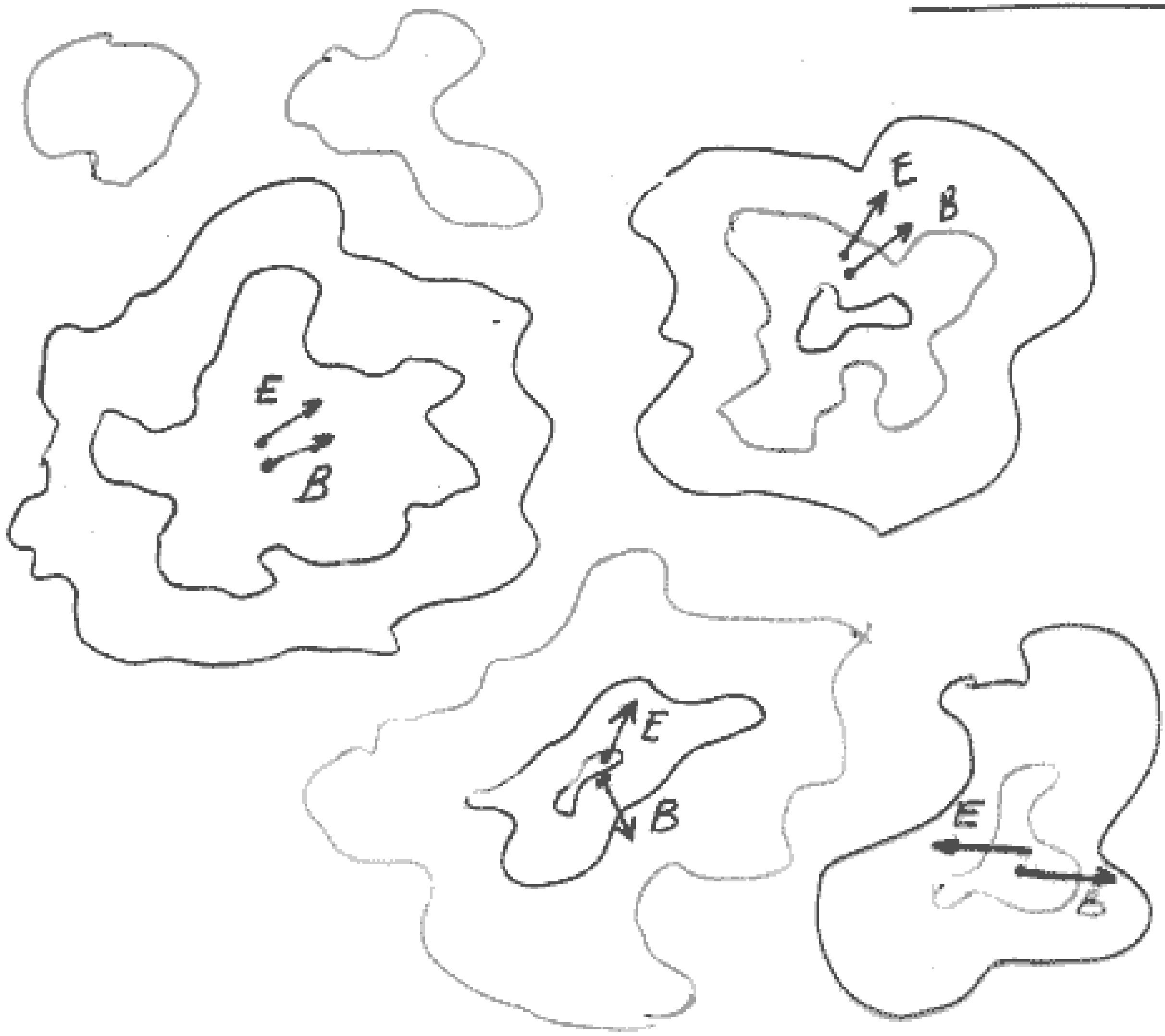
③

Vacuum strong ($\langle G^a G^a \rangle \sim (600\ \text{MeV})^4$)
and homogeneous (Lorentz - inv.) $\rightarrow$ stochastic
in Euclidean space



top. charge $Q = \frac{g^2}{32\pi^2} \int d^4x\, \underset{\vec{E}\vec{B}}{\underset{\shortparallel}{F\tilde{F}}}$; $\approx$ 1 per fm^4 !

from η' mass

(lattice) top. susceptibility

General characteristics of nonpert. vacuum

$$\langle F_{\mu\nu}(x_1)\Phi F_{\rho\sigma}(x_2)\Phi \dots F_{\lambda\xi}(x_n)\Phi \rangle$$

$\Phi(x_1, x_2)$ - parallel transporter

Di Giacomo + Panagopoulos PL '92

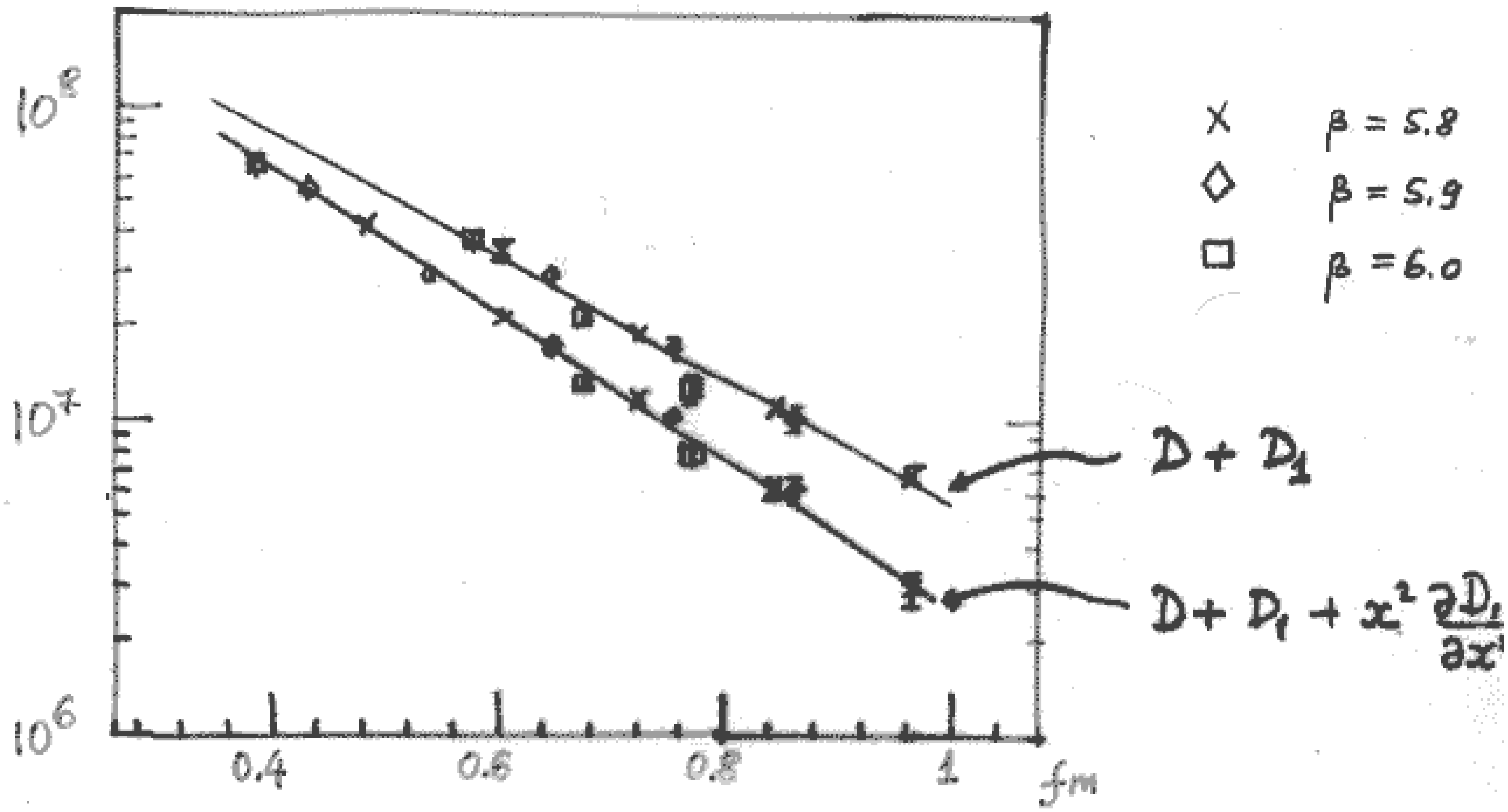
$$D = A \exp\left(-\frac{|x|}{T_g}\right)$$

$$D_1 = A_1 \exp\left(-\frac{|x|}{T_g'}\right)$$

$$\boxed{T_g \simeq T_g' \simeq 0.2 \text{ fm}}$$

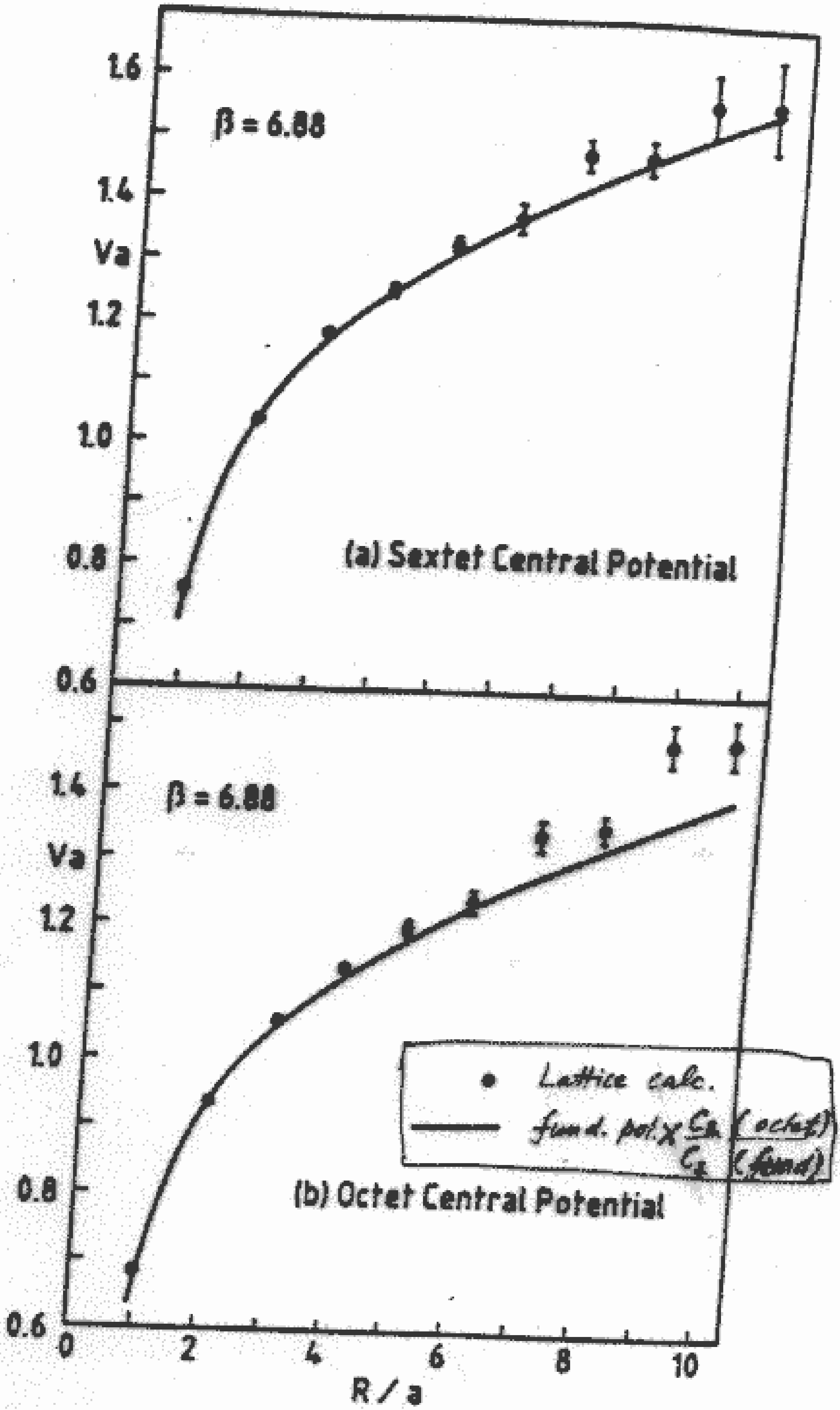
$$\frac{A_1}{A} \approx \frac{1}{3}$$

$$A \simeq 3.6 \cdot 10^8 \Lambda^4$$



Check of $V(R) \sim C_2(j)$

Ford, Hoek, Dalitz 32^4



[2] Wilson loop and Field Correlators

$$\langle W(C)\rangle = \frac{1}{N_c} \,\mathrm{tr}\, \mathcal{P} \exp ig \int_C A_\mu dz_\mu$$

Nonabelian Stokes theorem using as a tool the new gauge

The general coordinate gauge (V. Shevchenko & Yu. S.) Ivanov, Korchemsky, Rad

$$A_\mu(x) = \int_{C(x,x_0)} du_\alpha \, F_{\alpha\beta}(u) \frac{du_\beta}{dz_\mu}$$

arbitrary contour $C: \underline{u_\mu(x,s)}$

$$\boxed{\langle W(C)\rangle = \frac{1}{N_c} \,\mathrm{tr}\, \mathcal{P}_s \exp ig \int_S F_{\mu\nu}(u,x_0)\, d\sigma_{\mu\nu}}$$

$$F_{\mu\nu}(u,x_0) \equiv \Phi(x_0,u)\, F_{\mu\nu}(u)\, \Phi(u,x_0)$$

Properties:

1. Gauge invariance of r.h.s

   Gauge covariance of $\exp ig \int_S F_{\mu\nu}(u,x_0) d\sigma_{\mu\nu} \to$

   $$\to U^+(x_0) \exp ig \int_S F_{\mu\nu}(u,x_0) d\sigma_{\mu\nu} \cdot U(x_0)$$

2. Surface ordering (not affected by gauge transf.) by contour ordering.

3. $F_{\mu\nu}(u,x_0)$ depends on surface S, and $x_0 \in S$; $\langle W(C)\rangle$ does not depend.

Identity : Cluster expansion theorem.

$$\langle W(C)\rangle = \frac{1}{N_c}\,\mathrm{tr}\,\exp\sum_{n=1}^{\infty}\frac{(ig)^n}{n!}\int_S d\sigma(1)\ldots\int d\sigma(n)\,\langle\langle F(1,x_0)\ldots F(n,x_0)\rangle\rangle$$

Properties :

* $\langle\langle F\ldots F\rangle\rangle$ is a unit matrix in color space
Hence P_s not needed.

** Each term $\langle\langle F(1,x_0)\ldots F(n,x_0)\rangle\rangle$ depends on points $x_1,\ldots x_n$ and x_0, i.e. on the surface S

*** The sum $\sum_{n=1}$ on the l.h.s. does not depend on the shape of surface S.

New object : Global Field correlators Λ

$$\langle W(C)\rangle = \frac{1}{N_c}\,\mathrm{tr}\,\exp\left\{\int d\sigma_{\mu_1\nu_1}(u_1)\,d\sigma_{\mu_2\nu_2}(u_2)\,\Lambda_{\mu_1\nu_1,\mu_2\nu_2}(u_1,u_2,C)\right\}$$

$$\Lambda_{\mu_1\nu_1,\mu_2\nu_2}(u_1,u_2,C) \equiv \boxed{\frac{g^2}{2}\langle F_{\mu_1\nu_1}(u_1,x_0)\,F_{\mu_2\nu_2}(u_2,x_0)\rangle} -$$

$$-\left\{\sum_{n=3}^{\infty}\frac{(ig)^n}{n!}\int\langle\langle F_{\mu_1\nu_1}(u_1,x_0)\,F_{\mu_2\nu_2}(u_2,x_0)\,F_{\mu_3\nu_3}(u_3,x_0)\ldots F_{\mu_n\nu_n}(u_n,x_0)\rangle\rangle\right.$$

$$\left.\cdot\, d\sigma_{\mu_3\nu_3}(u_3)\ldots d\sigma_{\mu_n\nu_n}(u_n) + \text{perm}\,(1\,2\,\ldots\,n)\right\}$$

Property :

* $\int\Lambda$ does not depend on the shape of S and x_0

** Λ depends on the contour C.

$$\boxed{\langle W \rangle = \exp\left(-\iint_S d\sigma(1)\, d\sigma(2)\, \Lambda(1,2)\right)}$$

$$\Lambda(1,2) = \frac{g^2}{2}\langle F(1)\, F(2)\rangle - \frac{g^4}{4!}\int d\sigma(3)\, d\sigma(4) \langle\langle F(1) F(2) F(3) F(4) \rangle\rangle + \ldots$$

$\int \Lambda$ does not depend on S | depends on S | depends on S

Choose $S = S_{min}$ to minimize contribution of $O(g^4, \ldots)$

analogy:

$$R(\alpha_s) = R_0 \left(1 + c_1 \alpha_s(\mu) + c_2 \alpha_s^2(\mu^2) + \ldots \right)$$

$$\alpha_s(\mu) = \frac{4\pi}{b_0 \ln \mu/\Lambda}$$ depends on μ

R does not!

Choose $\mu = \mu_{optimal}$ to minimize contribution of $O(\alpha_s^3, \ldots)$

$$\boxed{\mu \sim \text{shape of } S}$$

Lattice data provide two important and surprising facts:

$\boxed{1}$ Both $D(x)$ and $D_1(x)$ have the same asymptotics $\exp(-x/T_g)$; $\boxed{D_1 \ll D}$!

$\boxed{2}$ The gluon correlation length T_g is rather small, $T_g \approx 1\ \text{GeV}^{-1} \approx 0.2$ fm

This suggests a parameter

$$\bar{F} T_g^2 \simeq 0.1 \div 0.2 \ll 1$$

$\frac{\alpha_s}{\pi}\langle (F^a_{\mu\nu})^2 \rangle \sim 0.01\ \text{GeV}^4$

where $\bar{F} = \sqrt{F^a_{ik} F^a_{ik}}$, i,k fixed e.g. $\approx \sqrt{E^a_1 E^a_1} \approx \sqrt{0.01\ \text{GeV}^2}$

$\boxed{3}$ Therefore cluster expansion

$$\int \frac{d^2u}{2!} \langle FF \rangle + \frac{1}{3!}\int d^2u\, d^2v \langle FFF \rangle + \int \frac{d^2u\, d^2v\, d^2w}{4!} \langle\langle FFFF \rangle\rangle$$

is well converging, as $\frac{(\bar{F} T_g^2)^n}{n!}$

Additional support from • Casimir scaling

•• lattice data (Pisa group)

quantities through it, e.g. $\Lambda_{QCD}, \sigma, < qq >$ etc., we shall come ... point in the next Sections.

Let us turn now to the nonlocal field correlator (1.4); the simplest one for $n = 2$ is bilocal and can be expressed in terms of two Lorentz–invariant functions, D and $D_1(z)$ [18]

Dosch + Yu.S

$$\boxed{\begin{aligned} g^2 tr < F_{\mu\nu}(x_1)\Phi(x_1 x_2)F_{\lambda\sigma}(x_2)\Phi(x_2, x_1) > &= N_c[(\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\lambda})D(u) + \\ &+ \frac{1}{2}(\frac{\partial}{\partial x_{1\mu}} u_\lambda \cdot \delta_{\nu\sigma} + \frac{\partial}{\partial x_{1\lambda}} u_\mu \delta_{\nu\sigma} + perm)D_1(u)] \end{aligned}} \tag{1.8}$$

where $u \equiv x_1 - x_2$.

It is clear that $D(u), D_1(u)$ contain two types of masses (lengths). The condensate $< F^2(0) >$ is expressed through $D(0) + D_1(0)$ and it was discussed above. But the dependence on u implies another length (mass), typically one can expect a nonperturbative dependence like

$$D(u) = D(0) exp(-|u|/T_g) \tag{1.9}$$

$$D_1(u) = D_1(0) exp(-|u|/T'_g) \tag{1.10}$$

in addition to the perturbative contribution which appears in $D_1(u)$, to lowest order [19]

$$\boxed{D_1^{pert}(u) = \frac{16\alpha_s(u)}{3\pi u^4}} \tag{1.11}$$

The notion of T_g, which we shall call the gluonic correlation length of the vacuum, was first introduced in [20] in the treatment of heavy quarkonia.

Actually it has much more universal meaning and the value of T_g is crucial in our understanding of all nonperturbative dynamics [21].