The structure functions

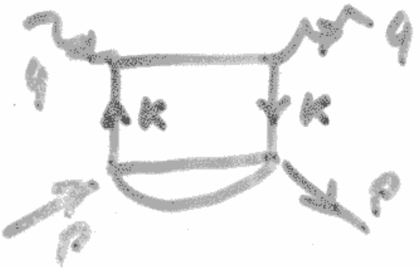
Proton

Four-point correlator

$$T^{\pm}_{\mu\nu}(p,q) = -i\int d^4x\, d^4y\, d^4z\, e^{iqx} e^{ip(y-z)}$$

$$\times \langle 0| T\{\eta(y), j^{\mp}_{\mu}(x), j^{\pm}_{\nu}(0), \bar{\eta}(z)\}|0\rangle$$

$$j^{+}_{\mu} = \bar{u}\gamma_{\mu}(1+\gamma_5)d$$



$q^2 = -Q^2 \quad Q^2 \gg |p^2|$

$p^2 < 0 \quad |p^2| \gg \frac{1}{R_c^2}$

$Q^2 \gg |p^2|$

$$\mathrm{Im}\, T_{\mu\nu} = \frac{1}{2i}\left[T_{\mu\nu}(p^2, q^2, s+i\varepsilon) - T_{\mu\nu}(p^2, q^2, s-i\varepsilon)\right]$$

OPE for $\mathrm{Im}\, T_{\mu\nu}$

$x = \frac{Q^2}{2\nu}$ $\quad k^2 = p^2 x - \frac{k_\perp^2}{1-x}$ $\quad k_\perp^2 \sim |p^2| x(1-x)$

$k^2 \sim p^2 x$ $\quad$ OPE works at not small x

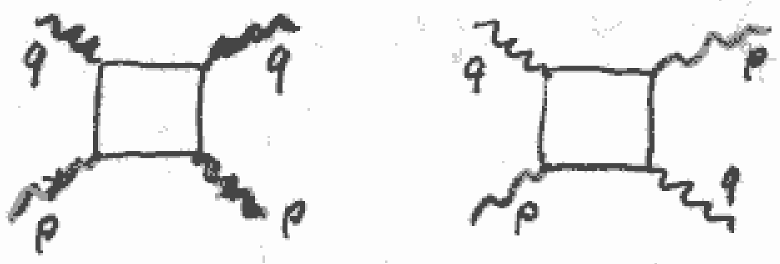
general proof: Mandelstam curve $\quad t = 4\frac{p^2 q^2}{s} = -4\frac{x}{1-x}p^2.$

Small x - Regge region

Larg x - resonance region

Photon structure function

$d=0$

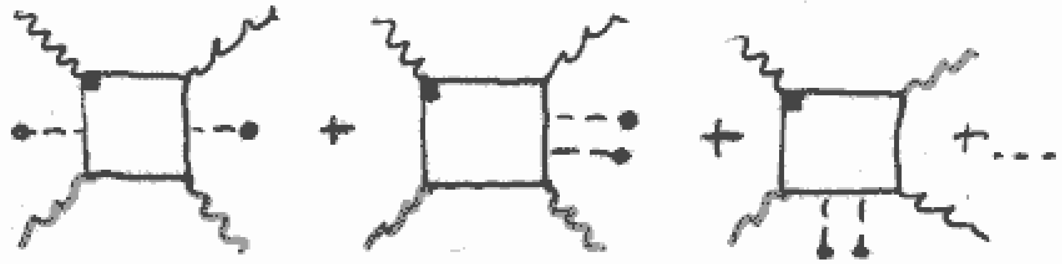


q - hard photon

p - soft photon

$|q^2| \gg |p^2| \gg R_c^{-2}$

gluonic condensate corrections $d=4$



7 diagrams

The contributions of quark condensate are zero up to dimension 8.

$$F_2^T(x,p^2) = \frac{3\alpha}{\pi}\sum_q e_q^4 x\Big\{-2+8x(1-x)+[x^2+(1-x)^2]\ln\frac{Q^2}{-x^2p^2} - \frac{\pi^2}{27}\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}^2|0\rangle\frac{4}{p^4x^2}\Big\} \qquad (1)$$

At small x the expansion breaks down!

In terms of the physical states

$$F_2^T(x,p^2)_{phys} = d(x) + \int_0^\infty \frac{\varphi(x,p'^2)}{p'^2-p^2}dp'^2 + \int_0^\infty dp_1'^2\int_0^\infty dp_2'^2\, \frac{\rho(x,p_1'^2,p_2'^2)}{(p_1'^2-p^2)(p_2'^2-p^2)} \qquad (2)$$

Model: vector meson + continuum

$$\varphi = v(x)\,\delta(p'^2-m_V^2)+\gamma(x,p'^2)\,\theta(p'^2-p_0^2)$$

$$\rho(x,p_1'^2,p_2'^2) = C_V(x)\,\delta(p_1'^2-m_V^2)\,\delta(p_2'^2-m_V^2)+\beta(x,p_1'^2,p_2'^2)\,\theta(p_1'^2-p_0^2)\times\theta(p_2'^2-p_0^2)$$

By comparison of (1) and (2) at large $|p^2|$ (up to $\frac{1}{p^4}$) $v(x)$, $\gamma(x,p'^2)$, $C_V(x)$, $\beta(x,p_1'^2,p_2'^2)$ are determined.

$F_2^T(x,p^2)_{phys}$ has no singularities at $p^2=0$, unlike $F_2^T(x,p^2)_{QCD}$

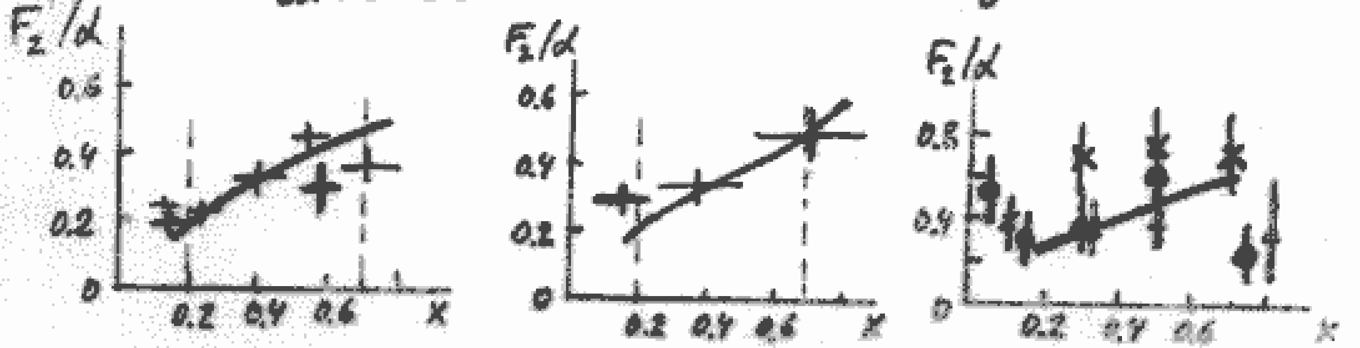
The expansion holds at $\underline{0.2<x<0.7}$ and $|p^2|>0.5$ GeV^2

Below $p^2=0.5$ GeV^2 there are no resonances →
→ the extrapolation to $p^2=0$ is possible.

$$F_2^T(x) = \frac{3\alpha}{\pi}\, x \sum e_q^4 \Big\{ -1+6x(1-x) + [x^2+(1-x)^2]\ln\frac{Q^2}{x^2 p_0^2} + \frac{1}{2}\frac{p_0^4}{m_V^4}\Big[x^2+(1-x)^2 - \underline{\underline{\frac{8\pi^2}{27 p_0^4 x^2}}}\langle 0|\frac{\alpha_s}{\pi}G^2_{\mu\nu}|0\rangle\Big]\Big\}$$

No α_s corrections were accounted → $Q^2\sim 5-10$ GeV^2

Nonperturbative effects at small x: interaction with vacuum gluon fields !!



+ PLUTO, 5.3 GeV^2 9.2 GeV^2 24 GeV^2 • JADE
+ TPC × TASSO
Twist-4 corrections 20 GeV^2 ▲TPC

Proton structure functions and valence quark distributions in proton.

$a>0$

Gluonic condensate $\sim \langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle\left(\frac{-a}{x^2}+\frac{b}{x}+\dots\right)$

Quark condensate $\sim \alpha_s \langle 0|\bar{\psi}\psi|0\rangle^2 \frac{1}{x}\frac{1}{1-x}$

$\downarrow$ $x\leq 0.2$

$0.2<x<0.7$ u-quark
$0.15<x<0.4$ d-quark $\Big\}$ EMC effect!

The price: appearance of new vacuum expectation values.

Proton and neutron magnetic moments

Quarks in constant electromagnetic field $F_{\mu\nu}$ ($F_{\mu\nu}$ is weak) $\Delta L = \sum_q e_q \bar{q}\gamma_\mu q A_\mu$

$d=3$ $$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F = \sqrt{4\pi\alpha}\,\chi_q F_{\mu\nu}\langle 0|\bar{q}q|0\rangle$$

χ_q - magnetic susceptibility of quark condensate.

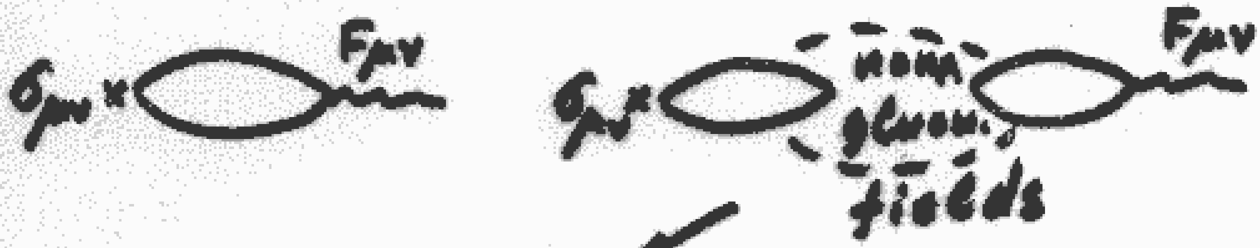
The method

$$\Pi(p) = \Pi^{(0)}(p) + \sqrt{4\pi\alpha}\,\Pi_{\mu\nu}(p)F_{\mu\nu}$$

Basic assumption

$$\chi_q = e_q\chi, \qquad q = u, d$$

χ - is an universal constant



zero in the instanton field analogues to ρ-ω mixing amplitude

$\Pi_{\mu\nu}(p)$ is composed of three tensor structures

$$\boxed{\hat{p}\sigma_{\mu\nu} + \sigma_{\mu\nu}\hat{p}}, \quad \boxed{i(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p}, \ \sigma_{\mu\nu}}$$

chirality conserv. — chirality violating

for sum rules

Chiral. cons.

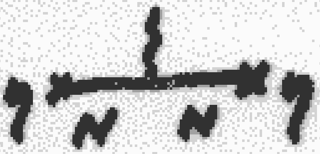
$d=6$

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F \langle 0|\bar{q}q|0\rangle$$

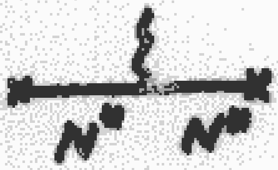
Chir. vol.

$$\longrightarrow \langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F \quad d=3$$

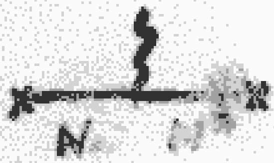
Contributions of physical states



$$\sim \mu_N \frac{1}{(p^2-m^2)^2} \to \underline{\mu_N e^{-\frac{m^2}{M^2}} \frac{1}{M^2}}$$



$$\sim \frac{1}{(p^2-m^{*2})^2} \to e^{-\frac{m^{*2}}{M^2}} \frac{1}{M^2}$$



$$\sim \frac{A}{(p^2-m^2)(p^2-m^{*2})} \to \underline{A' e^{-\frac{m^2}{M^2}}} \ldots$$

The general form of vertex function $\Gamma(p_1^2, p_2^2; q^2)$ dispersion representation:

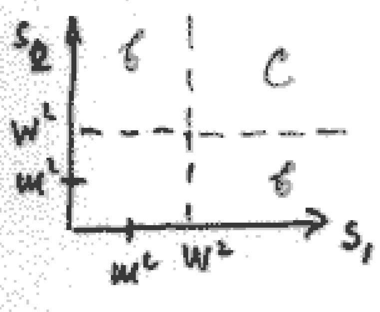
$$\Gamma(p_1^2, p_2^2; q^2) = \int_0^\infty ds_1 \int_0^\infty ds_2 \, \frac{\rho(s_1, s_2; q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)} +$$

$$+ \mathcal{P}_1(p_1^2) f_2(p_2^2, q^2) + \mathcal{P}_2(p_2^2) f_2(p_1^2, q^2)$$

$\mathcal{P}_1(p^2)$, $\mathcal{P}_2(p^2)$ - are polynomials. In symmetrical case $\mathcal{P}_1 = \mathcal{P}_2$, $f_1 = f_2$, $\rho(s_1, s_2; q^2) = \rho(s_2, s_1; q^2)$

Our case: $q = 0$. Even in this case double dispersion relation cannot be reduced to one-variable dispersion relation, because the integrals over the other variable (s_1 or s_2) are divergent and this divergence cannot be killed by Borel transformation.

$$\rho(s_1, s_2; q^2) = a(q^2)\,\delta(s_1 - m^2)\,\delta(s_2 - m^2) + b(s_1, q^2)\,\delta(s_2 - m^2) + $$
$$+ b(s_2, q^2)\,\delta(s_1 - m^2) + C(s_1, s_2, q^2)\,\theta(s_1 - W^2)\,\theta(s_2 - W^2)$$



Bare loop: $\rho(s_1, s_2) = a s_1 s_2 \, \delta(s_1 - s_2)$

Representation of hadronic spectrum: (after Borel)

$$\mu e^{-m^2/M^2} \frac{1}{M^2} + A e^{-m^2/M^2} + \text{continuum}$$

suppressed by $e^{-\frac{W^2}{M^2}}$

<u>Proton</u>

$$e_u M^4 E_1\left(\frac{W^2}{M^2}\right) L^{-4/9} + \frac{a^2}{3M^2} L^{4/9}\left[-\left(e_d + \frac{2}{3} e_u\right) + \frac{1}{3} e_u (\varkappa - 2\xi)\right] - \qquad \text{I}$$

$$- 2 e_u \chi \left(M^2 L^{-16/27} - \frac{1}{8} m_0^2 L^{-28/27}\right) = \frac{1}{4} \tilde{\lambda}_N^2 e^{-\frac{m^2}{M^2}} \left(\frac{\mu_p}{M^2} + A_p\right)$$

$$m_s a\left\{e_u+\frac{1}{2}e_d+\frac{1}{3}e_d\chi M^2\left[E_0(M^2)+\frac{b}{24M^4}\right]L^{-16/27}\right\}= \qquad \mathrm{II}$$

$$=\frac{1}{4}\tilde{\lambda}_N^2 e^{-m^2/M^2}\left(\frac{\mu_p^a}{M^2}+B_p\right)$$

For neutron $e_u\to e_d$ $e_d\to e_u$, $\mu_p\to\mu_n$ $\mu_p^a\to\mu_n$

Multiply I for proton by e_d, for neutron by e_u and subtract one from another.

Multiply II for proton by e_u, for neutron by e_d and subtract one from another.

$$\mu_p e_d-\mu_n e_u+M^2(A_p e_d-A_n e_u)=\frac{4a^2}{3\tilde{\lambda}_N^2}\exp\left(\frac{m^2}{M^2}\right)(e_u^2-e_d^2)L^{4/9}$$

$$\mu_p^a e_u-\mu_n e_d+M^2(B_p e_u-B_n e_d)=\frac{4amM^2}{\tilde{\lambda}_N^2}e^{m^2/M^2}(e_u^2-e_d^2)$$

$$\mu_p e_d-\mu_n e_u=\frac{4a^2}{3\tilde{\lambda}_N^2}(e_u^2-e_d^2)\left(1-M^2\frac{\partial}{\partial M^2}\right)e^{m^2/M^2}L^{4/9}$$

$$(\mu_p-1)e_u-\mu_n e_d=\frac{4am}{\tilde{\lambda}_N^2}(e_u^2-e_d^2)\left(1-M^2\frac{\partial}{\partial M^2}\right)M^2e^{m^2/M^2}$$

Approximately: $M=m$, $L=1$, $\tilde{\lambda}_N^2=\frac{2aM^4}{m}e^{m^2/M^2}\Big|_{M^2=m^2}$

$$\left.\begin{aligned}\mu_p&=\frac{8}{3}\left(1+\frac{1}{6}\frac{a}{m^3}\right)=2.96\\ \mu_n&=-\frac{4}{3}\left(1+\frac{2}{3}\frac{a}{m^3}\right)=-1.93\end{aligned}\right\}\begin{array}{l}\text{Exp.}\\ 2.79\\ -1.91\end{array}$$

Quark condensate magnetic susceptibility

$$\chi=-(5.7\pm0.6)\ \mathrm{GeV}^{-2}$$

Baryon magnetic moments

	QCD sum rules	Quark model	Exper.
p	3.0	—	2.79
n	−2.0	—	−1.91
Σ^+	2.40	2.67	2.46 ± 0.08
Σ^0	0.70	0.78	—
Σ^-	−1.0	−1.09	−1.157 ± 0.025
Λ	−0.70	—	−0.61
$\Sigma^0\Lambda$	1.55	1.63	1.61 ± 0.08
Ξ^0	−1.40	−1.44	−1.25 ± 0.014
Ξ^-	−0.90	−0.49	−0.651 ± 0.025
Ω^-	−2.9 ± 0.5	−3.5	−1.94 ± 0.22 −2.024 ± 0.056