TPOL η reolution studies

Formulas to parametrize the TPOL calorimeter response as a function of U, D, E, η are derived. Comparisons to GEANT predictions and testbeam data are presented.

1 Model for the calorimeter resultion

The aim of this study is to find a way how to parametrize in a general way the (correlated) response function of the upper (U) and lower (D) half of the calorimeter. The total energy and the energy asymmetry are defined as

$$E = U + D, \quad \eta = \frac{U - D}{U + D}. \tag{1}$$

For an incident particle with known energy and position, the probability to measure an energy E is described by a Gaussian of width σ_e around the average energy response E_0. The energy resolution σ_e possibly depends on E_0 and on the average asymmetry response η_0.

Similarly to the total energy E it is a reasonable assumption that the probability to measure energies U and D in the upper and lower half of the calorimeter again are described by Gaussians of width σ_u and σ_d around the average values $U_0 = \frac{E_0}{2}(1+\eta_0)$ and $D_0 = \frac{E_0}{2}(1-\eta_0)$.

In general, the response of the U and D channel are correlated. The corresponding correlation coefficient is denoted ρ_{ud}. The energy response is guided by the covariance matrix

$$V_{ud} = \begin{pmatrix} \sigma_u^2 & \rho_{ud}\sigma_u\sigma_d \\ \rho_{ud}\sigma_u\sigma_d & \sigma_d^2 \end{pmatrix}. \tag{2}$$

It is also interesting to look at the properties of the covariance matrix for the total energy E and the asymmetry η, including the η resolution σ_η and the correlation parameter $\rho_{e\eta}$. However, one should note that the response in η is not expected to be a Gaussian.

$$V_{e\eta} = \begin{pmatrix} \sigma_e^2 & \rho_{e\eta}\sigma_e\sigma_\eta \\ \rho_{e\eta}\sigma_e\sigma_\eta & \sigma_\eta^2 \end{pmatrix} \tag{3}$$

Using standard error propagation, one can express all these variables by the resolution parameters σ_e, σ_u, σ_d:

$$\rho_{ud} = \frac{\sigma_e^2 - \sigma_u^2 - \sigma_d^2}{2\sigma_u\sigma_d}, \tag{4}$$

$$\sigma_\eta^2 = \frac{\sigma_e^2}{E^2}\left((1-\eta_0^2) + 2(\frac{\sigma_u^2}{\sigma_e^2} + \frac{\sigma_d^2}{\sigma_e^2} - 1) - 2\eta_0(\frac{\sigma_u^2}{\sigma_e^2} - \frac{\sigma_d^2}{\sigma_e^2} - \eta_0)\right).,\tag{5}$$

$$\rho_{e\eta} = \frac{\sigma_e}{\sigma_\eta E_0}(\frac{\sigma_u^2}{\sigma_e^2} - \frac{\sigma_d^2}{\sigma_e^2} - \eta_0). \tag{6}$$

By definition, the correlation coefficients ρ_{ud} and $\rho_{e\eta}$ are bound to be in the range -1 to 1. For parametrizing the calorimeter response it is thus useful to find expressions for σ_u and σ_d as a function of σ_e, ρ_{ud} and $\rho_{e\eta}$, because the latter functions can be parametrized more easily than σ_u and σ_d. The results are

$$f = \sqrt{\frac{1-\rho_{e\eta}^2}{1-\rho_{ud}^2}} \tag{7}$$

$$p_{1/2} = \eta_0 \rho_{e\eta} + f\rho_{ud} \tag{8}$$

$$\tilde{\sigma} = -p_{1/2} + \sqrt{1-\eta_0^2+p_{1/2}^2} \tag{9}$$

$$\sigma_\eta = \frac{\sigma_e}{E_0}\tilde{\sigma} \tag{10}$$

$$\sigma_u = \frac{\sigma_e}{\sqrt{2}}\sqrt{1+\eta_0-\tilde{\sigma}(f\rho_{ud}-\rho_{e\eta})} \tag{11}$$

$$\sigma_d = \frac{\sigma_e}{\sqrt{2}}\sqrt{1-\eta_0-\tilde{\sigma}(f\rho_{ud}+\rho_{e\eta})}. \tag{12}$$

The formulas exhibit the following feature: if there are no correlations, the well-known relation $\sigma_\eta \approx \frac{\sigma_e}{E_0}\sqrt{1-\eta_0^2}$ is found back. Similarly, the U and D channels simplify to the known relation $\sigma_{u,d} \approx \sigma_e\sqrt{(1\pm\eta_0)/2}$.

2 GEANT studies

For tuning these parameters one can to the following:

- Run the GEANT simulation for particles with fixed energy and fixed position repeated times and produce a "table-scan".
- For each table position, derive the following five numbers: mean asymmetry $\eta_0 = \langle\eta_0\rangle$, mean energy $E_0 = \langle E\rangle$, energy spread σ_e, up channel spread σ_u, down channel spread σ_d. These number are derived from Gaussians fitted to distributions $E - E_0$, $U - U_0$, $D - D_0$, similar to the testbeam data analysis shown below. However, instead of using the silicon detector event by event, the mean energies U_0 and D_0 are determined globally for each GEANT run.
- Plot the six resolution parameters σ_e, σ_u, σ_d, σ_η, $\rho_{e\eta}$, ρ_{ud} as a function of η_0, where the latter three parameters are obtained from equations (4), (5), (6).
- Try to fit these points by functions $\sigma_E(E_0, \eta_0)$, $\rho_{ud}(E_0, \eta_0)$, $\rho_{e\eta}(E_0, \eta_0)$. Equations (10), (11), (12) may be used to compare the prediction to σ_η, σ_u and σ_d.

Examples from small statistics test runs at a fixed energy of 10 GeV are shown in figure 1 and 2. The following parametrisations were used:

$$\sigma_e = \sqrt{a^2E_0 + b^2E_0^2}. \tag{13}$$

$$\rho_{ud} = c_0 + c_1|\eta_0| + c_2\sqrt{1-\eta_0^2} \tag{14}$$

$$\rho_{e\eta} = c_3\eta_0\sqrt{1-\eta_0^2} \tag{15}$$

with numerical values for a, b, c_0, c_1, c_2 and c_3 as collected in table 1.

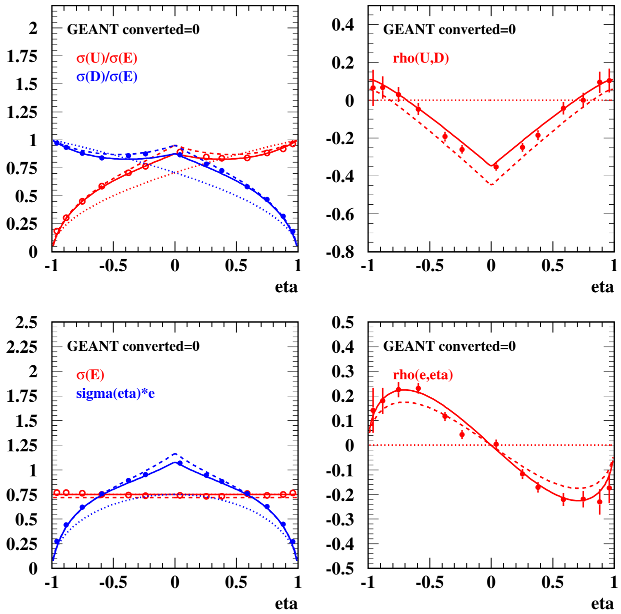

Figure 1: Resolution and correlation of U, D and E, η, obtained from GEANT runs at 10 GeV, non converted photons. The solid line corresponds to the parametrisation for non-converted photons. The dashed line corresponds to the parametrisation for converted photons. The dotted line is the naive model with all parameters $c_i = 0$.

	non-converted	converted
a	0.25	0.24
b	0.	0.
c_0	-0.45	-0.55
c_1	0.55	0.6
c_2	0.1	0.1
c_3	-0.45	-0.35

Table 1: Parametrisations obtained from a small GEANT run

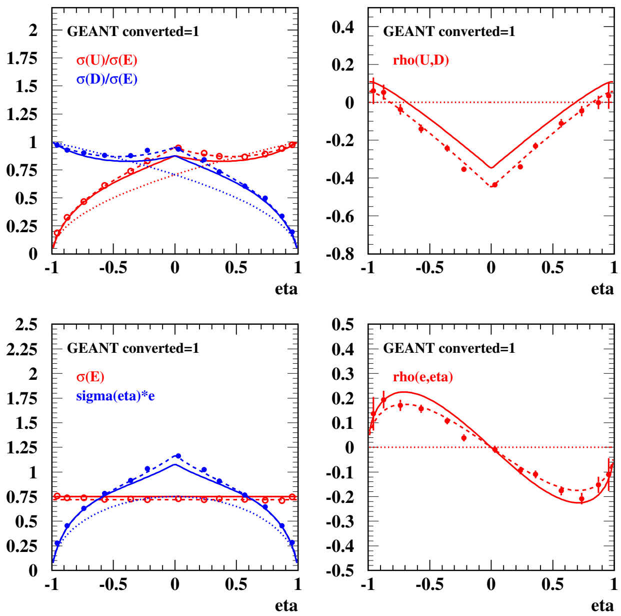

Figure 2: Resolution and correlation of U, D and E, η, obtained from GEANT runs at 10 GeV, converted photons. The solid line corresponds to the parametrisation for non-converted photons. The dashed line corresponds to the parametrisation for converted photons. The dotted line is the naive model with all parameters $c_i = 0$.

3 Testbeam results

The energy and position resolution can also be determined from CERN testbeam data. For analyzing the CERN testbeam data, certain data quality cuts have to be applied. These are summarized in table 2. The events are analyzed in the following way:

- The asymmetry η_0 is predicted from the silicon data, using a simple ηy transformation

$$x = \frac{|y - y_{\rm si}|}{r_0/2 + \sqrt{r_0^2/4 + |y - y_{\rm si}| r_1}} \tag{16}$$

$$\eta_0 = \text{sign}(1.0 - \exp[-x], y_0 - y_{\rm si}), \tag{17}$$

  where $y_{\rm si}$ is the silicon cluster position `cypos(1)` and the parameters $y_0 = 31.1$ mm, $r_0 = 2.2638$ mm, $r_1 = 1.4035$ mm are used. Because the analysis is done in bins of η_0, it is not very sensitive to details of the ηy transformation used.

- The residuals $E - E_0$, $U - E_0/2(1 + \eta_0)$, $D - E_0/2(1 - \eta_0)$ are filled in histograms, for given bins in η_0.

- The width of these residuals is determined, using a $\pm 3\sigma$ truncated Gaussian fit. This is necessary to suppress off-momentum background, producing tails in the histograms.

The results are shown in Figure 3 for 10 GeV and in Figures 4 to 7 for energies from 6 to 15 GeV. They are compared to the parametrisations also shown with the GEANT study.

cut	explaination
`iand(ioreg,512).ne.0`	Finger trigger (horiz. position)
`abs(elr/energy-1.).lt.0.8`	Reject empty events
`ncy.eq.1`	Events with one silicoin cluster
`cychg(1).gt.25`	Significant charge in silicon detector
`tdc.gt.0`	Reject events with bad timing

Table 2: analysis cuts applied to the testbeam data.

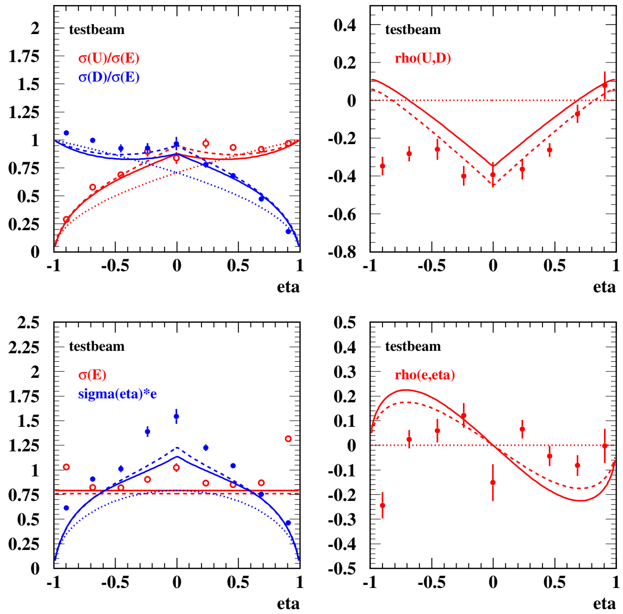

Figure 3: Resolution and correlation of U, D and E, η, obtained from testbeam data at 10 GeV. The solid line corresponds to the parametrisation for non-converted photons. The dashed line corresponds to the parametrisation for converted photons. The dotted line is the naive model with all parameters $c_i = 0$.

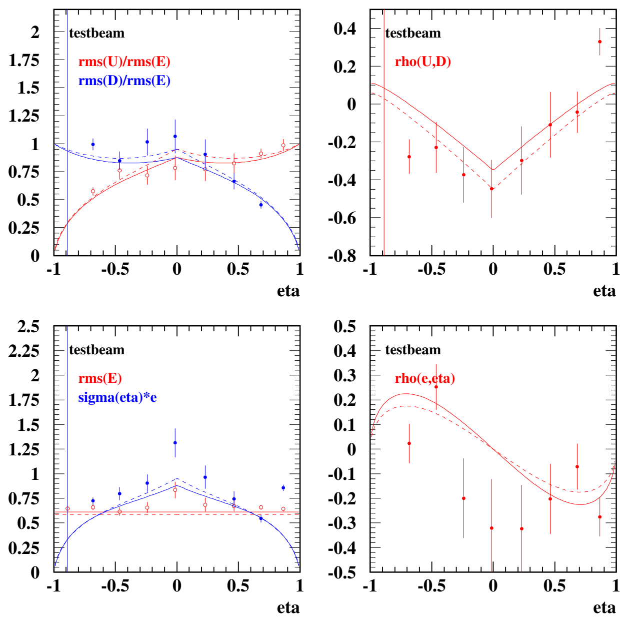

Figure 4: Resolution and correlation of U, D and E, η, obtained from testbeam data at 6 GeV. The thin lines corresponds to the parametrisation for converted and non-converted photons.

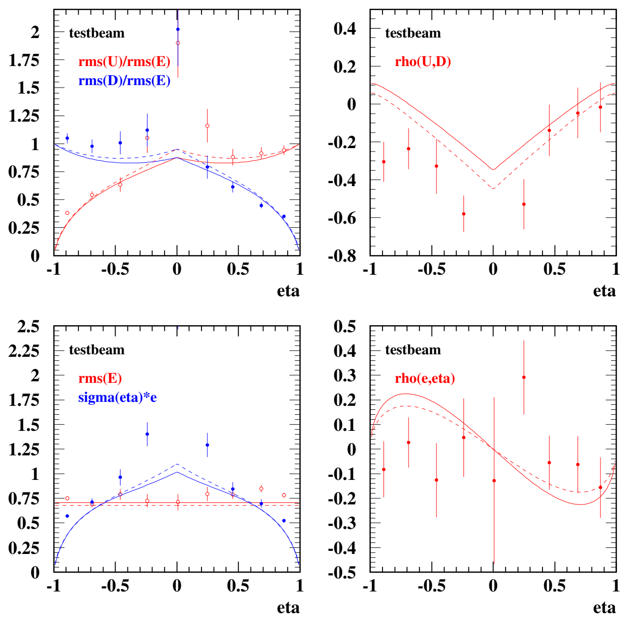

Figure 5: Resolution and correlation of U, D and E, η, obtained from testbeam data at 8 GeV. The thin lines corresponds to the parametrisation for converted and non-converted photons.

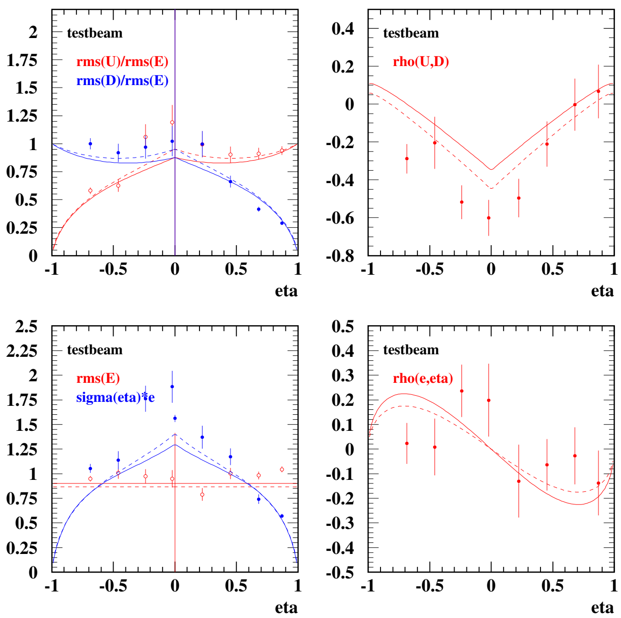

Figure 6: Resolution and correlation of U, D and E, η, obtained from testbeam data at 12.5 GeV. The thin lines corresponds to the parametrisation for converted and non-converted photons.

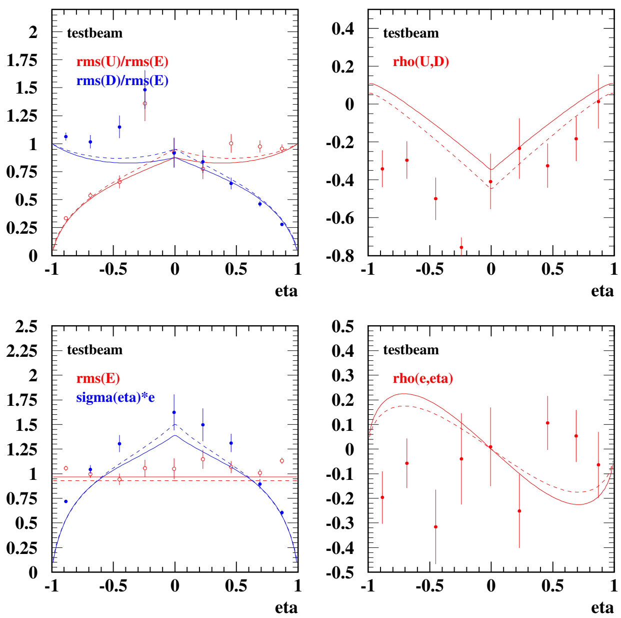

Figure 7: Resolution and correlation of U, D and E, η, obtained from testbeam data at 15 GeV. The thin lines corresponds to the parametrisation for converted and non-converted photons.