(Mostly) Model-Independent Searches for New Physics in Vector Boson Scattering

Jürgen R. Reuter, DESY

based on work with A. Alboteanu, W. Kilian, T. Ohl, M. Sekulla

PRD93(16),3. 036004 [1511.00022], PRD91(15),096007 [1408.6207],
US Snowmass Summer Study 1310.6708, 1309.7890, 1307.8180,
JHEP 0811.010 [0806.4145], EPJC48(06)353 [hep-ph/0604048]

Why [EW] vector boson scattering ?

- **Discovery of a light Higgs boson leaves still open questions:**

1. Nature of Electroweak Symmetry Breaking
2. Does the H(125) fulfill the US-fermion/Europe-boson rule?
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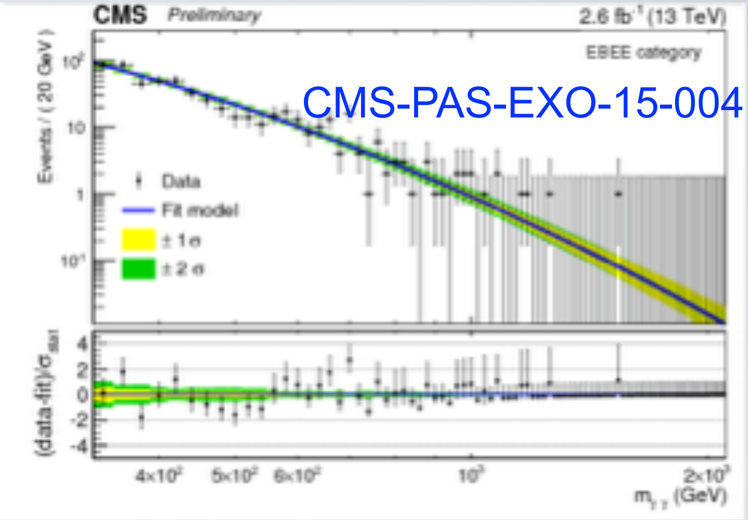

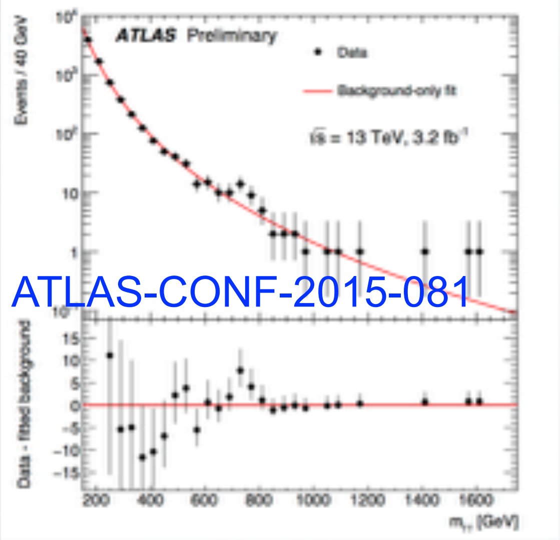

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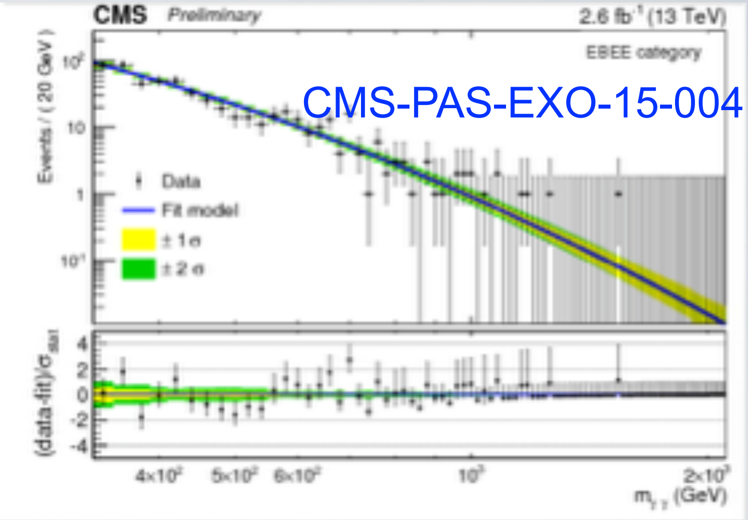

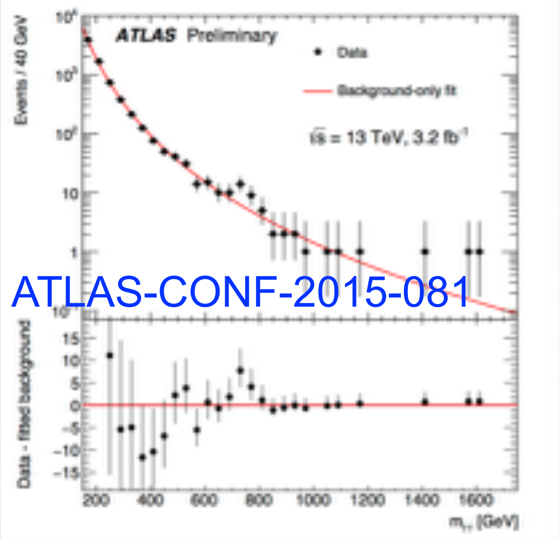

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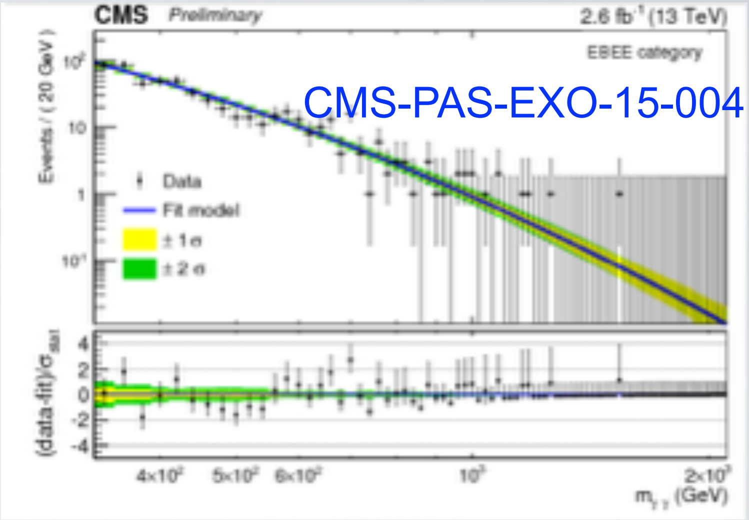

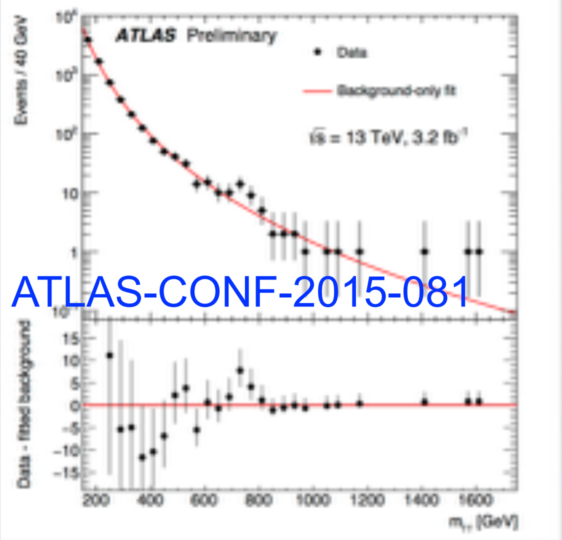

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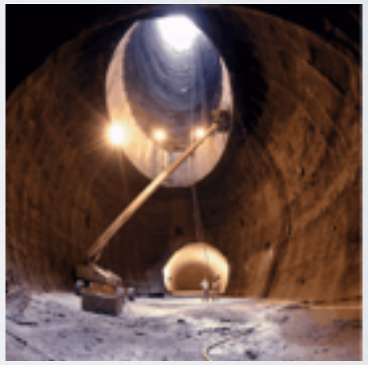

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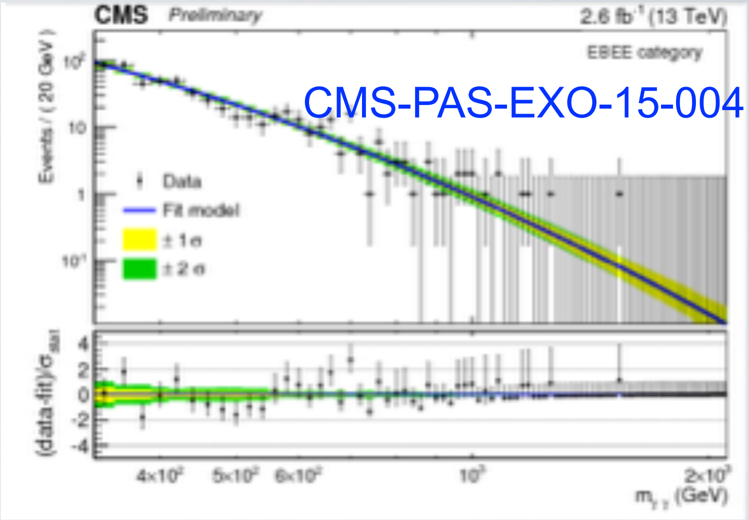

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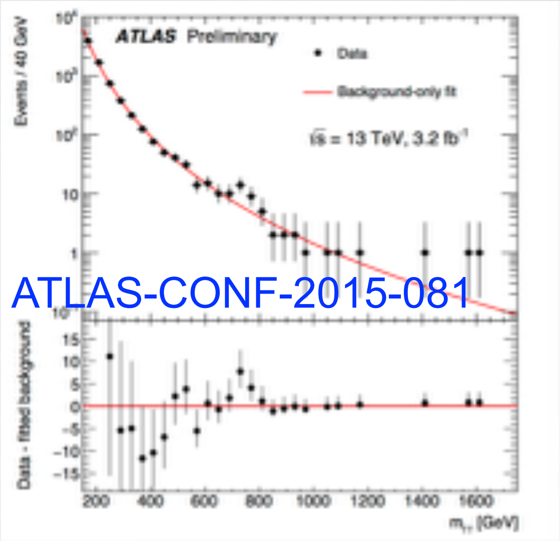

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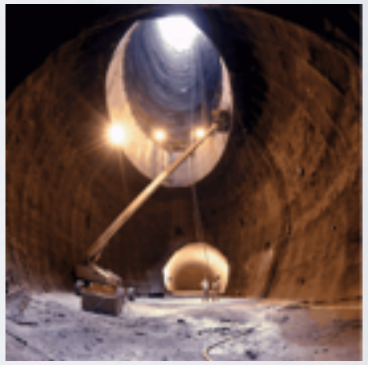

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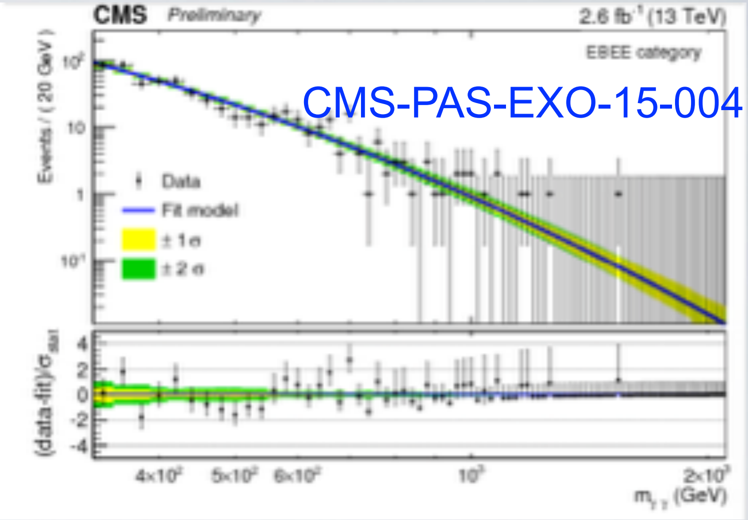

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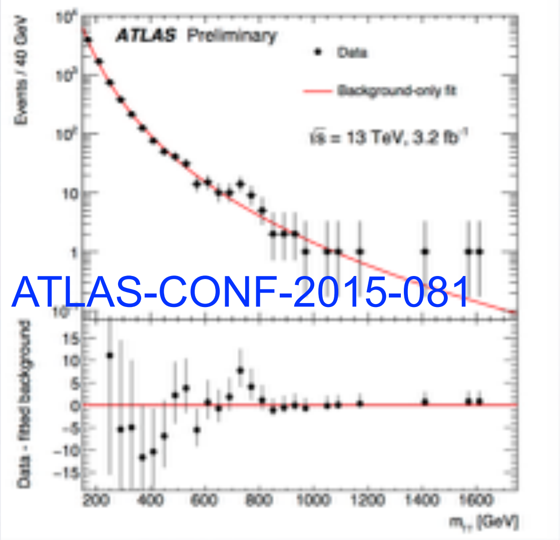

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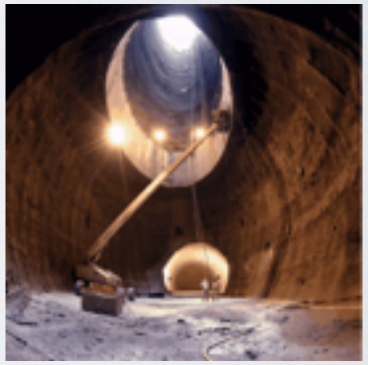

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Exploration of E-frontier → look for heavy objects, including high-mass $V_L V_L$ scattering:
❑ requires as much integrated luminosity as possible (cross-section goes like 1/s)

F. Gianotti, 01/2014

Anatomy of Vector Boson Scattering (VBS)

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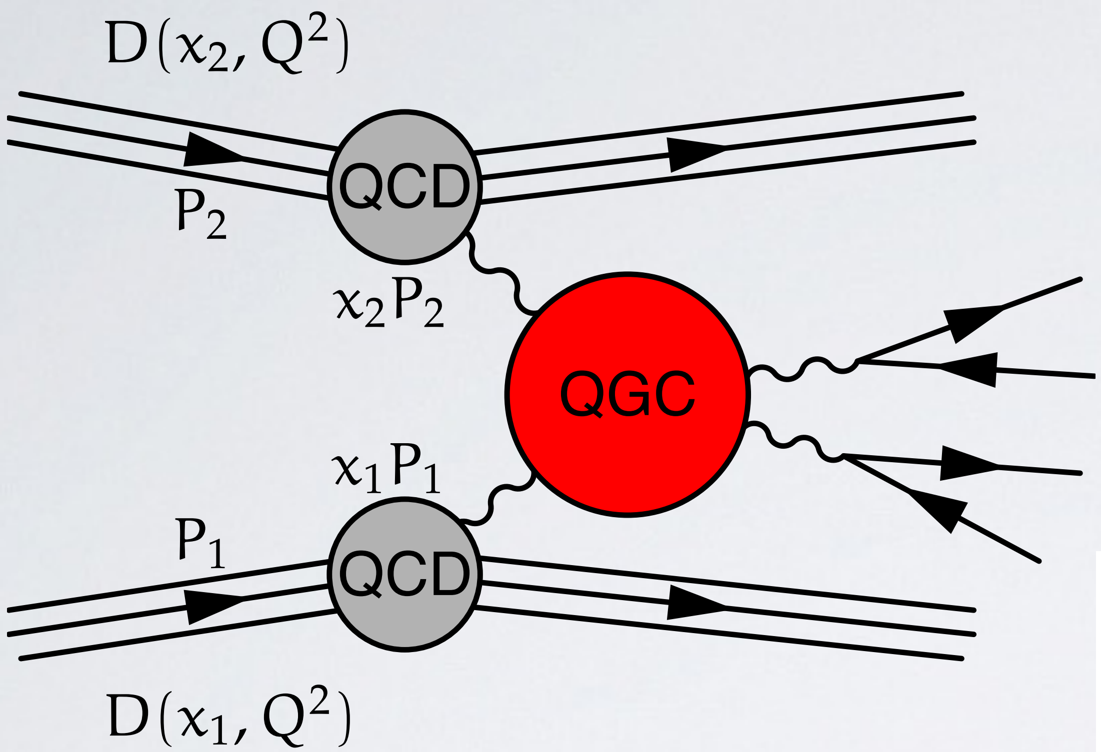
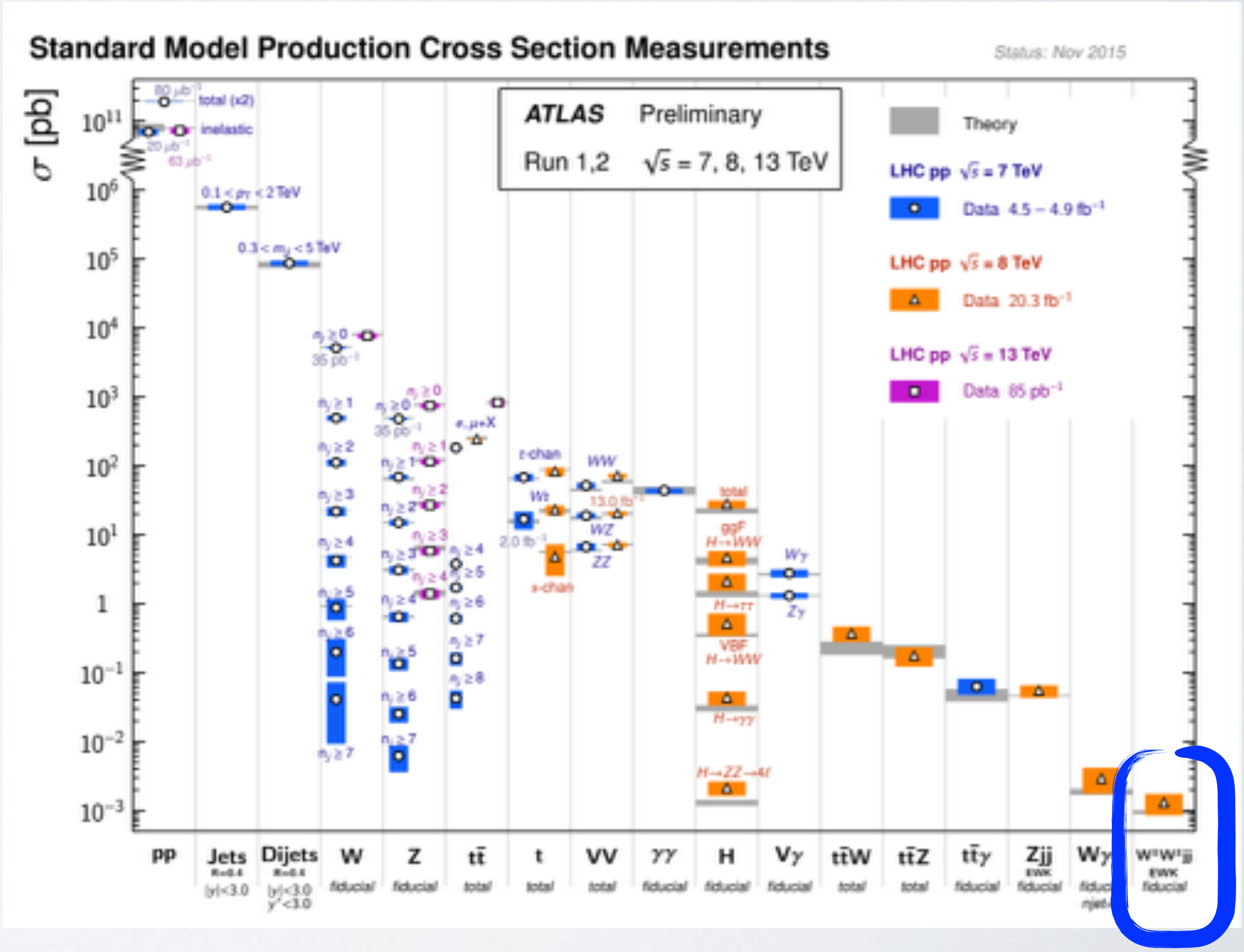


Backgrounds [+ $V_T V_T$ bkgd.]:

- $tt \to WbWb$
- W + *jets*
- single top, misreconstructed jet
- *WWjj* QCD production
- *ll* + X + Emiss ("prompt")



Fiducial phase space volume:

- *lljj* tag
- m_{jj} > 500 GeV ("jet recoil")
- $y_{j,1} \cdot y_{j,2} < 0$ ("collinear beams")
- $|\Delta y_{jj}| > 2.4$ ("rapidity distance")
- Cuts on E_j , p_T^j
- No mini jet vetoes

LHC Run I: First time evidence for VBS

- Evidence for W^+W^+jj (electroweak production)

  ATLAS PRL 113(2014)14, 141803 [1405.6241]; CMS PRL 114(2015), 051801 [1410.6315]

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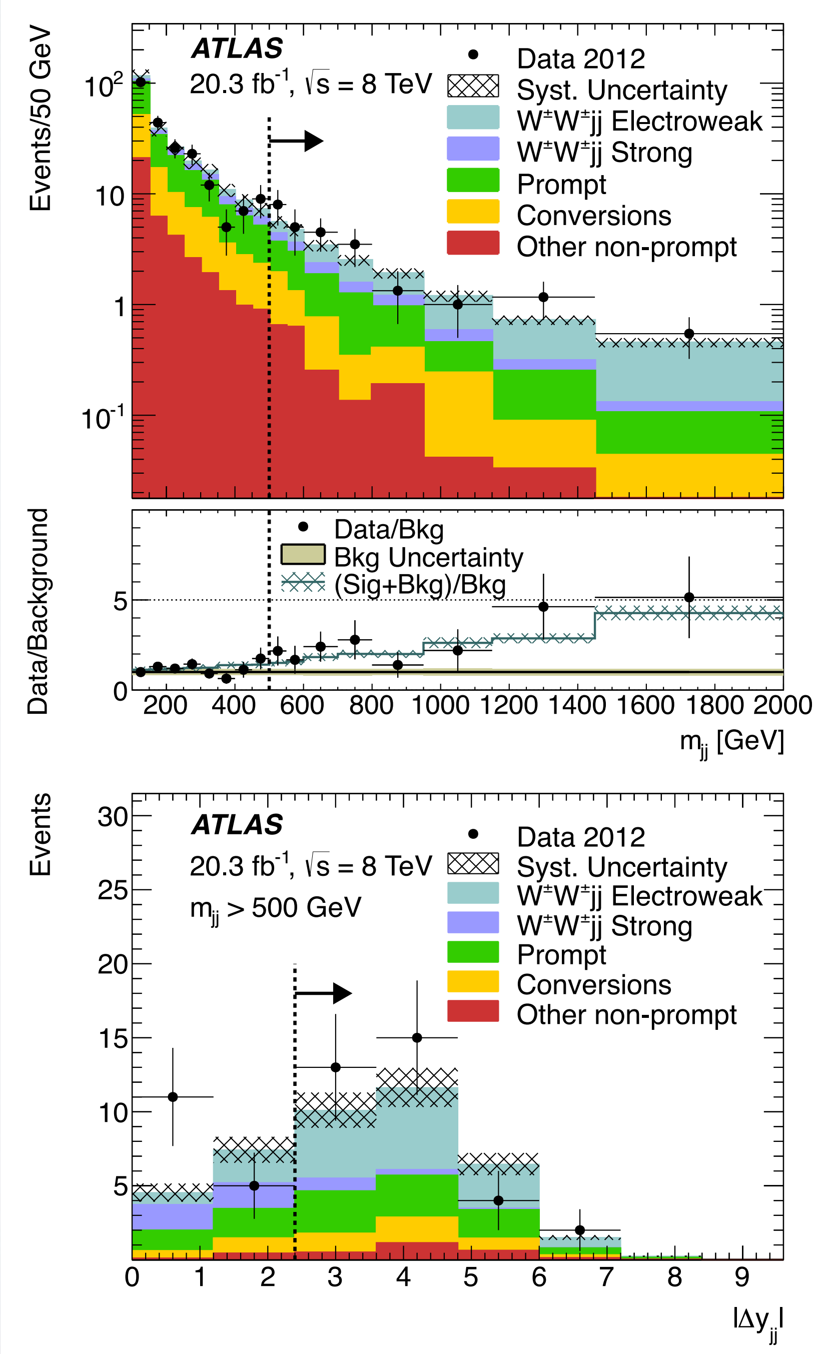

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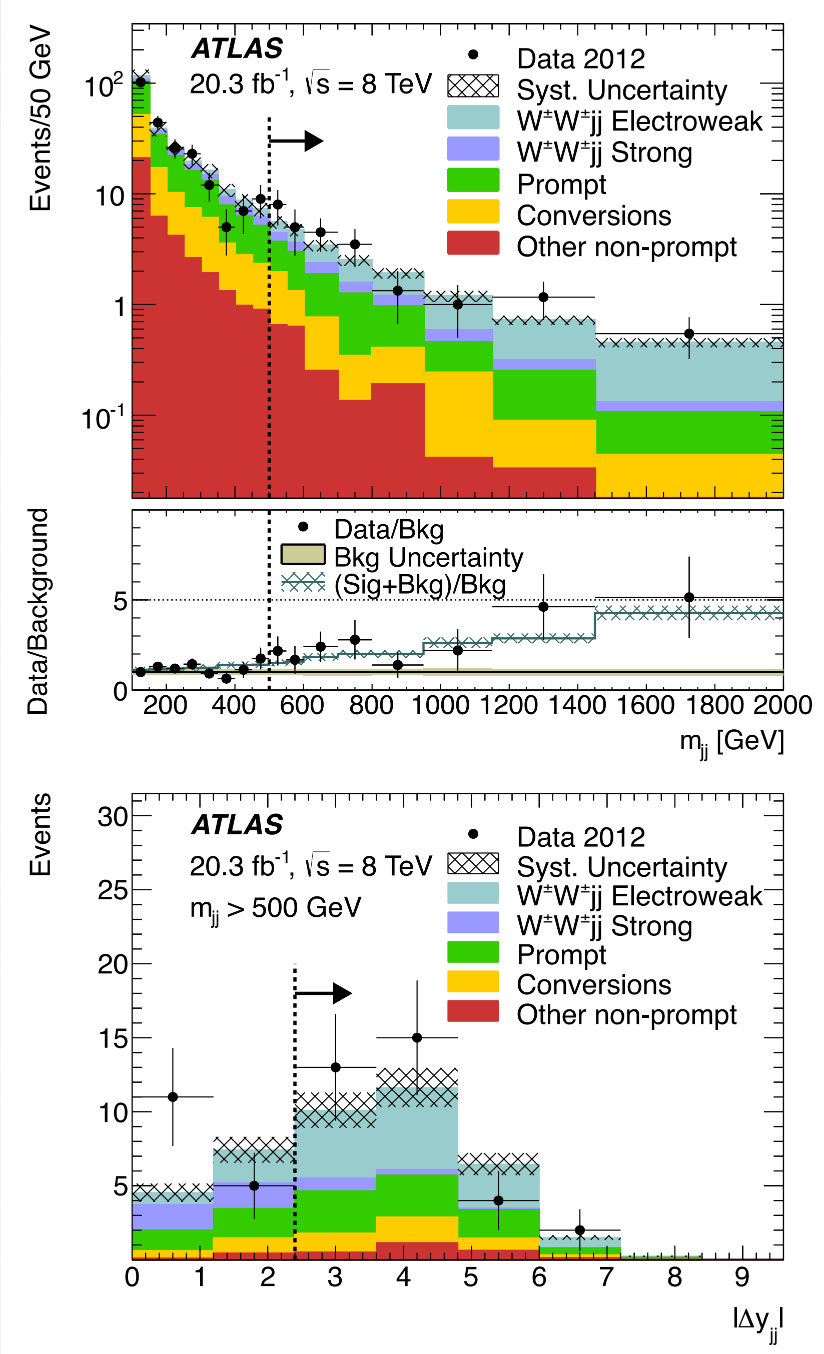

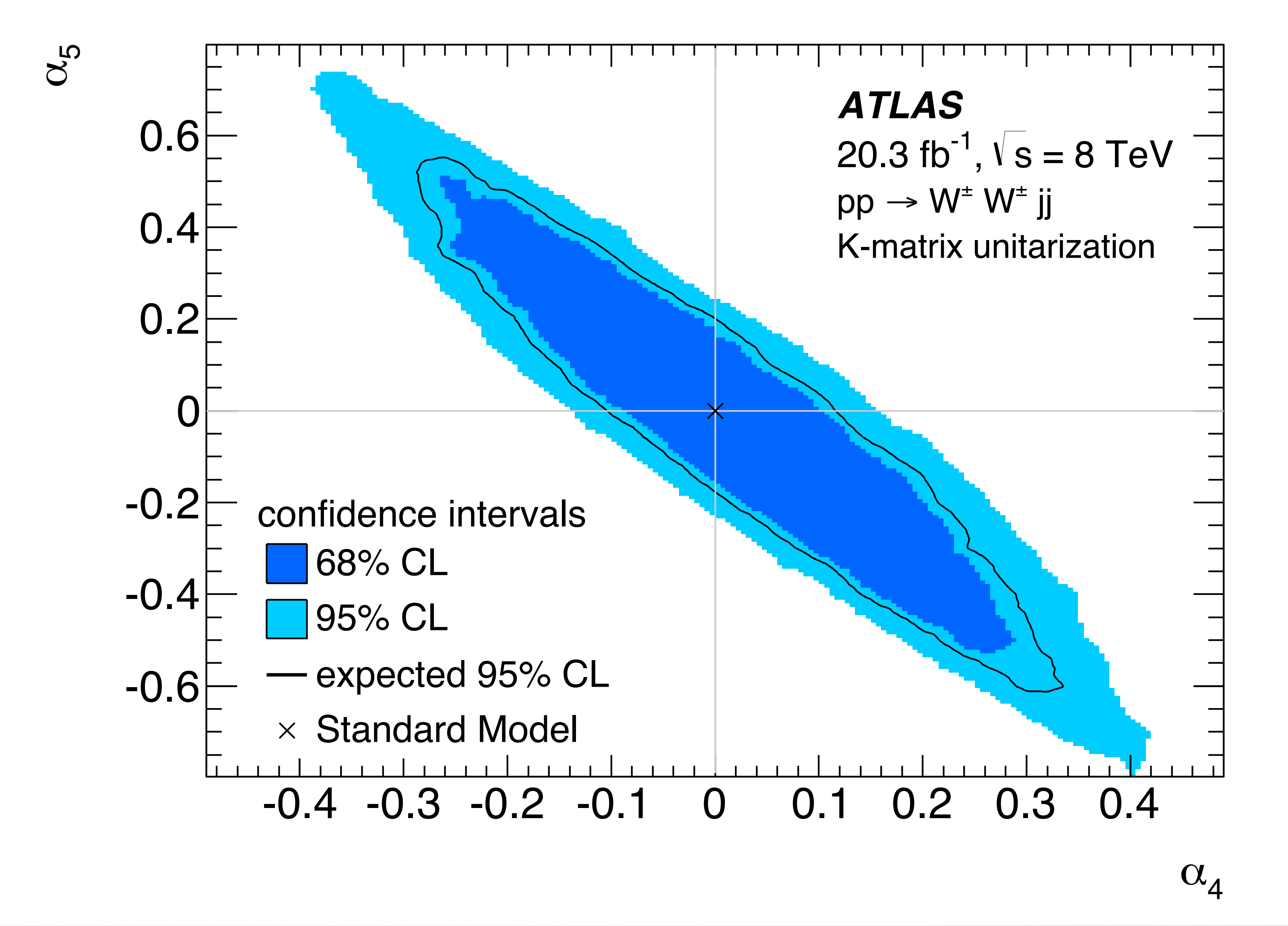

Unitarity in vector boson scattering

Optical Theorem (Unitarity of the S(cattering) Matrix):

$$\sigma_{\text{tot}} = \text{Im}\left[\mathcal{M}_{ii}(t=0)\right]/s \qquad t = -s(1-\cos\theta)/2$$

Partial wave amplitudes:

$$\mathcal{M}(s,t,u) = 32\pi \sum_\ell (2\ell+1)\mathcal{A}_\ell(s) P_\ell(\cos\theta) \qquad \text{("Power spectrum")}$$

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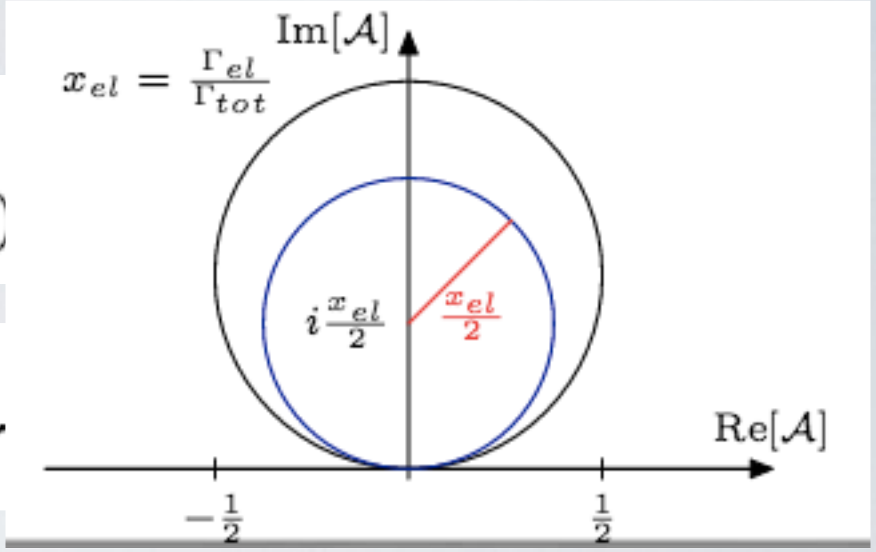

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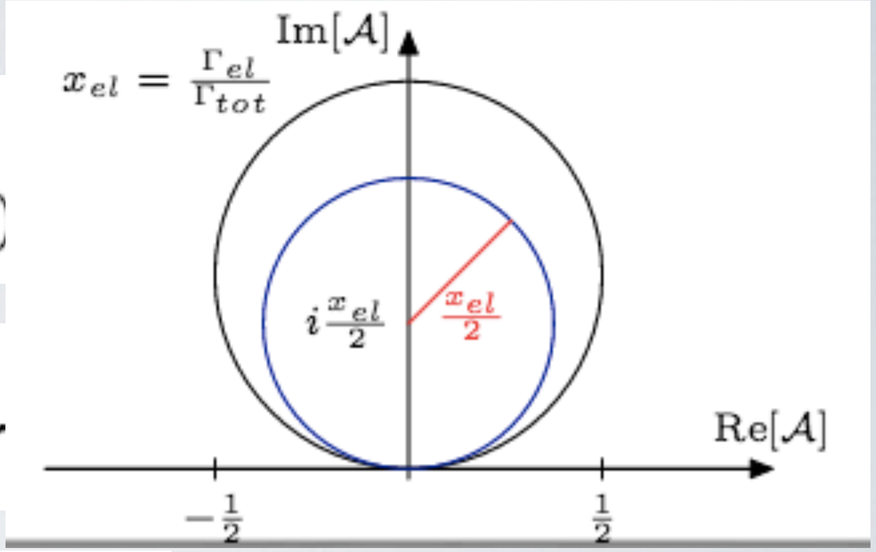

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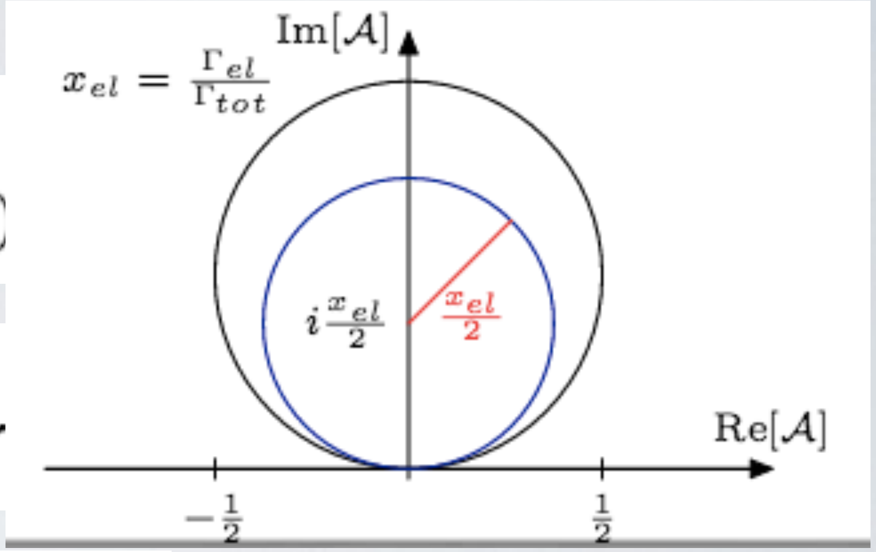

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SM longitudinal isospin eigenamplitudes ($\mathcal{A}_{I,\text{spin}=J}$):

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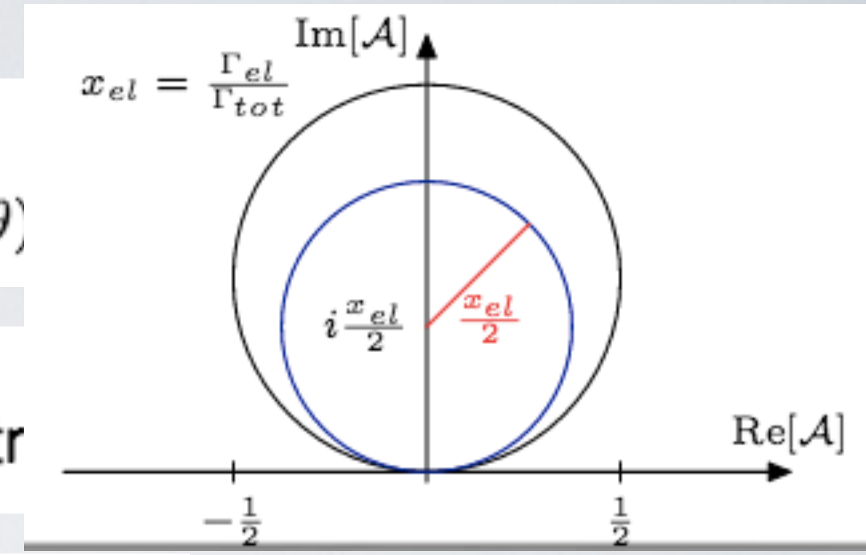

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Lee/Quigg/Thacker, 1973

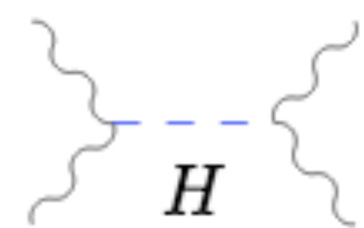
exceeds unitarity bound $|\mathcal{A}_{IJ}| \lesssim \frac{1}{2}$ at:

$$I = 0: \qquad E \sim \sqrt{8\pi}v = 1.2\,\text{TeV}$$

$$I = 1: \qquad E \sim \sqrt{48\pi}v = 3.5\,\text{TeV}$$

$$I = 2: \qquad E \sim \sqrt{16\pi}v = 1.7\,\text{TeV}$$

Higgs exchange:

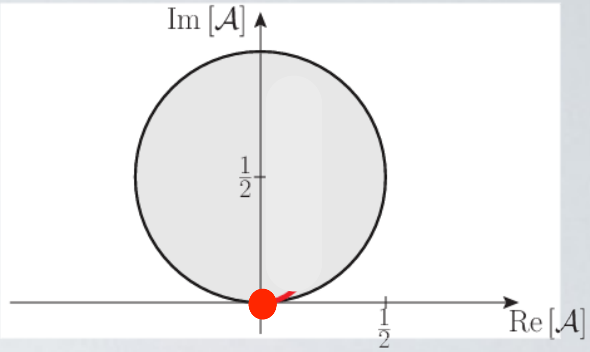


$$\mathcal{A}(s,t,u) = -\frac{M_H^2}{v^2}\frac{s}{s-M_H^2}$$

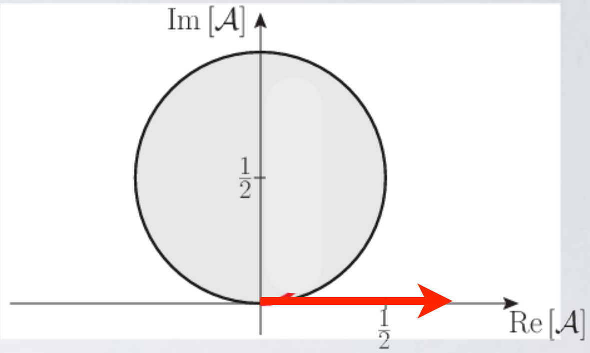
Unitarity: $M_H \lesssim \sqrt{8\pi}v \sim 1.2\,\text{TeV}$

Scenarios for New Physics in VBS

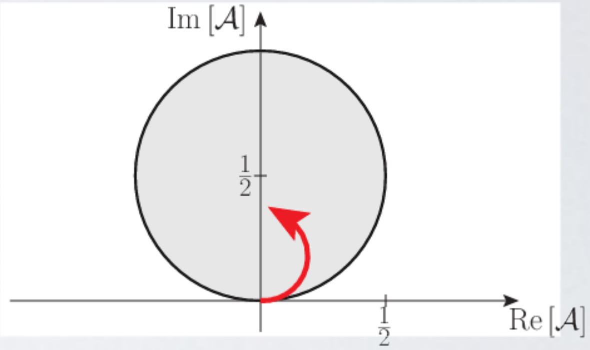
1. SM or weakly coupled physics (e.g. 2HDM):
   amplitude remains close to origin



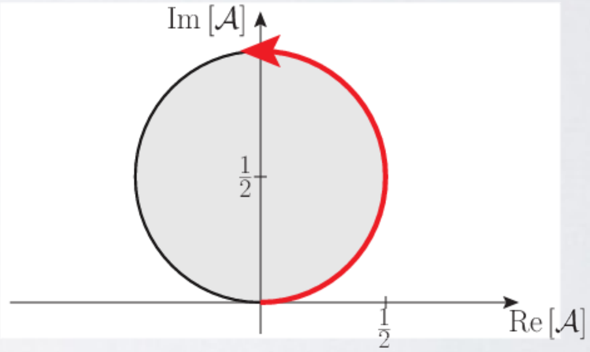
2. Rising amplitude (at least one dim-8 operator): rise beyond unitarity circle [unphys.], strongly interacting regime



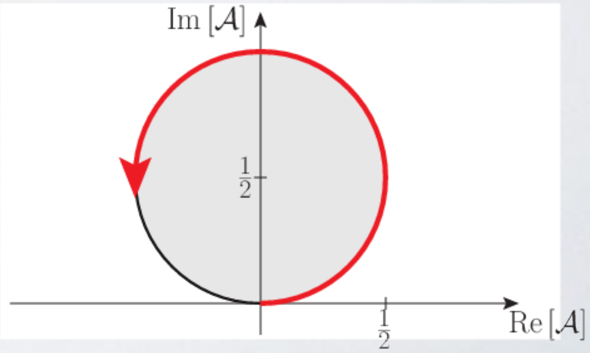
3. Inelastic channel opens (form-factor description): new channels open out, multi-boson final states



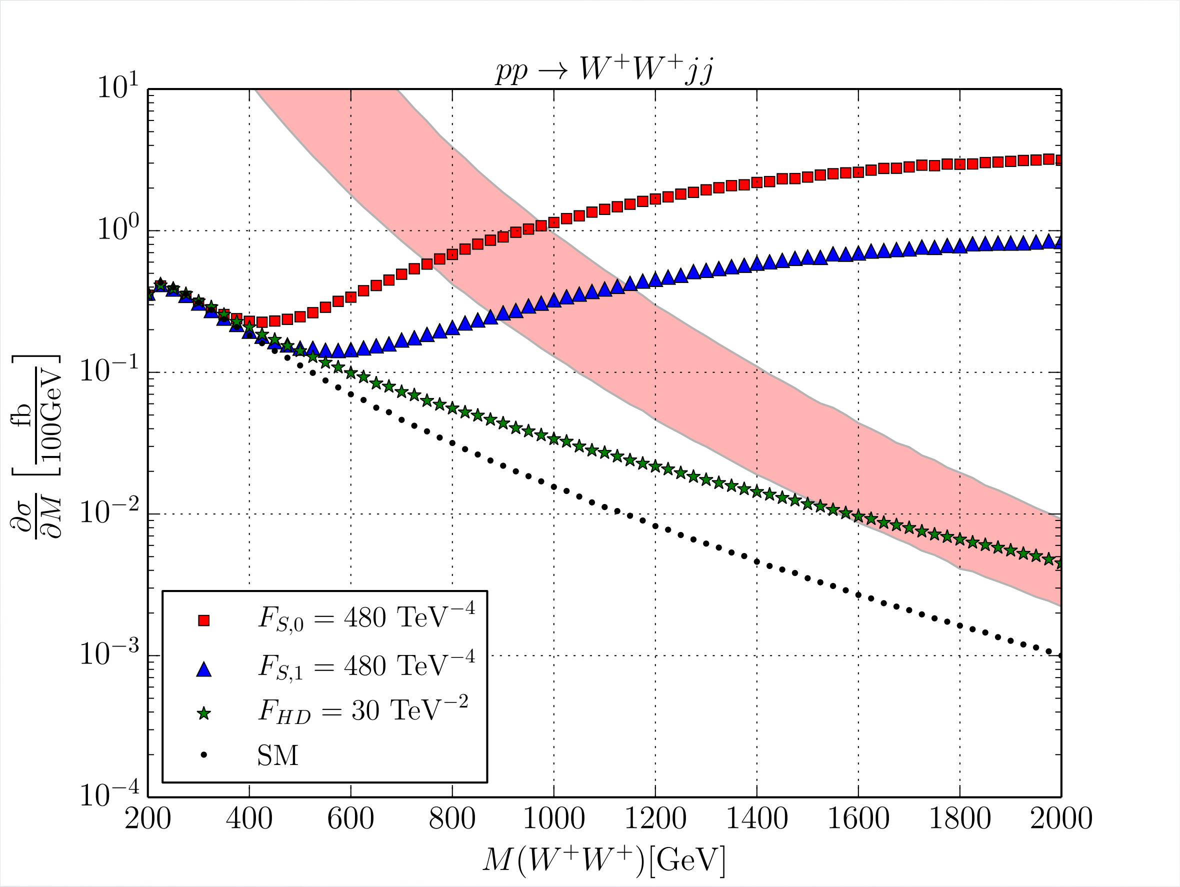
4. Saturation of amplitude: maximal amplitude, strongly interacting continuum, K-/T-matrix unitarization



5. New resonance: amplitude turns over



VBS diboson spectra



General cuts: $M_{jj} > 500\,\text{GeV};\ \Delta\eta_{jj} > 2.4;\ \ p_T^j > 20\,\text{GeV};\ \ |\Delta\eta_j| < 4.5$

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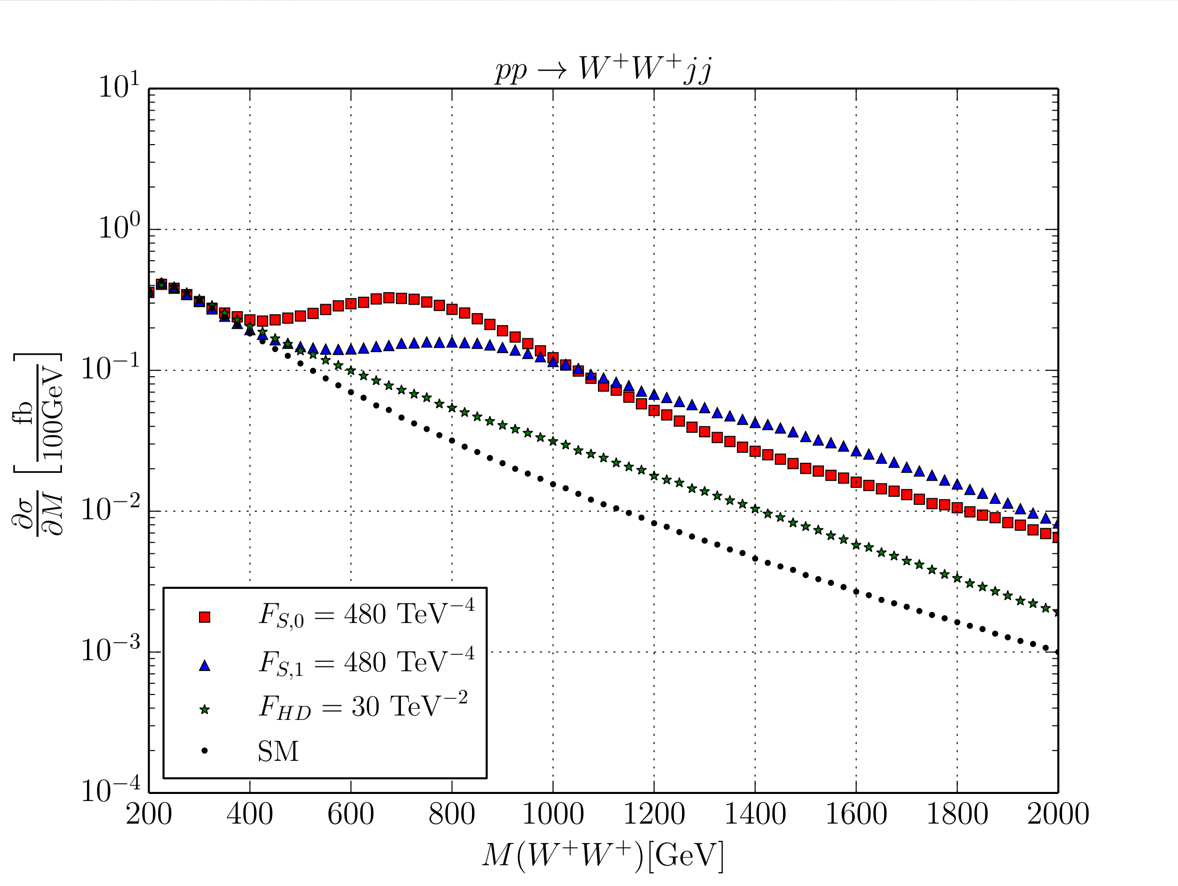

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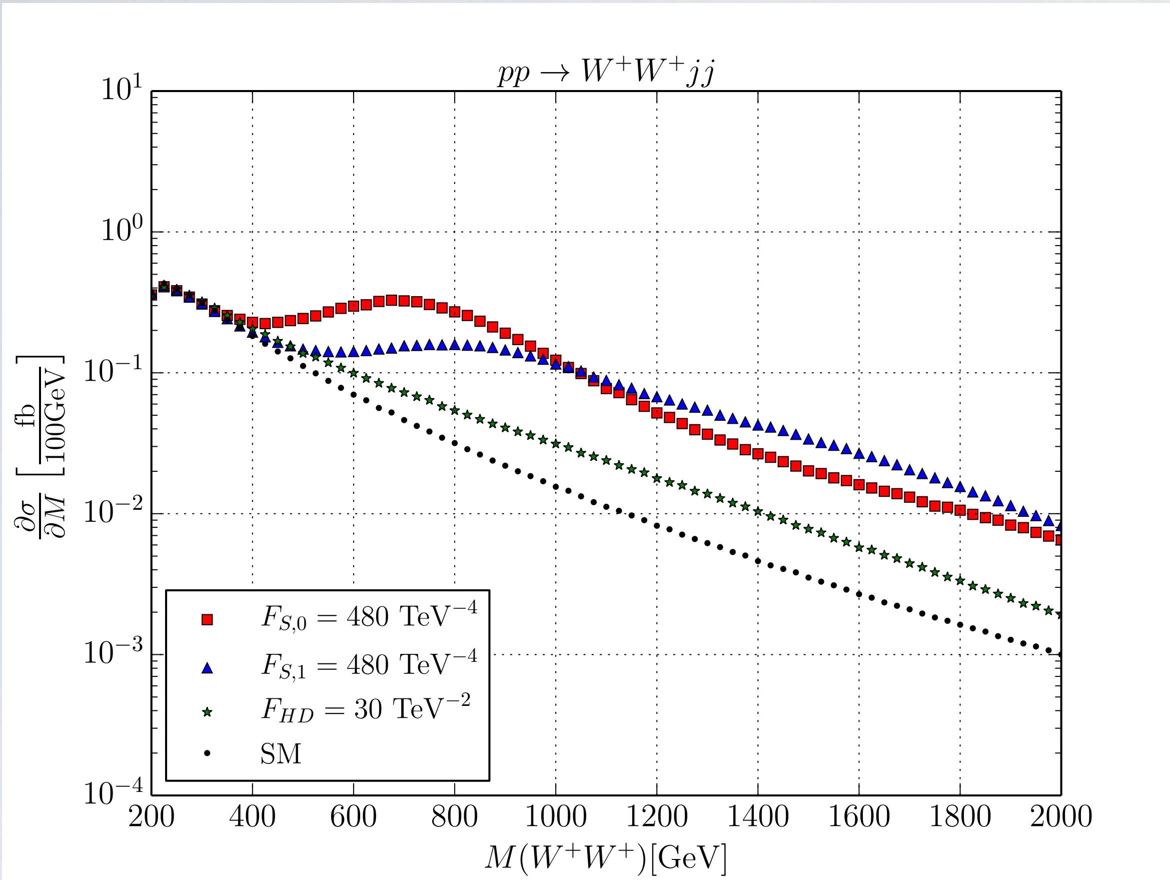

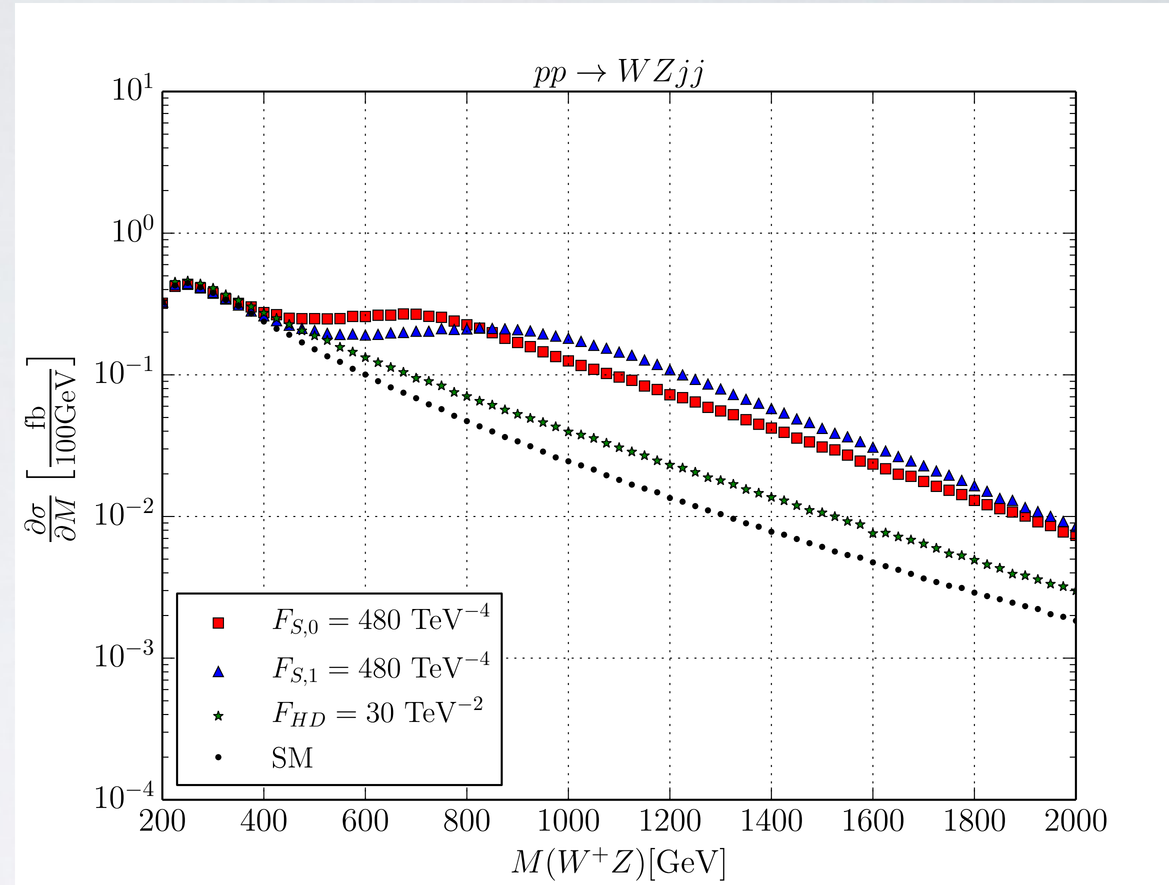

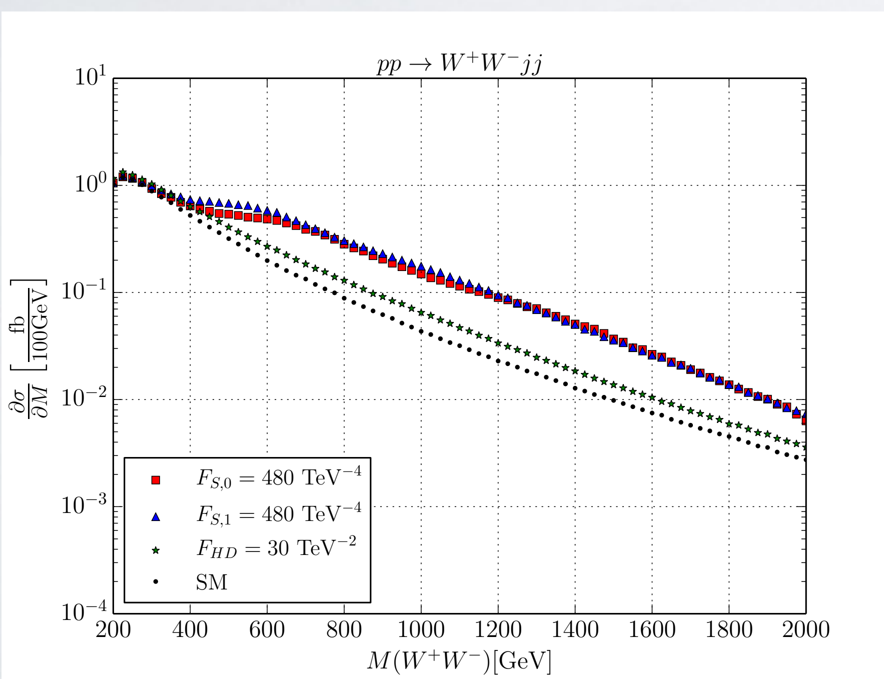

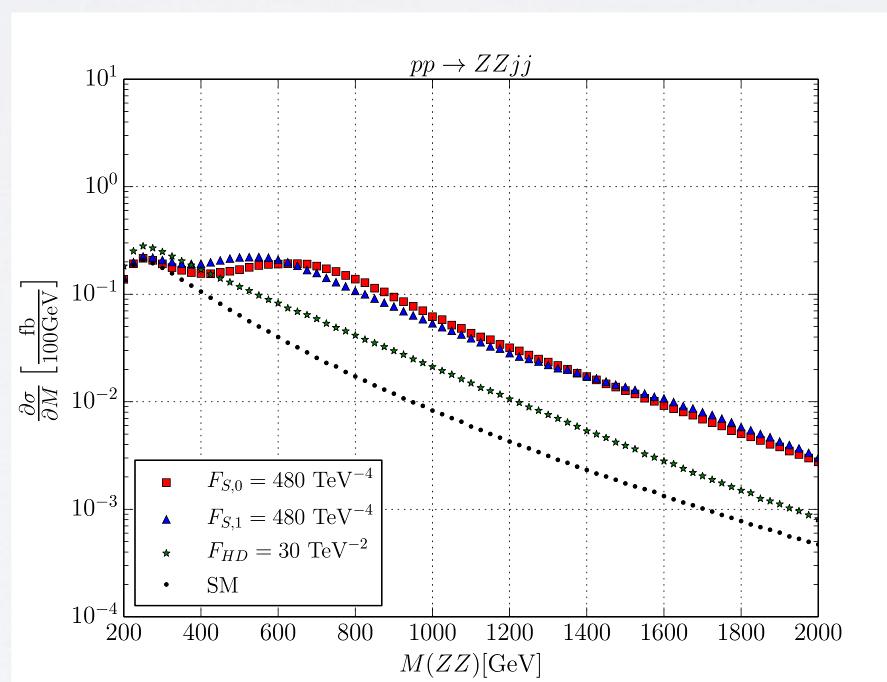

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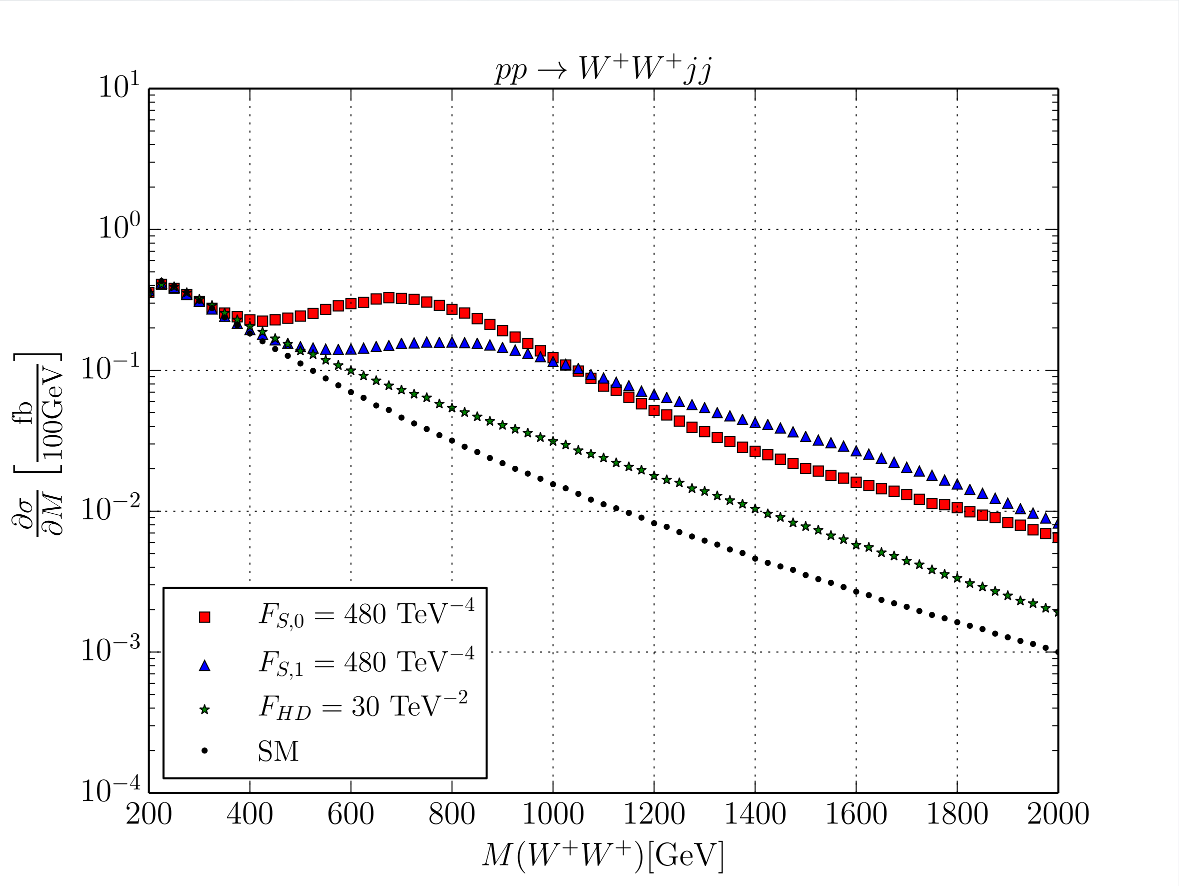

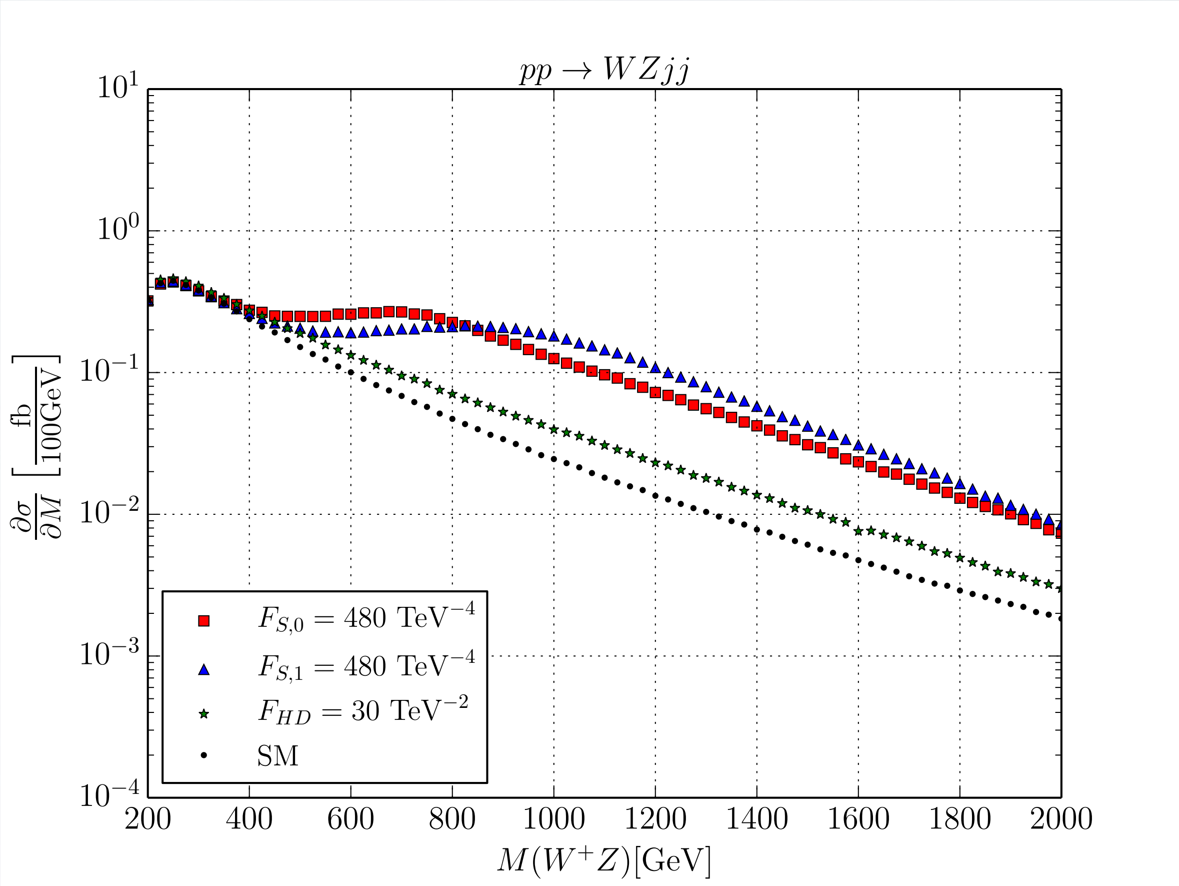

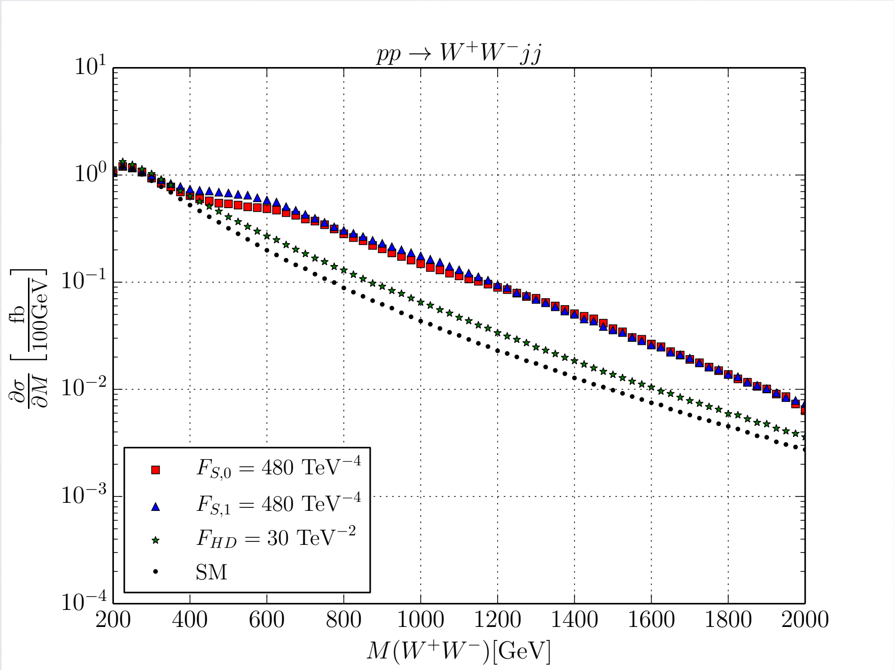

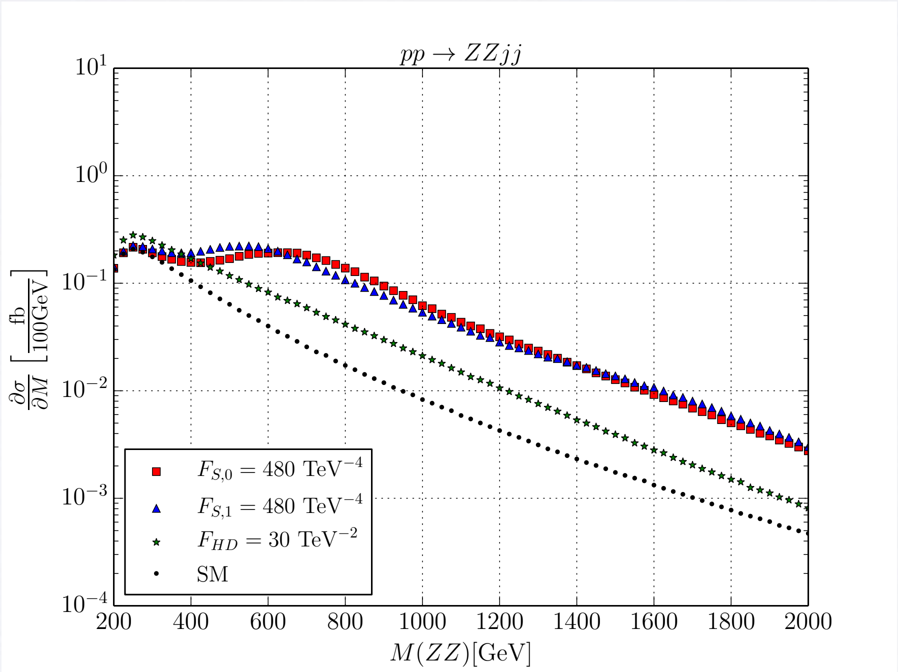

WWWW-Vertex:
$$\alpha_4 = \frac{f_{S,0}}{\Lambda^4}\frac{v^4}{8}$$
$$\alpha_4 + 2\cdot\alpha_5 = \frac{f_{S,1}}{\Lambda^4}\frac{v^4}{8}$$

WWZZ-Vertex:
$$\alpha_4 = \frac{f_{S,0}}{\Lambda^4}\frac{v^4}{16}$$
$$\alpha_5 = \frac{f_{S,1}}{\Lambda^4}\frac{v^4}{16}$$

ZZZZ-Vertex:
$$\alpha_4 + \alpha_5 = \left(\frac{f_{S,0}}{\Lambda^4} + \frac{f_{S,1}}{\Lambda^4}\right)\frac{v^4}{16}$$

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Differential spectra in VBS

$$pp \to e^+\mu^+\nu_e\nu_\mu jj \qquad \sqrt{s} = 14\,\mathrm{TeV} \qquad \mathcal{L} = 1\,\mathrm{ab}^{-1}$$

Simulations with WHIZARD [http://whizard.hepforge.org, Kilian/Ohl/JRR]

$$M_{jj} > 500\,\mathrm{GeV}; \quad \Delta\eta_{jj} > 2.4; \quad p_T^j > 20\,\mathrm{GeV}; \quad |\Delta\eta_j| < 4.5; \quad p_T^\ell > 20\,\mathrm{GeV}$$

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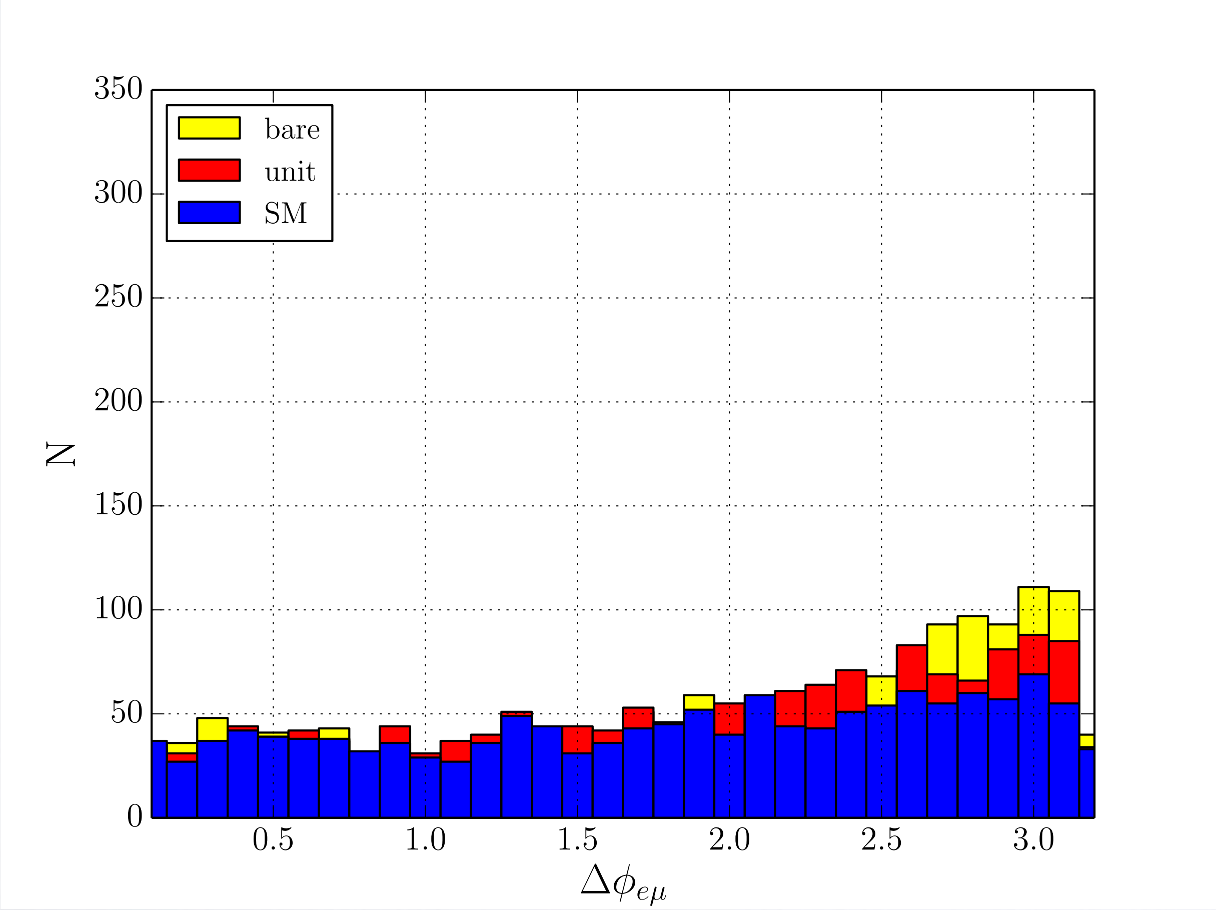
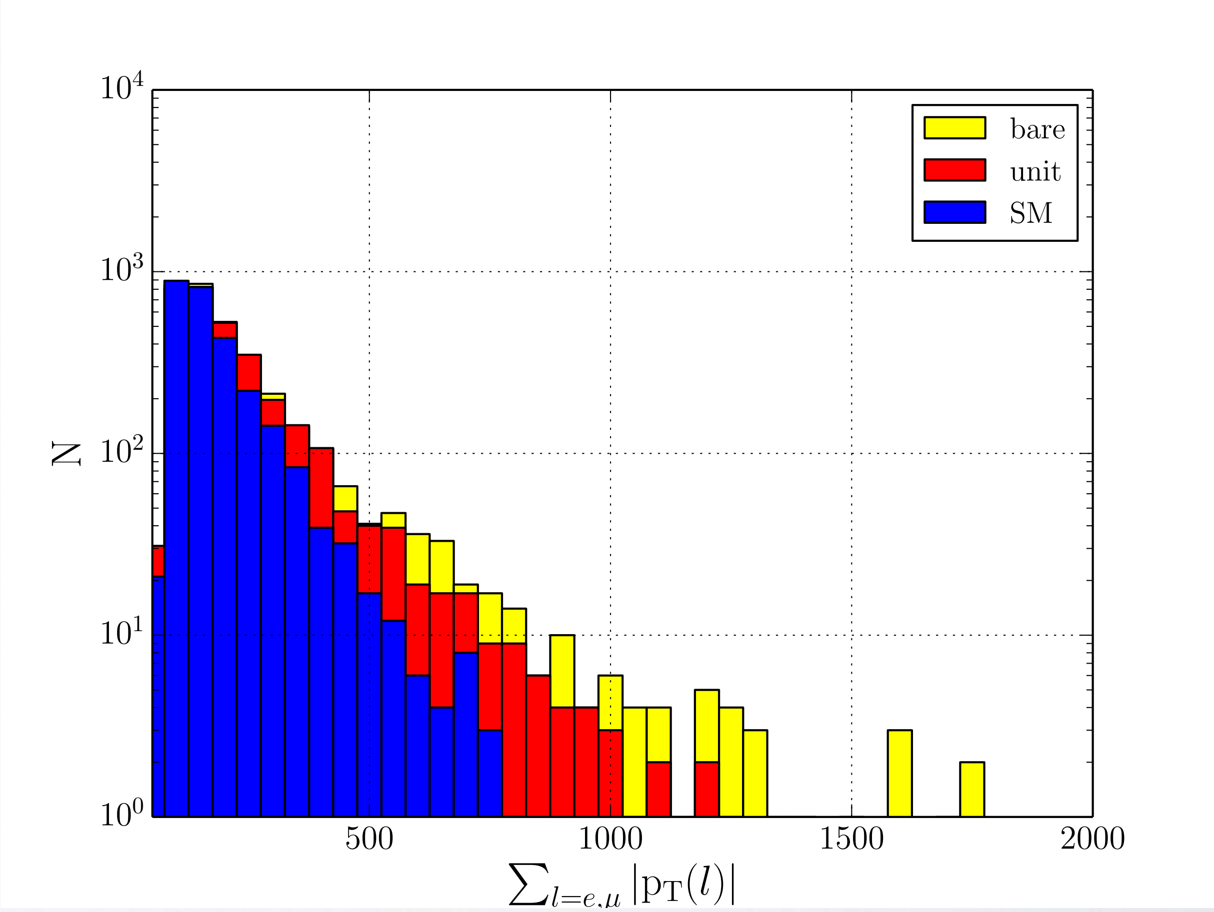
Simulations with `WHIZARD` [http://whizard.hepforge.org, Kilian/Ohl/JRR]

$$F_{HD} = 30\ \mathrm{TeV}^{-2}$$





$$M_{jj} > 500\,\mathrm{GeV}; \quad \Delta\eta_{jj} > 2.4; \quad p_T^j > 20\,\mathrm{GeV}; \quad |\Delta\eta_j| < 4.5; \quad p_T^\ell > 20\,\mathrm{GeV}$$

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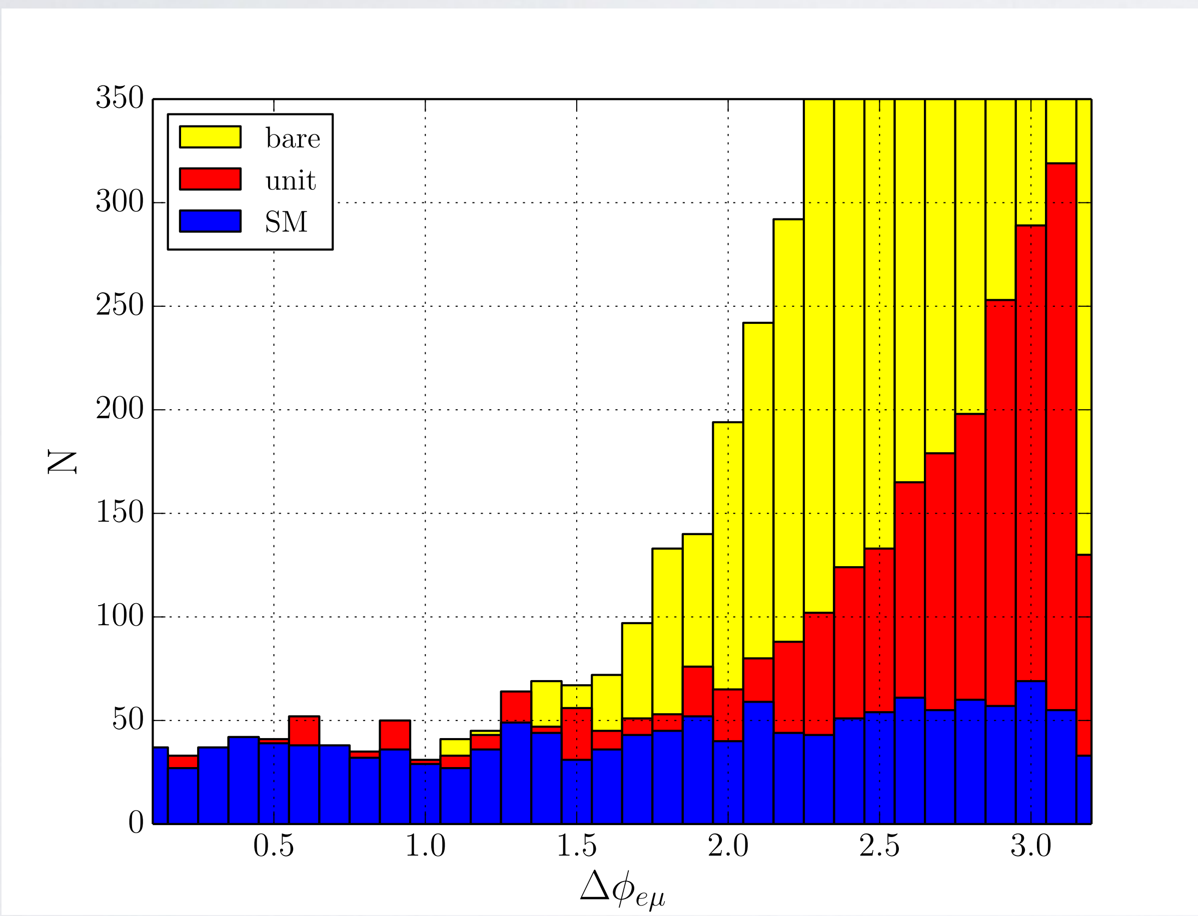

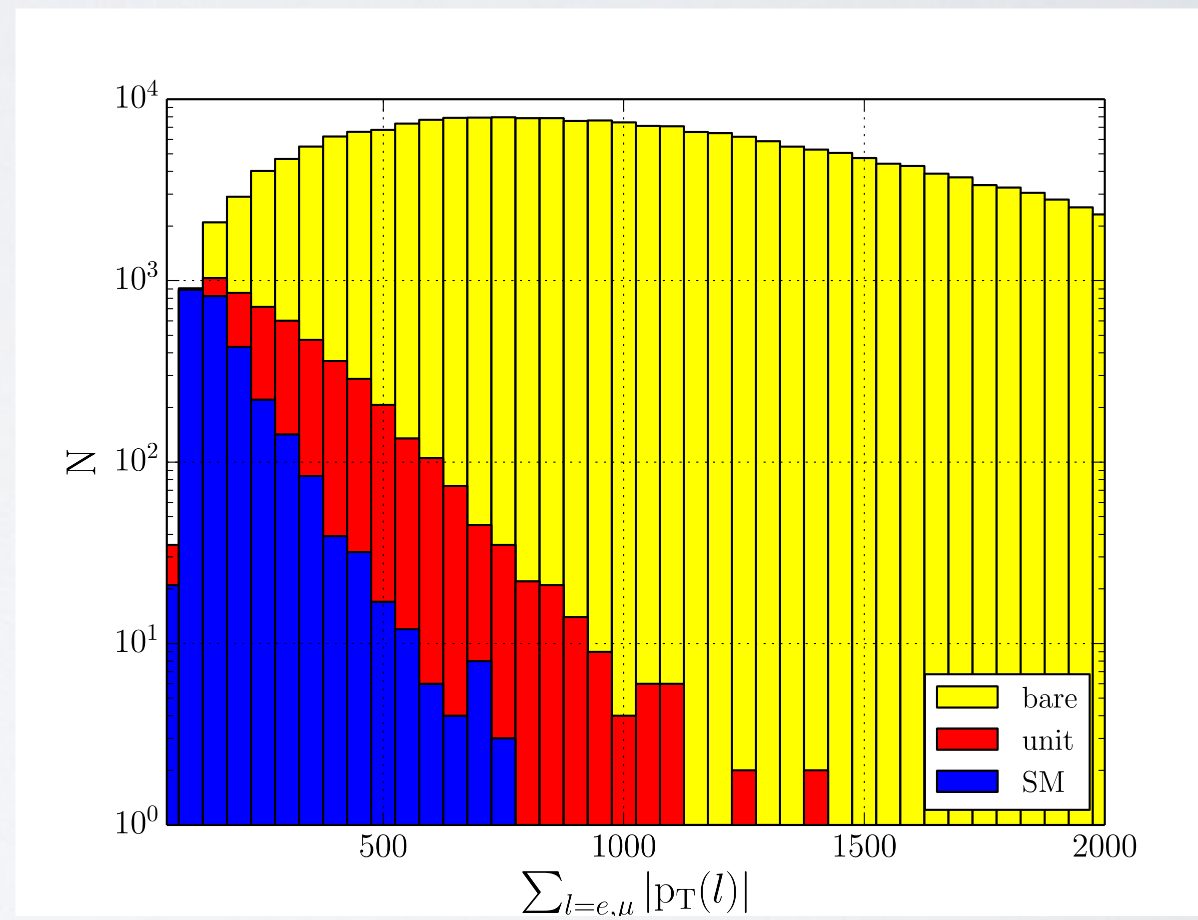

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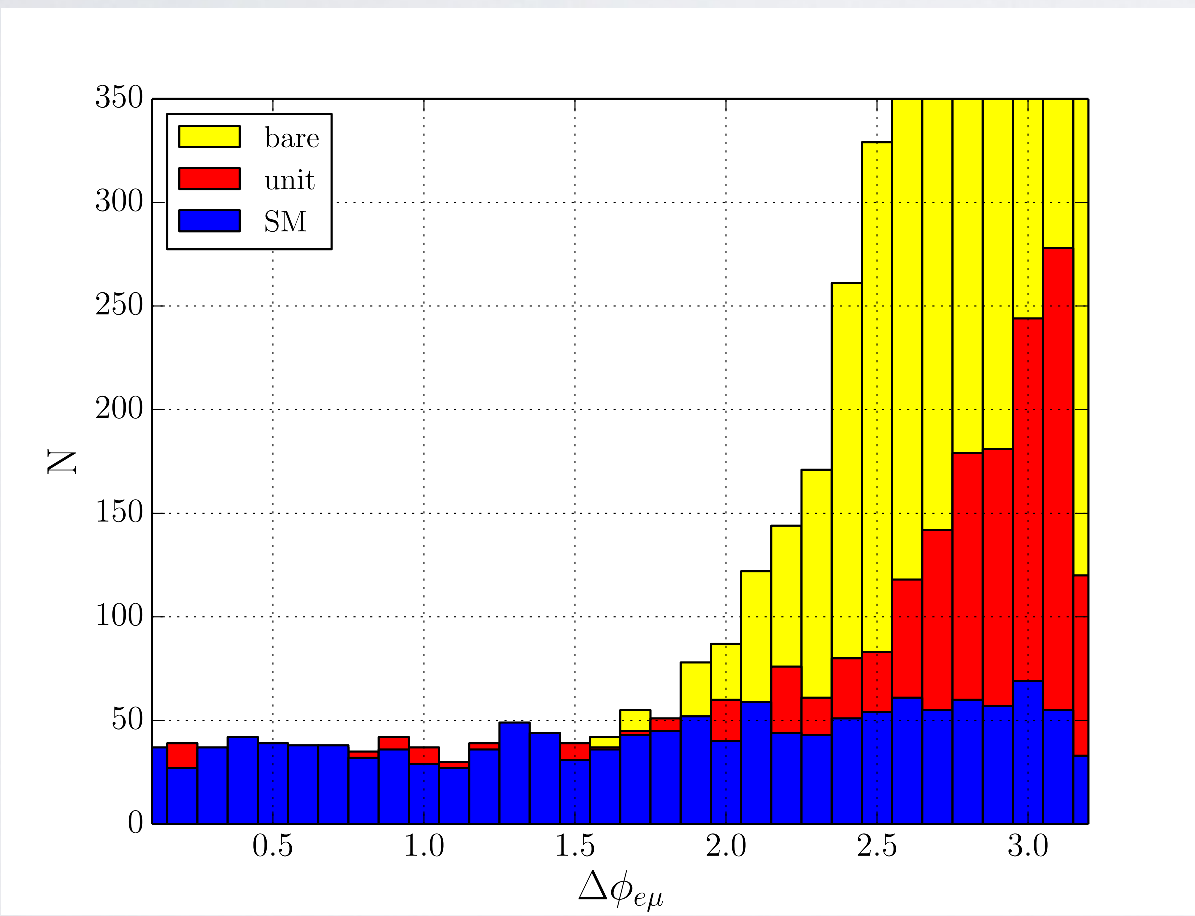

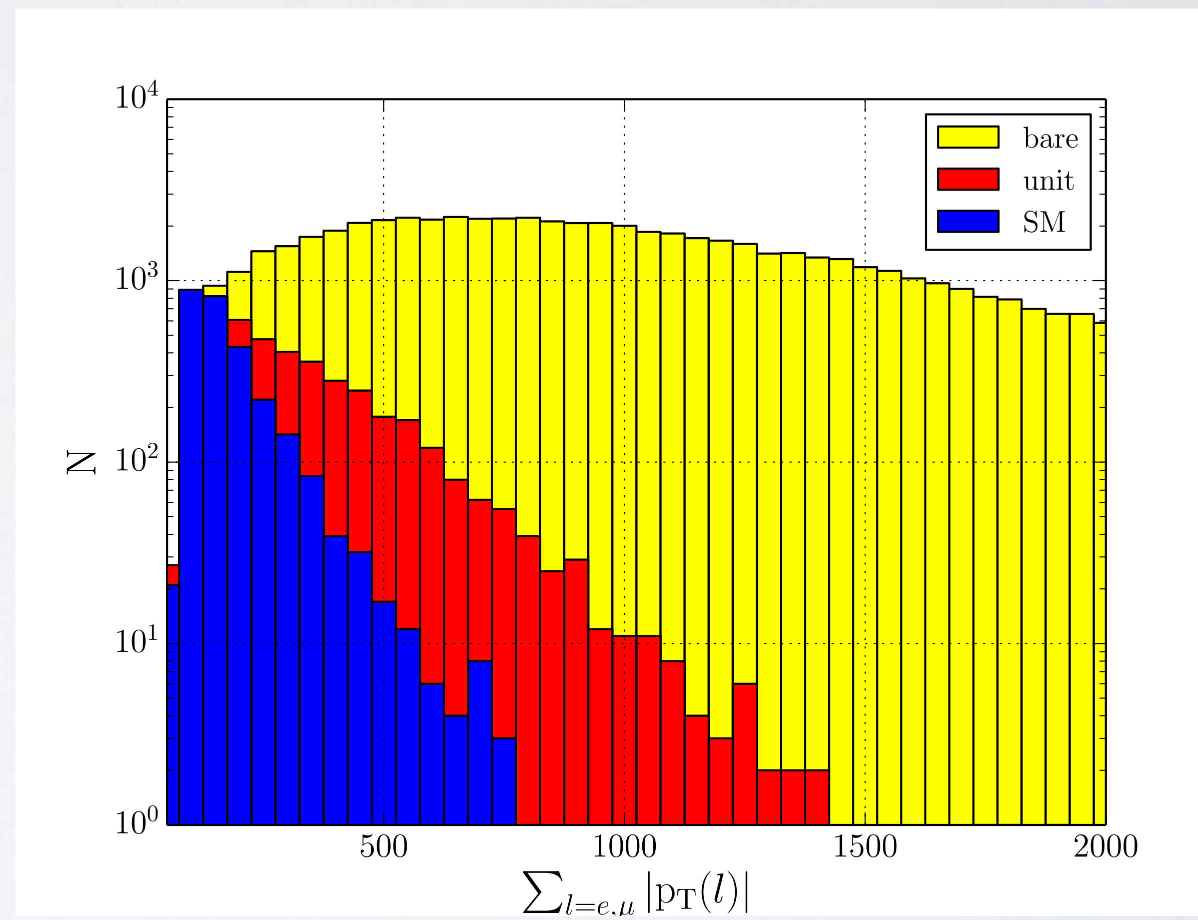

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- Resonances might be in direct reach of LHC
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...	...	...

Tensor resonances

- Symmetric tensor $f_{\mu\nu}$
- On-shell conditions: 10 → 5 components
- Tracelessness: $f_\mu{}^\mu = 0$
- Transversality: $\partial_\mu f^{\mu\nu} = 0$

How to deal with *off-shell* tensor in realistic processes?

Tensor resonances: Fierz-Pauli vs. Stückelberg

- Start with Fierz-Pauli Lagrangian for symmetric tensor

$$\mathcal{L}_{\text{FP}} = \frac{1}{2}\partial_\alpha f_{\mu\nu}\partial^\alpha f^{\mu\nu} - \frac{1}{2}m^2 f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}\partial_\alpha f^\mu_{\ \mu}\partial^\alpha f^\nu_{\ \nu} + \frac{1}{2}m^2 f^\mu_{\ \mu} f^\nu_{\ \nu} \\ - \partial^\alpha f_{\alpha\mu}\partial_\beta f^{\beta\mu} - f^\alpha_{\ \alpha}\partial^\mu\partial^\nu f_{\mu\nu} + f_{\mu\nu}J^{\mu\nu}_f$$

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  - $f^{\mu\nu}$: on-shell $f^{\mu\nu}$
  - ϕ: $\partial_\mu\partial_\nu f^{\mu\nu}$
  - A^μ: $\partial_\nu f^{\mu\nu}$
  - σ: $f^\mu_{\ \mu}$

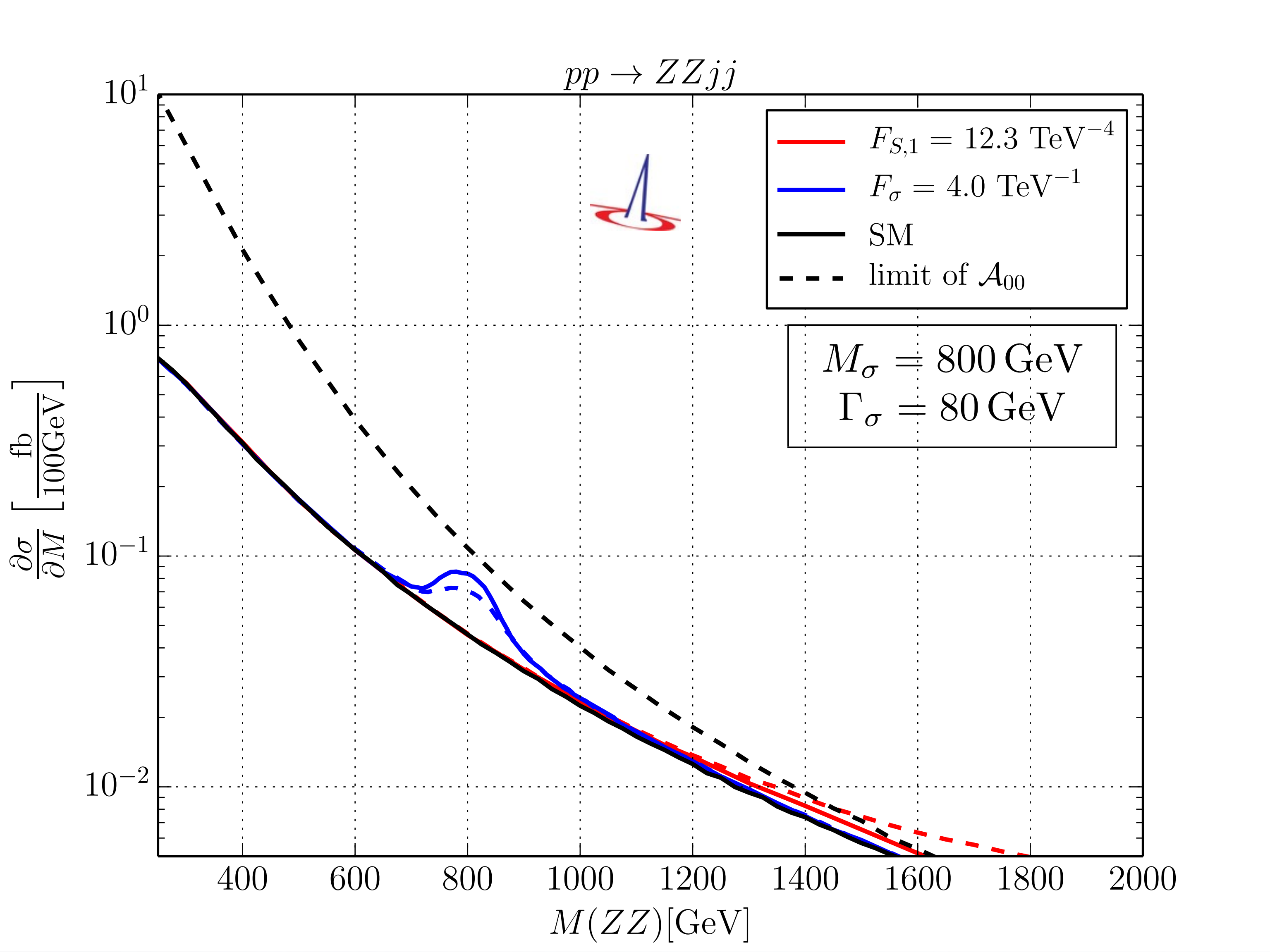
  Gauge fixing: $\sigma = -\phi$

$$\mathcal{L} = \frac{1}{2} f_{f\,\mu\nu}\left(-\partial^2 - m_f^2\right) f_f^{\mu\nu} + \frac{1}{2} f^\mu_{f\ \mu}\left(-\frac{1}{2}\left(-\partial^2 - m_f^2\right)\right) f^\nu_{f\ \nu} + \frac{1}{2}A_{f\,\mu}\left(\partial^2 + m_f^2\right)A_f^\mu + \frac{1}{2}\sigma_f\left(-\partial^2 - m_f^2\right)\sigma_f + \left(f_{\mu\nu} - \frac{1}{\sqrt{6}}\sigma_f g_{\mu\nu}\right)J_f^{\mu\nu} - \left(\frac{1}{\sqrt{2}m_f}\left(A_{f\mu}\partial_\nu + A_{f\nu}\partial_\mu\right) - \frac{\sqrt{2}}{\sqrt{3}m_f^2}\sigma_f\partial_\mu\partial_\nu\right)J_f^{\mu\nu}$$

Comparison: Simplified Models & EFT

Kilian/Ohl/JRR/Sekulla: PRD93(16),3. 036004 [1511.00022]

Black dashed line:
saturation of $\mathcal{A}_{22}(W^+W^+)/\mathcal{A}_{00}(ZZ)$



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$32\pi\Gamma/M^5$

	σ	ϕ	f	X
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ATLAS PRL 113(2014)14, 141803 [1405.6241]

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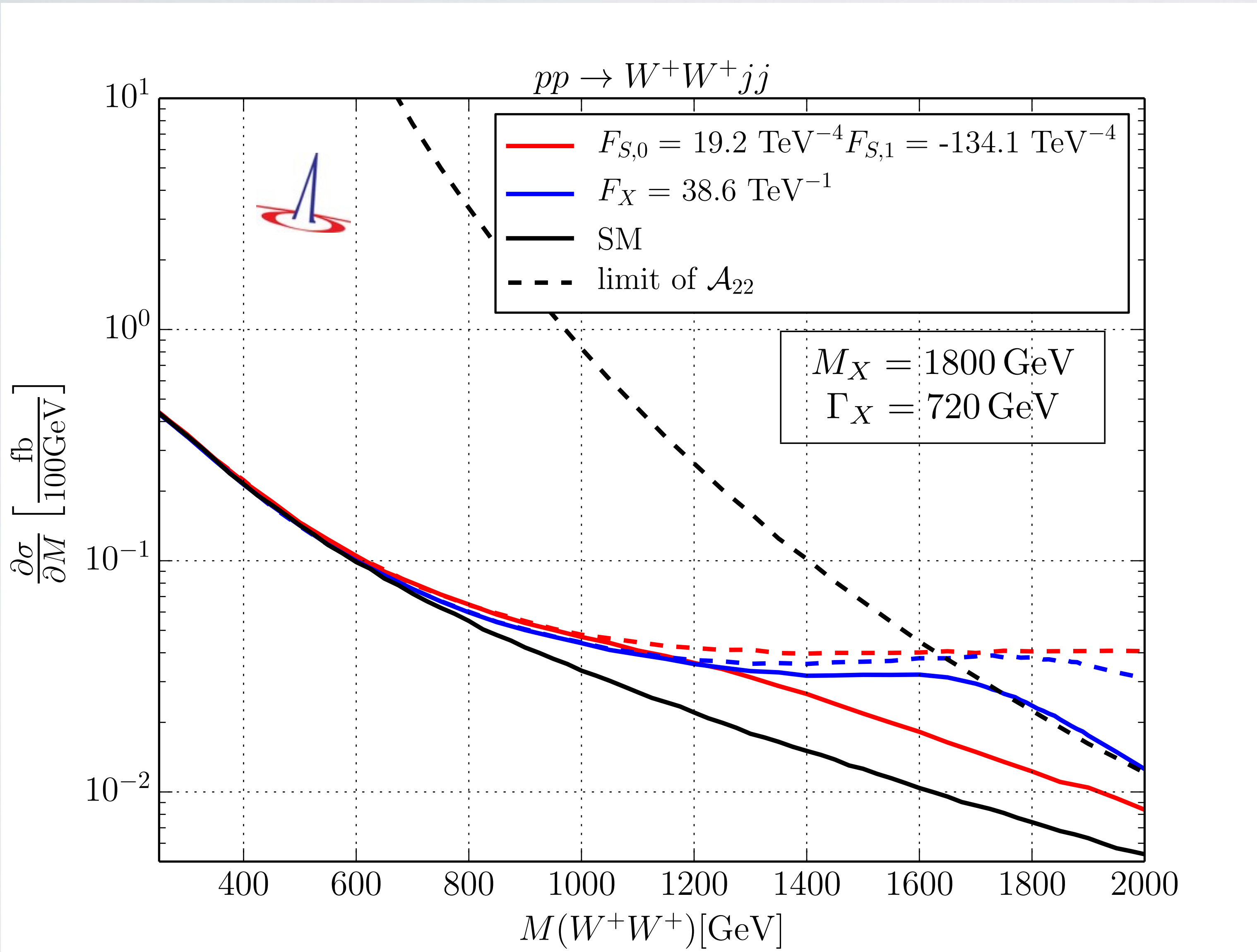

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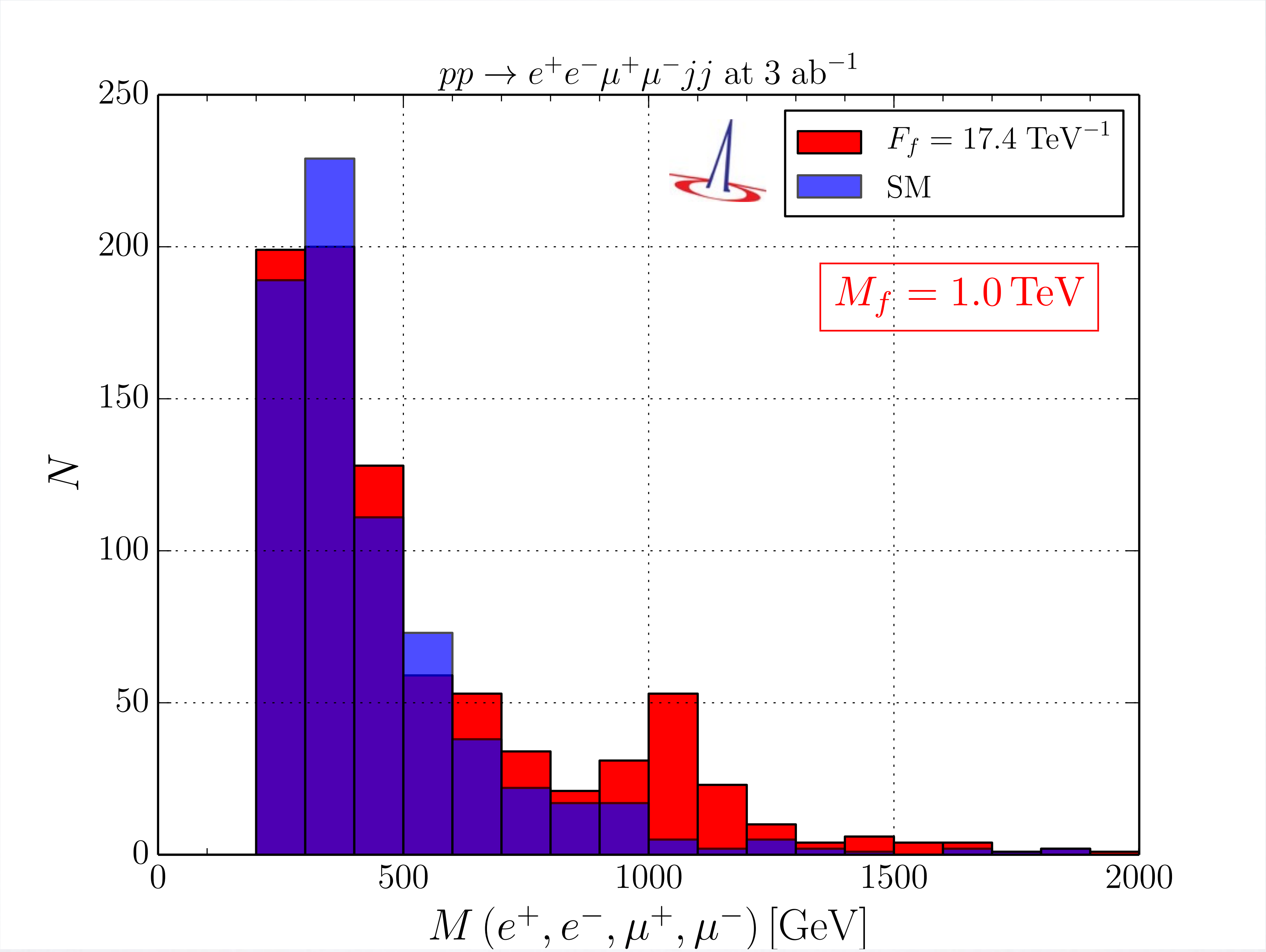
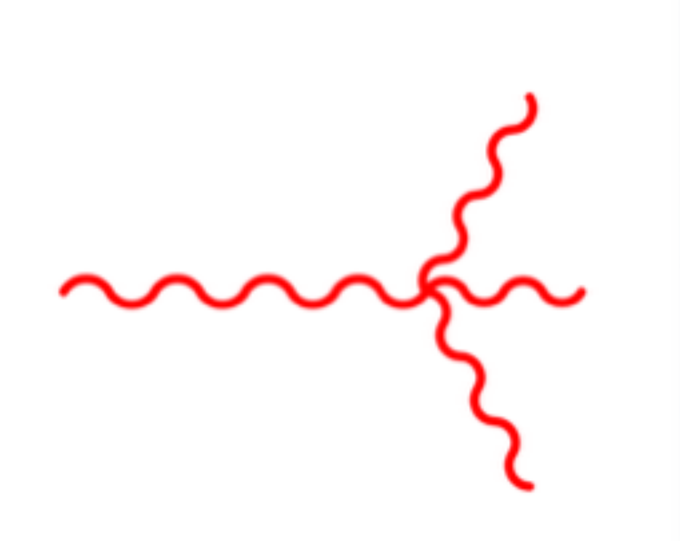
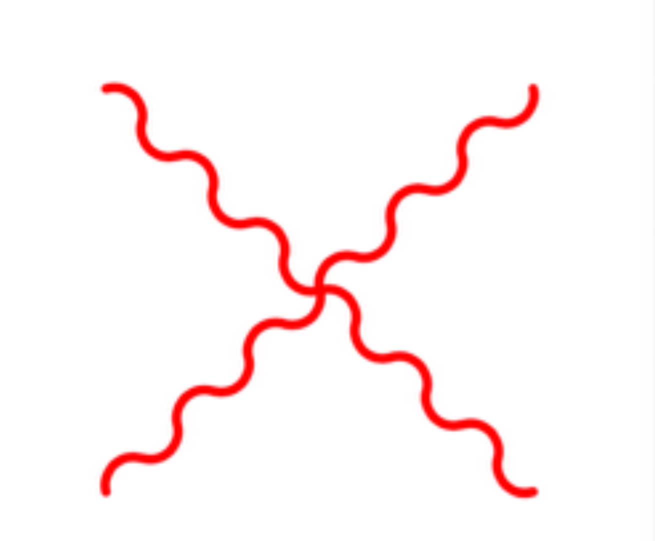
Complete LHC process at 14 TeV



Triple [multiple] Vector Boson Production ?

Relate  to  ?

- Yes, same Feynman rule as in VBS, but …
- one external $W/Z/\gamma$ always far off-shell
- Unitarization formalism not available (would need $2 \to 3$ unitarizations)
- Different Wilson coefficients dominate (particularly for resonances)
- Important physics (partially) independent from VBS

Conclusions / Summary

- VBS (one of) flagship measurements of LHC Runs II/III *and* a 100 TeV machine
- EFT provides *a* (!) [not *the*] consistent framework for SM deviations
- Very well-defined (and limited) range of applicability
- Accounts for access to New Physics in VBS and Di-/Triboson channels

- Unitarization for theoretically sane description (allows to calculate 'best limit')
- *T*-matrix unitarization universal scheme for EFT and resonances
- Unitarization: Not just a theory tool $\Longrightarrow$ "composite continuum saturation"

- Simplified models: generic electroweak resonances
- Vectors/tensors: high-energy behavior tricky [vectors special: *W/Z* mixing]

- Limits from LHC still incredibly puny: $M \sim$ 200-300 GeV
- Make sure that actual limits are meaningful and comparable
- Lots of space/work for improvement: V_L / V_T separation, backgrounds etc.

MBI 2016 [4th Multi-Boson Interactions]
Madison, WI, U.S., Aug. 24-26 2016

BACKUP SLIDES

Effective Field Theory (EFT) for Weak Interactions

- SppS: discovery of W, Z (on-shell)
- SLC/LEP: proof of non-Abelian weak structure, failure to find (very) light Higgs
- Measurement of longitudinal Ws: $ee \to WW$ (LEP), $t \to Wb$ (Tevatron)
- Using all known d.o.f., parameterizing all possible interactions

Building blocks for EFT:

$$\psi \quad , \quad \mathbf{W}_\mu \quad , \quad \mathbf{B}_\mu \quad , \quad \Sigma = \exp\left[\frac{-i}{v}\mathbf{w}\boldsymbol{\tau}\right]$$

SM fermions — weak bosons — hypercharge boson — longitudinal d.o.f.

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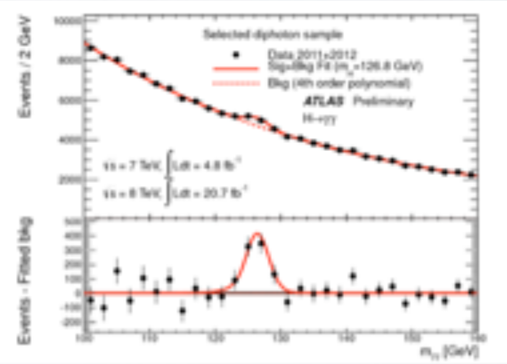
Ruled out by LHC data (Higgs discovery)



EFTs: Higher-dimensional operators

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- Redundancy of operators $\Longrightarrow$ minimal set of operators (in principle)
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- No unique basis exists (more in a second)
- Well-known in *B* physics: different experimental measurements constrain different operators

Parameterizing SM deviations

★ Specific models (SUSY, Compositeness, Little Higgses, 2HDM, Modified Higgses, Xdim, ...)

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★ Anomalous Couplings

- Usually first "model-independent" proposal
- At the moment applied by HXSWG (but under debate)
- Only modifications of SM couplings or introduction of new (Lorentz) structures ?
- Allows fits of coupling strengths
- Possible deviations difficult to interpret in terms of quantum field theory, unitarity!!

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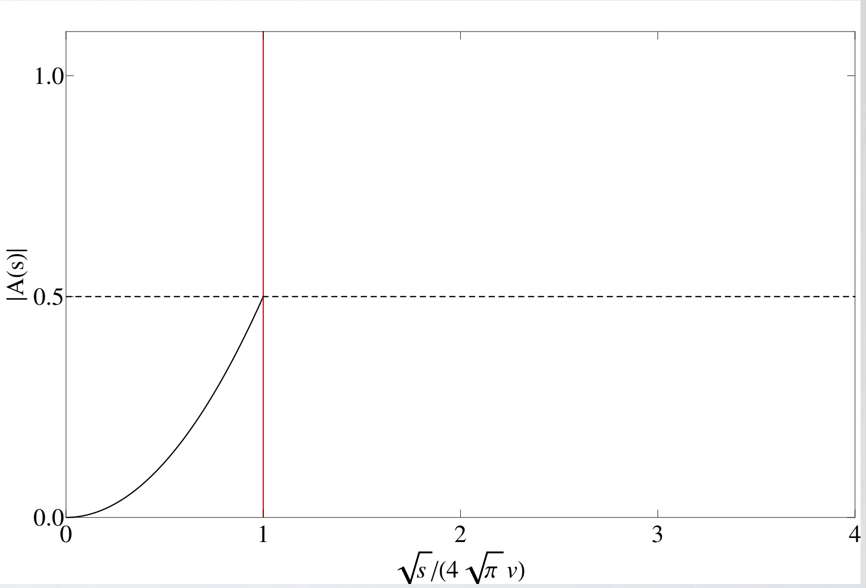
★ Effective Field Theory

- (Almost) model-independent, consistent calculation of perturbative corrections (power counting !?)
- Depends on (possibly) many free parameters
- Requires decoupling of New Physics
- Range of applicability strongly depends on couplings and scales (unitarity issue)

Procedures to treat unitarity violations

Cut-off (a.k.a. "Event clipping") $\theta(\Lambda_C^2 - s)$

unitarity bound (0th partial wave) at Λ_C
no continuous transition beyond



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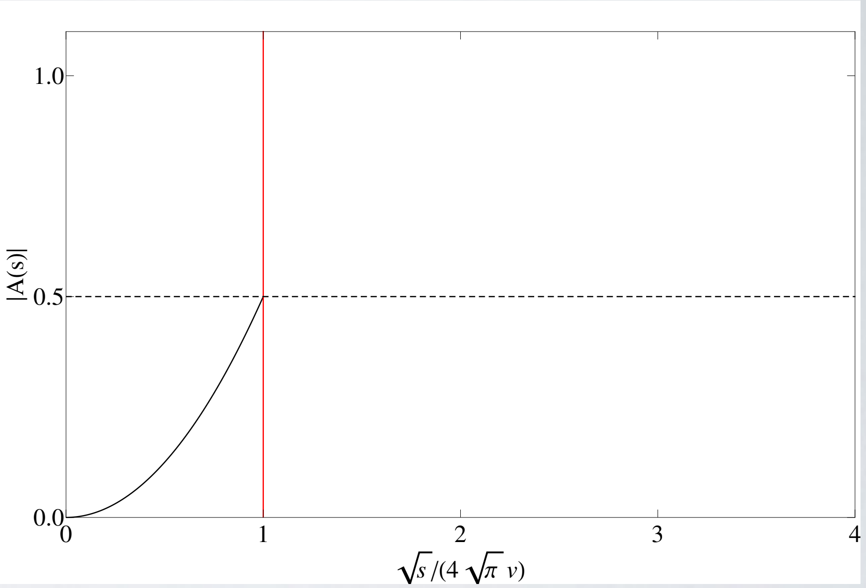

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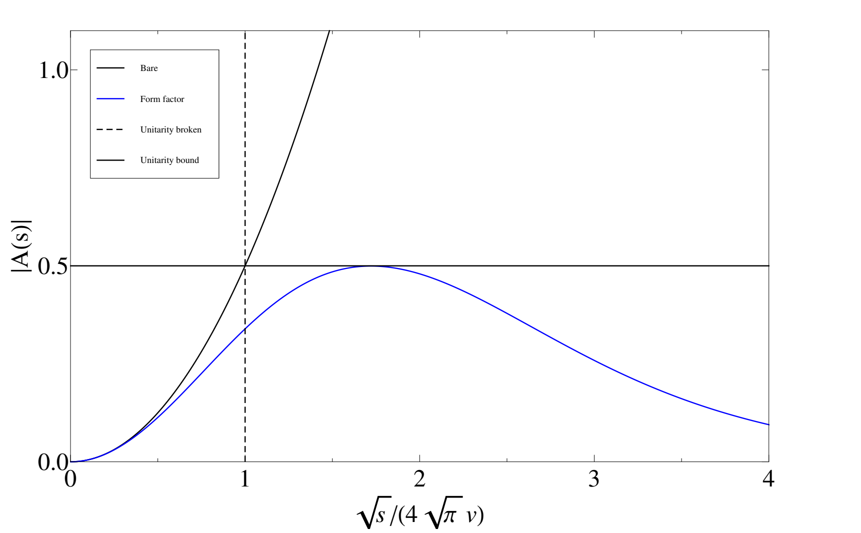

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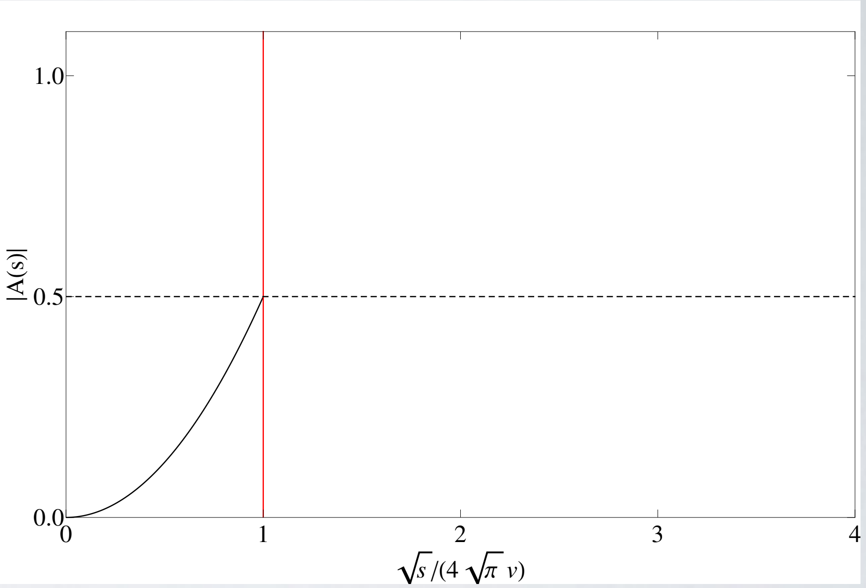

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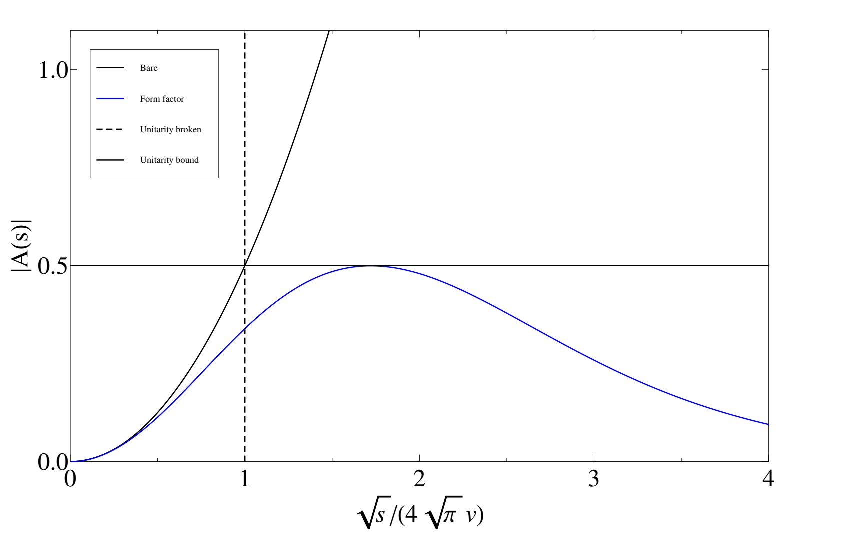

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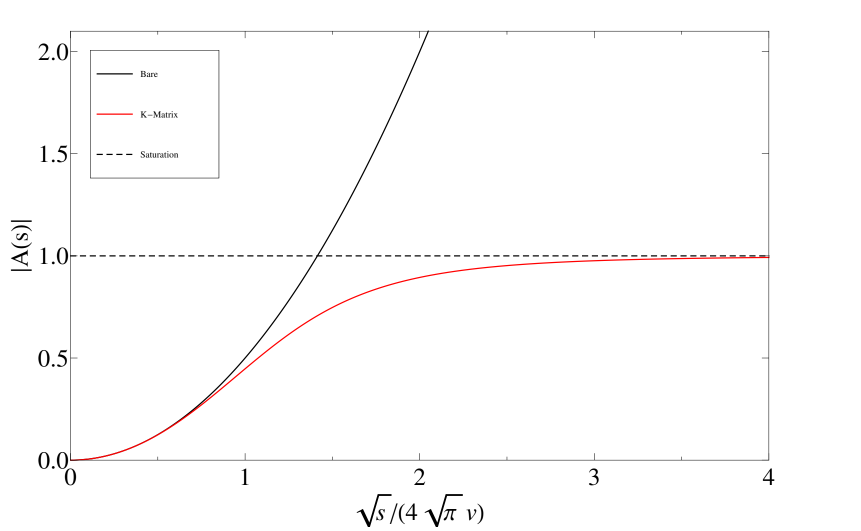

K-/T-matrix saturation

saturates the amplitude, usable for complex amplitudes, no additional parameters

Different unitarity projections

- *K*-matrix: Cayley transform of *S*-matrix Heitler, 1941; Schwinger, 1949; Gupta, 1950
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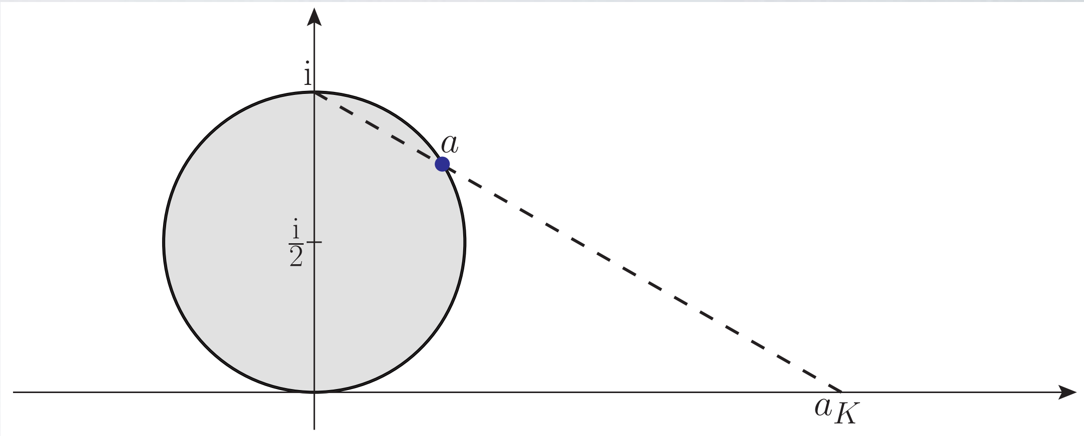

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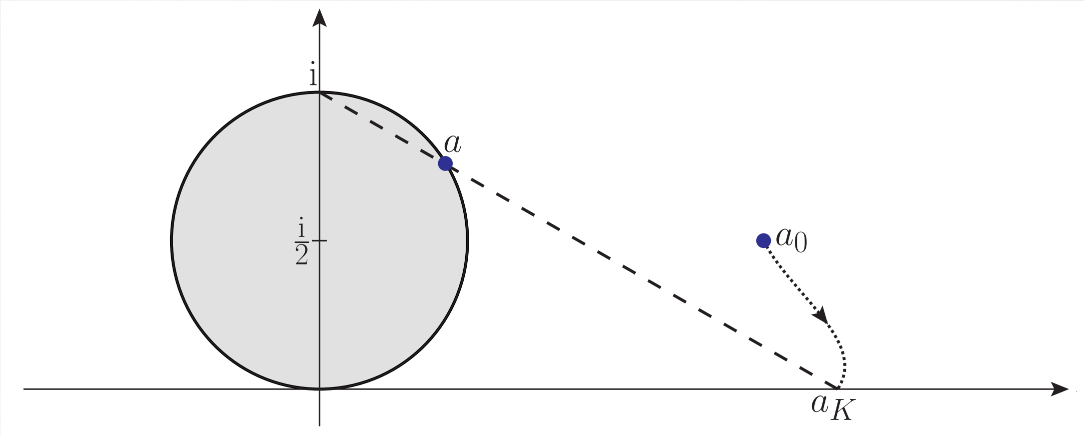

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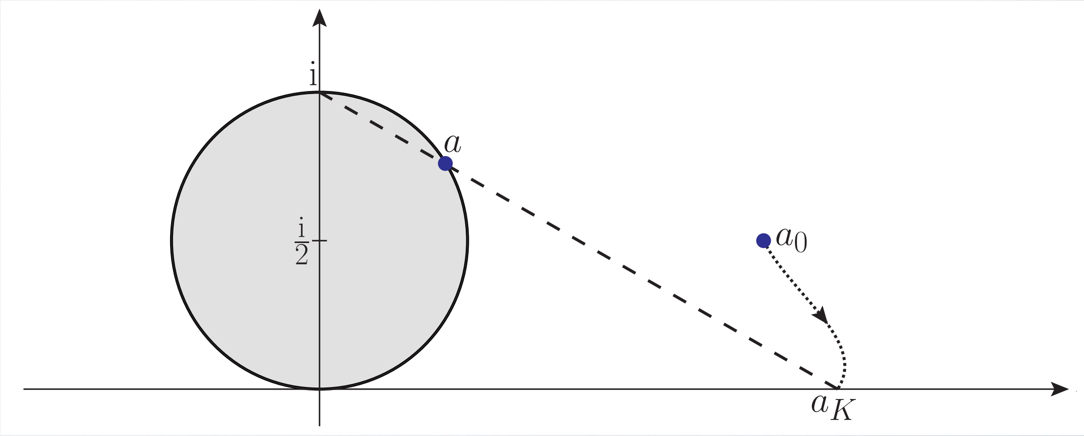

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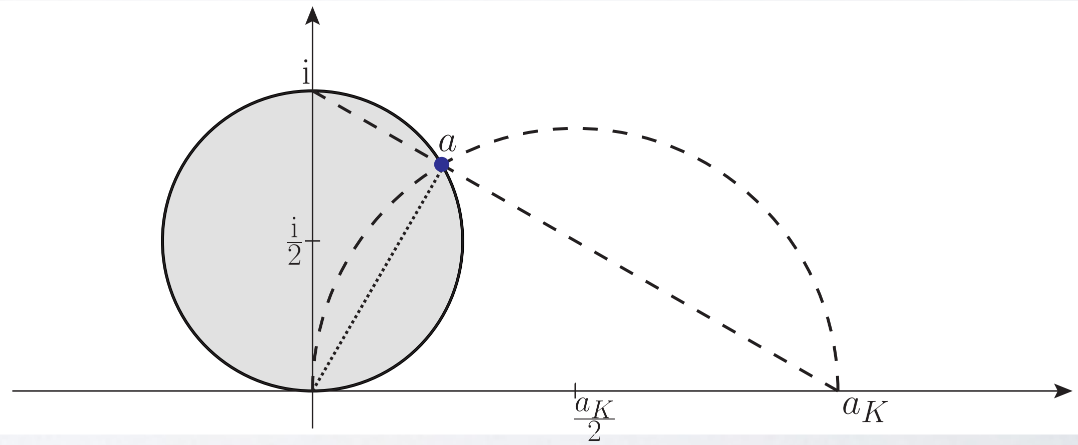

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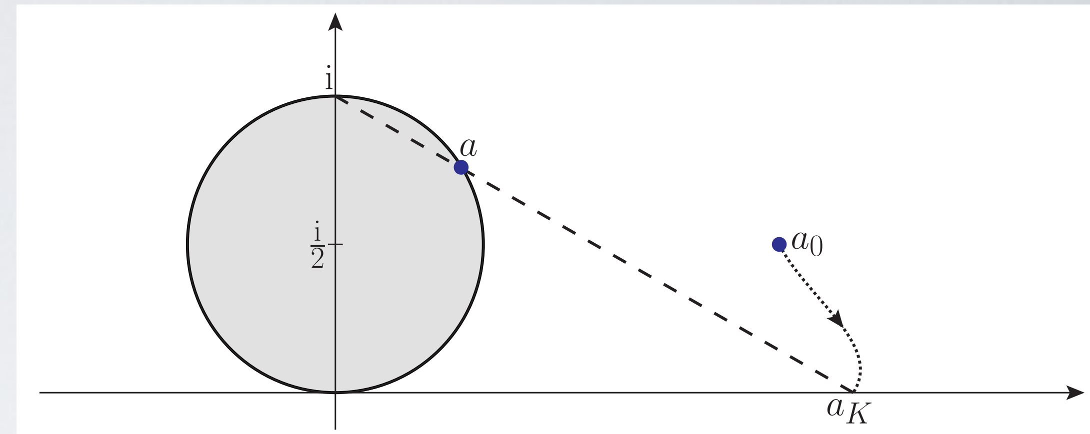

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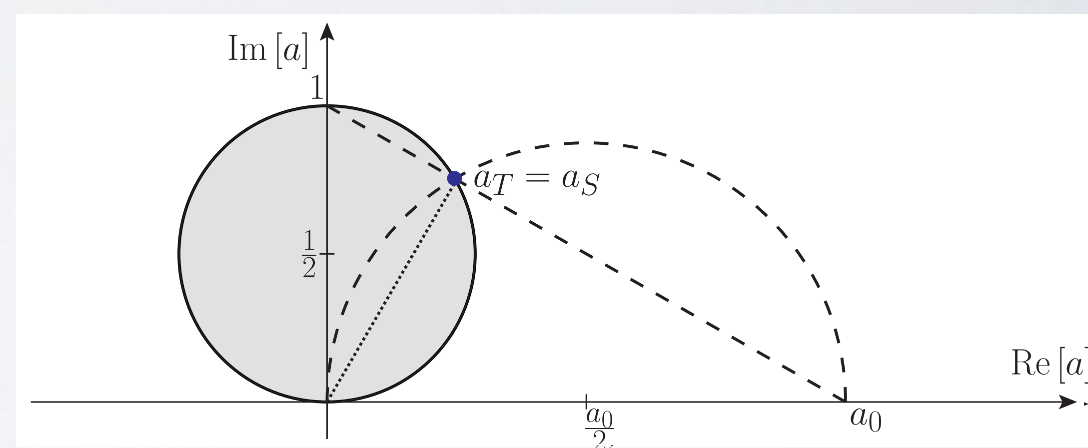

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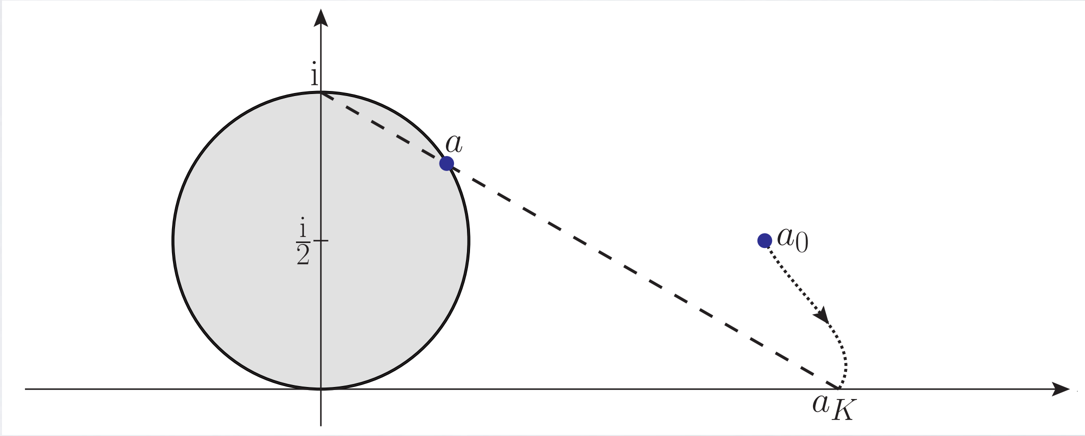

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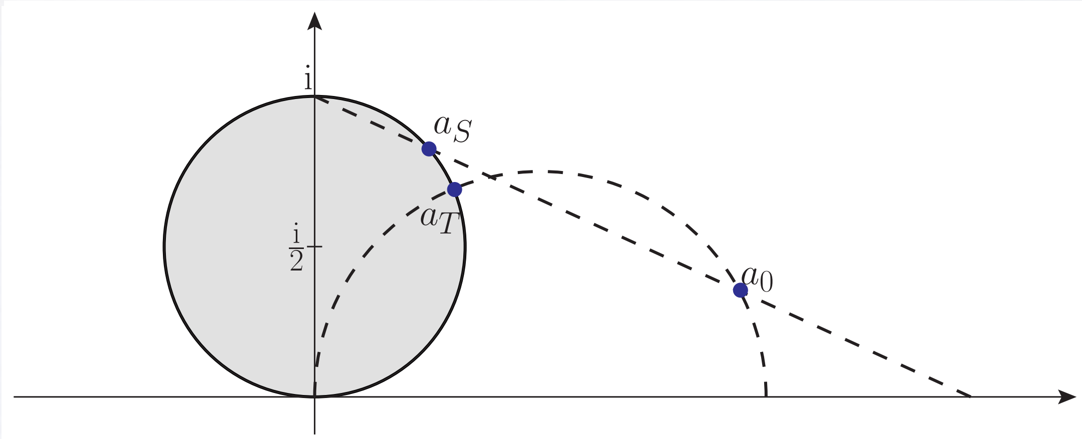

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Generation of Higher-dimensional Operators

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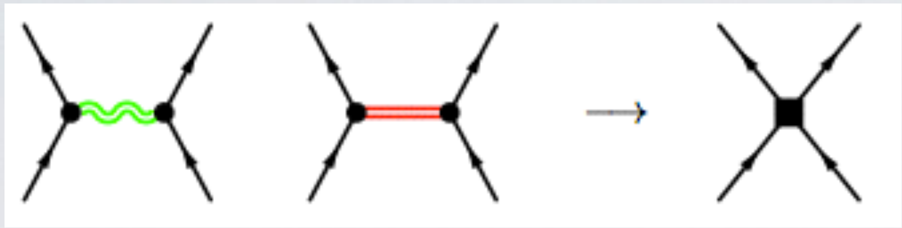

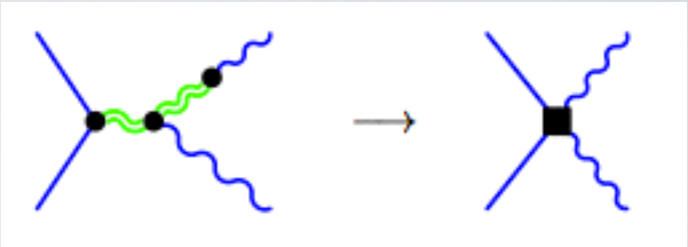

$$\mathcal{O}'_{WW} = -\frac{1}{\Lambda^2}\frac{1}{2}(\Phi^\dagger\Phi - v^2/2)\mathrm{tr}\left[W_{\mu\nu}W^{\mu\nu}\right]$$

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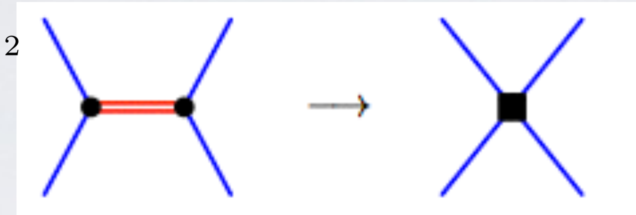

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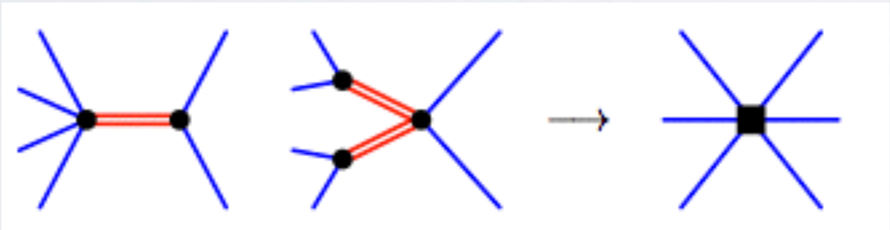

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Couplings of new states to the longitudinal / transversal diboson system

	$J=0$	$J=1$	$J=2$
$I=0$	σ^0 (Higgs singlet?)	ω^0 (γ'/Z' ?)	f^0 (Graviton ?)
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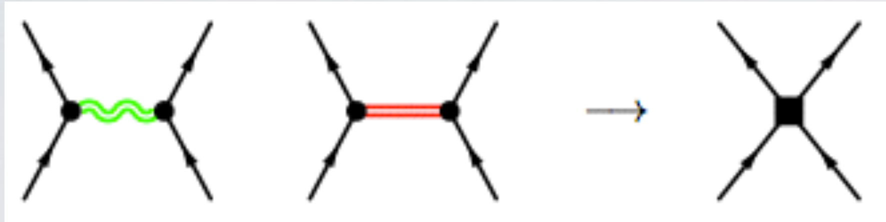

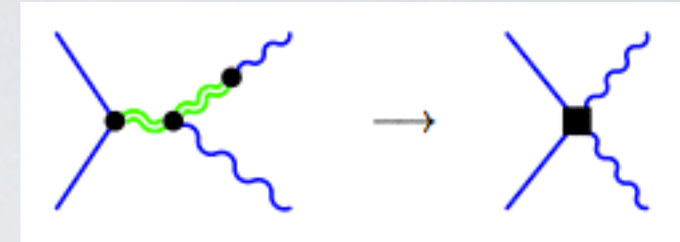

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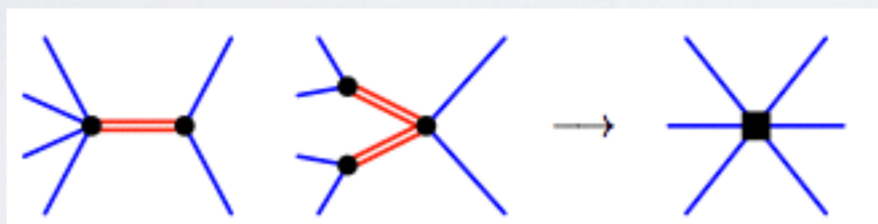

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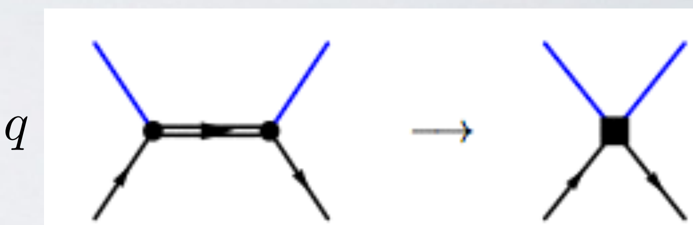

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Different power counting for weakly and strongly interacting theories

$$\frac{c_i}{\Lambda} \sim \frac{g}{4\pi\Lambda} \quad \text{vs.} \quad \frac{c_i}{\Lambda} \sim \frac{g}{\Lambda}$$