## Anomalous Quartic Gauge Couplings - Theory Overview

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### LHCEWWG Meeting, CERN, Apr. 16th, 2013

• Light Higgs boson found

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- Deviations from the SM: where? what? how?
- Anomalous Triple Gauge Couplings: dibosons
- Anomalous Quartic Gauge Couplings: tribosons, VV scattering
- Remark: no CP-violating operators in the talk
- Hot topic: Snowmass BNL 04/13, SM@LHC Freiburg 04/13, Dresden workshop 10/13

### Extensions of the SM

Lagrangian of the EW SM (no fermions/QCD here):

$$\mathcal{L}_{EW} = -\frac{1}{2} \text{tr} \left[ W_{\mu\nu} W^{\mu\nu} \right] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + (D_{\mu}\phi)^{\dagger} (D^{\mu}\phi) + \mu^{2} \phi^{\dagger}\phi - \lambda (\phi^{\dagger}\phi)^{2}$$

with building blocks:

$$D_{\mu} = \partial_{\mu} + \frac{i}{2}g\tau^{I}W_{\mu}^{I} + \frac{i}{2}g'B_{\mu}$$
$$W_{\mu\nu} = \frac{i}{2}g\tau^{I}(\partial_{\mu}W_{\nu}^{I} - \partial_{\nu}W_{\mu}^{I} + g\epsilon_{IJK}W_{\mu}^{J}W_{\nu}^{K})$$
$$B_{\mu\nu} = \frac{i}{2}g'(\partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu})$$

Any EFT has higher-dimensional operators:

Weinberg, 1979

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{i} \left[ rac{a_i}{\Lambda} \mathcal{O}_i^{(5)} + rac{c_i}{\Lambda^2} \mathcal{O}_i^{(6)} + rac{e_i}{\Lambda^4} \mathcal{O}_i^{(8)} \cdots 
ight]$$

► without more fundamental theory ⇒ no clue on the scale (neither on the coefficients)

### Effective EW Dim. 6 Operators

Hagiwara/Hikasa/Peccei/Zeppenfeld, 1987; Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993

$$\longrightarrow \qquad \qquad \mathcal{O}_{h,1}' = \frac{1}{F^2} \left( (D\phi)^{\dagger} \phi \right) \cdot \left( h^{\dagger} (D^{\phi}) \right) - \frac{v^2}{2} |D\phi|^2$$
$$\qquad \qquad \mathcal{O}_{hh}' = \frac{1}{\Lambda^2} (\phi^{\dagger} \phi - v^2/2) (D\phi)^{\dagger} \cdot (D\phi)$$





$$\mathcal{O}'_{WW} = -\frac{1}{\Lambda^2} \frac{1}{2} (\phi^{\dagger} \phi - v^2/2) \text{tr} [W_{\mu\nu} W^{\mu\nu} \\ \mathcal{O}_B = \frac{1}{\Lambda^2} \frac{i}{2} (D_{\mu} \phi)^{\dagger} (D_{\nu} \phi) B^{\mu\nu} \\ \mathcal{O}'_{BB} = -\frac{1}{\Lambda^2} \frac{1}{4} (\phi^{\dagger} \phi - v^2/2) B_{\mu\nu} B^{\mu\nu}$$



### Effective Dim. 8 Operators

Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993

$$\begin{array}{ccc} & & & \\ & & & & \\ & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ &$$

- operators linked through e.o.m.
- SM: 59 independent operators (1 fermion gen.)

Buchmüller/Wyler, 1986;

Grzadkowski/Iskrzynski/Misiak/Rosiek, 2010

- Renormalization mixes operators
- Beware of power counting

### Electroweak Chiral Lagrangian

Originally for heavy Higgses or Higgsless models

$$\begin{aligned} \mathcal{L}_{\chi} &= -\sum_{\psi} \overline{\psi}_{L} \Sigma M \psi_{R} + \beta_{1} \mathcal{L}_{0}' + \sum_{i} \alpha_{i} \mathcal{L}_{i} + \frac{1}{v} \sum_{i} \alpha_{i}^{(5)} \mathcal{L}^{(5)} + \frac{1}{v^{2}} \sum_{i} \alpha_{i}^{(6)} \mathcal{L}^{(6)} + \dots \\ \mathcal{L}_{0}' &= \frac{v^{2}}{4} \operatorname{tr} \left[ \mathbf{T} \mathbf{V}_{\mu} \right] \operatorname{tr} \left[ \mathbf{T} \mathbf{V}^{\mu} \right] \\ \mathcal{L}_{1} &= \operatorname{tr} \left[ \mathbf{B}_{\mu \nu} \mathbf{W}^{\mu \nu} \right] & \mathcal{L}_{6} &= \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}_{\nu} \right] \operatorname{tr} \left[ \mathbf{T} \mathbf{V}^{\mu} \right] \operatorname{tr} \left[ \mathbf{T} \mathbf{V}^{\nu} \right] \\ \mathcal{L}_{2} &= \operatorname{itr} \left[ \mathbf{B}_{\mu \nu} \left[ \mathbf{V}^{\mu}, \mathbf{V}^{\nu} \right] \right] & \mathcal{L}_{7} &= \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}^{\mu} \right] \operatorname{tr} \left[ \mathbf{T} \mathbf{V}^{\nu} \right] \operatorname{tr} \left[ \mathbf{T} \mathbf{V}^{\nu} \right] \\ \mathcal{L}_{3} &= \operatorname{itr} \left[ \mathbf{W}_{\mu \nu} \left[ \mathbf{V}^{\mu}, \mathbf{V}^{\nu} \right] \right] & \mathcal{L}_{8} &= \frac{1}{4} \operatorname{tr} \left[ \mathbf{T} \mathbf{W}_{\mu \nu} \right] \operatorname{tr} \left[ \mathbf{T} \mathbf{W}^{\mu \nu} \right] \\ \mathcal{L}_{4} &= \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}_{\nu} \right] \operatorname{tr} \left[ \mathbf{V}^{\mu} \mathbf{V}^{\nu} \right] & \mathcal{L}_{9} &= \frac{1}{2} \operatorname{tr} \left[ \mathbf{T} \mathbf{W}_{\mu \nu} \right] \operatorname{tr} \left[ \mathbf{T} (\mathbf{V}^{\mu}, \mathbf{V}^{\nu} \right] \right] \\ \mathcal{L}_{5} &= \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}^{\mu} \right] \operatorname{tr} \left[ \mathbf{V}_{\nu} \mathbf{V}^{\nu} \right] & \mathcal{L}_{10} &= \frac{1}{2} \left( \operatorname{tr} \left[ \mathbf{T} \mathbf{V}_{\mu} \right] \operatorname{tr} \left[ \mathbf{T} \mathbf{V}^{\mu} \right] \right)^{2} \end{aligned}$$

Indirect info on new physics in  $\beta_1, \alpha_i, \ldots$  (Flavor physics only in *M*)

## EW Chiral Lagragian $\rightarrow$ Eff. Building Blocks

Originally for heavy Higgses or Higgsless models

$$\mathcal{L}_{\text{eff}} = -\sum_{\psi} \overline{\psi}_L \Sigma M \psi_R + \beta_1 \mathcal{L}'_0 + \sum_i \alpha_i \mathcal{L}_i$$

$$\begin{aligned} \mathcal{L}'_{0} &= \frac{v^{2}}{4} \mathrm{tr} \left[ \mathbf{T} \mathbf{V}_{\mu} \right] \mathrm{tr} \left[ \mathbf{T} \mathbf{V}^{\mu} \right] \\ \mathcal{L}_{1} &= \mathrm{tr} \left[ \mathbf{B}_{\mu\nu} \mathbf{W}^{\mu\nu} \right] \\ \mathcal{L}_{2} &= \mathrm{itr} \left[ \mathbf{B}_{\mu\nu} [\mathbf{V}^{\mu}, \mathbf{V}^{\nu} \right] \right] \\ \mathcal{L}_{3} &= \mathrm{itr} \left[ \mathbf{W}_{\mu\nu} [\mathbf{V}^{\mu}, \mathbf{V}^{\nu} \right] \right] \\ \mathcal{L}_{4} &= \mathrm{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}_{\nu} \right] \mathrm{tr} \left[ \mathbf{V}^{\mu} \mathbf{V}^{\nu} \right] \\ \mathcal{L}_{5} &= \mathrm{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}^{\mu} \right] \mathrm{tr} \left[ \mathbf{V}_{\nu} \mathbf{V}^{\nu} \right] \\ \mathcal{L}_{10} &= \frac{1}{2} \mathrm{tr} \left[ \mathbf{T} \mathbf{V}_{\mu} \right] \mathrm{tr} \left[ \mathbf{T} \mathbf{V}^{\mu} \right]^{2} \end{aligned}$$

Indirect info on new physics in  $\beta_1, \alpha_i, \ldots$  (Flavor physics only in *M*) Electroweak precision observables (LEP I/II, SLC):

$\Delta S = -16\pi\alpha_1$	$\alpha_1 = 0.0026 \pm 0.0020$
$\Delta T = 2\beta_1/\alpha_{QED}$	$\beta_1 = \ -0.00062 \pm 0.00043$
$\Delta U = -16\pi\alpha_8$	$\alpha_8 = -0.0044 \pm 0.0026$

### The Fundamental Building Blocks

- $\mathbf{V} = \Sigma (\mathbf{D}\Sigma)^{\dagger}$  (longitudinal vectors),  $\mathbf{T} = \Sigma \tau^3 \Sigma^{\dagger}$  (neutral component)
- Unitary gauge (no Goldstones):  $\mathbf{w} \equiv 0$ , i.e.,  $\Sigma \equiv 1$ .

$$\begin{split} \mathbf{V} &\longrightarrow -\frac{\mathrm{i}g}{2} \left[ \sqrt{2} (W^+ \tau^+ + W^- \tau^-) + \frac{1}{c_\mathrm{w}} Z \tau^3 \right] \\ \mathbf{T} &\longrightarrow \tau^3 \end{split}$$

• Gaugeless limit (only Goldstones)  $(g, g' \rightarrow 0)$ :

$$\begin{split} \mathbf{V} &\longrightarrow \frac{\mathrm{i}}{v} \bigg\{ \sqrt{2} \partial w^+ \tau^+ + \sqrt{2} \partial w^- \tau^- + \partial z \tau^3 \bigg\} + O(v^{-2}) \\ \mathbf{T} &\longrightarrow \tau^3 + 2\sqrt{2} \frac{\mathrm{i}}{v} \left( w^+ \tau^+ - w^- \tau^- \right) + O(v^{-2}) \end{split}$$

So T projects out the neutral part:

$$\operatorname{tr}\left[\mathbf{T}\mathbf{V}\right] = \frac{2\mathrm{i}}{v} \left[\partial z + \frac{\mathrm{i}}{v} \left(w^{+}\partial w^{-} - w^{-}\partial w^{+}\right)\right] + O(v^{-3})$$

### Anomalous triple and quartic gauge couplings

$$\begin{aligned} \mathcal{L}_{TGC} &= \mathrm{i}e\left[g_{1}^{\gamma}A_{\mu}\left(W_{\nu}^{-}W^{+\mu\nu} - W_{\nu}^{+}W^{-\mu\nu}\right) + \kappa^{\gamma}W_{\mu}^{-}W_{\nu}^{+}A^{\mu\nu} + \frac{\lambda^{\gamma}}{M_{W}^{2}}W_{\mu}^{-\nu}W_{\nu\rho}^{+}A^{\rho\mu}\right] \\ &+ \mathrm{i}e\frac{c_{\mathrm{w}}}{s_{\mathrm{w}}}\left[g_{1}^{Z}Z_{\mu}\left(W_{\nu}^{-}W^{+\mu\nu} - W_{\nu}^{+}W^{-\mu\nu}\right) + \kappa^{Z}W_{\mu}^{-}W_{\nu}^{+}Z^{\mu\nu} + \frac{\lambda^{Z}}{M_{W}^{2}}W_{\mu}^{-\nu}W_{\nu\rho}^{+}Z^{\rho\mu}\right] \end{aligned}$$

$$\begin{aligned} & \text{SM values: } g_1^{\gamma, Z} = \kappa^{\gamma, Z} = 1, \lambda^{\gamma, Z} = 0 \text{ and } \delta_Z = \frac{\beta_1 + g'^2 \alpha_1}{c_w^2 - s_w^2} \quad g_1^{VV'} = 1, h^{ZZ} = 0 \\ & \Delta g_1^{\gamma} = 0 & \Delta \kappa^{\gamma} = g^2 (\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2 (\alpha_9 - \alpha_8) \\ & \Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2} \alpha_3 & \Delta \kappa^Z = \delta_Z - g'^2 (\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2 (\alpha_9 - \alpha_8) \\ & \Delta g_1^{\gamma\gamma} = \Delta g_2^{\gamma\gamma} = 0 & \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4} (\alpha_5 + \alpha_7) \\ & \Delta g_1^{\gamma Z} = \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2} \alpha_3 & \Delta g_1^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2 (\alpha_9 - \alpha_8) + g^2 \alpha_4 \\ & \Delta g_1^{ZZ} = 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4} (\alpha_4 + \alpha_6) & \Delta g_2^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2 (\alpha_9 - \alpha_8) - g^2 (\alpha_4 + 2\alpha_5) \end{aligned}$$

$$h^{ZZ} = g^{2} \left[ \alpha_{4} + \alpha_{5} + 2 \left( \alpha_{6} + \alpha_{7} + \alpha_{10} \right) \right]$$

### Anomalous triple and quartic gauge couplings

$$\begin{aligned} \mathcal{L}_{QGC} &= e^2 \left[ g_1^{\gamma\gamma} A^{\mu} A^{\nu} W_{\mu}^{-} W_{\nu}^{+} - g_2^{\gamma\gamma} A^{\mu} A_{\mu} W^{-\nu} W_{\nu}^{+} \right] \\ &+ e^2 \frac{c_{w}}{s_{w}} \left[ g_1^{\gamma Z} A^{\mu} Z^{\nu} \left( W_{\mu}^{-} W_{\nu}^{+} + W_{\mu}^{+} W_{\nu}^{-} \right) - 2 g_2^{\gamma Z} A^{\mu} Z_{\mu} W^{-\nu} W_{\nu}^{+} \right] \\ &+ e^2 \frac{c_{w}^2}{s_{w}^2} \left[ g_1^{ZZ} Z^{\mu} Z^{\nu} W_{\mu}^{-} W_{\nu}^{+} - g_2^{ZZ} Z^{\mu} Z_{\mu} W^{-\nu} W_{\nu}^{+} \right] \\ &+ \frac{e^2}{2 s_{w}^2} \left[ g_1^{WW} W^{-\mu} W^{+\nu} W_{\mu}^{-} W_{\nu}^{+} - g_2^{WW} \left( W^{-\mu} W_{\mu}^{+} \right)^2 \right] + \frac{e^2}{4 s_{w}^2 c_{w}^4} h^{ZZ} (Z^{\mu} Z_{\mu})^2 \end{aligned}$$

 $\text{SM values: } g_1^{\gamma,Z} = \kappa^{\gamma,Z} = 1, \\ \lambda^{\gamma,Z} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2} \quad g_{1/2}^{VV'} = 1, \\ h^{ZZ} = 0 \text{ and } \delta_Z = \frac{\beta_1 + {g'}^2 \alpha_1}{c_{\rm w}^2 - s_{\rm w}^2}$ 

$$\Delta g_1^{\gamma} = 0 \qquad \qquad \Delta \kappa^{\gamma} = g^2 (\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2 (\alpha_9 - \alpha_8)$$
  
$$\Delta g_1^Z = \delta_Z + \frac{g^2}{c_w^2} \alpha_3 \qquad \qquad \Delta \kappa^Z = \delta_Z - g'^2 (\alpha_2 - \alpha_1) + g^2 \alpha_3 + g^2 (\alpha_9 - \alpha_8)$$

$$\begin{split} \Delta g_1^{\gamma\gamma} &= \Delta g_2^{\gamma\gamma} = 0 & \Delta g_2^{ZZ} = 2\Delta g_1^{\gamma Z} - \frac{g^2}{c_w^4} (\alpha_5 + \alpha_7) \\ \Delta g_1^{\gamma Z} &= \Delta g_2^{\gamma Z} = \delta_Z + \frac{g^2}{c_w^2} \alpha_3 & \Delta g_1^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2 (\alpha_9 - \alpha_8) + g^2 \alpha_4 \\ \Delta g_1^{ZZ} &= 2\Delta g_1^{\gamma Z} + \frac{g^2}{c_w^4} (\alpha_4 + \alpha_6) & \Delta g_2^{WW} = 2c_w^2 \Delta g_1^{\gamma Z} + 2g^2 (\alpha_9 - \alpha_8) - g^2 (\alpha_4 + 2\alpha_5) \\ h^{ZZ} &= g^2 [\alpha_4 + \alpha_5 + 2 (\alpha_6 + \alpha_7 + \alpha_{10})] \end{split}$$

## Classification of approaches

• Translation between operator bases:

Hagiwara/Ishihara/Szalapski/Zeppenfeld, 1993; Wudka,

1994; Degrande/Greiner/Kilian/Mattelaer/Mebane/Stelzer/Willenbrock/Zhang, 2012



- Effective Field Theory description valid, if
  - $\hat{s} \ll \Lambda^2$ : new physics out of direct LHC reach
  - Operator coefficients rather smallish, e.g.  $c_{WWW} \lesssim 1$
  - No large logarithms in the game (resummation)
- Relation  $\Delta g_1^Z = \Delta \kappa_Z + \tan^2 \theta_W \Delta \kappa_\gamma$  invalidated by dim 8 operators

### Classification of approaches

### **Remarks:**

- EFT approach leads to new interaction vertices
- Coupling constants are EFT Lagrangian parameters
- Framework for higher-order corrections straightforward (though rarely needed)
- ► Threshold/soft-collinear resummation ⇒ momentum-dependent couplings/form factors
- Anomalous couplings understood as effective vertices/vertex functions
- Nevertheless: Lagrangian for new physics reconstructable
- Parameterize new physics effects as new resonances/particles

### Parameters and Scales, Resonances

 $\alpha_i$ /operator coefficients measurable at LHC (and LC)

- $\alpha_i \ll 1$  (LEP)
- $\alpha_i \gtrsim 1/16\pi^2 \approx 0.006$  (renormalize divergencies,  $16\pi^2 \alpha_i \gtrsim 1$ )

Translation of parameters into new physics scale  $\Lambda$ :  $\alpha_i = v^2/\Lambda^2$ 

- Operator normalization is arbitrary
- Power counting can be intricate

To be specific: consider resonances that couple to EWSB sector Resonance mass gives detectable shift in the  $\alpha_i$ 

- Narrow resonances  $\Rightarrow$  particles
- Wide resonances  $\Rightarrow$  continuum

 $eta_1 \ll 1 \ \Rightarrow SU(2)_c \ {\rm custodial\ symmetry}$  (weak isospin, broken by hypercharge g' 
eq 0 and fermion masses)



accounts for weakly and strongly interacting models

### Integrating out resonances

Consider leading order effects of resonances on EW sector:

 $\mathcal{L}_{\Phi} = z \left[ \Phi \left( M_{\Phi}^2 + DD \right) \Phi + 2 \Phi J \right] \qquad \Rightarrow \qquad \mathcal{L}_{\Phi}^{\text{eff}} = -\frac{z}{M^2} J J + \frac{z}{M^4} J (DD) J + \mathcal{O}(M^{-6})$ 

Simplest example: scalar singlet *σ*:

$$\mathcal{L}_{\sigma} = -\frac{1}{2} \left[ \boldsymbol{\sigma} (M_{\sigma}^2 + \partial^2) \boldsymbol{\sigma} - g_{\sigma} v \boldsymbol{\sigma} \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}^{\mu} \right] - h_{\sigma} \operatorname{tr} \left[ \mathbf{T} \mathbf{V}_{\mu} \right] \operatorname{tr} \left[ \mathbf{T} \mathbf{V}^{\mu} \right] \right]$$

- Effective Lagrangian  $\mathcal{L}_{\sigma}^{\text{eff}} = \frac{v^2}{8M_{\sigma}^2} \left\{ g_{\sigma} \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}^{\mu} \right] + h_{\sigma} \operatorname{tr} \left[ \mathbf{T} \mathbf{V}_{\mu} \right] \operatorname{tr} \left[ \mathbf{T} \mathbf{V}^{\mu} \right] \right\}^2$
- leads to anomalous quartic couplings

$$\alpha_{5} = g_{\sigma}^{2} \left( \frac{v^{2}}{8M_{\sigma}^{2}} \right) \qquad \alpha_{7} = 2g_{\sigma}h_{\sigma} \left( \frac{v^{2}}{8M_{\sigma}^{2}} \right) \qquad \alpha_{10} = 2h_{\sigma}^{2} \left( \frac{v^{2}}{8M_{\sigma}^{2}} \right) \qquad a^{\alpha_{5}}$$

$$\frac{\text{Resonance}}{\Delta \alpha_{4}[(16\pi\Gamma/M)(v^{4}/M^{4})]} \qquad b^{\alpha_{10}} = \frac{\sigma}{4} + \frac{\sigma}{3} + \frac{\sigma}{5} + \frac{1}{30} + \frac{\sigma}{5} +$$

## Anomalous Gauge Couplings at LHC

Anomalous quartic gauge couplings, by chiral EW Lagrangian:

$$\begin{split} \mathcal{L}_{4} &= \alpha_{4} \frac{g^{2}}{2} \left\{ \left[ (W^{+}W^{+})(W^{-}W^{-}) + (W^{+}W^{-})^{2} \right] + \frac{2}{c_{W}^{2}} (W^{+}Z)(W^{-}Z) + \frac{1}{2c_{W}^{4}} (ZZ)^{2} \right\} \\ \mathcal{L}_{5} &= \alpha_{5} \frac{g^{2}}{2} \left\{ (W^{+}W^{-})^{2} + \frac{2}{c_{W}^{2}} (W^{+}W^{-})(ZZ) + \frac{1}{2c_{W}^{4}} (ZZ)^{2} \right\} \end{split}$$

(all leptons, incl.  $\tau$ ):  $D(x_2, Q^2)$ 



 $pp \to jj(ZZ/WW) \to jj\ell^-\ell^+\nu_\ell\bar{\nu}_\ell$ 

 $\sigma\approx 40\,{\rm fb}$ 

### Background:

- $t\bar{t} \rightarrow WbWb, \sigma \approx 52 \, \text{pb}$
- Single t, misrec. jet:  $\sigma \approx 4.8 \, \mathrm{pb}$
- QCD:  $\sigma \approx 0.21 \, \text{pb}$

Mertens, 2006

## **Tagging and Cuts:**

- ▶  $\ell\ell jj$ -Tag,  $\eta_{tag}^{min} < \eta_{\ell} < \eta_{tag}^{max}$ , b-Veto
- ►  $|\Delta \eta_{jj}| > 4.4$ ,  $M_{jj} > 1080 \, {\rm GeV}$
- Minijet-Veto:  $p_{T,j} < 30 \,\text{GeV}$
- ▶  $E_j > 600, 400 \,\text{GeV}, \quad p_{T,j}^1 > 60, 24 \,\text{GeV}$

Events / Bin 5200 5000 WHIZARDIEWI α. = 0.003 . x5 WHIZARD[EW] a4 = 0.003 , x100 # Events / ttbat Singletop Singletop WHIZARD[QCD], x10 WHIZARDIOCDI 2000 30000 1500 20000 1000 10000 500 0 1500 10 20 70 80 90 10 P. (Minijet,max) [GeV] M (jet,jet,) [GeV] Events / Bi Events / Bii α<sub>4</sub> = 0 (SM) 600 350 ····· a, = 0.006  $\alpha_{4} = 0 (SM)$  $-\alpha_{.} = 0.01$ ····· α<sub>4</sub> = 0.006 300 - α. = 0.01 300 200 100 100 1500 2000 2500 3000 M (jet, jet,) [GeV] P. (Minijet,max) [GeV]

Improves  $S/\sqrt{B}$  from 3.3 to 29.7



**Results:** (1 $\sigma$  Sensitivity to  $\alpha$ s)

Coupl.	LHC ( $100  \text{fb}^{-1}$ )	$   LC (1 ab^{-1})  $
$\alpha_4$	0.00160	0.0088
$lpha_5$	0.00098	0.0071

Limits for  $\Lambda$  [TeV]:

Spin	I = 0	I = 1	I=2
0	1.39	1.55	1.95
1	1.74	2.67	_
2	3.00	3.01	5.84



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### **Different Selection Criteria**

- General selection criteria
  - exactly 2 leptons within detector acceptance,
  - ▶ 2 tag jets with  $2 < |\eta_j| < 5$  and opposite directions,
  - no b-tag
  - $M_{j_1l_2}, M_{j_2l_1} > 200 \text{ GeV}$
  - ▶ M<sub>jj</sub> > 400 GeV
  - $\Delta R_{jl} > 0.4$
  - ▶  $p_T^{l_1}, p_T^{l_2} > 40 \text{ GeV}$
  - ▶  $|\eta_{l_1}|, |\eta_{l_2}| < 1.5$
  - $\Delta \phi_{ll} > 2.5$
  - ▶ M<sub>ll</sub> > 200 GeV
- Proposal of new variable

Doroba/Kalinowski/Kuczmarski/Pokorski/Rosiek/Szleper/Tkaczyk, 1201.2768

$$R_{p_T} = (p_T^{\ l_1} \cdot p_T^{\ l_2}) \ / \ (p_T^{\ j_1} \cdot p_T^{\ j_2})$$

Proposal of new variable

Doroba/Kalinowski/Kuczmarski/Pokorski/Rosiek/Szleper/Tkaczyk, 1201.2768

$$R_{p_T} = (p_T^{\ l_1} \cdot p_T^{\ l_2}) / (p_T^{\ j_1} \cdot p_T^{\ j_2})$$

• Works well for  $W^{\pm}W^{\pm}$ , not feasible for  $W^+W^-$ 



- Might allow to relax jet vetoes: gain for high pile-up!
- Remark: EWA works for selection, but shapes need not be the same

## Unitarity of Amplitudes

### UV-incomplete theories could violate unitarity

Cross section: 
$$\sigma = \int d\Omega \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^{2}s} |\mathcal{M}|^{2}$$

Optical Theorem (Unitarity of the S(cattering) Matrix):  $\sigma_{tot} = \text{Im} \left[ \mathcal{M}_{ii}(t=0) \right] / s$   $t = -s(1 - \cos \theta)/2$ 

Partial wave amplitudes:  $\mathcal{M}(s,t,u) = 32\pi \sum_{\ell} (2\ell+1)\mathcal{A}_{\ell}(s)P_{\ell}(\cos\theta)$ 

### Assuming only elastic scattering:

$$\sigma_{\rm tot} = \sum_{\ell} \frac{32\pi(2\ell+1)}{s} |\mathcal{A}_{\ell}|^2 \stackrel{!}{=} \sum_{\ell} \frac{32\pi(2\ell+1)}{s} {\rm Im}\left[\mathcal{A}_{\ell}\right] \quad \Rightarrow \quad \left[|\mathcal{A}_{\ell}|^2 = {\rm Im}\left[\mathcal{A}_{\ell}\right]\right]$$



Argand circle $|\mathcal{A}(s) - \frac{i}{2}| = \frac{1}{2}$ Resonance: $\mathcal{A}(s) = \frac{-M\Gamma_{el}}{s - M^2 + iM\Gamma_{tot}}$ Counterclockwise circle, radius  $\frac{x_{el}}{2}$ Pole at  $s = M^2 - iM\Gamma_{tot}$ 

## Unitarity in the EW sector: SM

Project out isospin eigenamplitudes

Lee, Quigg, Thacker, 1973

$$\mathcal{A}_{\ell}(s) = \frac{1}{32\pi} \int_{-s}^{0} \frac{dt}{s} \mathcal{A}(s, t, u) P_{\ell}(1 + 2t/s) \qquad \cos \theta = 1 + 2t/s$$

Remember Legendre polynomials:  $P_0(s) = 1$   $P_1(s) = \cos \theta$   $P_2(s) = (3\cos^2 \theta - 1)/2$ 

SM longitudinal isospin eigenamplitudes  $(A_{I,spin=J})$ :

$$\mathcal{A}_{I=0} = 2\frac{s}{v^2} P_0(s) \qquad \mathcal{A}_{I=1} = \frac{t-u}{v^2} = \frac{s}{v^2} P_1(s) \qquad \mathcal{A}_{I=2} = -\frac{s}{v^2} P_0(s)$$
$$\boxed{\mathcal{A}_{0,0} = \frac{s}{16\pi v^2}} \qquad \boxed{\mathcal{A}_{1,1} = \frac{s}{96\pi v^2}} \qquad \boxed{\mathcal{A}_{2,0} = -\frac{s}{32\pi v^2}}$$

exceeds unitarity bound  $|A_{IJ}| \lesssim \frac{1}{2}$  at:

$$I = 0: \qquad E \sim \sqrt{8\pi}v = 1.2 \,\mathrm{TeV}$$

$$I = 1: \qquad E \sim \sqrt{48\pi}v = 3.5 \,\mathrm{TeV}$$

$$I = 2:$$
  $E \sim \sqrt{16\pi}v = 1.7 \text{ TeV}$ 

Higgs exchange:  $\mathcal{A}(s,t,u) = -\frac{M_H^2}{v^2} \frac{s}{s - M_H^2}$ Unitarity:  $M_{TC} \leq \sqrt{8\pi} v \approx 1.2 \text{ Tc}$ 

Unitarity:  $M_H \lesssim \sqrt{8\pi} v \sim 1.2 \text{ TeV}$ 

Isospin decomposition
 ► Lowest order chiral Lagrangian (incl. anomalous couplings)

$$\mathcal{L} = -\frac{v^2}{4} \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}^{\mu} \right] + \frac{\alpha_4}{4} \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}_{\nu} \right] \operatorname{tr} \left[ \mathbf{V}^{\mu} \mathbf{V}^{\nu} \right] + \frac{\alpha_5}{4} \left( \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}^{\mu} \right] \right)^2$$

Leads to the following amplitudes:  $s = (p_1 + p_2)^2$   $t = (p_1 - p_3)^2$   $u = (p_1 - p_4)^2$ 

$$\begin{array}{c} \hline \mathcal{A}(s,t,u) =: \\ \hline \mathcal{A}(w^+w^- \to zz) = & \frac{s}{v^2} & +8\alpha_5 \frac{s^2}{v^4} + 4\alpha_4 \frac{t^2 + u^2}{v^4} \\ \\ \mathcal{A}(w^+z \to w^+z) = & \frac{t}{v^2} & +8\alpha_5 \frac{t^2}{v^4} + 4\alpha_4 \frac{s^2 + u^2}{v^4} \\ \\ \mathcal{A}(w^+w^- \to w^+w^-) = -\frac{u}{v^2} & +(4\alpha_4 + 2\alpha_5) \frac{s^2 + t^2}{v^4} + 8\alpha_4 \frac{u^2}{v^4} \\ \\ \mathcal{A}(w^+w^+ \to w^+w^+) = -\frac{s}{v^2} & +8\alpha_4 \frac{s^2}{v^4} + 4(\alpha_4 + 2\alpha_5) \frac{t^2 + u^2}{v^4} \\ \\ \mathcal{A}(zz \to zz) = & 8(\alpha_4 + \alpha_5) \frac{s^2 + t^2 + u^2}{v^4} \end{array}$$

(Clebsch-Gordan) Decomposition into isospin eigenamplitudes 

$$\begin{aligned} \mathcal{A}(I=0) &= \ 3\mathcal{A}(s,t,u) + \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t) \\ \mathcal{A}(I=1) &= \ \mathcal{A}(t,s,u) - \mathcal{A}(u,s,t) \\ \mathcal{A}(I=2) &= \ \mathcal{A}(t,s,u) + \mathcal{A}(u,s,t) \end{aligned}$$

## BSM (Unitarized) Resonances: e.g. Scalar Singlet

### Assumptions:

- LHC is able to detect a resonance in the EW sector
- Further resonances might exist, but out of reach or not detectable
- Describe 1st resonance by correct amplitude
- Use K-matrix unitarization to define a consistent model

### Example: Scalar Singlet

$$\boldsymbol{\mathcal{L}}_{\sigma} = -\frac{1}{2}\sigma \left( M_{\sigma}^2 + \partial^2 \right) \sigma + \frac{g_{\sigma} v}{2} \sigma \operatorname{tr} \left[ \mathbf{V}_{\mu} \mathbf{V}^{\mu} \right]$$

Feynman rules:  $\sigma w^+ w^- : -\frac{2ig_\sigma}{v}(k_+ \cdot k_-) \qquad \sigma zz: -\frac{2ig_\sigma}{v}(k_1 \cdot k_2)$ 

$$\mathcal{A}^{\sigma}(s,t,u) = \frac{g_{\sigma}^2}{v^2} \frac{s^2}{s - M^2}$$

Isospin eigenamplitudes:

$$\begin{aligned} \mathcal{A}_{0}^{\sigma}(s,t,u) &= \frac{g_{\sigma}^{2}}{v^{2}} \left( 3\frac{s^{2}}{s-M^{2}} + \frac{t^{2}}{t-M^{2}} + \frac{u^{2}}{u-M^{2}} \right) \\ \mathcal{A}_{1}^{\sigma}(s,t,u) &= \frac{g_{\sigma}^{2}}{v^{2}} \left( \frac{t^{2}}{t-M^{2}} - \frac{u^{2}}{u-M^{2}} \right) \\ \mathcal{A}_{2}^{\sigma}(s,t,u) &= \frac{g_{\sigma}^{2}}{v^{2}} \left( \frac{t^{2}}{t-M^{2}} + \frac{u^{2}}{u-M^{2}} \right) \end{aligned}$$

### K-Matrix Unitarization and friends K-Matrix unitarization

$$\mathcal{A}_K(s) = \frac{\mathcal{A}(s)}{1 - i\mathcal{A}(s)} = \mathcal{A}(s)\frac{1 + i\mathcal{A}(s)}{1 + \mathcal{A}(s)^2}$$

Unitarization by infinitely heavy and wide resonance



Padé unitarization separates higher chiral orders  $\mathcal{A}_P(s) = \frac{\mathcal{A}^{(0)}(s)^2}{\mathcal{A}^{(0)}(s) - \mathcal{A}^{(1)}(s) - i\mathcal{A}^{(0)}(s)^2}$ each partial wave dominated by single resonance



- Low-energy theorem (LET):  $\frac{s}{v^2}$
- ► K-Matrix amplitude:  $|\mathcal{A}(s)|^2 = \frac{s^2}{s^2 + v^4} \xrightarrow{s \to \infty} 1$
- Poles  $\pm iv$ :  $M_0$ ,  $\Gamma$  large

"Naive" Unitarization

$$\mathcal{A}_N(s) = e^{i\mathcal{A}(s)}\sin\mathcal{A}(s)$$

Infinitely many resonances becoming denser for  $s \to \infty$ 

### Unitarizing the scalar singlet

### Alboteanu/Kilian/JRR, 2008

$$\begin{aligned} \mathcal{A}_{00}^{\sigma}(s) &= 3\frac{g_{\sigma}^{2}}{v^{2}}\frac{s^{2}}{s-M^{2}} + 2\frac{g^{2}}{v^{2}}\mathcal{S}_{0}(s) \qquad \qquad \mathcal{A}_{02}^{\sigma}(s) &= 2\frac{g_{\sigma}^{2}}{v^{2}}\mathcal{S}_{2}(s) = A_{22}^{\sigma}(s) \\ \mathcal{A}_{11}^{\sigma}(s) &= 2\frac{g_{\sigma}^{2}}{v^{2}}\mathcal{S}_{1}(s) \qquad \qquad \mathcal{A}_{13}^{\sigma}(s) = 2\frac{g_{\sigma}^{2}}{v^{2}}\mathcal{S}_{3}(s) \\ \mathcal{A}_{20}^{\sigma}(s) &= 2\frac{g_{\sigma}^{2}}{v^{2}}\mathcal{S}_{0}(s) \end{aligned}$$

S-wave coefficients no longer polynomial, e.g.:

$$S_0(s) = M^2 - \frac{s}{2} + \frac{M^4}{s} \log \frac{s}{s+M^2}$$

s-channel pole must be explicitly subtracted:

$$A_{IJ}(s) = A_{IJ}^{(0)}(s) + F_{IJ}(s) + \frac{G_{IJ}(s)}{s - M^2},$$

- $F_{IJ}(s)$  is finite
- $G_{IJ}(s) \propto s$  (vector),  $\propto s^2$  (scalar, tensor)

$$\hat{A}_{IJ}(s) = \frac{A_{IJ}(s)}{1 - \frac{i}{32\pi} A_{IJ}(s)} = A_{IJ}^{(0)}(s) + 32\pi i \Delta A_{IJ}(s),$$

$$\Delta A_{IJ}(s) = 32\pi i \left( 1 + \frac{i}{32\pi} A_{IJ}^{(0)}(s) + \frac{s - M^2}{\frac{i}{32\pi} G_{IJ}(s) - (s - M^2) \left[ 1 - \frac{i}{32\pi} (A_{IJ}^{(0)}(s) + F_{IJ}(s)) \right]} \right)$$

### Eigenamplitudes



### "Partonic" cross sections



- $\sigma(\mathcal{V}\mathcal{V}\to\mathcal{V}\mathcal{V})$  in nb  $M_R=500~{\rm GeV}$
- all amplitudes K-matrix unitarized
- ▶ Cut of 15° around the beam axis

## LHC Example: Vector Isovector

### Alboteanu/Kilian/JRR,

### 2008

- Example: 850 GeV vector resonance, coupling g<sub>ρ</sub> = 1
- (Theory) Cuts:
  - $p_{\perp}(\ell\nu) > 30 \text{ GeV}$
  - $|\delta R(\ell\nu)| < 1.5$
  - $\theta(u/d) > 0.5^\circ$
- Integrated luminosity: 225 fb<sup>-1</sup>
- Discriminator: angular correlations  $\Delta \phi(\ell \ell)$
- Ongoing ATLAS study
  - More kinematic observables
  - Comparison and validation phase
  - first reproduce SM
  - then anom. couplings/BSM resonances



### **Including Higgs Operators**

- Higgs has been discovered (sic!)
- Include more operators, e.g. (D<sub>µ</sub>φ)<sup>†</sup>W<sup>µν</sup>(D<sub>ν</sub>φ), (∂(φ<sup>†</sup>φ))<sup>2</sup>: usually called O<sub>B</sub>, O<sub>W</sub>, O<sub>WW</sub>, O<sub>WWW</sub> etc.
- ▶ both anom.  $V^3 + V^4$  and HVV etc. couplings !
- Implemented for an ATLAS study in WHIZARD



- Ongoing theoretical study
- Very preliminary results

Kilian/JRR/Sekulla, 2013

## Matrix Element Method (MEM)

Kondo, 1988; Dalitz/Goldstein, 1992; CDF; DØ; Freitas/Gainer, arXiv:1212.3598

- Construct a likelihood from the squared matrix elements
- Marginalize over invisible particles
- Case study for  $pp \rightarrow jjW^+W^+$

Freitas/Gainor, 2012

Up to now only compared to dilepton mass: m<sub>ll</sub>



 Important cross-check for experimentalists: Cut-based vs. MVA vs. MEM

## Summary/Conclusions

- New Physics in EW effective Lagrangian (SM + higher-dim. op.)
- Triple/Quartic gauge couplings measured either
  - via diboson production
  - via triple boson production
  - via vector boson scattering
- Unified description for different channels difficult
- EFT approach for low-energy regime, unitarized by form factors in resonance scheme at high energies
- interpreted as resonances coupled to EW bosons
- "Correct" description for first resonance (also [very] broad)
- Beyond that: assure unitarity (K matrix)
- Approach includes standard EFT ansatz
- Sensitivity rises with number of intermediate states:
  - LHC sensitivity limited in pure EW sector: 0.8 3 TeV (???)
  - ILC
- More and intensive studies needed

 $: 1.5 - 6 \,\text{TeV}$ 

### AQGC Workshop Dresden 30.9.-2.10.2013



_	
20/20	Bouter
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# Backup: The Effective W approximation

•  $M_{\mathcal{V}}, \hat{t}_i$  small corrections,  $\mathcal{V}$  nearly onshell:

$$\sigma(q_1 q_2 \to q_1' q_2' \mathcal{V}_1' \mathcal{V}_2') \approx \sum_{\lambda_1, \lambda_2} \int dx_1 \, dx_2 \, F_{q_1 \to q_1' \mathcal{V}_1}^{\lambda_1}(x_1) \, F_{q_2 \to q_2' \mathcal{V}_2}^{\lambda_2}(x_2) \, \sigma_{\mathcal{V}_1 \mathcal{V}_2 \to \mathcal{V}_1' \mathcal{V}_2'}^{\lambda_1 \lambda_2}(x_1 x_2 s)$$

In addition to Weizsäcker-Williams: longitudinal polarisation

$$\begin{split} F_{q \rightarrow q' \mathcal{V}}^+(x) &= \frac{(V-A)^2 + (V+A)^2(1-x)^2}{16\pi^2 x} \left[ \ln \left( \frac{p_{\perp,\max}^2 + (1-x)m_{\mathcal{V}}^2}{(1-x)m_{\mathcal{V}}^2} \right) - \frac{p_{\perp,\max}^2}{p_{\perp,\max}^2 + (1-x)m_{\mathcal{V}}^2} \right] \\ F_{q \rightarrow q' \mathcal{V}}^-(x) &= \frac{(V+A)^2 + (V-A)^2(1-x)^2}{16\pi^2 x} \left[ \ln \left( \frac{p_{\perp,\max}^2 + (1-x)m_{\mathcal{V}}^2}{(1-x)m_{\mathcal{V}}^2} \right) - \frac{p_{\perp,\max}^2}{p_{\perp,\max}^2 + (1-x)m_{\mathcal{V}}^2} \right] \\ F_{q \rightarrow q' \mathcal{V}}^0(x) &= \frac{V^2 + A^2}{8\pi^2} \frac{2(1-x)}{x} \frac{p_{\perp,\max}^2}{p_{\perp,\max}^2 + (1-x)m_{\mathcal{V}}^2} \\ \\ \text{Dominant contribution from small } \mathcal{V} \text{ virtualities} \\ \\ \text{Transverse momentum cutoff } p_{\perp,\max} \leq (1-x)\sqrt{s}/2; \\ & \models \text{ longitudinal pol.: finite for } p_{\perp,\max} \rightarrow \infty \\ & \models \text{ Transversal pol.: logarithmic singularity} \end{split}$$

► EWA structure functions: W (left) and Z (right)



- Emission from  $u, \sqrt{s} = 2 \text{ TeV}$ emission

preferred at high energy: transversal

Problem: Irreducible background to weak-boson scattering



- Double ISR/FSR *t*-channel like diagrams
- Coulomb-singularity (peak): cut on  $p_{T,V} \gtrsim 30 \text{ GeV}$



- Effective W approx. vs. WHIZARD full matrix elements
- Shapes/normalization of distributions heavily affected
- EWA: Sideband subtraction completely screwed up!

## Backup: ILC example: Triboson production

 $e^+e^- \rightarrow WWZ/ZZZ$ , dep. on  $(\alpha_4 + \alpha_6)$ ,  $(\alpha_5 + \alpha_7)$ ,  $\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})$ Polarization populates longitudinal modes, suppresses SM bkgd.



Simulation with WHIZARD Kilian/Ohl/JR 1 TeV, 1 ab<sup>-1</sup>, full 6-fermion final states, SIMDET fast simulation Observables:  $M_{WW}^2$ ,  $M_{WZ}^2$ ,  $\triangleleft(e^-, Z)$ A) unpol., B) 80%  $e_R^-$ , C) 80%  $e_R^-$ , 60%  $e_L^+$ 

	WWZ			ZZZ	best
$16\pi^2 \times$	no pol.	$e^-$ pol.	both pol.	no pol.	
$\Delta \alpha_4^+$	9.79	4.21	1.90	3.94	1.78
$\Delta \alpha_4^-$	-4.40	-3.34	-1.71	-3.53	-1.48
$\Delta \alpha_5^+$	3.05	2.69	1.17	3.94	1.14
$\Delta \alpha_5^{-}$	-7.10	-6.40	-2.19	-3.53	-1.64

32 % hadronic decays Durham jet algorithm Bkgd.  $t\bar{t} \rightarrow 6$  jets Veto against  $E_{\rm mis}^2 + p_{\perp,\rm mis}^2$ No angular correlations yet

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## Vector Boson Scattering

1 TeV,  $1 \text{ ab}^{-1}$ , full 6f final states, 80 %  $e_R^-$ , 60 %  $e_L^+$  polarization, binned likelihood

Contributing channels:  $WW \rightarrow WW, WW \rightarrow ZZ, WZ \rightarrow WZ, ZZ \rightarrow ZZ$ 

Process	Subprocess	σ [fb]
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q \bar{q} q \bar{q}$	$WW \rightarrow WW$	23.19
$e^+e^- \rightarrow \nu_e \bar{\nu}_e q \bar{q} q \bar{q}$	$WW \rightarrow ZZ$	7.624
$e^+e^- \rightarrow \nu \bar{\nu} q \bar{q} q \bar{q}$	$V \rightarrow VVV$	9.344
$e^+e^- \rightarrow \nu e q \bar{q} q \bar{q}$	$WZ \rightarrow WZ$	132.3
$e^+e^- \rightarrow e^+e^-q\bar{q}q\bar{q}$	$ZZ \rightarrow ZZ$	2.09
$e^+e^- \rightarrow e^+e^-q\bar{q}q\bar{q}$	$ZZ \rightarrow W^+W^-$	414.
$e^+e^- \rightarrow b\bar{b}X$	$e^+e^- \rightarrow t\bar{t}$	331.768
$e^+e^- \rightarrow q\bar{q}q\bar{q}$	$e^+e^- \rightarrow W^+W^-$	3560.108
$e^+e^- \rightarrow q\bar{q}q\bar{q}$	$e^+e^- \rightarrow ZZ$	173.221
$e^+e^- \rightarrow e\nu q\bar{q}$	$e^+e^- \rightarrow e\nu W$	279.588
$e^+e^- \rightarrow e^+e^-q\bar{q}$	$e^+e^- \rightarrow e^+e^-Z$	134.935
$e^+e^- \rightarrow X$	$e^+e^- \rightarrow q\bar{q}$	1637.405

 $SU(2)_c$  conserved case, all channels

coupling	$\sigma-$	$\sigma +$
$16\pi^2 \alpha_4$	-1.41	1.38
$16\pi^2 \alpha_5$	-1.16	1.09

### $SU(2)_c$ broken case, all channels

coupling	$\sigma -$	$\sigma +$
$16\pi^2 \alpha_4$	-2.72	2.37
$16\pi^2 \alpha_5$	-2.46	2.35
$16\pi^2 \alpha_6$	-3.93	5.53
$16\pi^2 \alpha_7$	-3.22	3.31
$16\pi^{2}\alpha_{10}$	-5.55	4.55



200

### Backup: Interpretation as limits on resonances

Consider the width to mass ratio,  $f_{\sigma} = \Gamma_{\sigma}/M_{\sigma}$ 

### SU(2) conserving scalar singlet

### SU(2) broken vector triplet

needs input from TGC covariance matrix



f = 1.0 (full), 0.8 (dash), 0.6 (dot-dash), 0.3 (dot)



upper/lower limit from  $\lambda_Z$ , grey area: magnetic moments

	Spin	I = 0	I = 1	I=2	Spin	I = 0	I = 1	I=2
Final	0	1.55	_	1.95	0	1.39	1.55	1.95
result:	1	-	2.49	_	1	1.74	2.67	_
	2	3.29	_	4.30	2	3.00	3.01	5.84