Position Response of the Polarimeter Transversely Segmented Calorimeter

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Hera Compton transverse polarimetry is based on measurements of scattered photons azimuthal distributions using a segmented calorimeter where vertical displacement y of an incoming photon is introducing an up-down energy asymmetry

$$\eta = \frac{E_u - E_d}{E_u + E_d} \tag{1}$$

with energies E_u, E_d measured by the calorimeter upper and lower parts respectively [1]. Therefor, spatial response $S(y, \eta)$ of the detector is essential for unfolding parent distribution $\Sigma(y)$ and calculating beam polarization. For a measured distribution M one can write

$$M(\eta) = \int \Sigma(y) S(y, \eta) dy \tag{2}$$

which actually is a definition of the response function.

Relationship between y and η has been measured at DESY and CERN test beams beginning of 1990s for the first and in 1997 for the second calorimeter [2], the latter was tested also with a preshower in 2001 [3]. A $y - \eta$ distribution in energy range of 8-14 GeV, measured at CERN X1 beam in 1997 is shown on Fig.1.

Following an approach described in [1] the authors of [2] and [3] approximated y-slices of the η vs y 2dim histogram by normal distributions and then the obtained gaussian means are parameterized by a function f(y) being sum of exponents with number of parameters 3 or more.

Open points of this approach are:

- all but central y-slices have asymmetric forms of distribution

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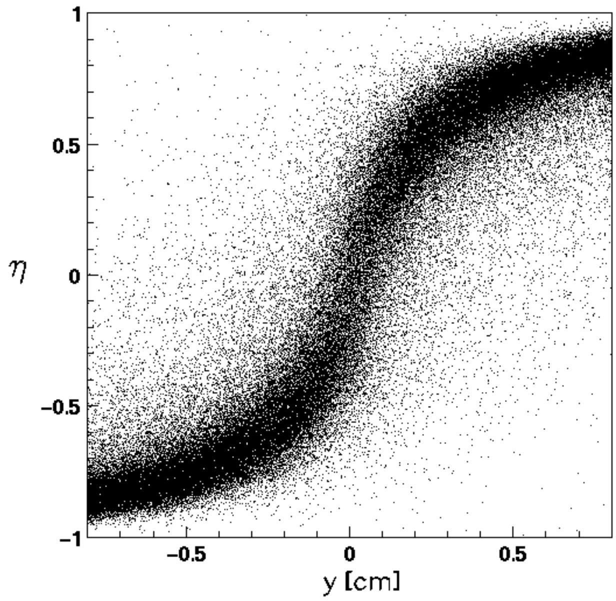

Fig. 1. 1997 CERN X1 test beam η-y measurements for energies from 8 to 14 GeV.

- it is difficult to associate exponential parameters with physics of shower
- using only the $\eta = f(y)$ transformation is a leading order approximation equivalent to substituting the response function by Dirac delta function $S(y, \eta) = \delta(y - f^{-1}(\eta))$ where $f^{-1}(\eta) \equiv g(\eta)$ is inverse of the f.

We will address these questions utilizing data shown on Fig.1. Let start from fitting the asymmetric y-slices by logarithmic gaussian function of the form

$$s(\eta) = \frac{N}{\sigma_\eta} exp\Big(-\frac{\sigma_0^2}{2} - \frac{1}{2\sigma_0^2}\ln\big(1 - \frac{\eta - \eta_0}{\sigma_\eta}\alpha\big)\Big) \tag{3}$$

with

$$\sigma_0 = \frac{1}{\sqrt{\ln 4}} \sinh^{-1}(\alpha\sqrt{\ln 4})$$

and free parameters N, η_0, σ_η, α as normalization factor, average, width, asymmetry coefficient respectively. The function is adopted from ref. [4] where authors use it as the BELLE CsI(Tl) calorimeter energy response, the only change is reduction of the N by factor $1/\sigma_\eta$ which we add to account for η distributions height dependence on y.

Fitted samples of the 0.5 mm y-slices for y range from -8 mm to 8 mm are shown on Fig.2. The Log-Normal function is well reproducing shapes of η distributions and collecting the fitting parameters at minimized χ^2 values we proceed to next step to parameterize η_0, σ_η and α dependence on y (see Fig.3).

Following an electromagnetic shower radial distribution formula described in

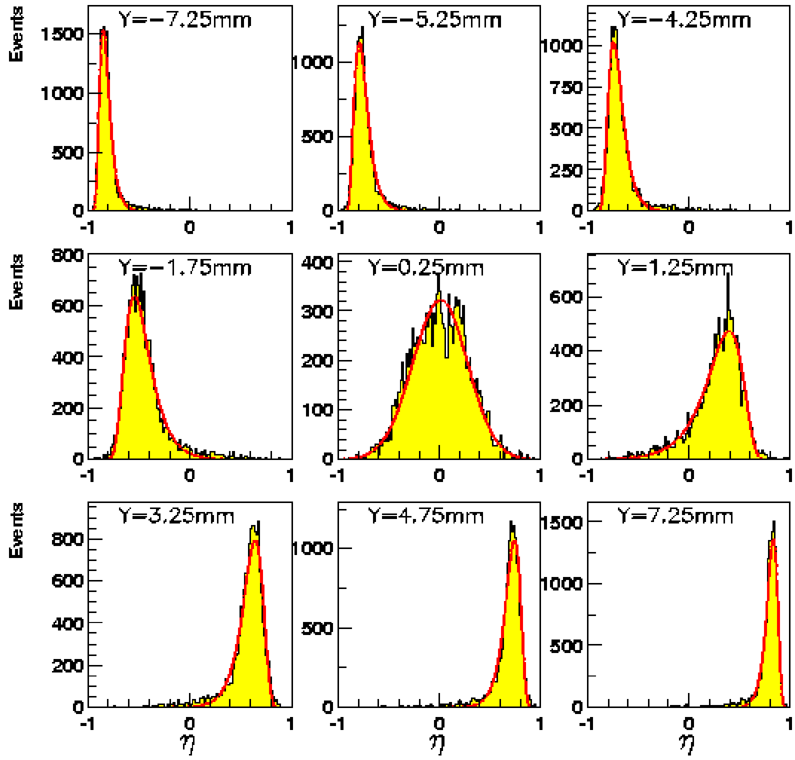

Fig. 2. Log-Normal fits to y-slices. Center of the y bin is shown on each plot.

ref. [5a–b] one easily derives a functional form for the f

$$\eta_0 = f(y) = p\frac{y}{\sqrt{y^2 + R_C^2}} + (1-p)\frac{y}{\sqrt{y^2 + R_T^2}} \tag{4}$$

with $0 \leq p \leq 1$, where R_C (R_T) is median of the shower core (tail) contributing with a probability p $(1-p)$. Best fitted function has parameters $R_C = (6.67 \pm 0.04)$mm, $R_T = (1.01 \pm 0.02)$mm, $p = 0.568 \pm 0.004$ where errors are calculated by minimization algorithm [6].

Since the (4) does not have analytic solution to find the inverse transformation $y = g(\eta_0)$, we apply a polynomial fit to η_0 vs y distribution which shows that data can be satisfactory fitted varying only 2 parameters a and b

$$y = a\eta_0 + b\eta_0^5 \tag{5}$$

with final $a = (2.53 \pm 0.06)$mm and $b = (11.88 \pm 0.16)$mm.

In a fit of the σ_η vs η_0 we use a heuristic function

$$\sigma_\eta = \frac{S_1}{\sqrt{S_2 + \eta_0^2}} + \frac{S_4}{S_3 + \eta_0^2} \tag{6}$$

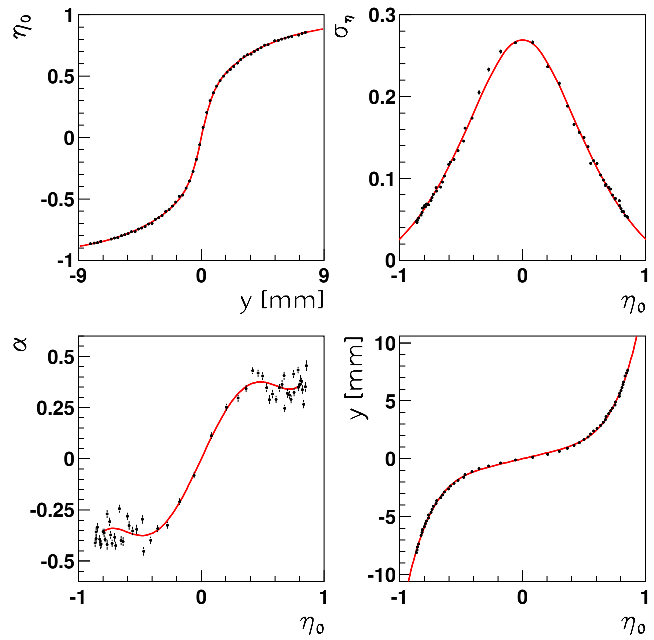

Fig. 3. Parameterization of the $\eta_0 = f(y)$, σ_η, α and $y = g(\eta_0)$ for 0.25 mm y-slices. Fitted function is plotted in red.

with converged parameters

$$S_1 = -0.23 \pm 0.01$$

$$S_2 = 3.8 \pm 0.5$$

$$S_3 = 0.509 \pm 0.011$$

$$S_4 = 0.197 \pm 0.006$$

while for the α vs η_0 a polynomial

$$\alpha = A_1\eta_0 + A_2\eta_0^3 + A_3\eta_0^5 \tag{7}$$

with

$$A_1 = 1.285 \pm 0.027$$

$$A_2 = -2.678 \pm 0.121$$

$$A_3 = 2.142 \pm 0.127$$

Applying these parameterizations, for any given y, we derive the η_0, σ_η, α from the equations (4), (6) and (7) respectively and use eqn. (3) to approximate the calorimeter response function by

$$S(y, \eta) = s(\eta)/N \tag{8}$$

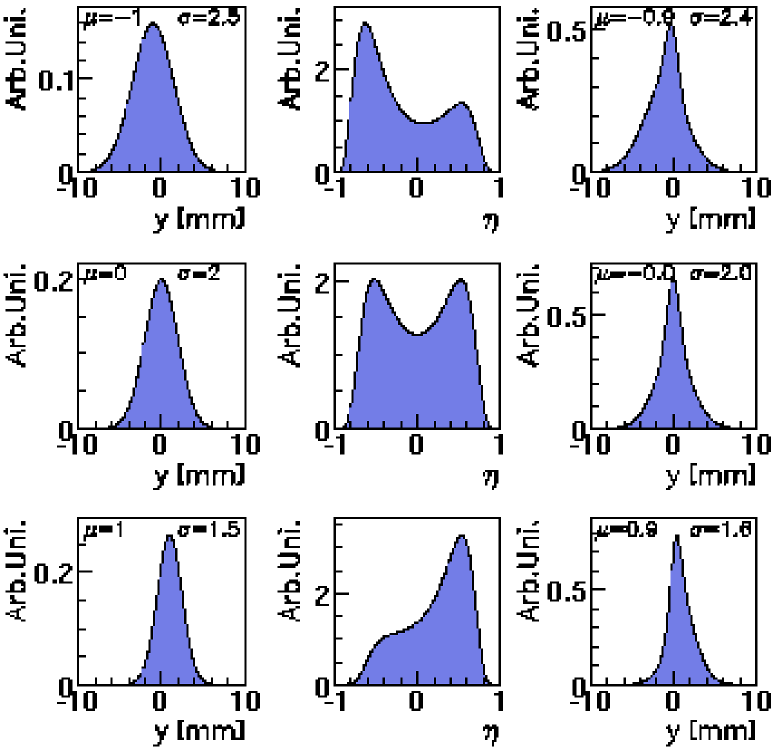

Fig. 4. Input parent distributions (first column) convoluted with calorimeter position response function (second column) and resulting unfolded spectra (third column). Average and standard deviation of the y-distributions are shown on each plot.

Calorimeter spatial response also depends on energy of the incoming particle which could be parameterized by subdividing the data into energy slices. However, one may account for this effect using a general $1/\sqrt{E}$ dependence of the position resolution (see [7], sec.28.10). In our case of an average 11 GeV energy we can simply scale σ_η in eqn.(6) by a factor $\sqrt{11/E[GeV]}$.

To see how the derived formulas are working, we simulate 3 different gaussian spatial spectra incident on the calorimeter with means μ=(-1, 0, 1)mm and standard deviations σ=(2.5, 2, 1.5)mm (Fig.4). After folding the gaussians according to eqn.(2) one has η distributions 'measured' by calorimeter. Though unfolding of the obtained spectra is beyond the scope of this note, here we briefly present three approaches to estimate the parent distribution's mean $\bar{y}$ and width σ_y from the measured η spectra. First method was used on the HERA-I data and derives $\bar{y} = g(\bar{\eta})$, $\sigma_y = g(\sigma_\eta)$ directly from an average $\bar{\eta}$ and width σ_η of the measured η distribution applying the $y = g(\eta)$ transformation. On the HERA-II, formula for the σ_y is more realistic and reads $\sigma_y = g(\sigma_\eta)f'(\bar{y})$ where $f' = d\eta/dy$ is a Jacobian of the y-η transformation. Third method is a

Distributions				Deconvolution Methods				
Parent		Folded		HERA I		II	δ-Response	
$\bar{y}$ mm	σ_y mm	$\bar{\eta}$	σ_η	$\bar{y}$ mm	σ_y mm	σ_y mm	$\bar{y}$ mm	σ_y mm
-1	2.5	-0.17	0.48	-0.44	1.50	0.62	-0.88	2.45
0	2.0	-0.00	0.45	0.00	1.36	0.70	0.00	2.01
1	1.5	0.22	0.38	0.55	1.04	0.39	0.87	1.62

Table 1

simple deconvolution assuming delta function response $S(y, \eta) = \delta(y - g(\eta))$ which rewrites the eqn.(2) as

$$\Sigma(y) = M(f(y))f'(y)$$

Reconstructed distributions by the third method are displayed on the Fig.4 where one can see how the δ-function approximation distorts the shape of initial distributions.

Resulting numbers from all the three methods are collected in Table 1 demonstrating advantage of the δ-response method (though still biased) which could be adapted for online reconstruction.

References

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