The QCD Zoo

quark : q^a $\quad \frac{\text{color}}{a =}$ 1 2 3 or • • •

flavor u d s c b t

Symbol:  quark 

 antiquark

gluon  + 8 other combinations of color

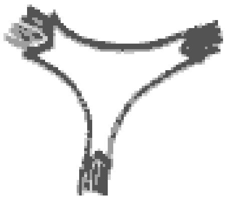
$F^{ab}_{\mu\nu}$

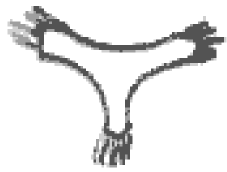
(− one condition: )

= 8 different (in color) gluons.

string junction

ϵ_{abc}





antistring-junction

parallel transporter

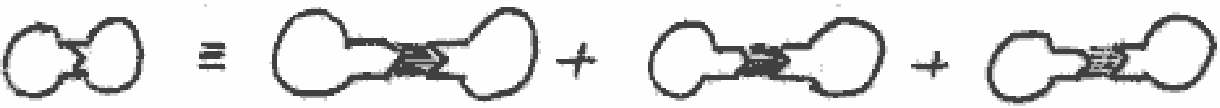
$P\exp\left(ig\int_x^y A_\mu dz_\mu\right)$

> The ruling principle – color gauge invariance : only colorless objects are admitted as physical objects

How to build colorless objects :

<u>Local objects</u>: at one point

<u>quark – antiquark</u> : $q_a(x)\bar{q}^a(x)$



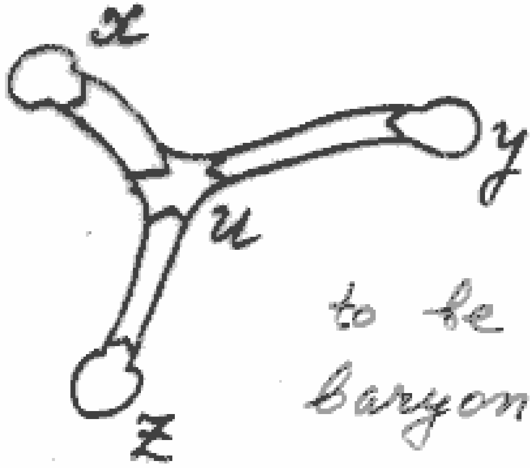
<u>gluon-gluon</u>

<u>3q</u> <u>at one point</u>

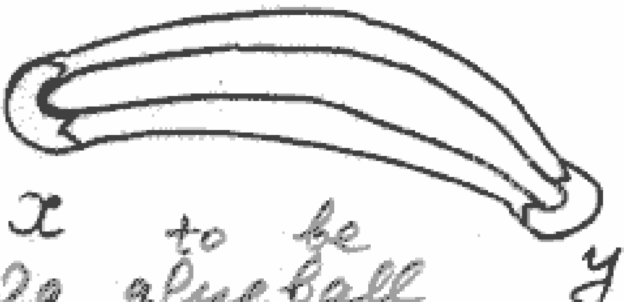
$q_a q_b q_c \epsilon^{abc}$

<u>Nonlocal objects</u>:

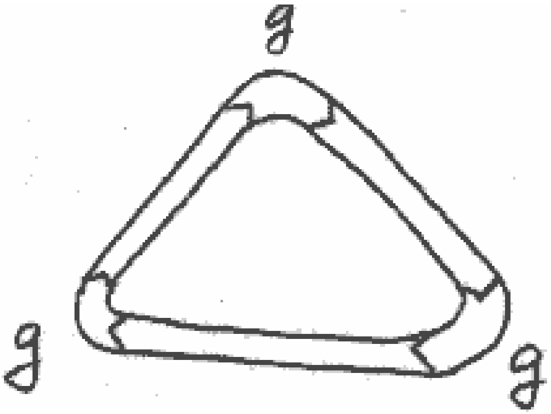
to be meson



to be baryon

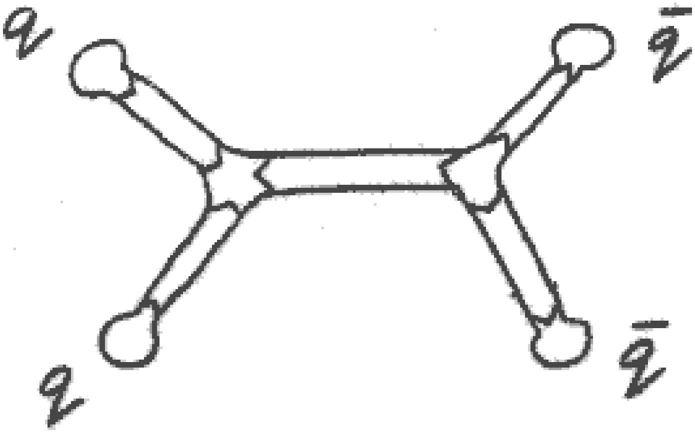


to be 2g glueball

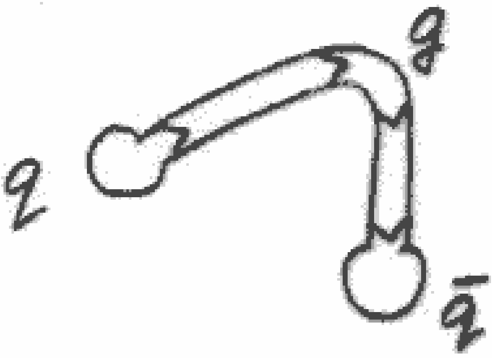


to be 3g glueball

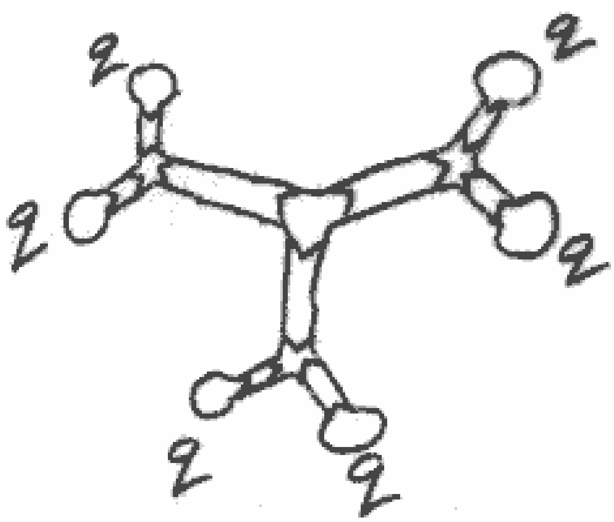
more exotic:



to be $(qq\bar{q}\bar{q})$ state



to be hybrid $qg\bar{q}$



to be exotic 6q state

1. Relativistic path integral

For relativistic particle of mass m_0 the path integral is (free case) (Polyakov)

$$G(x,x') = \int \left(\frac{Dx(\tau)}{Df(\tau)}\right) \exp\left(-m_0 \int_0^1 \sqrt{\dot{x}^2}\, d\tau\right)$$

Bad features: i) no limit for $m_0 \to 0$ ii) (important!) integral not defined

If $Dx(\tau) = \lim_{N\to\infty} \prod_{n=1}^{N} \frac{d^4 x(n)}{\varepsilon(n)}$

then no limit $N \to \infty$ exists!

because of $\sqrt{\dot{x}^2}$

The correct measure is $e^{-\int \dot{x}^2 d\tau}$

Therefore correct redefinition is the "einbein field formalism": Fock

a) Introduction of the proper time s and partial proper time τ, $\tau \le s$

then the free Green's function is (Fock; Feynman, Schwinger)

$$G(x,x') = N \int_0^\infty ds \int (\mathcal{D}z)_{xx'}\, e^{-s m_0^2 - \frac{1}{4}\int_0^s \dot{z}^2 d\tau}$$

$$\dot{z}_\mu \equiv \frac{d z_\mu(\tau)}{d\tau}, \quad (\mathcal{D}z)_{xx'} = \lim_{N\to\infty} \prod \frac{d^4 z(n)}{(4\pi\varepsilon)^2} \times$$

$$\times \frac{d^4 p}{(2\pi)^4} \exp\left(i\left(\sum \Delta z(n) - (x-x')\right)p\right)$$

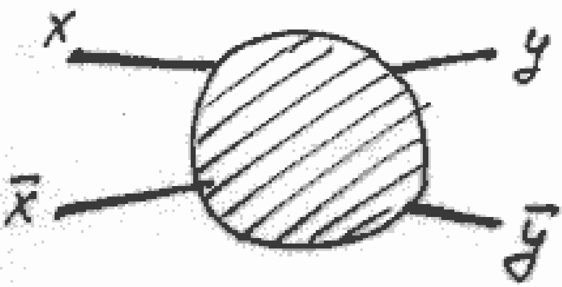
2. FFS for scalar particles (Simonov, Tjon, Ann. Phys.)

Consider charged scalar χ, χ^*, M (mass)
neutral scalar φ, m

$$S_E = \int d^4x \left[M^2 \chi^* \chi + |\partial_\mu \chi|^2 + g\chi^*\chi\varphi + \frac{m^2\varphi^2}{2} + \frac{1}{2}(\partial_\mu\varphi)^2 \right]$$

Two-body Green's function

$$G(x,\bar{x}\,|\,y,\bar{y}) = N' \int D\chi^* D\chi D\varphi\, e^{-S_E}$$



Particles χ, χ^* as |in> and |out>

$$G = N' \int D\varphi (\det \Lambda)^{-1} \left[\Lambda^{-1}(x,\bar{x})\Lambda^{-1}(\bar{y},y) + \Lambda^{-1}(x,y)\Lambda^{-1}(\bar{x},\bar{y}) \right]$$

$$\times \exp\left[-\frac{1}{2}\int d^4z\, \varphi(m^2-\partial^2)\varphi \right]$$

$$\Lambda^{-1}(x,y) \equiv \langle x | (M^2 - \partial_\mu^2 + g\varphi)^{-1} | y \rangle \equiv S_\chi(x,y)$$

FFS for S_χ

$$S_\chi = \int_0^\infty ds\, e^{-M^2 s} \int (\mathcal{D}z)_{xy}\, e^{-\int_0^s \frac{\dot{z}^2}{4} d\tau - g\int_0^s \varphi(z(\tau))d\tau}$$

$$(\det\Lambda)^{-1} = \int_0^\infty \left(\frac{ds}{s}\right)_{reg.} e^{-M^2 s} \int (\mathcal{D}z)_{xx}\, e^{-\int_0^s \frac{\dot{z}^2}{4} d\tau - g\int_0^s \varphi\, d\tau}$$

Quenched form

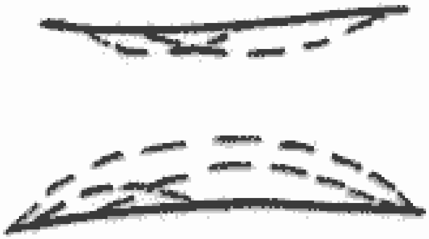
$$\boxed{G_{quen.} = \iint \mathcal{D}z\, \mathcal{D}\bar{z}\, \langle W_\varphi \rangle\, e^{-M^2(s_1+s_2) - \frac{1}{4}\int \dot{z}^2 d\tau - \frac{1}{4}\int \dot{\bar{z}}^2 d\tau}\; ds_1 ds_2}$$

$\langle W_\varphi \rangle$ - Interaction kernel

$$\langle W_\varphi \rangle = \exp\left\{\frac{g^2}{2}\sum_{\substack{i=1,2\\ j=1,2}} \int_0^{s_i} d\tau_i \int_0^{s_j} d\tau_j \, \Delta\left(z(\tau_i) - z(\tau_j)\right)\right\}$$

Δ – free propagator of φ

$$\Delta(x-y) = \langle x | (m^2 - \partial^2)^{-1} | y \rangle = \int \frac{d^4p}{(2\pi)^4} \frac{e^{ip(x-y)}}{p^2+m^2}$$



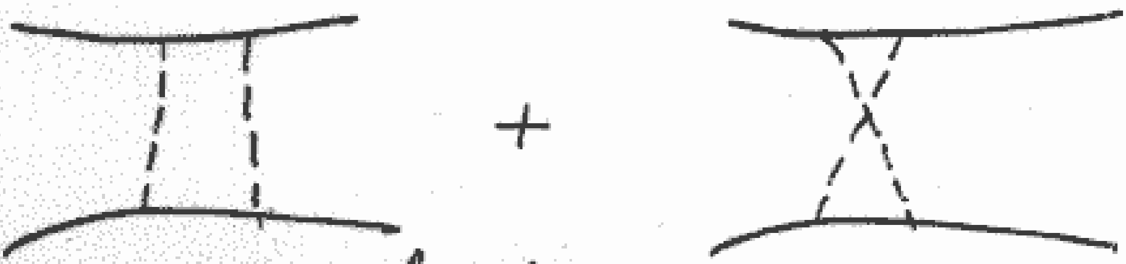
One can now take into account (dets)

<u>Perturbative expansion</u>

use $(Dz)_{xy} = (Dz)_{xu} \, d^4u \, (Dz)_{uy}$

$$\int_0^\infty ds_1 \int_0^{s_1} d\tau \, f(s_1, \tau) \equiv \int_0^\infty ds_1 \int_0^\infty d\tau \, f(s_1 + \tau, \tau)$$

Note the <u>crossed</u> graphs : in $O(g^4)$



<u>Not</u> ladder diagrams

3. FFS for gauge theory.

Consider scalar QED

$$S_E = \int d^4x \{ [M^2 \chi^* \chi + |(\partial_\mu - ieA_\mu)\chi|^2 + \frac{1}{4} F_{\mu\nu}^2 \}$$

Again the two-body Green's function

$$G(x\bar{x}|y\bar{y}) = N \int DA_\mu (\det S_A^{-1})^{-1} [S_A(x,\bar{x}) S_A(y,\bar{y}) + S_A(x,y) S_A(\bar{x},\bar{y})]$$

$$\psi_{in}(x,y) \psi_f^+(\bar{x},\bar{y}) \times \exp[-\frac{1}{2}\int A_\mu (\delta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu) A_\nu d^4z]$$

$$S_A(x,y) = \langle x | M^2 - D_\mu^2 | y \rangle =$$

$$= \int_0^\infty ds e^{-M^2 s} \mathcal{D}z e^{-K} \exp ie \int_y^x A_\mu dz_\mu ; \quad K = \int_0^s \frac{\dot{z}^2}{4} d\tau$$

Gauge-inv. Green's function – when initial and final states are gauge-inv

quenched E.g. $\psi_{in}(x,y) = \chi^+(x) \exp\left(ie \int_y^x A dz\right) \chi(y)$

$$\boxed{\begin{aligned} G_{inv} &= N' \int_0^\infty ds_1 \int_0^\infty ds_2 \, \mathcal{D}z \mathcal{D}\bar{z} \, e^{-M^2(s_1+s_2) + K_1 + K_2} \langle W(A) \rangle \\ \langle W(A) \rangle &= \int \mathcal{D}A_\mu e^{-S_0(A)} \exp ie \oint_C A_\mu dz_\mu \end{aligned}}$$

$$\langle W(A) \rangle = \exp\left(-\frac{e^2}{8\pi^2} \iint_{CC} \frac{dz_\mu dz'_\mu}{(z-z')^2}\right)$$

Inclusion of spin 1/2.

Spinor QED and QCD

$$S_E = \int d^4x \left\{ \psi^+ (m + \hat{D}) \psi + \frac{1}{4} F_{\mu\nu}^2 \right\}, \quad D_\mu = \partial_\mu - igA_\mu$$

$$S_A(x,y) = (m + \hat{D})^{-1} = (m - \hat{D})(m^2 - \hat{D}^2)^{-1} =$$

$$= (m - \hat{D})(m^2 - D_\mu^2 - g(\Sigma F))^{-1}$$

$$(\Sigma F) = -\frac{i}{4}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) F_{\mu\nu} = \begin{pmatrix} \vec{\sigma}\vec{B}, & \vec{\sigma}\vec{E} \\ \vec{\sigma}\vec{E}, & \vec{\sigma}\vec{B} \end{pmatrix}$$

$$S_A(x,y) = (m - \hat{D}) \int_0^\infty ds \, (Dz)_{xy} \, e^{-sm^2 - K} \, \overset{FS}{P} \exp\left(ig \int_y^x A_\mu dz_\mu + g \int_0^s \Sigma F d\tau\right)$$

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$$G_{inv}(x\bar{x}|y\bar{y}) = N' \Big\langle (m - \hat{D})(m - \hat{\bar{D}}) \int_0^\infty ds_1 ds_2 \, Dz \, D\bar{z} \, e^{-m^2(s_1+s_2) - K_1 - K_2} \times$$

$$\times P_1 P_2 \exp\left(g \int \Sigma F d\tau_1 - g \int \Sigma F d\tau_2\right) W(A) \Big\rangle_A$$