Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

Differential Results for off-shell e⁺e⁻ → *tt*







$$m_t^2 = m_W^2 + \frac{2\langle m_{\ell j_b}^2 \rangle}{1 - \langle \cos\theta_{\ell j_b} \rangle}$$

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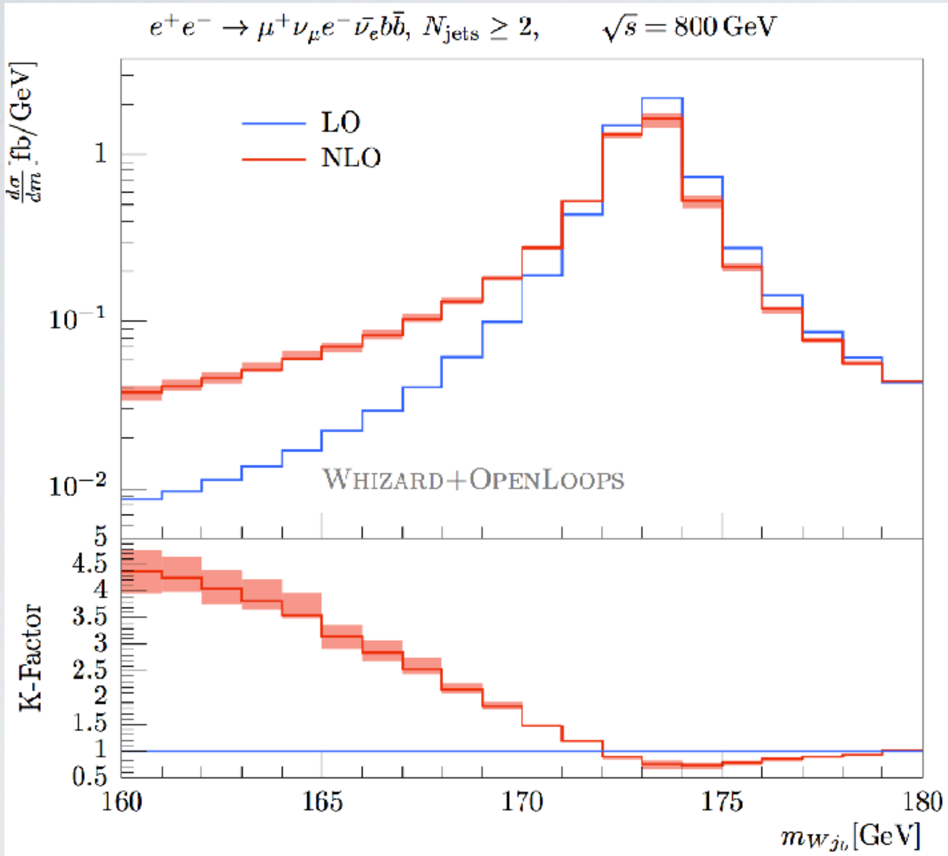
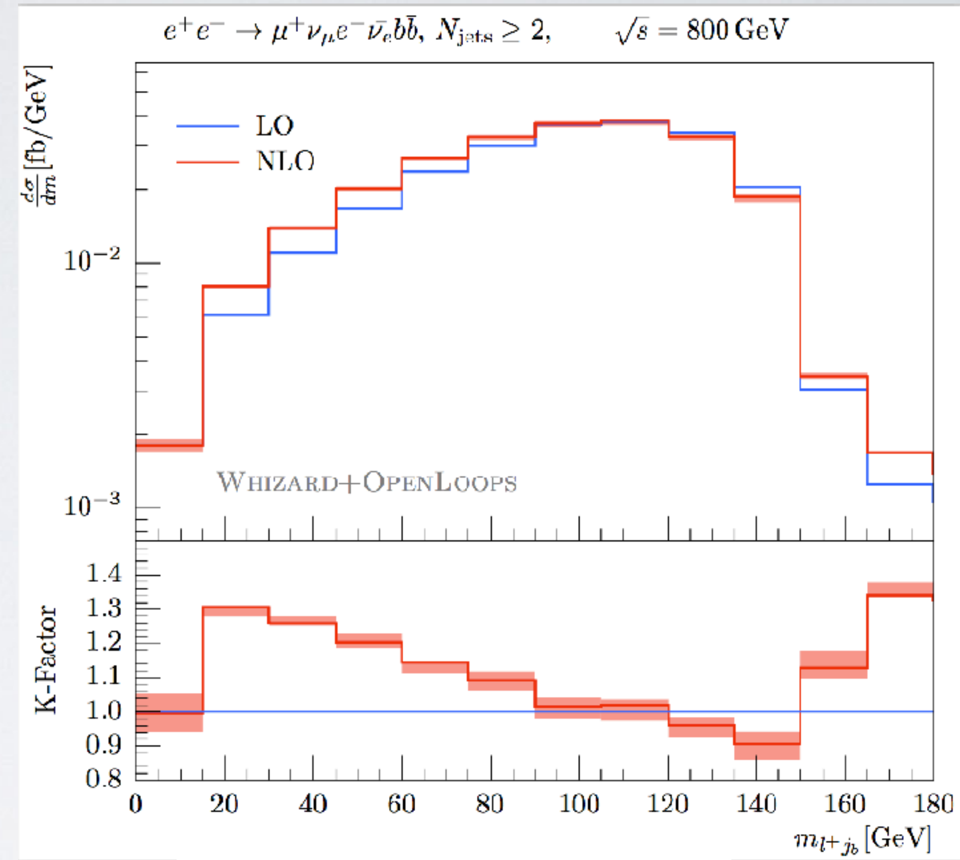
Differential Results for off-shell $e^+e^- \to tt$





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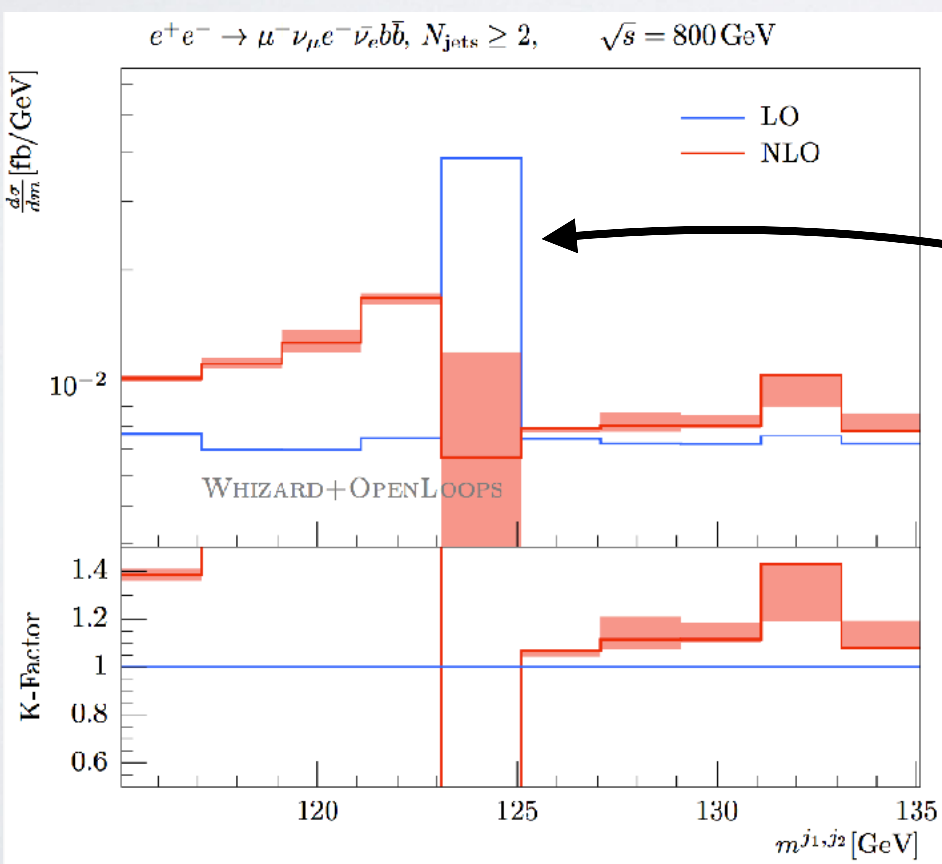


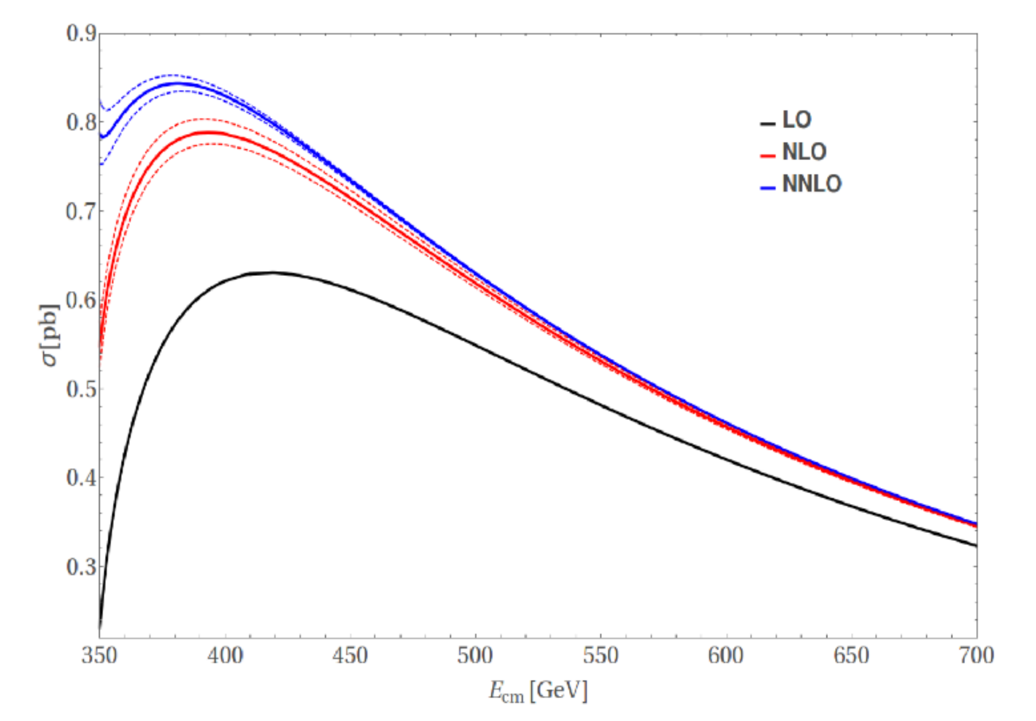
- Full process $e^+e^- \to \mu^+\nu_\mu e^-\nu_e bb$ contains also $e^+e^- \to W^+W^- H$ (!)

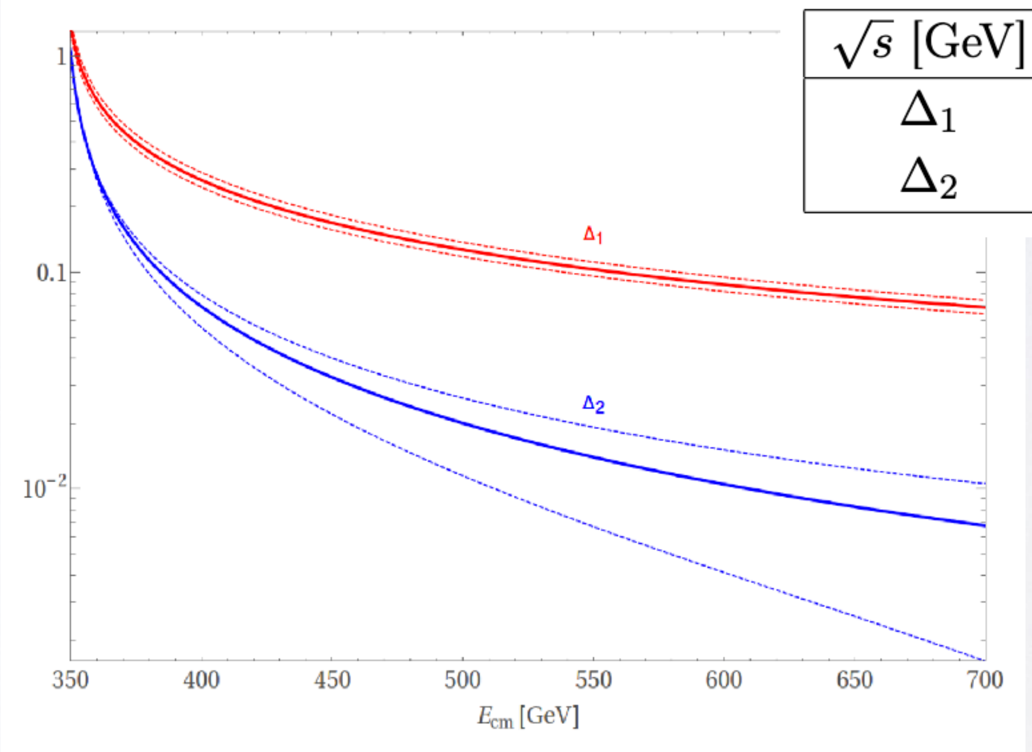
Chokoufé/Kilian/Lindert/Pozzorini/JRR/Weiss, 1609.03390

$e^+e^- \to tt$ on-shell in the continuum

- NNLO QCD on-shell for all infrared-safe observables
- Use of antenna subtraction

Chen/Dekkers/Heisler/ Bernreuther/Si, 1610.07897





$\sqrt{s}$ [GeV]	360	381.3	400	500
Δ_1	0.627	0.352	0.266	0.127
Δ_2	0.281	0.110	0.070	0.020

$$\sigma_{\text{NNLO}} = \sigma_{\text{LO}} (1 + \Delta_1 + \Delta_2)$$

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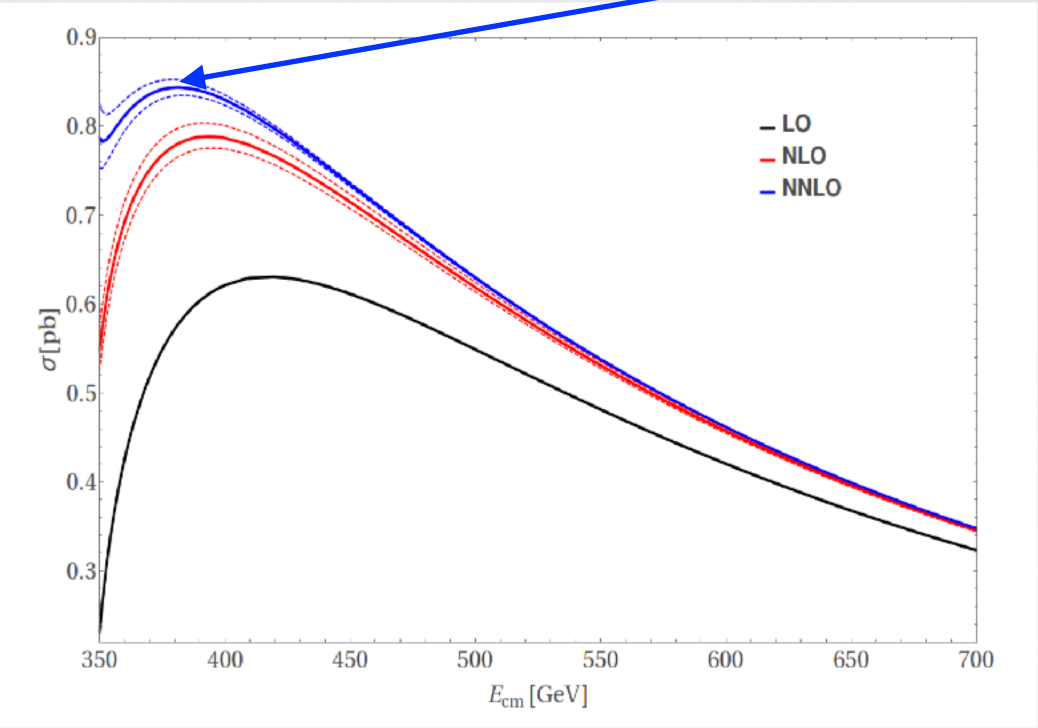
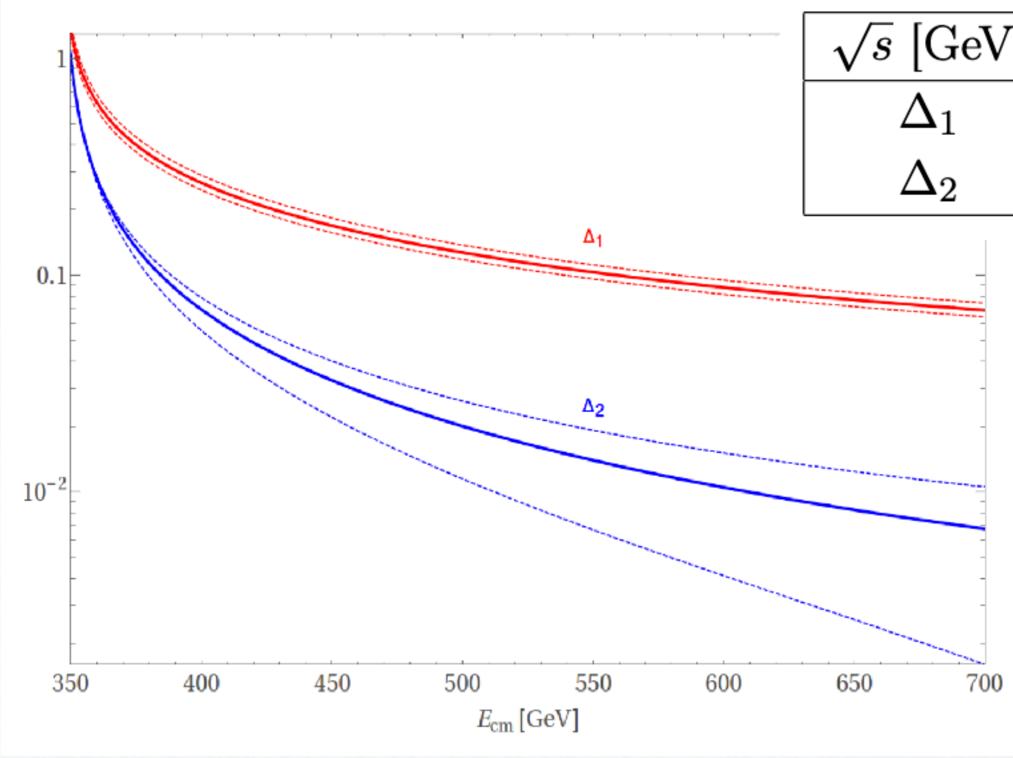
Maximal NNLO xsec. @ 381.3 GeV





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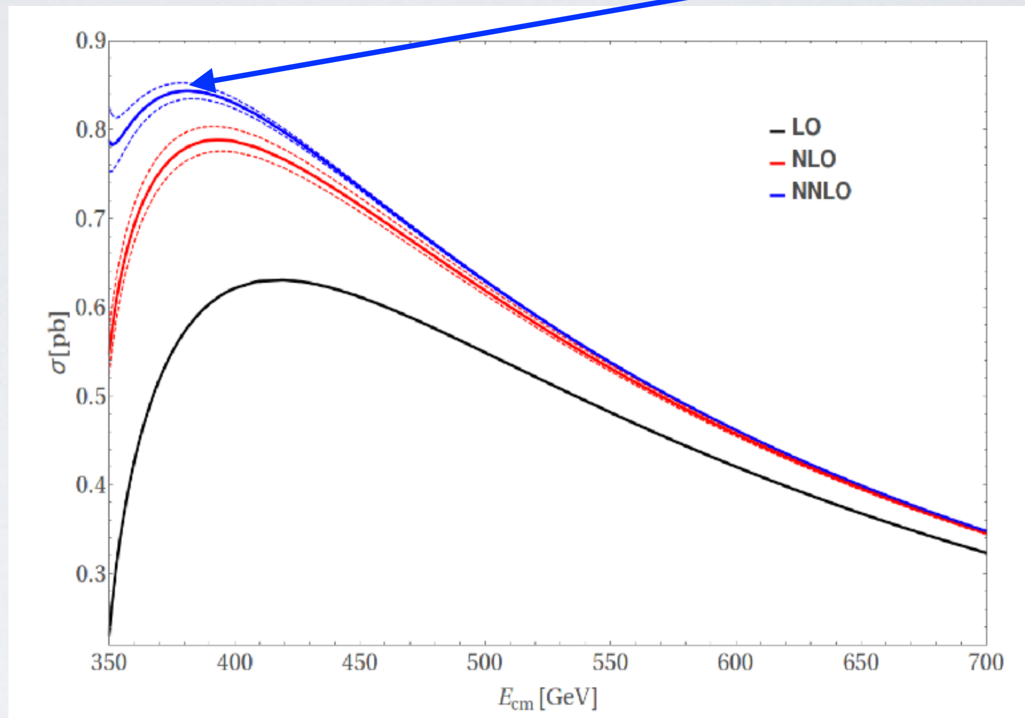

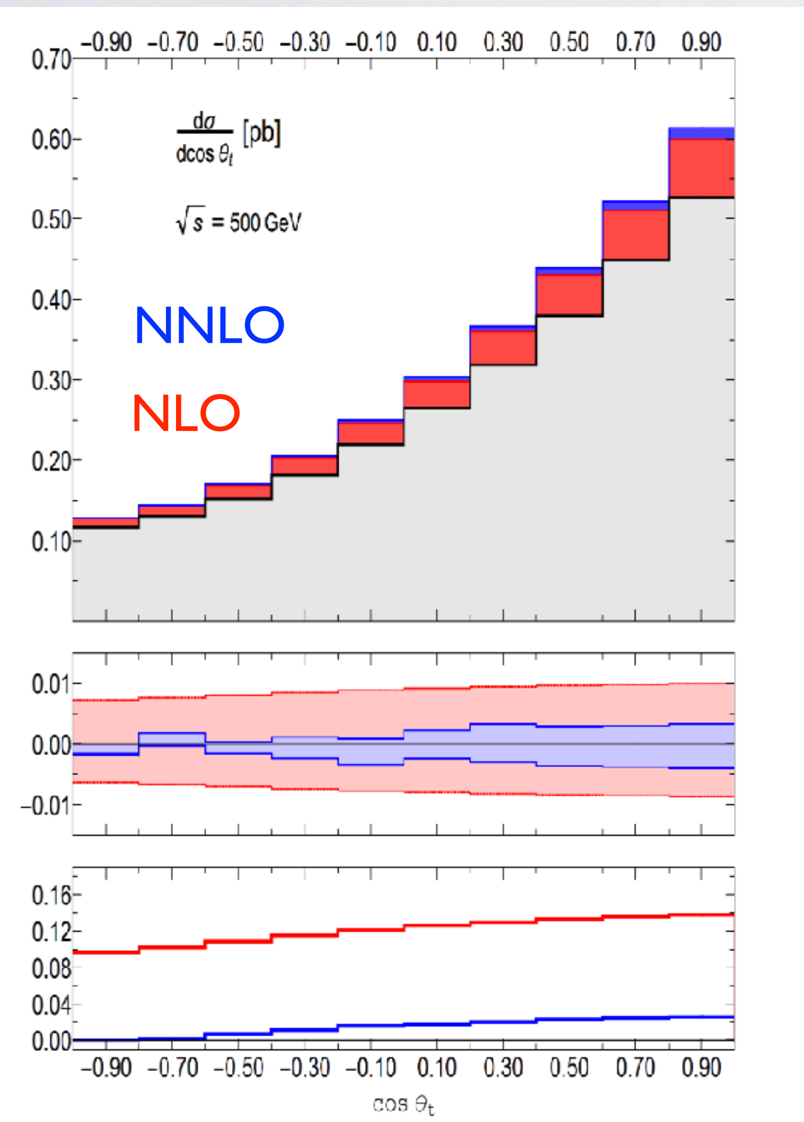

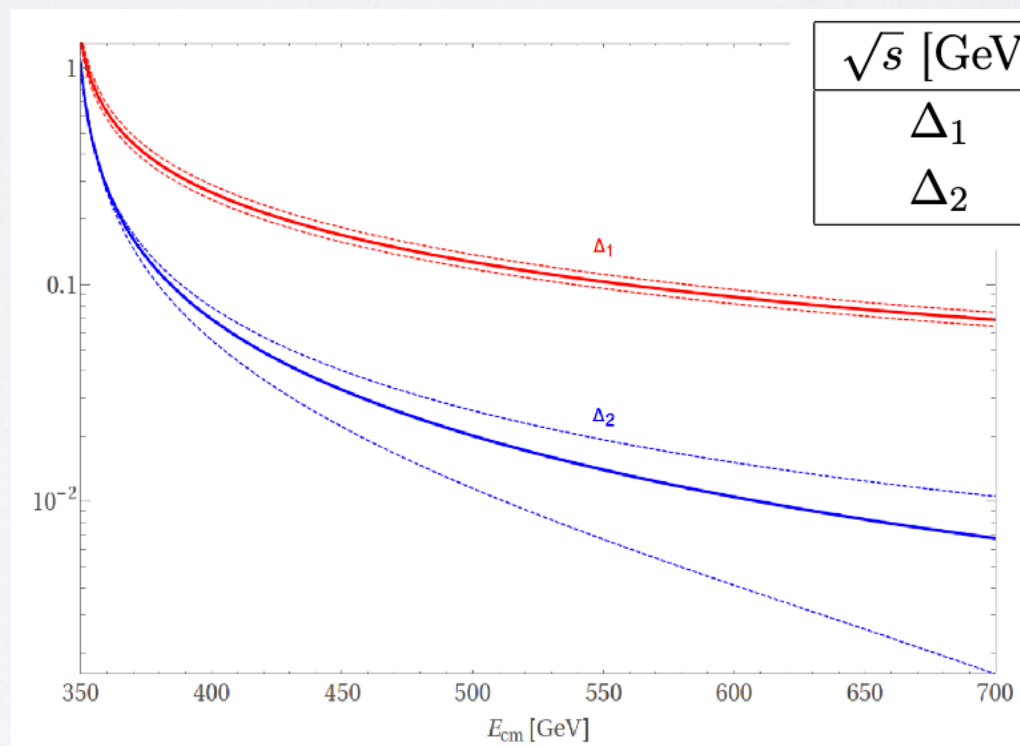

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NLO QCD top physics results

Process	MG5_aMC σ^{LO}[fb]	MG5_aMC σ^{NLO}[fb]	MG5_aMC K	WHIZARD σ^{LO}[fb]	WHIZARD σ^{NLO}[fb]	WHIZARD K
$e^+e^- \to t\bar{t}$	166.2(2)	174.5(3)	1.04994	166.4(1)	174.53(6)	1.04886
$e^+e^- \to t\bar{t}j$	48.13(5)	53.36(1)	1.10867	48.3(2)	53.25(6)	1.10248
$e^+e^- \to t\bar{t}jj$	8.614(9)	10.49(3)	1.21777	8.612(8)	10.46(6)	1.21458
$e^+e^- \to t\bar{t}jjj$	1.044(2)	1.420(4)	1.3601	1.040(1)	1.414(10)	1.3595
$e^+e^- \to t\bar{t}t\bar{t}$	$6.45(1) \cdot 10^{-4}$	$11.94(2) \cdot 10^{-4}$	1.85117	$6.463(2) \cdot 10^{-4}$	$11.91(2) \cdot 10^{-4}$	1.8428
$e^+e^- \to t\bar{t}t\bar{t}j$	$2.719(5) \cdot 10^{-5}$	$5.264(8) \cdot 10^{-5}$	1.93602	$2.722(1) \cdot 10^{-5}$	$5.250(14) \cdot 10^{-5}$	1.92873
$e^+e^- \to t\bar{t}b\bar{b}$	0.1819(3)	0.292(1)	1.60533	0.186(1)	0.293(2)	1.57527
$e^+e^- \to t\bar{t}H$	2.018(3)	1.909(3)	0.94601	2.022(3)	1.912(3)	0.9456
$e^+e^- \to t\bar{t}Hj$	$0.2533(3) \cdot 10^{-0}$	$0.2665(6) \cdot 10^{-0}$	1.05212	0.2540(9)	0.2664(5)	1.04889
$e^+e^- \to t\bar{t}Hjj$	$2.663(4) \cdot 10^{-2}$	$3.141(9) \cdot 10^{-2}$	1.1795	$2.666(4) \cdot 10^{-2}$	$3.144(9) \cdot 10^{-2}$	1.17928
$e^+e^- \to t\bar{t}\gamma$	12.7(2)	13.3(4)	1.04726	12.71(4)	13.78(4)	1.08418
$e^+e^- \to t\bar{t}Z$	4.642(6)	4.95(1)	1.06636	4.64(1)	4.94(1)	1.06467
$e^+e^- \to t\bar{t}Zj$	0.6059(6)	0.6917(24)	1.14168	0.610(4)	0.6927(14)	1.13565
$e^+e^- \to t\bar{t}Zjj$	$6.251(28) \cdot 10^{-2}$	$8.181(21) \cdot 10^{-2}$	1.30875	$6.233(8) \cdot 10^{-2}$	$8.201(14) \cdot 10^{-2}$	1.31573
$e^+e^- \to t\bar{t}W^{\pm}jj$	$2.400(4) \cdot 10^{-4}$	$3.714(8) \cdot 10^{-4}$	1.54747	$2.41(1) \cdot 10^{-4}$	$3.695(9) \cdot 10^{-4}$	1.5332
$e^+e^- \to t\bar{t}\gamma\gamma$	0.383(5)	0.416(2)	1.08618	0.382(3)	0.420(3)	1.09952
$e^+e^- \to t\bar{t}\gamma Z$	0.2212(3)	0.2364(6)	1.06873	0.220(1)	0.240(2)	1.09094
$e^+e^- \to t\bar{t}\gamma H$	$9.75(1) \cdot 10^{-2}$	$9.42(3) \cdot 10^{-2}$	0.96614	$9.748(6) \cdot 10^{-2}$	$9.58(7) \cdot 10^{-2}$	0.98277
$e^+e^- \to t\bar{t}ZZ$	$3.788(4) \cdot 10^{-2}$	$4.00(1) \cdot 10^{-2}$	1.05597	$3.756(4) \cdot 10^{-2}$	$4.005(2) \cdot 10^{-2}$	1.0663
$e^+e^- \to t\bar{t}W^+W^-$	0.1372(3)	0.1540(6)	1.1225	0.1370(4)	0.1538(4)	1.12257
$e^+e^- \to t\bar{t}HH$	$1.358(1) \cdot 10^{-2}$	$1.206(3) \cdot 10^{-2}$	0.888	$1.367(1) \cdot 10^{-2}$	$1.218(1) \cdot 10^{-2}$	0.8909
$e^+e^- \to t\bar{t}HZ$	$3.600(6) \cdot 10^{-2}$	$3.58(1) \cdot 10^{-2}$	0.99445	$3.596(1) \cdot 10^{-2}$	$3.581(2) \cdot 10^{-2}$	0.9958

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$e^+e^- \to t\bar{t}W^+W^-$	0.1372(3)	0.1540(6)	1.1225	0.1370(4)	0.1538(4)	[illegible]
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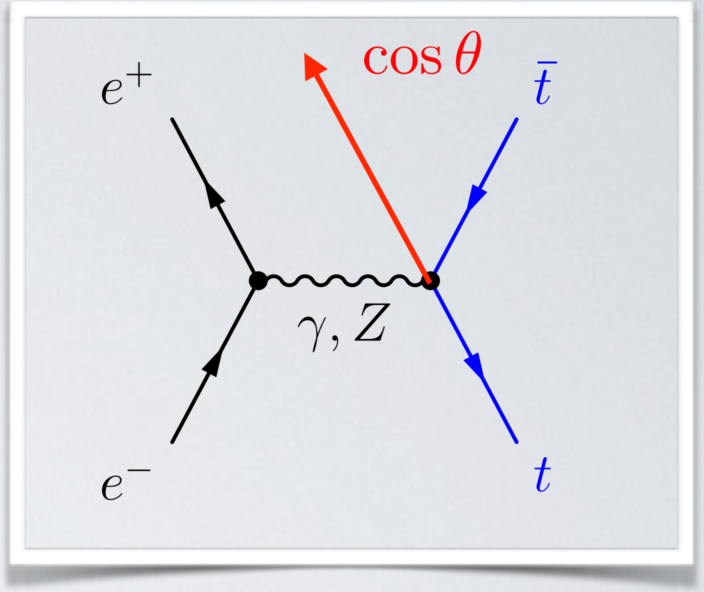
THE BEST OF BOTH SIDES OF THE DETECTOR: THE FORWARD-BACKWARD ASYMMETRY

Top-Forward Backward Asymmetry

$$\frac{d\sigma(e^+e^- \to t\bar{t})}{d\Omega_{\text{CM}}} = \frac{\alpha^2}{4s}\sqrt{1-\frac{4M_t^2}{s}}\left\{\left(1+\cos\theta^2+\frac{4M_t^2}{s}\sin^2\theta\right)G_1(s) - \frac{8M_t^2}{s}G_2(s) + \sqrt{1-\frac{4M_t^2}{s}}\,2\cos\theta G_3(s)\right\}$$



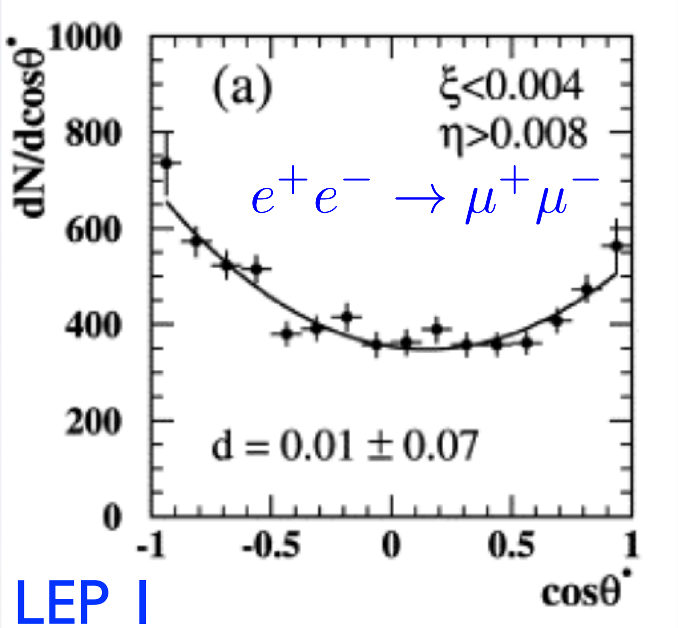
$$G_1(s) = Q_e^2Q_t^2 + 2Q_eQ_tV_eV_t\,\text{Re}\left[X_Z(s)\right] + \left(V_e^2+A_e^2\right)\left(V_t^2+A_t^2\right)\left|X_Z(s)\right|^2$$

$$G_2(s) = \left(V_e^2+A_e^2\right)A_t^2\left|X_Z(s)\right|^2$$

$$G_3(s) = 2\,Q_eQ_tA_eA_t\text{Re}\left[X_Z(s)\right] + 4\,V_eV_tA_eA_t\left|X_Z(s)\right|^2$$

$$X_Z(s) = \frac{s}{s-M_Z^2+\text{i}M_Z\Gamma_Z}$$

$$-\frac{g}{2c_W}\,\overline{f}\left[V_f\gamma^\mu - A_f\gamma^\mu\gamma^5\right]f\,Z_\mu$$



LEP I

- Axial vector photon-Z and vector—axial-vector interference
- Linearly dependent term generates Forward-Backward Asymmetry

$$A_{FB} = \frac{\sigma(\cos\theta_t > 0) - \sigma(\cos\theta_t < 0)}{\sigma(\cos\theta_t > 0) + \sigma(\cos\theta_t < 0)}.$$

Asymmetry is function of collider energy

Top-Forward Backward Asymmetry

	$e^+e^- \to$	A_{FB}^{LO}	A_{FB}^{NLO}	A_{FB}^{NLO}/A_{FB}^{LO}
A_{FB}	$t\bar{t}$	-0.535	-0.539	1.013
	$W^+W^-b\bar{b}$	-0.428	-0.426	0.995
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$	-0.415	-0.409	0.986
	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$, without neutrinos	-0.402	-0.387	0.964
$\bar{A}_{FB}$	$t\bar{t}$	0.535	0.539	1.013
	$W^+W^-b\bar{b}$	0.428	0.426	0.995
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	$\mu^+e^-\nu_\mu\bar{\nu}_e b\bar{b}$, without neutrinos	0.377	0.350	0.928

NLO QCD Corrections

Gluon emission symmetric in θ ⇒

NLO QCD corrections small

Djouadi/Lampe/Zerwas, hep-ph/9411386

Bardin/Christova/Jack/Kalinovskaya/Olchevski/S. Riemann/T. Riemann, hep-ph/9908433

Altarelli/Lampe, NPB391 (1993) 3

Ravindran/van Neerven, hep-ph/9809411

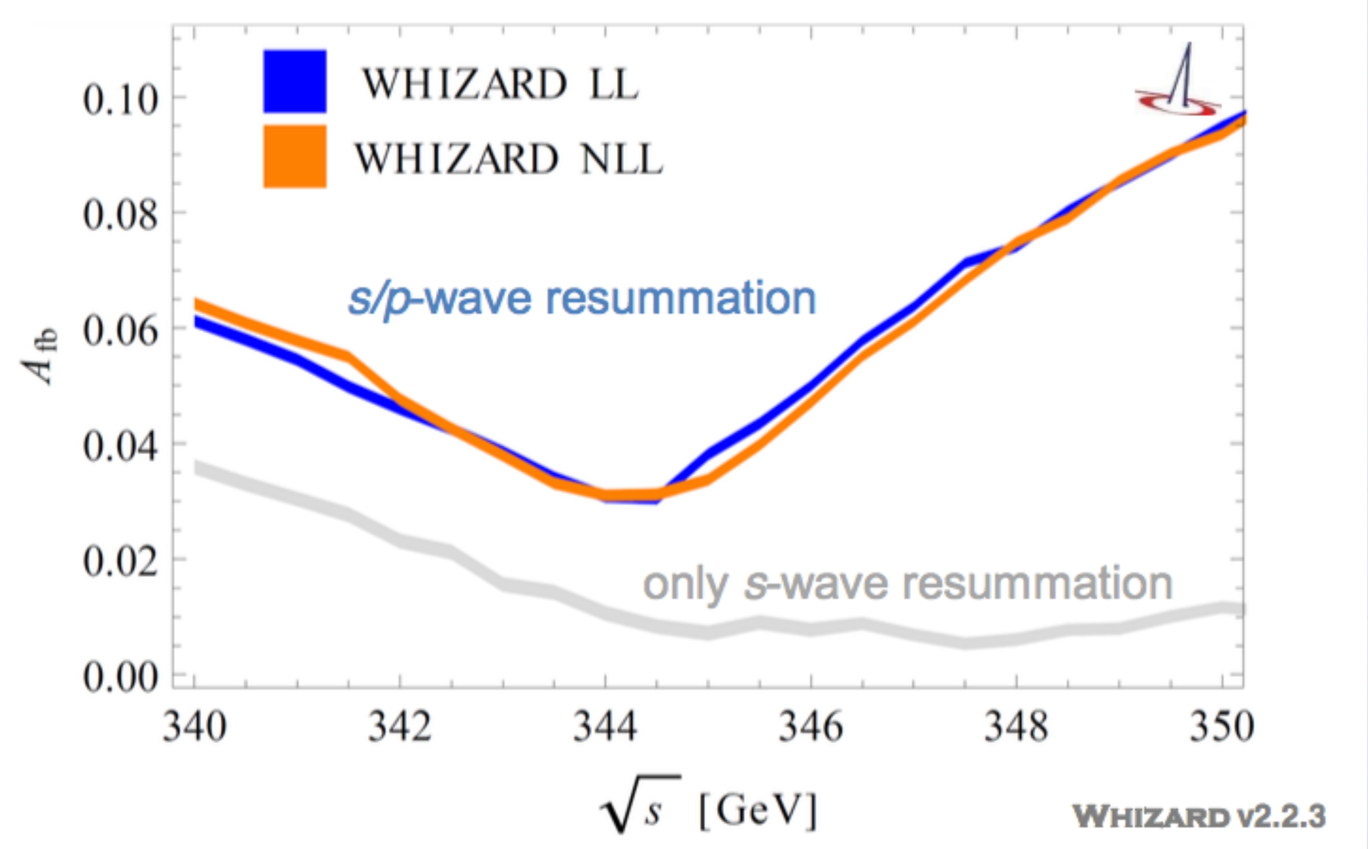
Catani/Seymour, hep-ph/9905424

A_{FB} of the top quark

Forward-backward asymmetry

$$A_{fb} := \frac{\sigma(p_z^t > 0) - \sigma(p_z^t) < 0)}{\sigma(p_z^t > 0) + \sigma(p_z^t < 0)}$$

Threshold region: *P*-wave (axial vector) resummation important



Top-Forward Backward Asymmetry

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Chen/Dekkers/Heisler/Bernreuther/Si, 1610.07897

A_{FB} of the top quark

$$A_{\rm FB}^{\rm NNLO} = A_{\rm FB}^{\rm LO}(1 + A_1 + A_2)$$

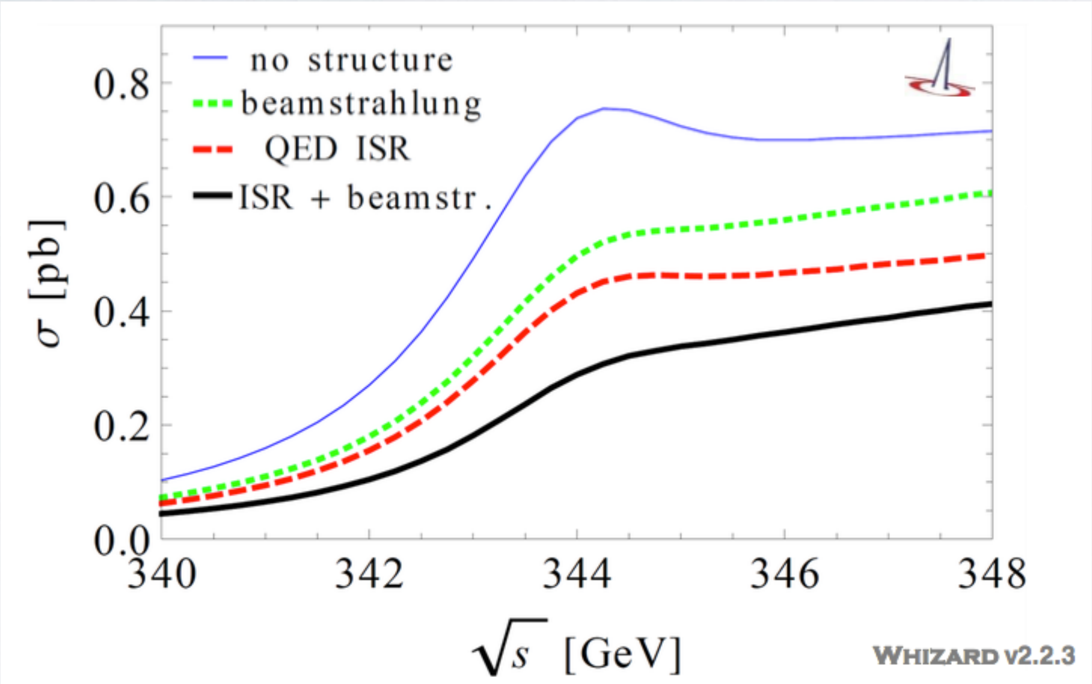
$\sqrt{s}$ [GeV]	$A_{\rm FB}^{\rm LO}$ [%]	$A_{\rm FB}^{\rm NLO}$ [%]	$A_{\rm FB}^{\rm NNLO}$ [%]	A_1 [%]	A_2 [%]	$\delta A_{\rm FB}^{\rm NNLO}$ [%]
360	14.94	$15.54^{+0.05}_{-0.04}$	$16.23^{+0.12}_{-0.10}$	$4.01^{+0.35}_{-0.29}$	$4.58^{+0.46}_{-0.38}$	± 0.59
400	28.02	$28.97^{+0.08}_{-0.07}$	$29.63^{+0.11}_{-0.10}$	$3.41^{+0.29}_{-0.25}$	$2.36^{+0.11}_{-0.11}$	± 0.27
500	41.48	$42.42^{+0.08}_{-0.07}$	$42.91^{+0.08}_{-0.07}$	$2.28^{+0.19}_{-0.16}$	$1.18^{+0.01}_{-0.01}$	± 0.13
700	51.34	$51.81^{+0.04}_{-0.03}$	$52.05^{+0.04}_{-0.04}$	$0.91^{+0.07}_{-0.06}$	$0.47^{+0.01}_{-0.01}$	± 0.06

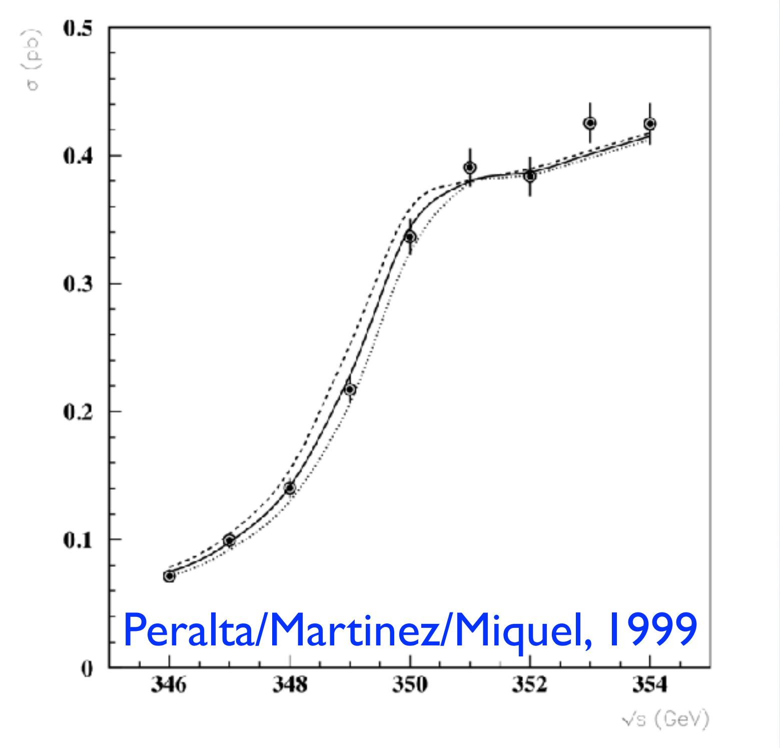
TOP THRESHOLD SCAN: THE "TOP" TOP MASS MEASUREMENT

Top Mass Measurement: Threshold

ILC top threshold scan best-known method to measure top quark mass, ΔM ~ 30-70 MeV

- Close to threshold: top quarks non-relativistic
- Very strong QCD attraction due to "Coulomb" gluon exchange
- Leads to a remnant 1*S* toponium (quasi-) bound state





error source	Δm_t^{PS} [MeV]
stat. error (200 fb^{-1})	13
theory (NNNLO scale variations, PS scheme)	40
parametric (α_s, current WA)	35
non-resonant contributions (such as single top)	< 40
residual background / selection efficiency	10 – 20
luminosity spectrum uncertainty	< 10
beam energy uncertainty	< 17
combined theory & parametric	30 – 50
combined experimental & backgrounds	25 - 50
total (stat. + syst.)	40 – 75

from 1702.05333

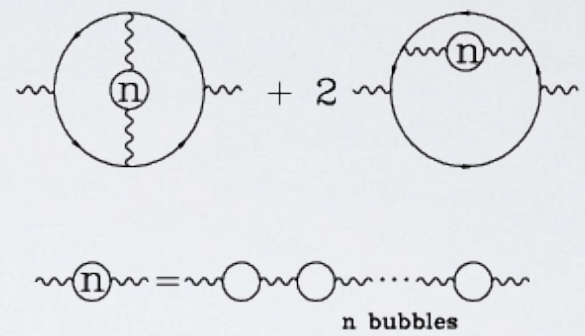
Top Mass Definitions

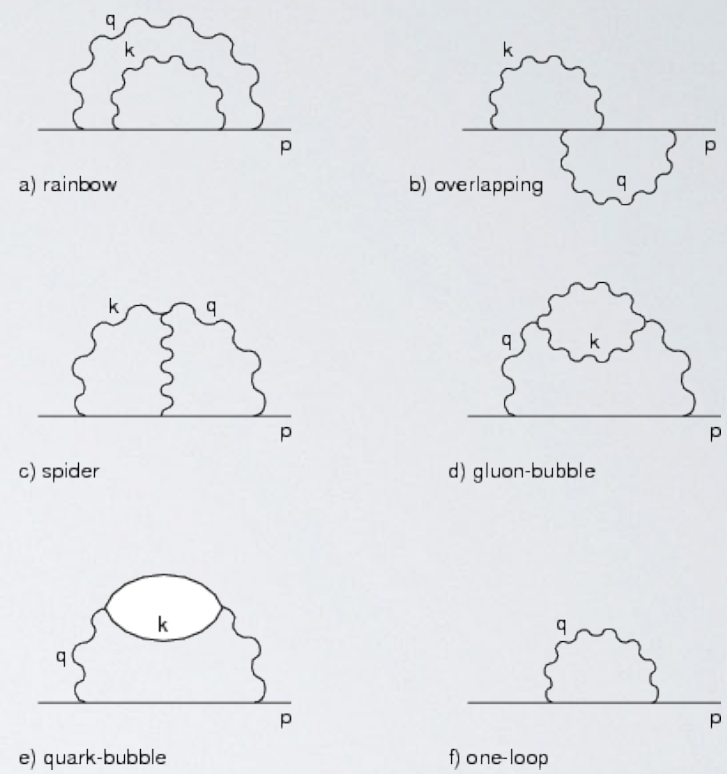
- On-shell mass *M*: inverse quark propagator has zero at on-shell mass
- $\overline{\mathrm{MS}}$ mass *m*: just divergent part is subtracted to achieve a finite quark propagator
- Short-distance masses: PS [potential subtr.], 1S, RS [renormalon subtr.] masses

$$m^0 = Z_m^{\overline{\mathrm{MS}}} m \qquad m^0 = Z_m^{\mathrm{OS}} M$$

$$Z_m^{\mathrm{OS}} = 1 + \Sigma_V(q^2 = M^2) + \Sigma_S(q^2 = M^2)$$

$$m^{\mathrm{PS}} = M - \frac{1}{2}\int_{|\vec{q}|<\mu_f} \frac{d^3 q}{(2\pi)^3} V(\vec{q})$$





- 1S mass is half the (pert.) mass of the $1\,{}^3S_1$ bound state

$$m^{1S} = M + \frac{1}{2} E_1^{pt}\Big|_{\alpha_s^n \to \alpha_s^n \epsilon^{n-1}}$$

- Relation between pole and $\overline{\mathrm{MS}}$ mass @ 4-loop

Marquard/Smirnov/Smirnov/Steinhauser, 1502.01030

$$\begin{aligned} M_t &= m_t \left(1 + 0.4244\alpha_s + 0.8345\alpha_s^2 + 2.375\alpha^3 + (8.49 \pm 0.25)\alpha_s^4\right) \\ &= 163.643 + 7.557 + 1.617 + 0.501 + 0.195 \pm .005 \text{ GeV} \end{aligned}$$

Final uncertainties from top mass conversions:

$$\Delta M_{PS} = 23\,\mathrm{MeV} \qquad \Delta M_{1S} = 7\,\mathrm{MeV} \qquad \Delta M_{RS} = 11\,\mathrm{MeV}$$

Top Threshold: a demanding theory calculation

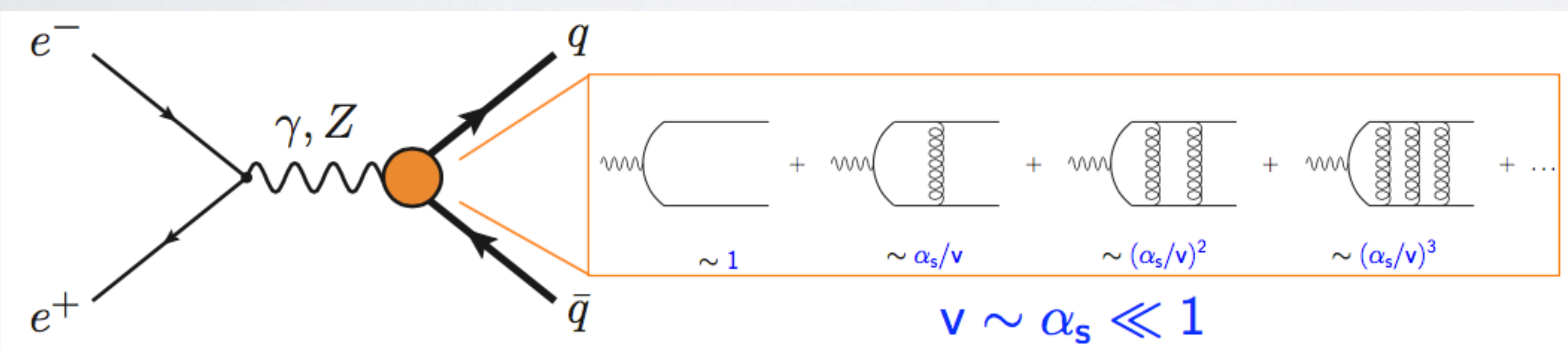
- NRQCD is EFT for non-relativistic quark-antiquark systems: separate $M \cdot v$ and $M \cdot v^2$
- Integrate out hard quark and gluon d.o.f. Hoang et al. '99-'01; Beneke et al., '13-'14
- Resummation of singular terms close to threshold (v = 0), NNNLO/NNLL available (!)

Phase space of two massive particles

(p/v)NRQCD EFT w/ RG improvement

$$R \equiv \frac{\sigma_{t\bar{t}}}{\sigma_{\mu\mu}} = v \sum_k \left(\frac{\alpha_s}{v}\right)^k \sum_i (\alpha_s \ln v)^i \times$$
$$\times \left\{ 1\,(\mathrm{LL});\ \alpha_s, v\,(\mathrm{NLL});\ \alpha_s^2, \alpha_s v, v^2\,(\mathrm{NNLL}) \right\}$$

Threshold region: top velocity $v \sim \alpha_s \ll 1$ non-relativistic EFT: (v)NRQCD



Continuum region: "standard" fixed-order QCD

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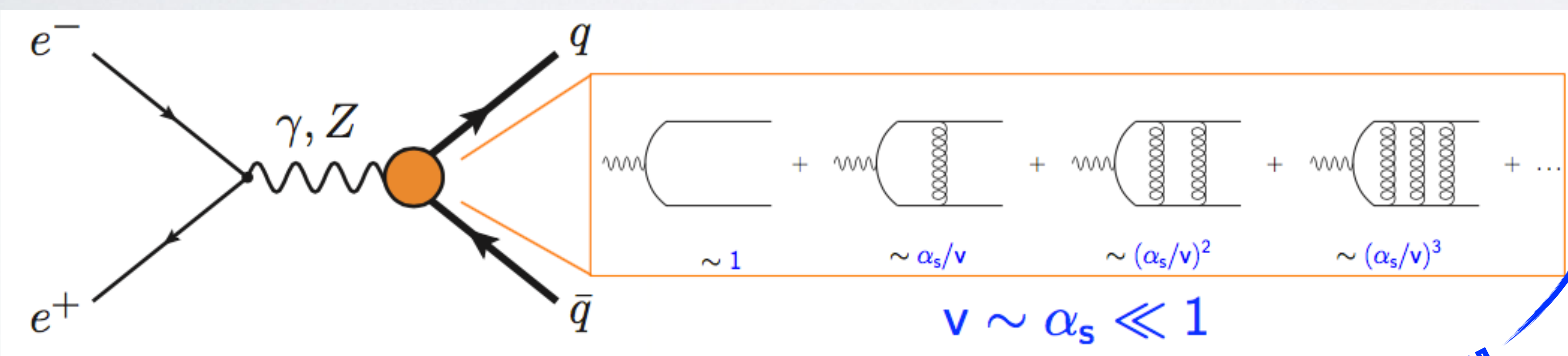


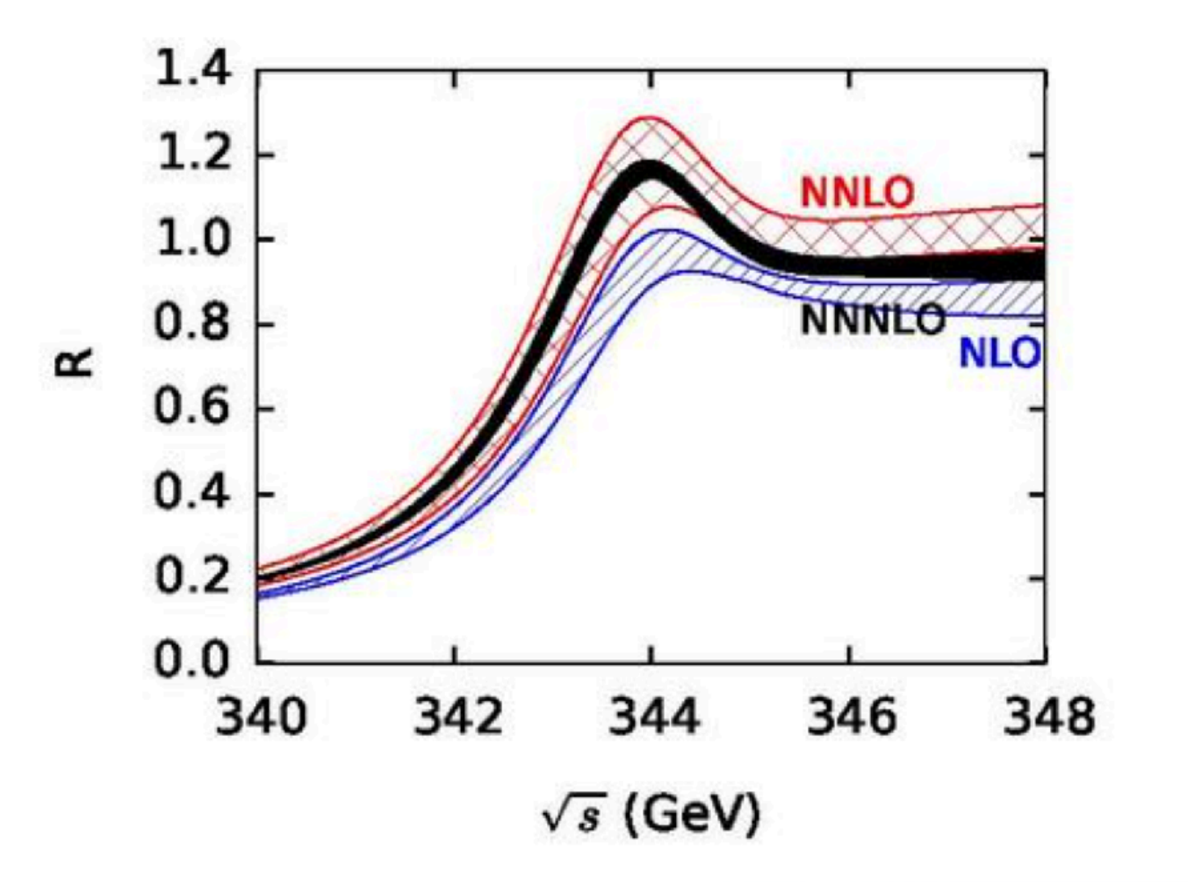
MATCH

Continuum region: "standard" fixed-order QCD

Fixed-order vs. resummation uncertainties

NRQCD NNNLO fixed order
+ α_s logarithms

Kiyo et al., 2005; Beneke et al., 2008-2015



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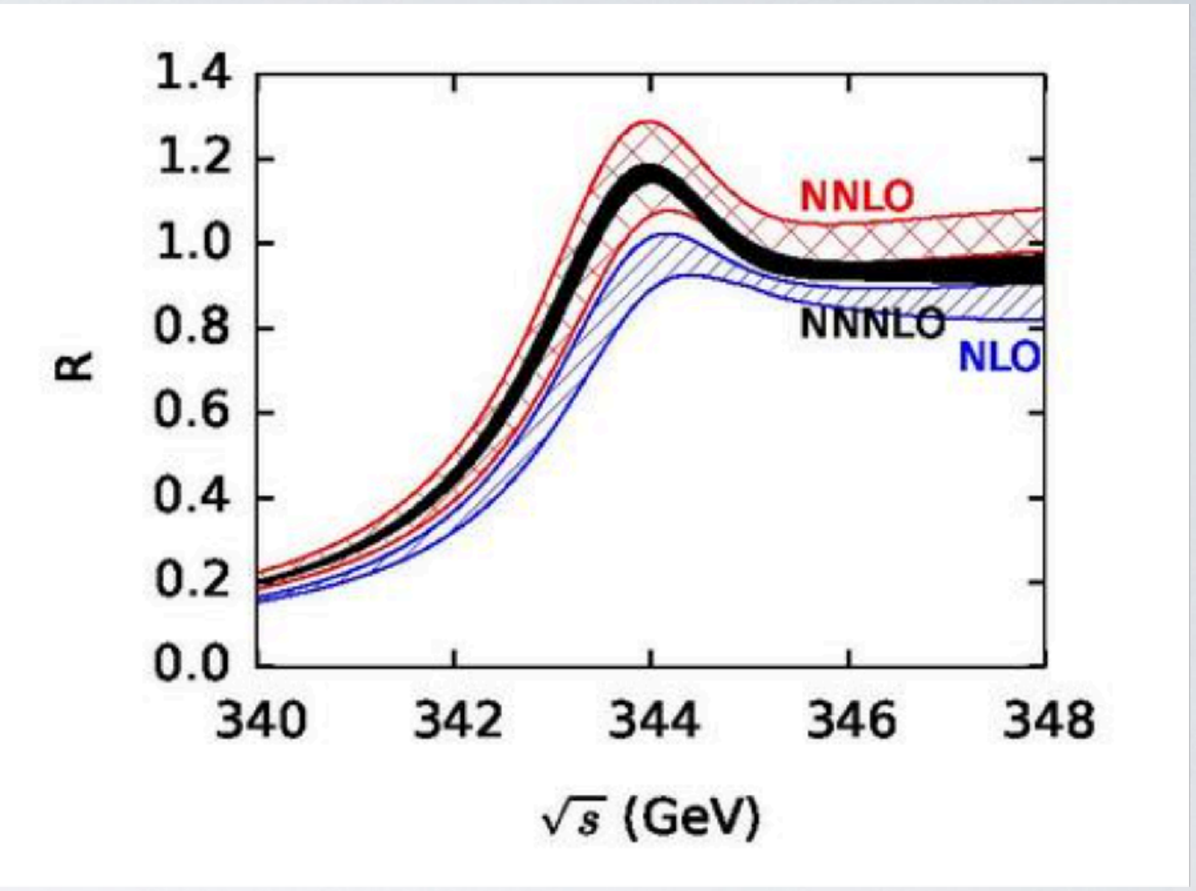
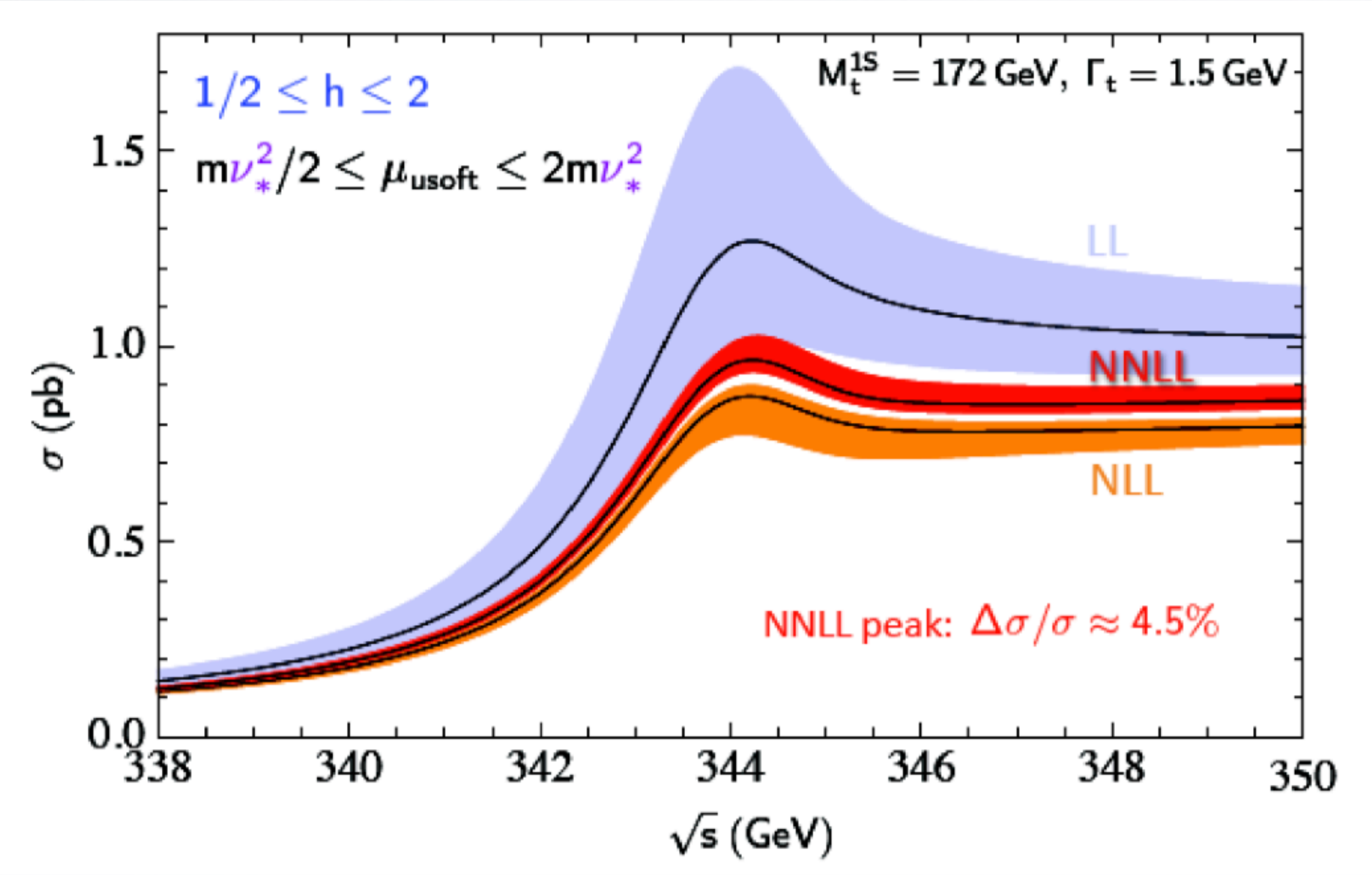
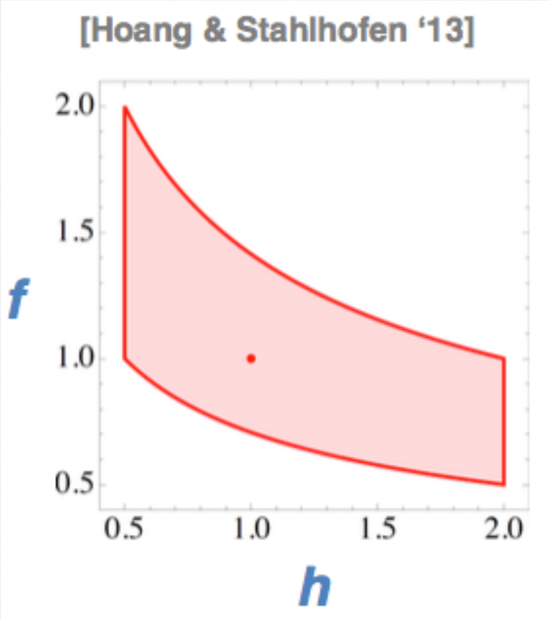


Resummation of
velocity logarithms

Hoang/Stahlhofen, 2012

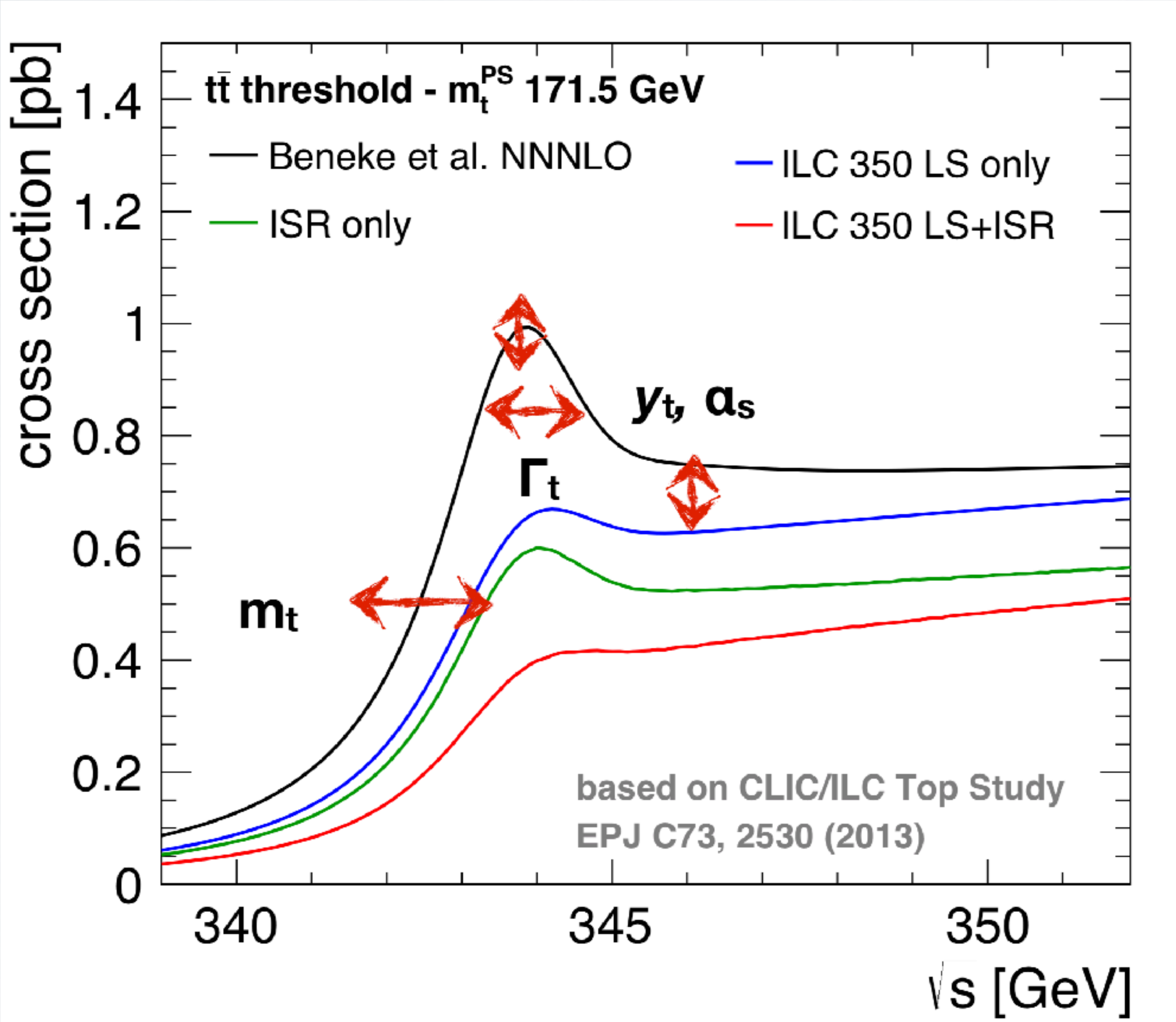


Theory uncertainties
from scale variations:
hard and soft scale



$\mu_h = h \cdot m_t \qquad \mu_s = f \cdot m_t v$

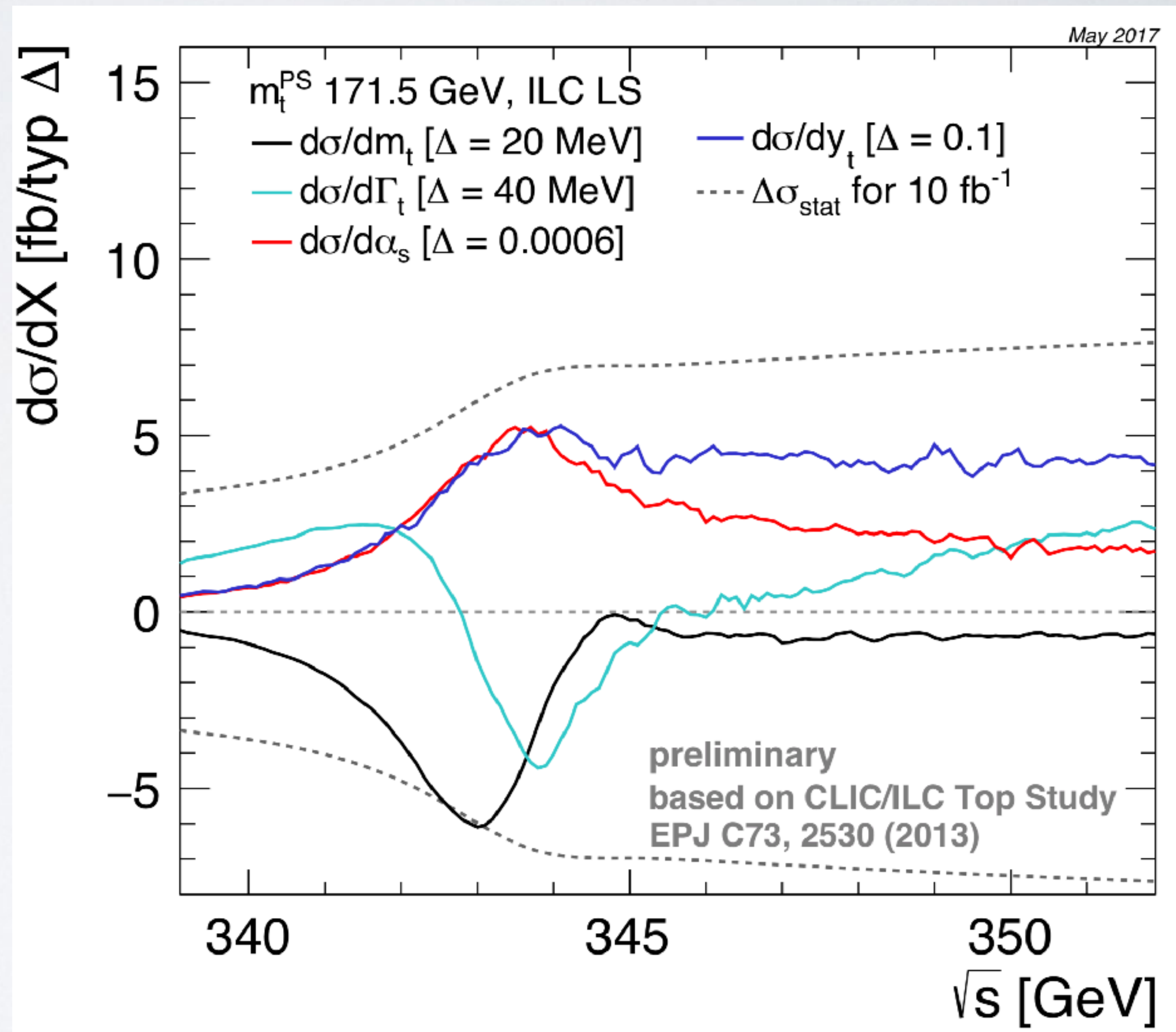
Top Threshold: parametric dependencies

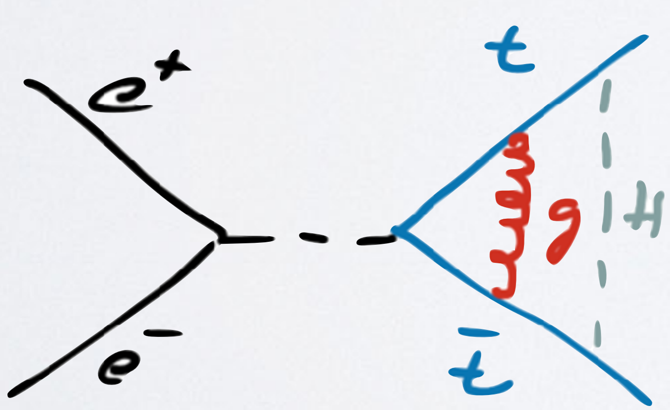


Dependence on M_t, Γ_t, α_s, y_t

based on:

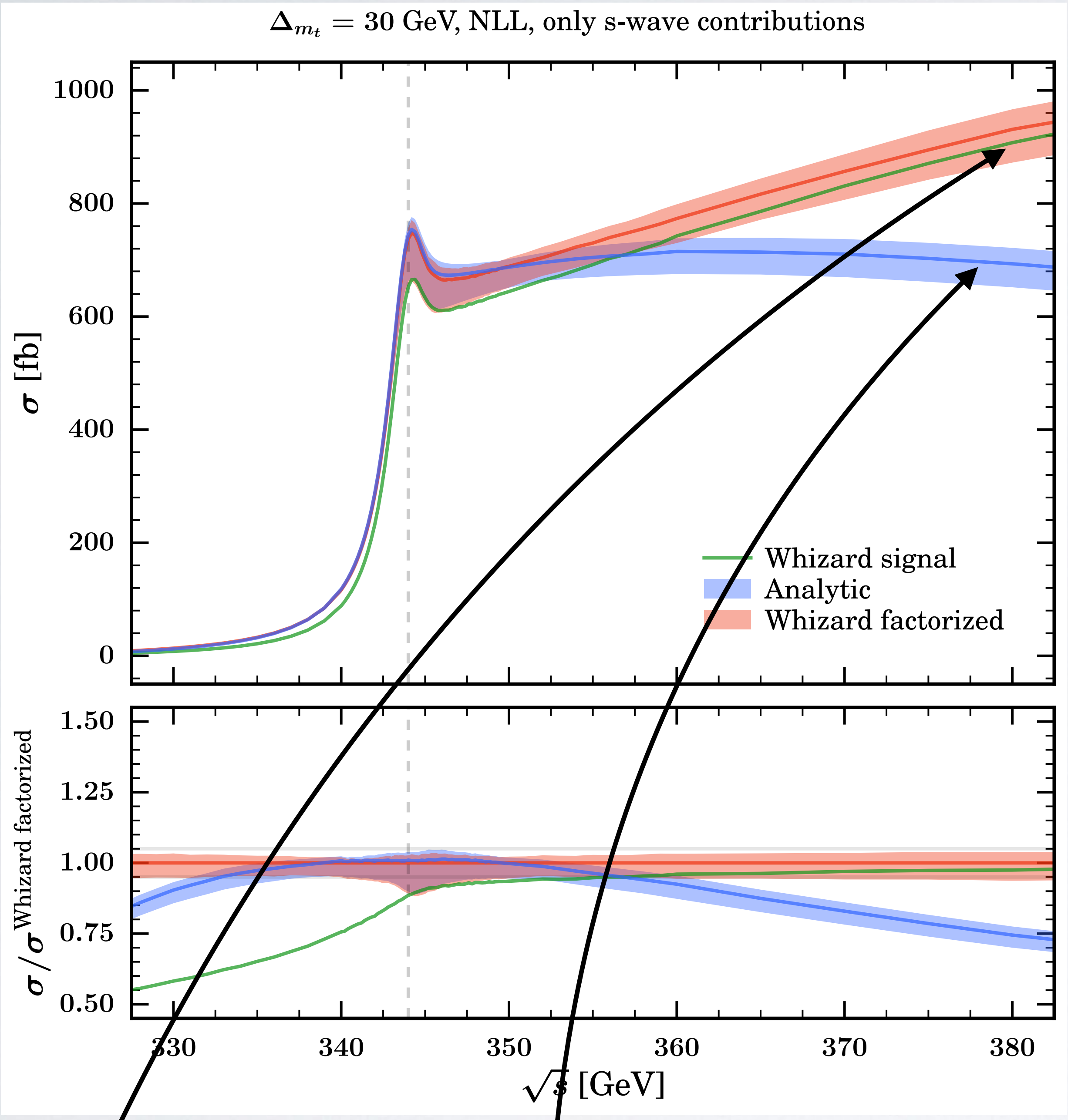
Beneke/Maier/Piclum/Rauh, 2015

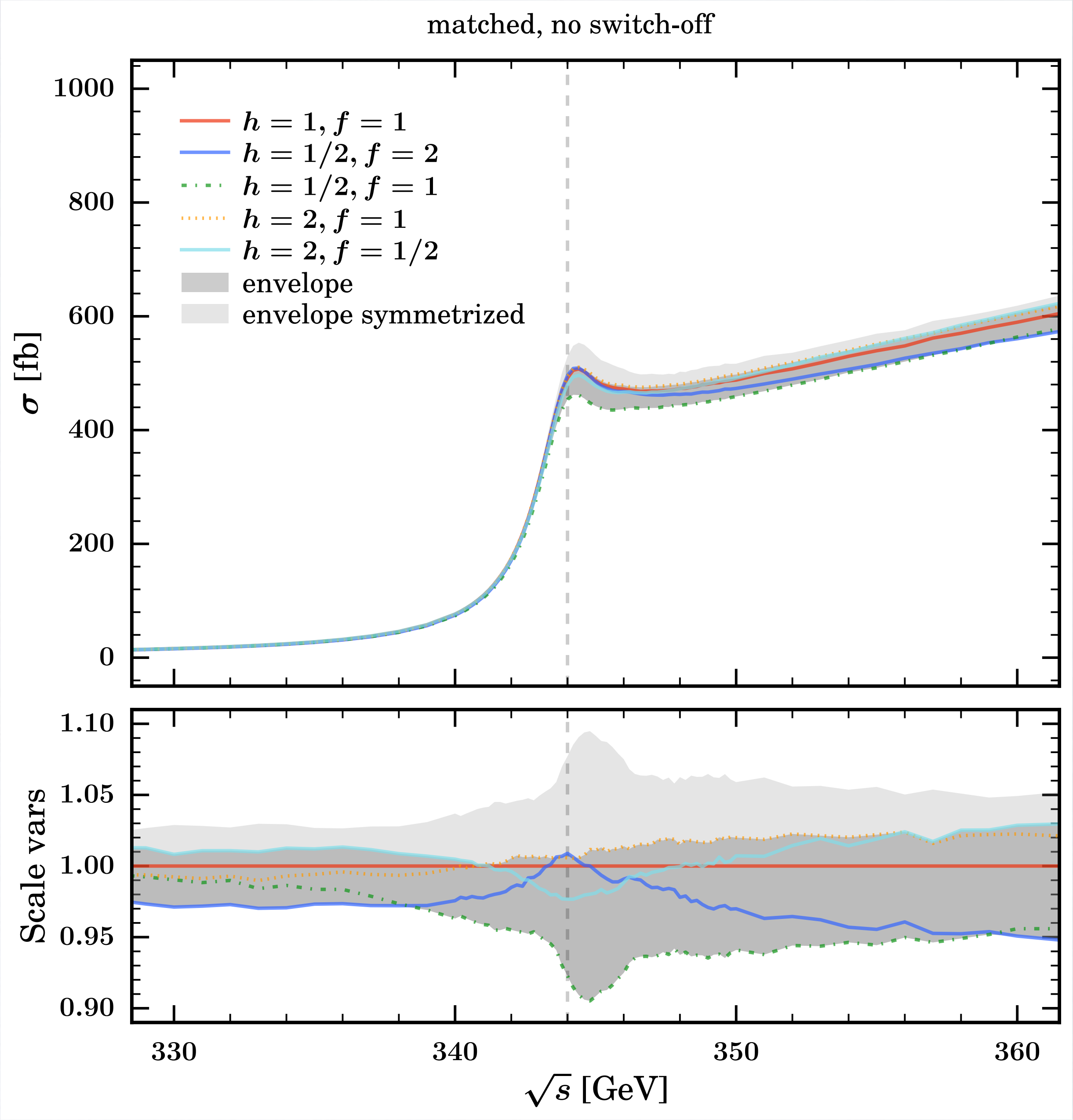




Fully exclusive Threshold in $e^+e^- \to W^+W^-bb$

Fully Exclusive Events: assess selection uncertainties



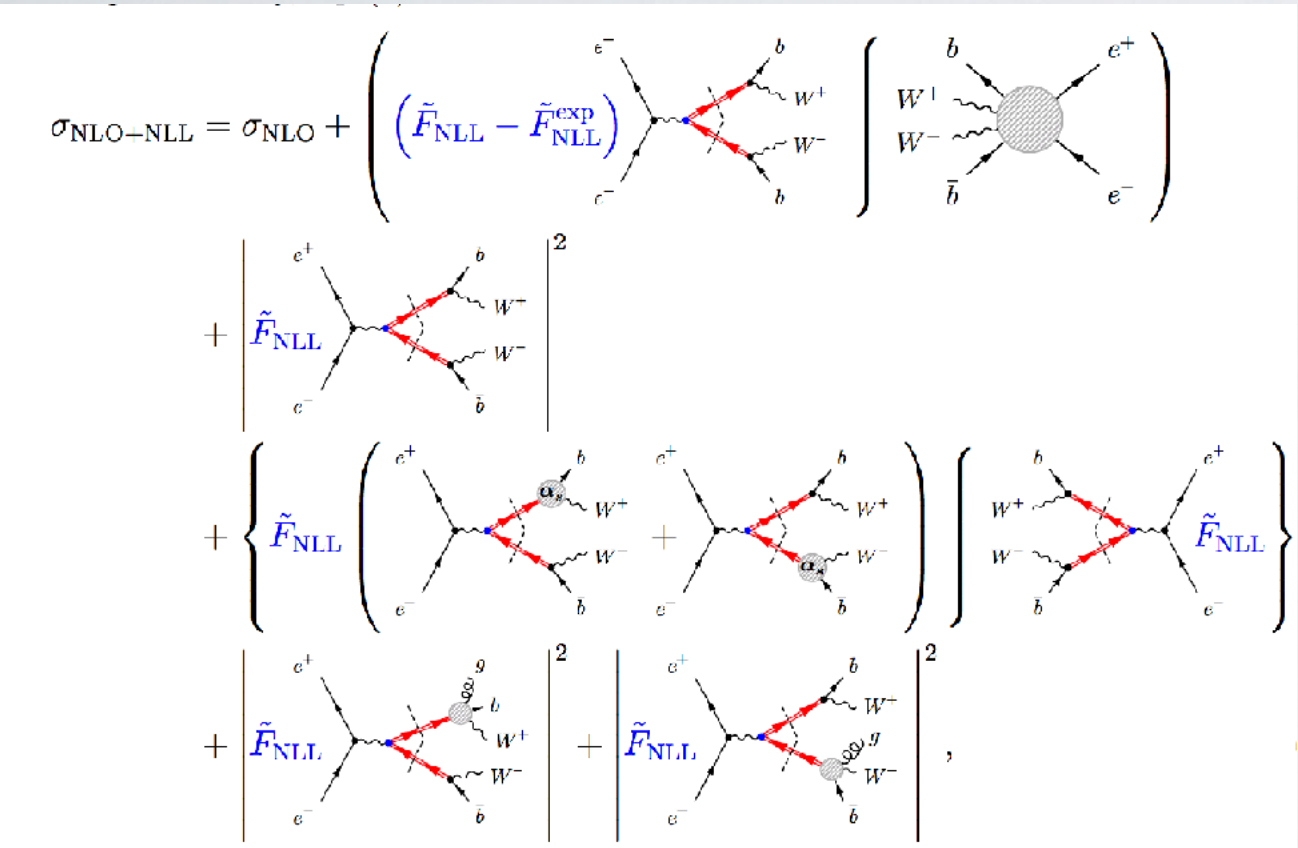


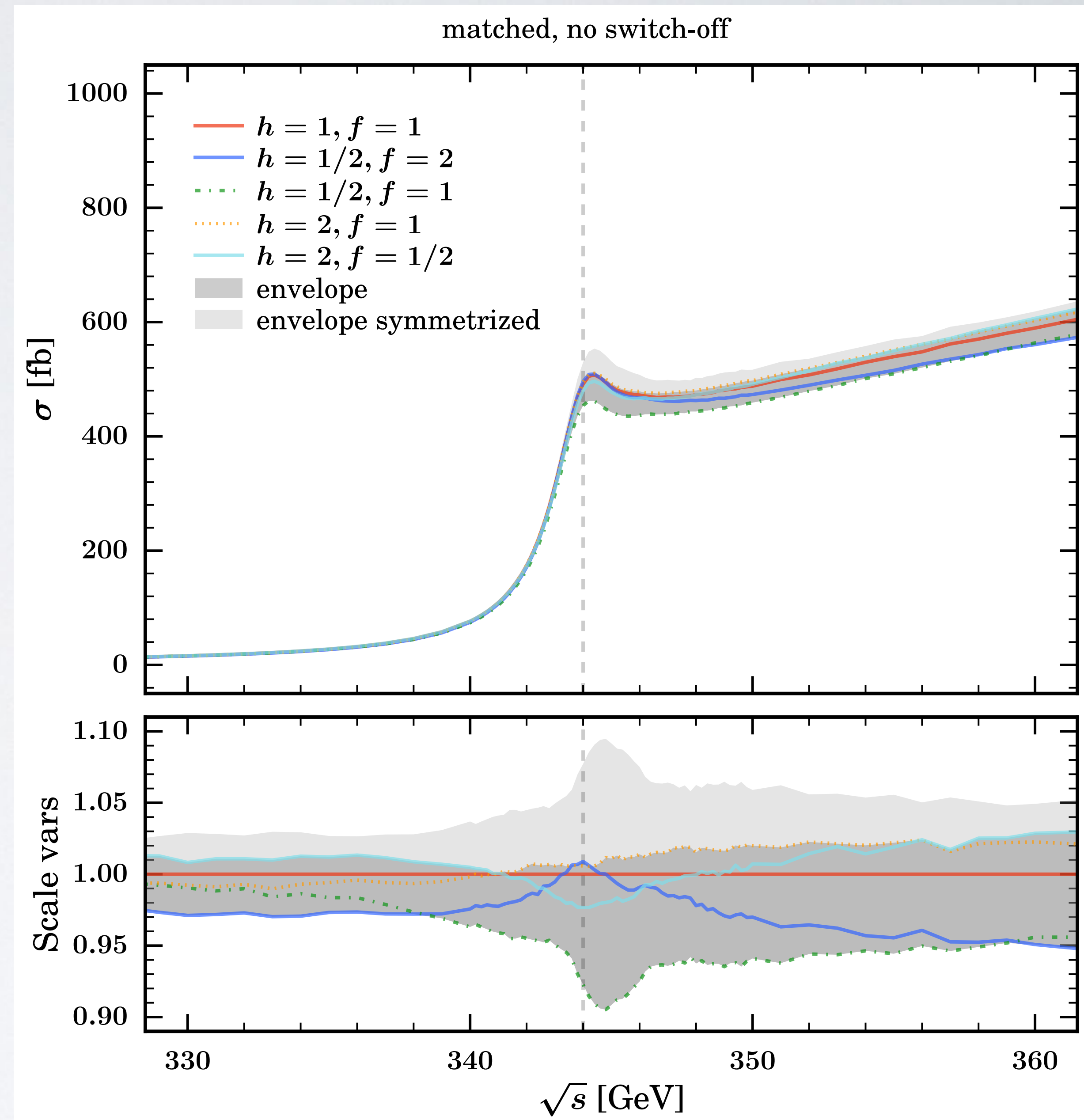
NRQCD result invalid away from threshold

Bach/Chokoufé/Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220

Fully exclusive Threshold in $e^+e^- \to W^+W^-bb$

Fully Exclusive Events: assess selection uncertainties

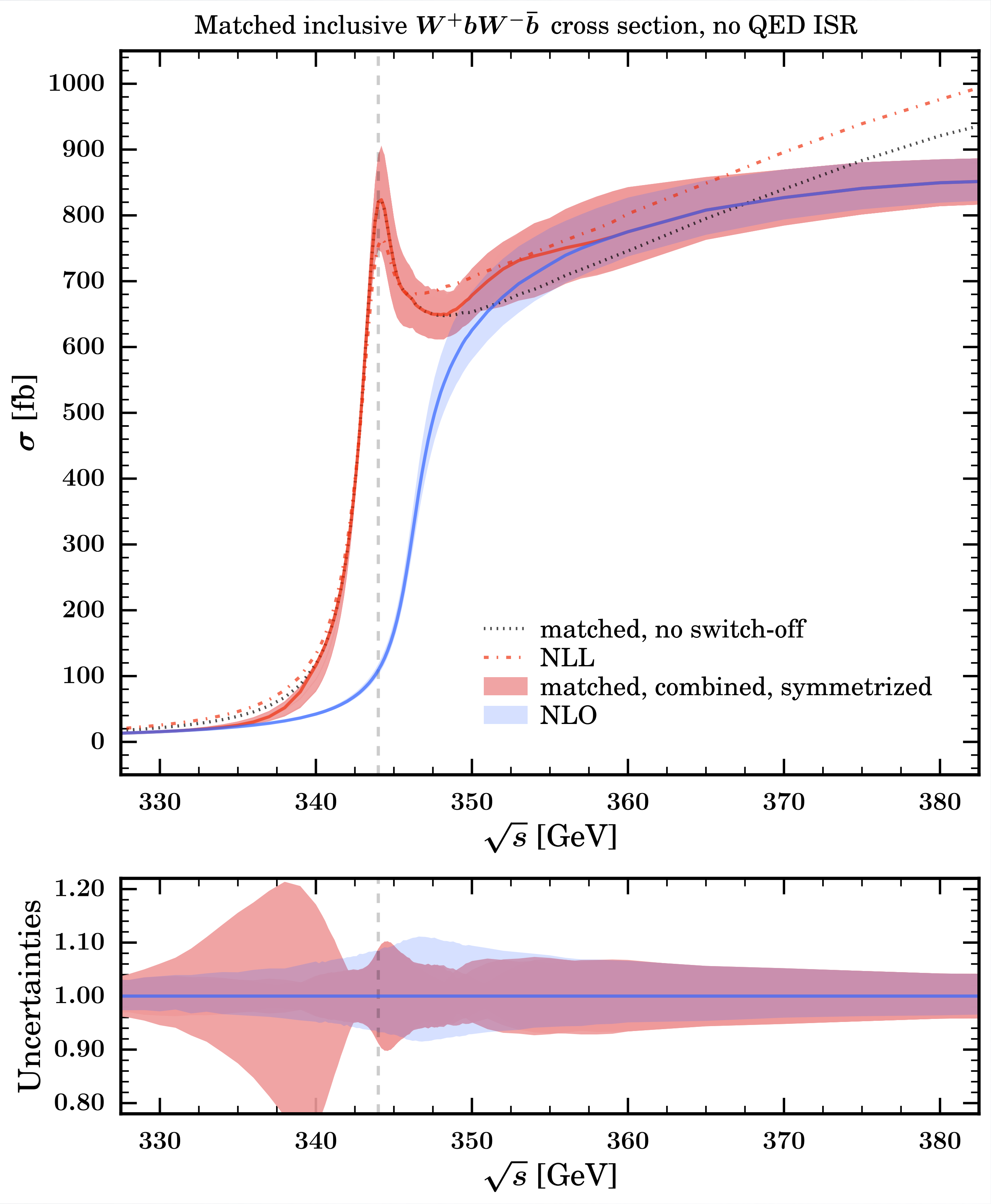




- Only factorized calculation feasible
- NRQCD result technically demanding
- Remove double-counting between NRQCD-NLL & QCD-NLO

Bach/Chokoufé/Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220

Threshold matching with QED ISR



Bach/Chokoufé/Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220

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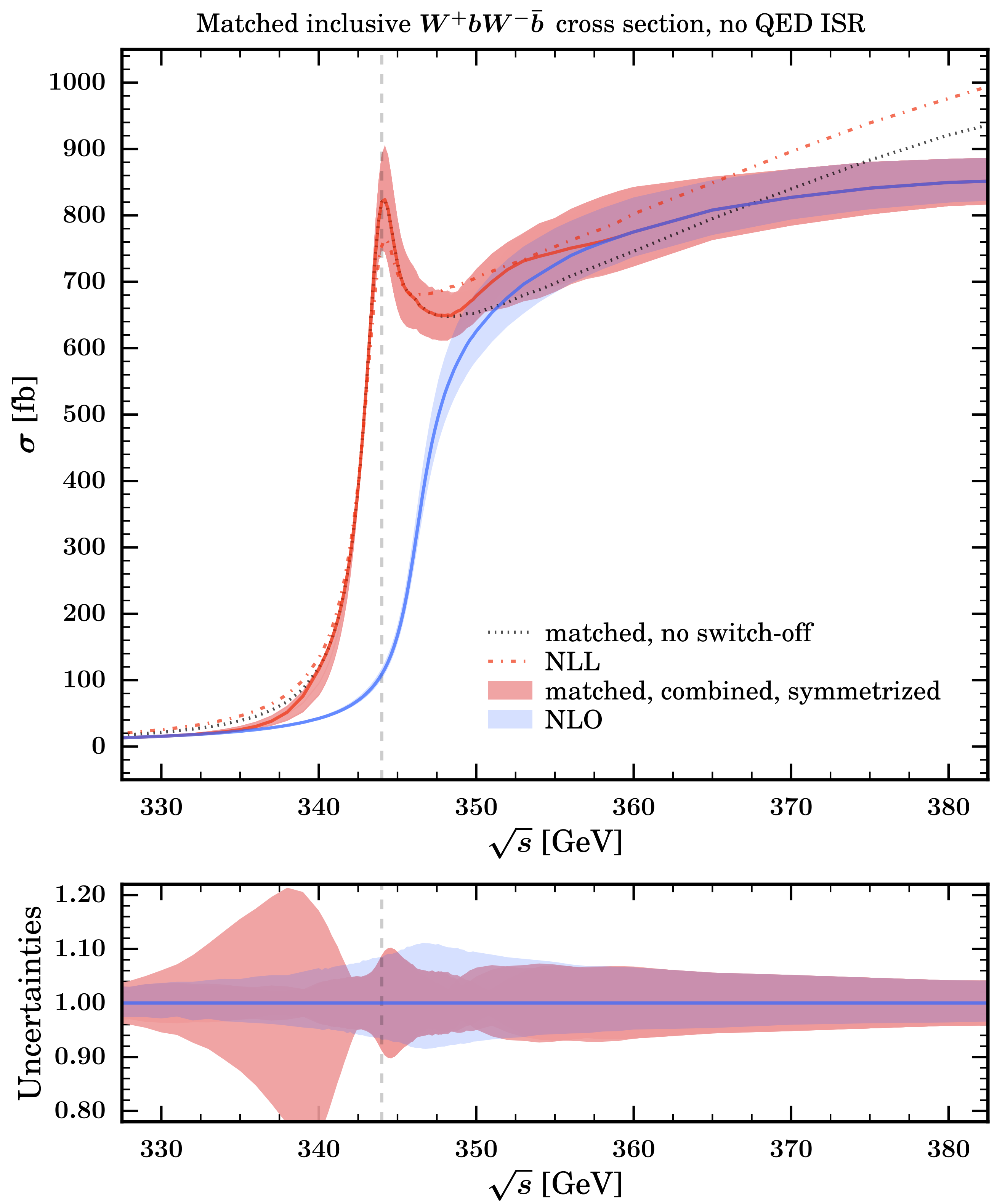
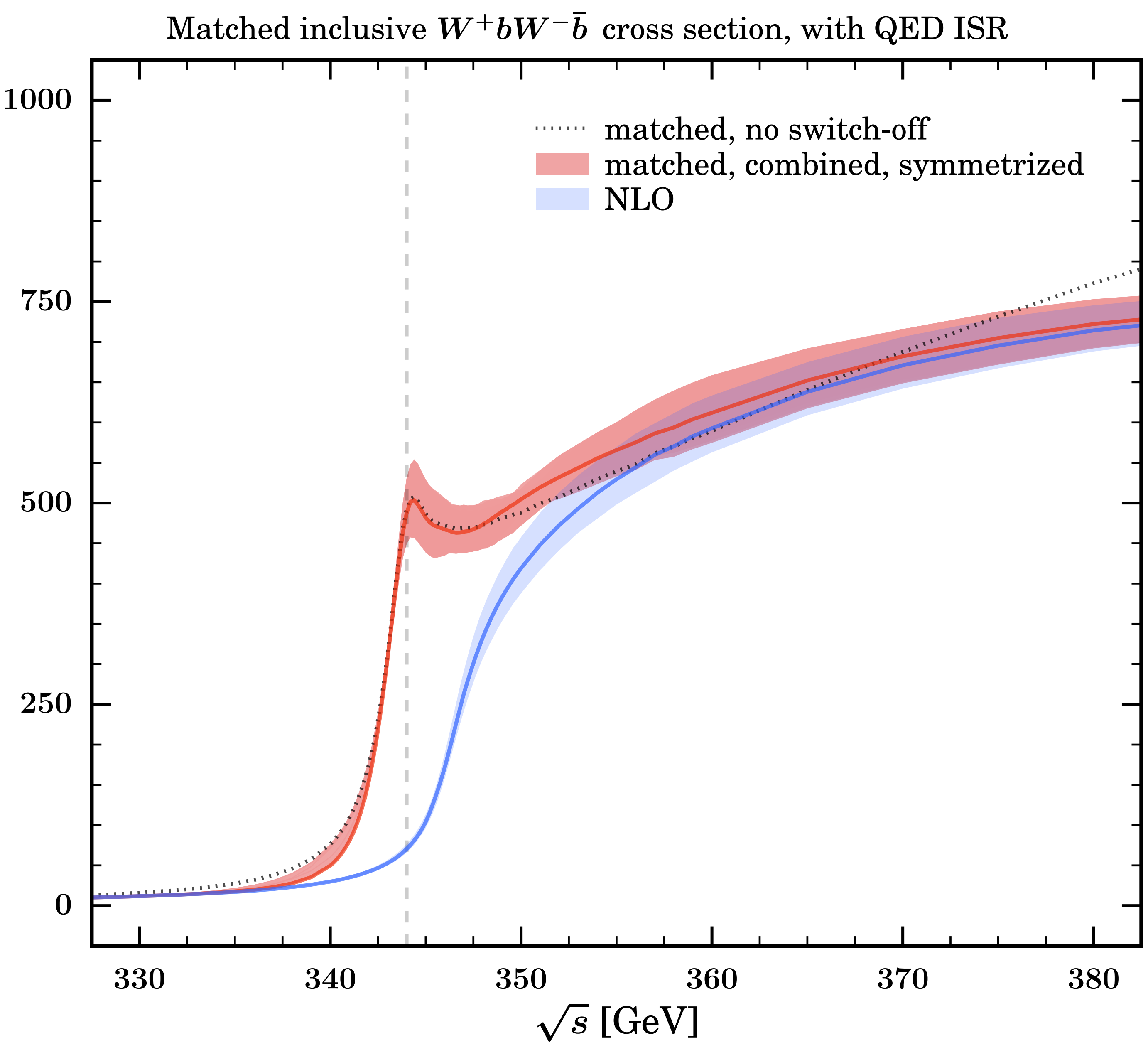
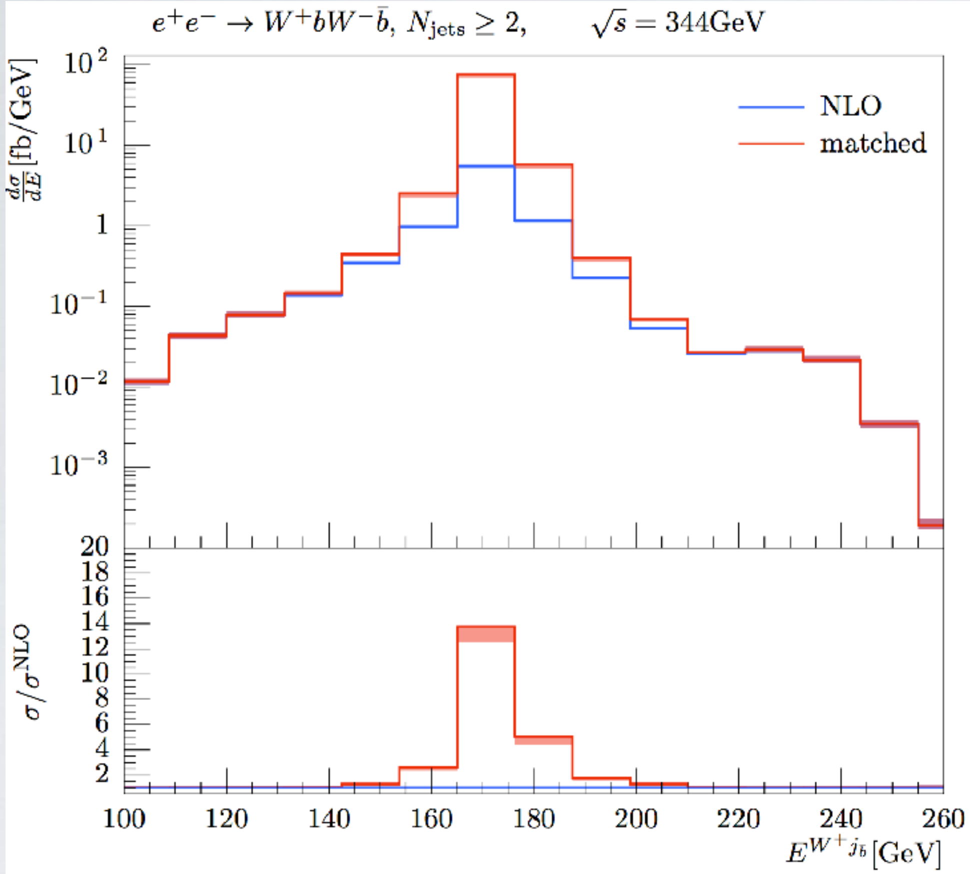
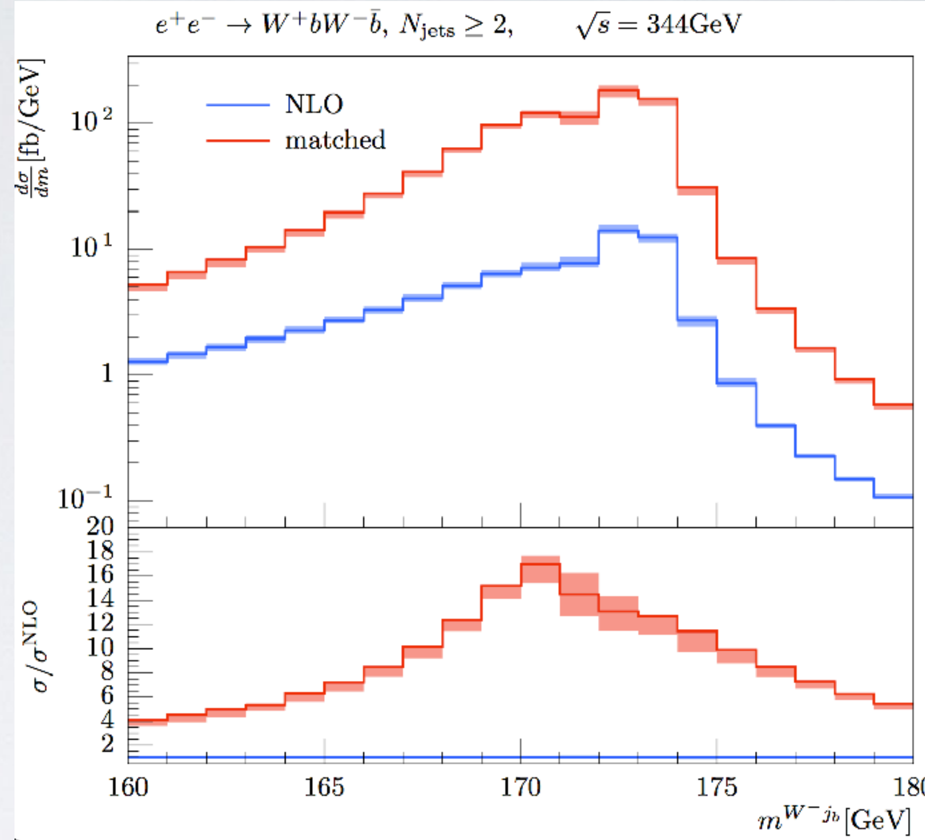


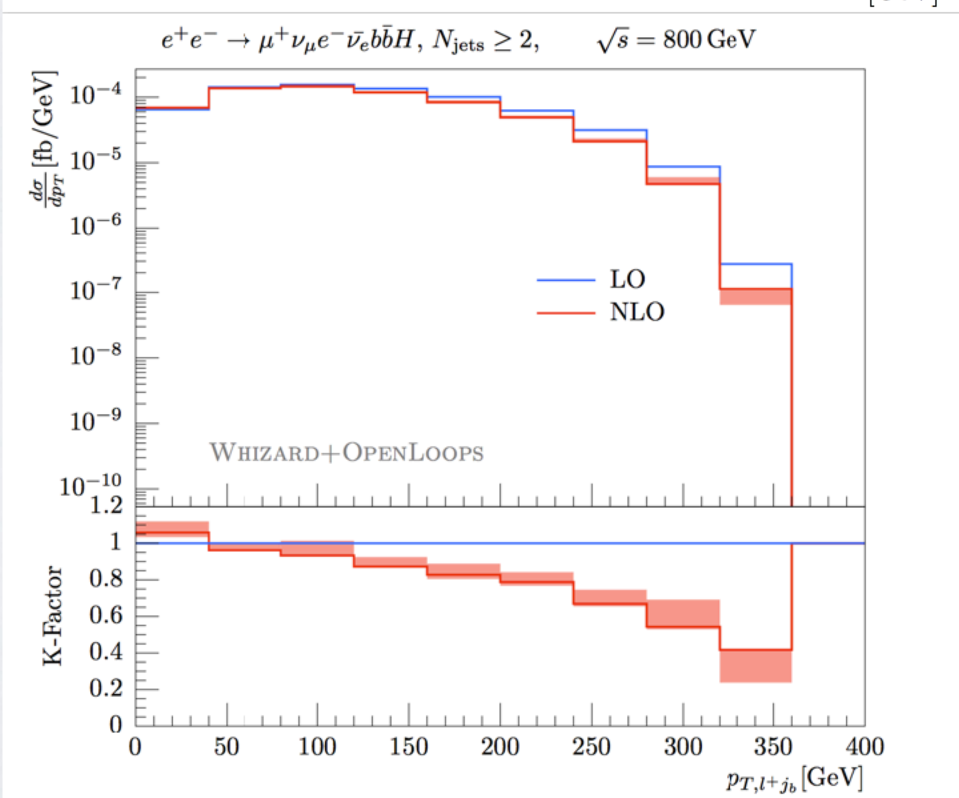


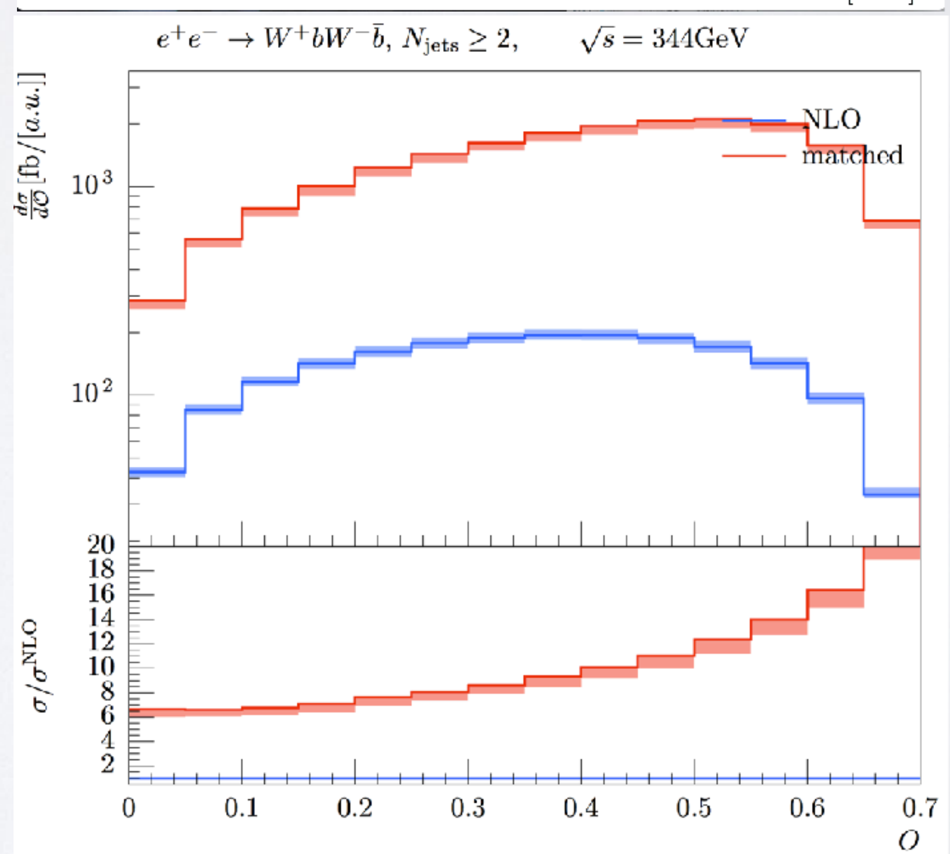
Bach/Chokoufé/Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220

Matched threshold differential distributions









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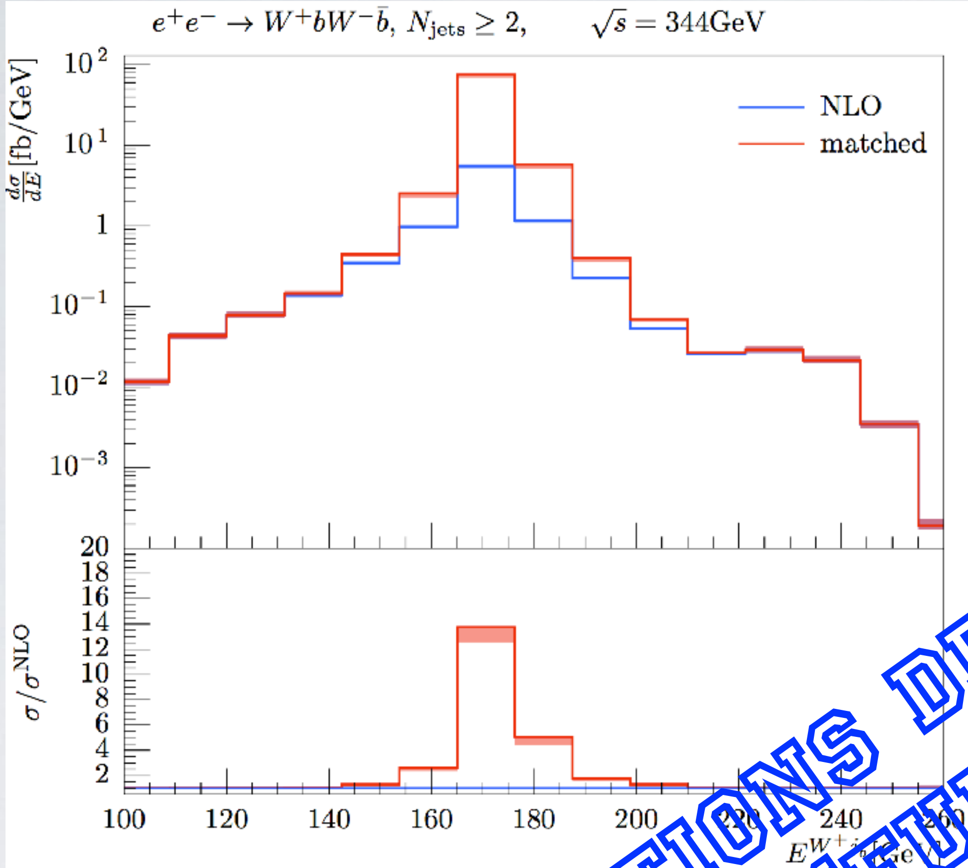
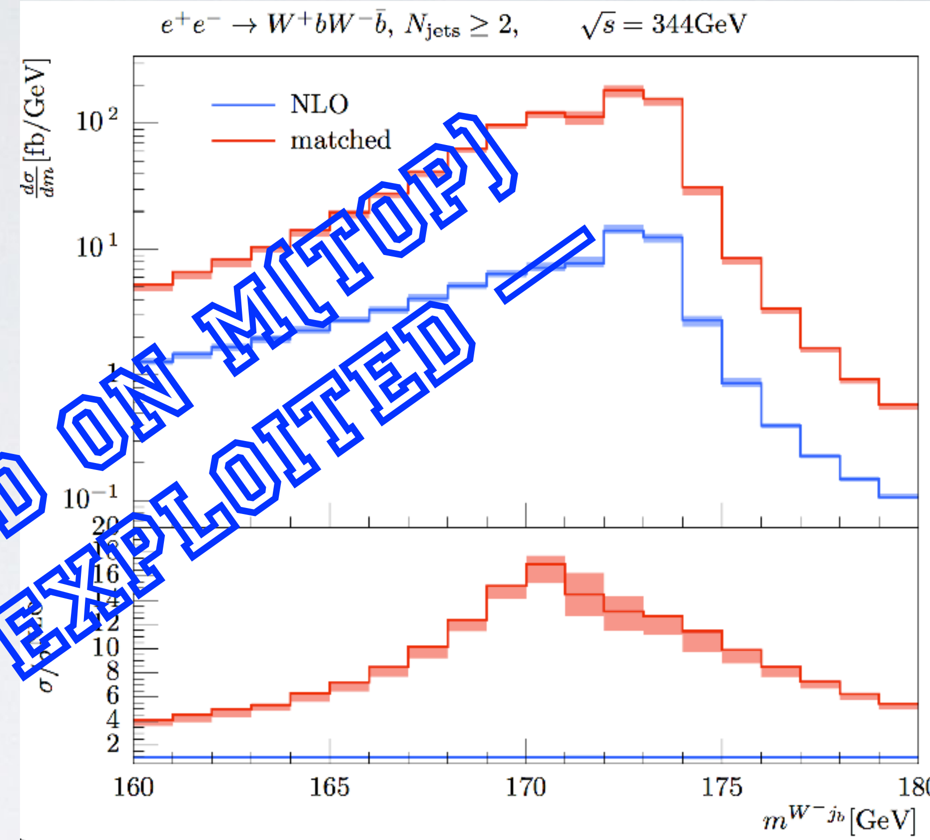
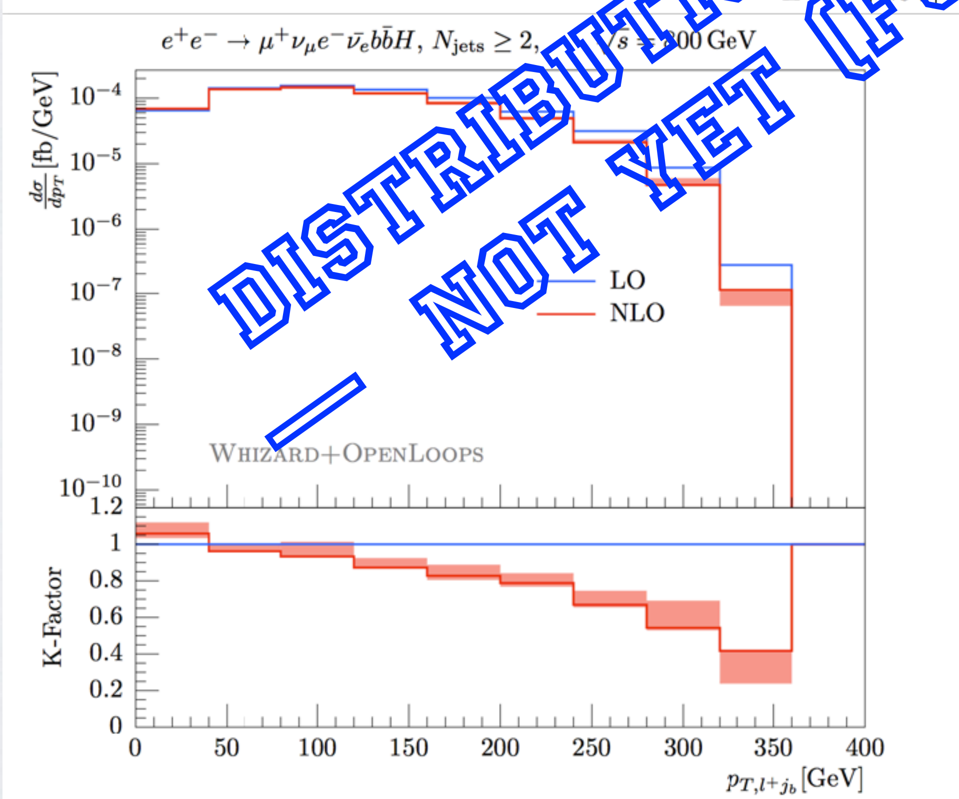
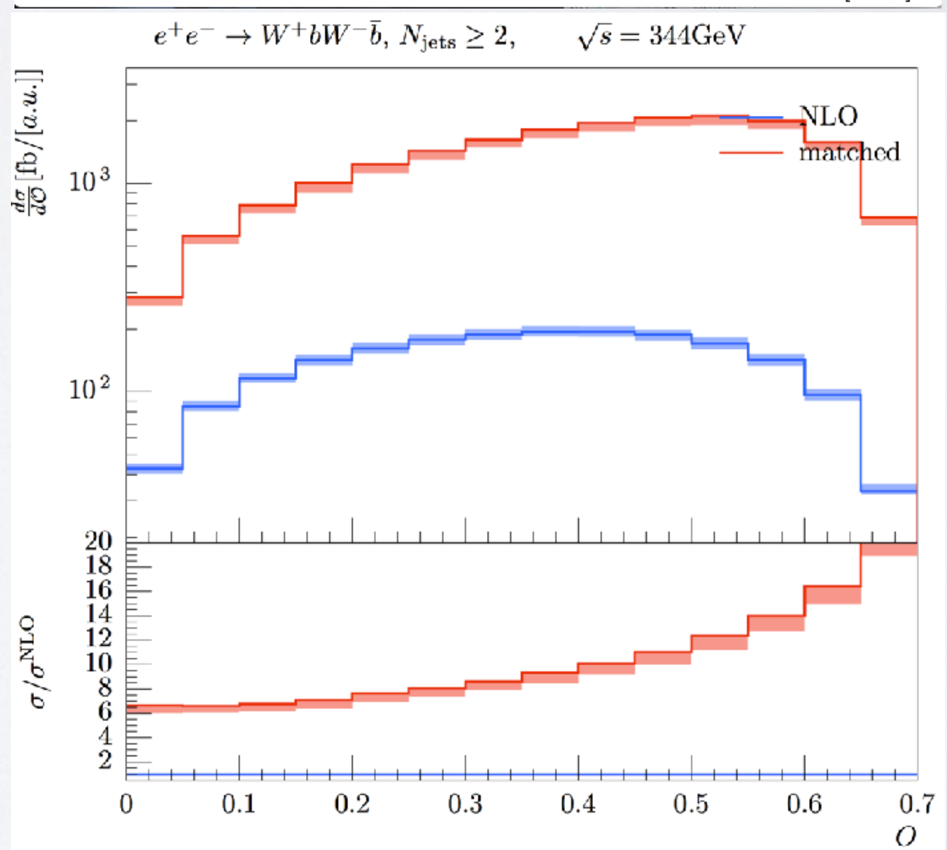






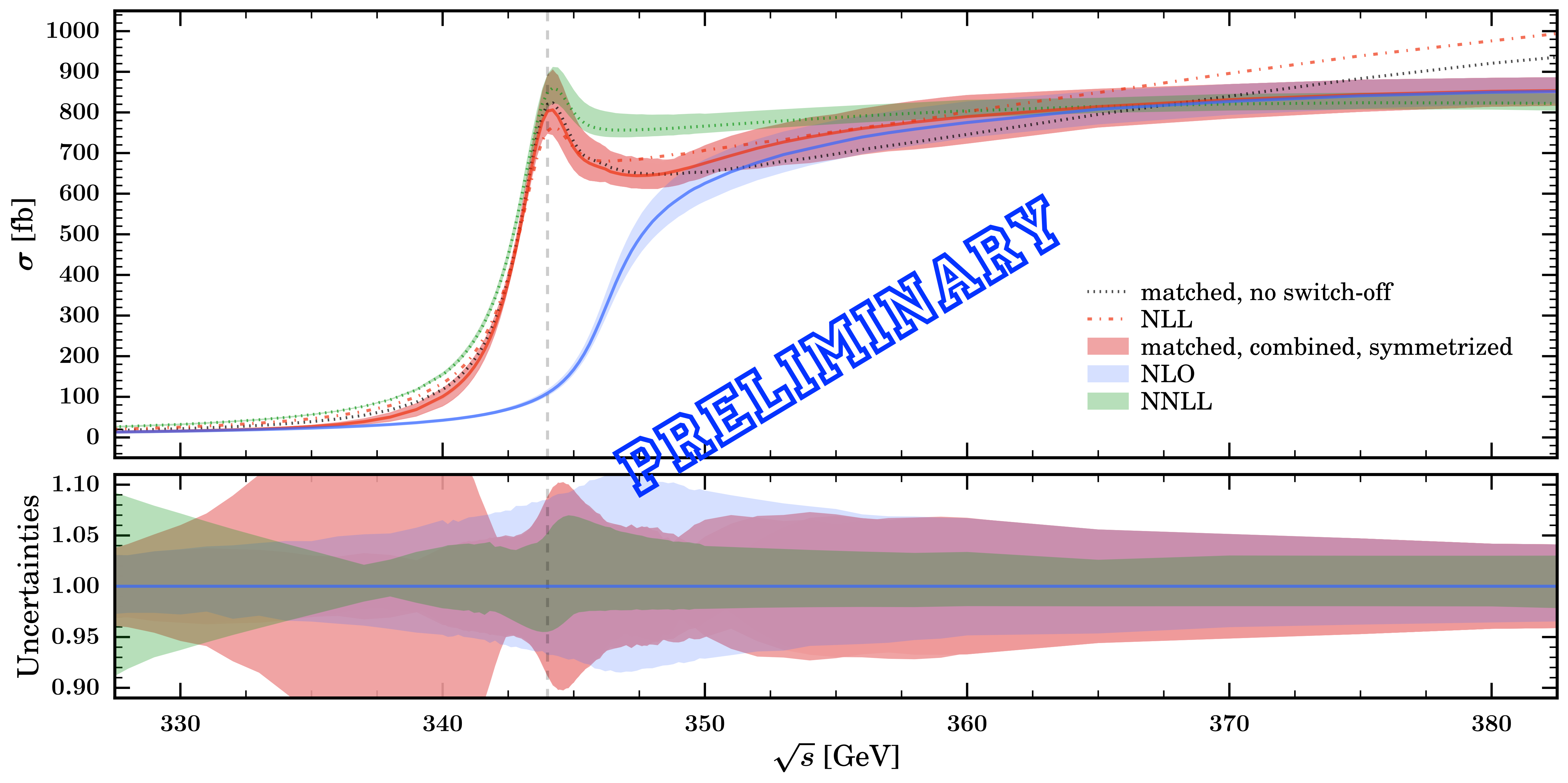


DISTRIBUTIONS DEPEND ON M(TOP) — NOT YET (FULLY) EXPLOITED —

Challenges for the off-shell threshold …

Next steps: higher QCD order, EW corrections (ISR matching!!), soft gluons … … …

$e^+e^- \to W^+bW^-\bar{b}$



Bach/Chokoufé/Hoang/Kilian/JRR/Stahlhofen/Teubner/Weiss, 1712.02220

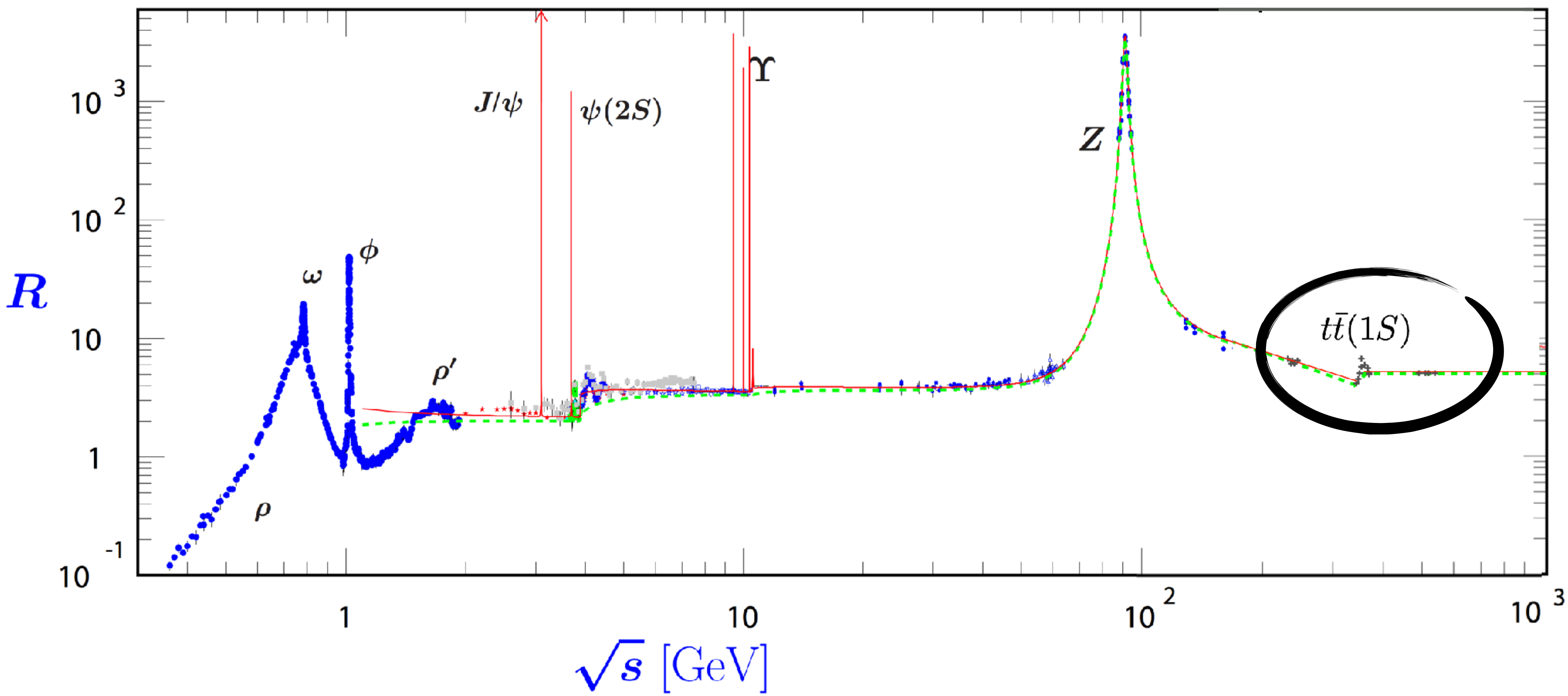
Summary & Outlook

- ☑ Top physics is cornerstone of future lepton collider program
- ☑ Leptonic top and associated Higgs fully off-shell at NLO QCD
- ☑ Inclusive processes: off-shell background grows with energy
- ☑ Complete NRQCD threshold / NLO continuum matching
- ☑ Can be reweighted to NNNLO QCD accuracy at threshold
- ☑ Offers framework for new differential top mass measurements
- ☑ Radiative return to the threshold at 365-380 GeV

Top Theorists To-Do List

- EW corrections (off-shell), effects of QED FSR
- Top threshold matched with EW corrections
- *ttH* threshold matching
- Systematic study of parton shower, logs, soft QCD effects
- … unforeseen to-dos …

Outlook to PDG 203x:



BACKUP

Resonance mappings for NLO processes

- Amplitudes (except for pure QCD/QED) contain resonances (Z, W, H, t)
- In general: resonance masses *not* respected by modified kinematics of subtraction terms
- Algorithm to include resonance histories

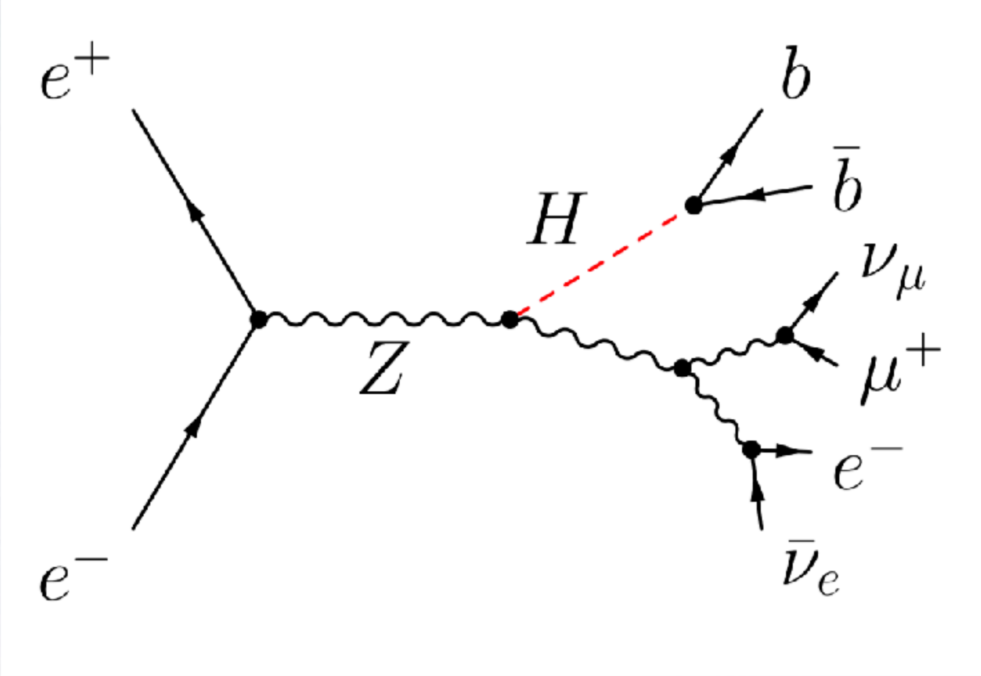
  [Ježo/Nason, 1509.09071]
- Most important for narrow resonances ($H \to bb$)
- Additional soft mismatch integration component

▸ $D_H^{\text{Born}} = \left[\left(\bar{p}_{bb}^2 - m_H^2\right)^2 + m_H^2 \Gamma_H^2\right]^{-1},$

▸ $D_H^{\text{Real}} = \left[\left(p_{bbg}^2 - m_H^2\right)^2 + m_H^2 \Gamma_H^2\right]^{-1}$

$$p_{bbg}^2 = \bar{p}_{bb}^2 + \Delta_{bbg}^2$$

$$\frac{D_H^{\text{Born}}}{D_H^{\text{Real}}} \overset{\bar{p}_{bb}^2 \to m_H^2}{=} 1 + \frac{\Delta_{bbg}^4}{m_H^2 \Gamma_H^2}$$



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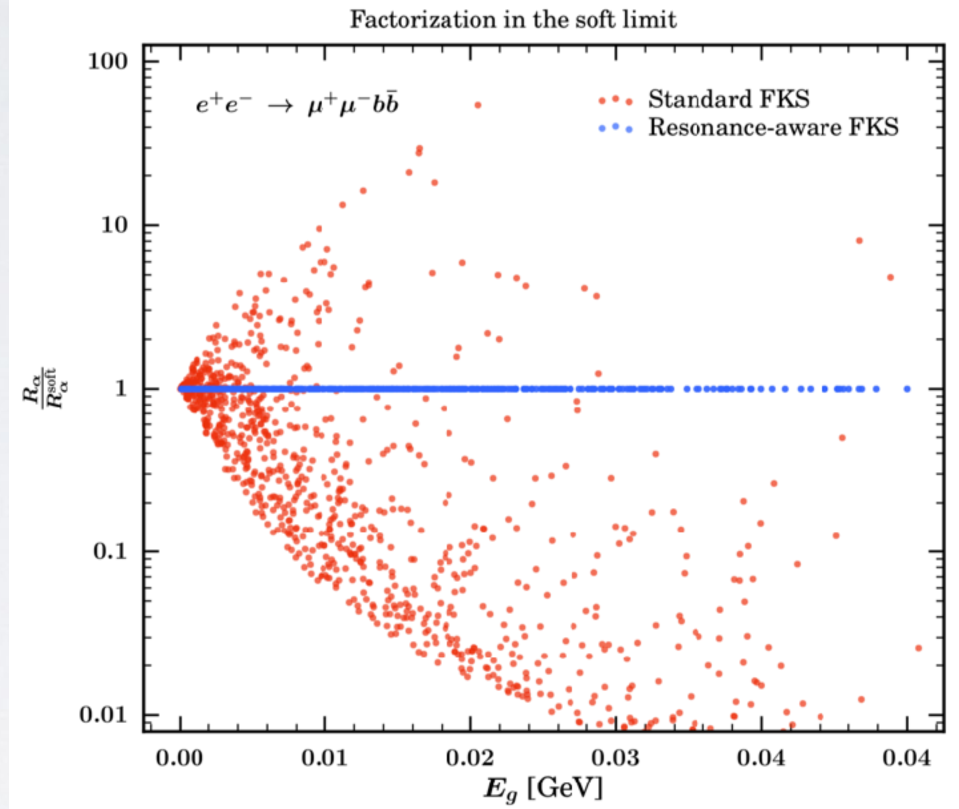

Resonance mappings for NLO processes

- Amplitudes (except for pure QCD/QED) contain resonances (*Z, W, H, t*)
- In general: resonance masses *not* respected by modified kinematics of subtraction terms
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  [Ježo/Nason, 1509.09071]
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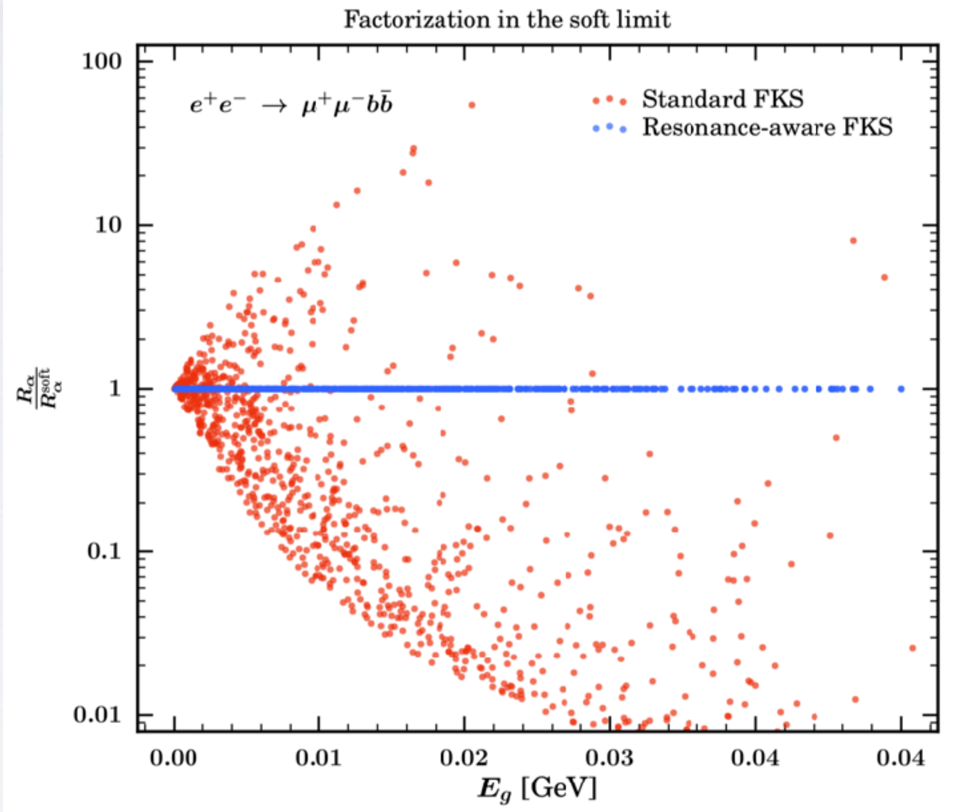

- WHIZARD complete automatic implementation: example $e^+ e^- \to \mu\mu bb$ (*ZZ, ZH* histories)

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	9.6811847E+00	6.42E+00	66.30	72.60*	0.65		
2	11959	2.8539703E+00	2.35E-01	8.25	9.02*	0.69		
3	11936	2.4907574E+00	6.54E-01	26.25	28.68	0.35		
4	11908	2.7695559E+00	9.67E-01	34.91	38.09	0.30		
5	11874	2.4346151E+00	4.82E-01	19.80	21.57*	0.74		
5	59665	2.7539078E+00	1.97E-01	7.15	17.47	0.74	0.49	5

standard FKS

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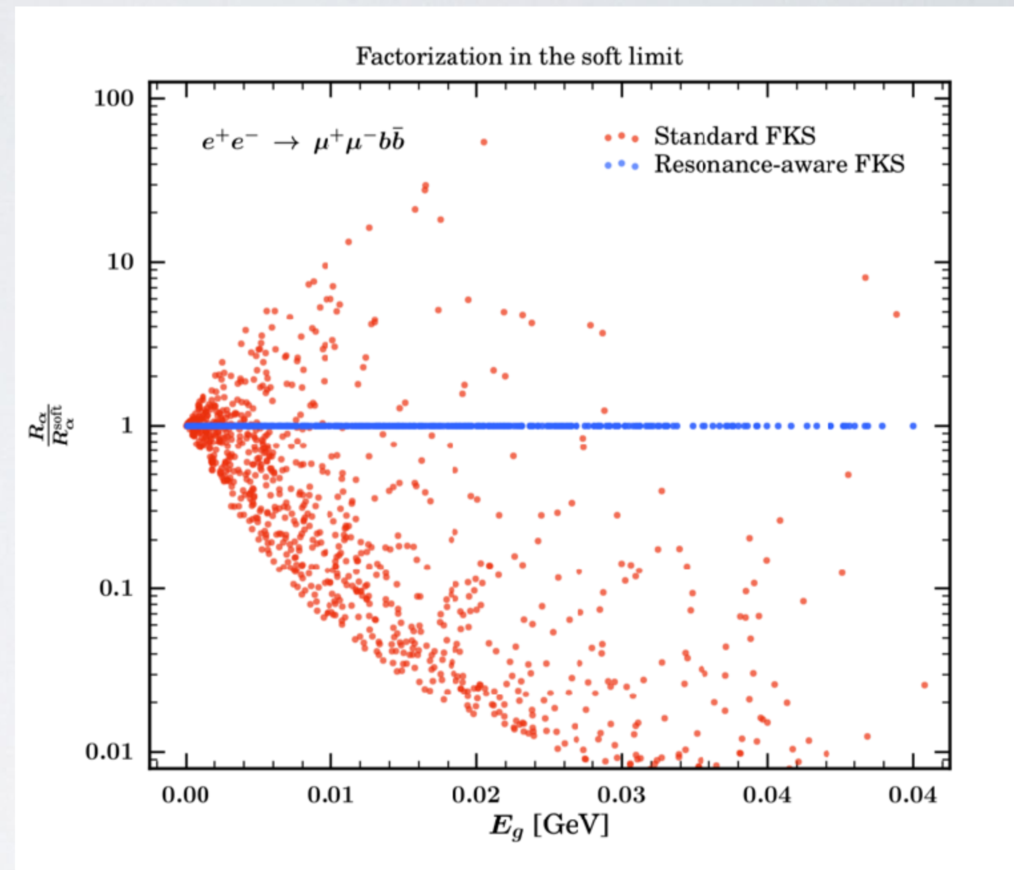

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standard FKS

It	Calls	Integral[fb]	Error[fb]	Err[%]	Acc	Eff[%]	Chi2	N[It]
1	11988	2.9057032E+00	8.35E-02	2.87	3.15*	7.90		
2	11962	2.8591952E+00	5.20E-02	1.82	1.99*	10.91		
3	11936	2.9277880E+00	4.09E-02	1.40	1.52*	14.48		
4	11902	2.8512337E+00	3.98E-02	1.40	1.52*	13.70		
5	11874	2.8855399E+00	3.87E-02	1.34	1.46*	17.15		
5	59662	2.8842006E+00	2.04E-02	0.71	1.72	17.15	0.53	5

FKS with resonance mappings