Spins involved

$$S_q(x,y) = (m - \gamma_\mu D_\mu)\, G_q(x,y)$$

$$(m^2 - D_\mu^2 - g\,\Sigma F)\, G_q(x,y) = \delta^{(4)}(x-y)$$

Euclidean γ:

$$\gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\sigma} \\ i\vec{\sigma} & 0 \end{pmatrix} \qquad \Sigma_{\mu\nu} = \frac{1}{2}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$$

$$(\Sigma F) = \begin{pmatrix} \vec{\sigma}\vec{B} & \vec{\sigma}\vec{E} \\ \vec{\sigma}\vec{E} & \vec{\sigma}\vec{B} \end{pmatrix}$$

F.-Sch. rep. for G_q:

$$G_q = P\int_0^\infty ds\; Dz\; \exp\left(-\int_0^s d\sigma \left(m^2 + \frac{1}{4}\dot{z}^2\right)\right).$$

$$\exp\int_0^s \left(-g\,(\Sigma F) - i g\, A_\mu \dot{z}_\mu\right) d\sigma$$

P orders colour and <u>spin</u> matrices

$$\left(P\exp\int (A+B)dz\right)^{\alpha}_{\beta} = \left(P\exp\int \tilde{B}dz\right)^{\alpha}_{\gamma}\left(P\exp\int A dz\right)^{\gamma}_{\beta}$$

$$\tilde{B} = \phi(x,z)\,B(z)\,\phi(z,x) \quad , \quad \phi = P\exp\int A dz$$

$$G_{q\bar{q}}(x,y) = \int_0^\infty ds \int_0^\infty d\bar{s}\, Dz\, D\bar{z}\, \exp(-K_q - K_{\bar{q}})$$

$$\langle W \rangle_B$$

$$\langle W \rangle_B = \langle \phi(x,y) \cdot P\exp\int_0^s d\tau_1\, g(z^{(1)}\tilde{F}) \cdot \phi(y,x) \cdot$$

$$\cdot P\exp\left(-\int_0^{\bar{s}} d\tau_2\, g\, \widetilde{z^{(2)} F^T}\right) \rangle_B$$

$$\langle Sp\, P\, \tilde{F}_{\mu_1\nu_1}(x_1, y) \ldots \tilde{F}_{\mu_n\nu_n}(x_n, y)\, W \rangle_B =$$

$$= \langle \frac{\delta}{ig\,\delta\sigma_{\mu_1\nu_1}(x_1)} \cdots \frac{\delta}{ig\,\delta\sigma_{\mu_n\nu_n}(x_n)}\, Sp\, P\exp\int A dz \rangle_B$$

$$Sp\, W(C) = Sp\, P\exp ig\oint A dz = Sp\, P\exp\int d\sigma_{\mu\nu} F_{\mu\nu}$$

Stokes theorem

Area law for $Sp\, W \to N\exp(-\sigma S)$

derivatives are taken out of $\langle\ldots\rangle_B$

$$G_{q\bar{q}}(x,y) = \int_0^\infty ds \int_0^\infty d\bar{s}\, Dz\, D\bar{z}\ \exp\{-K_q - K_{\bar{q}}\}\cdot$$

$$\cdot \exp\left(-\sigma S + \frac{e}{i}\Sigma^{(1)}_{\mu\nu}\frac{\delta}{\delta\sigma_{\mu\nu}} S\Big|_q - \frac{e}{i}\Sigma^{(2)}_{\rho\lambda}\frac{\delta}{\delta\sigma_{\rho\lambda}} S\Big|_{\bar{q}}\right)$$

$$S = \int_0^T d\tau \int_0^1 d\beta \sqrt{\frac{\sigma_{\mu\nu}\sigma_{\mu\nu}}{2}}$$

$$\sigma_{\mu\nu} = w'_\mu \dot{w}_\nu - \dot{w}_\mu w'_\nu \quad ; \quad \dot{w}_\mu = \frac{\partial w_\mu}{\partial\tau}, \quad w'_\mu = \frac{\partial w_\mu}{\partial\beta}.$$

$$\frac{\delta}{\delta\sigma_{\mu\nu}}\sqrt{\frac{\sigma_{\alpha\beta}\sigma_{\alpha\beta}}{2}} = \frac{\frac{1}{2}\sigma_{\mu\nu}}{\sqrt{\frac{\sigma_{\alpha\beta}\sigma_{\alpha\beta}}{2}}}$$

$$\Sigma^{(1)}_{\mu\nu}\frac{\delta}{\delta\sigma_{\mu\nu}} S\Big|_q = \Sigma^{(1)}_{\mu\nu}\sigma_{\mu\nu}\frac{1}{\sqrt{\frac{\sigma_{\alpha\beta}\sigma_{\alpha\beta}}{2}}}\Bigg|_{\tau,\beta=1}$$

$$\Sigma^{(2)}_{\mu\nu}\frac{\delta}{\delta\sigma_{\mu\nu}}\Big|_{\bar{q}} = \Sigma^{(2)}_{\mu\nu}\sigma_{\mu\nu}\frac{1}{\sqrt{\frac{\sigma_{\alpha\beta}\sigma_{\alpha\beta}}{2}}}\Bigg|_{\tau,\beta=0}$$

Straight-line string $w_\mu = \beta x_{1\mu} + (1-\beta)x_{2\mu}$

$x_{10} = x_{20} = \tau$

Back to classical action:

$$\mathcal{L} = \mathcal{L}_{\text{spinless}} + \Delta\mathcal{L}$$

$$H = H_{\text{spinless}} - \Delta\mathcal{L}(p,q)$$

$$H = m_1 + m_2 + \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + V$$

$$V = \sigma r - \frac{\sigma}{6}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2} - \frac{1}{m_1 m_2}\right)(\vec{q}_T^{\,2} r)$$

$$- \frac{\sigma}{2}\left(\frac{\vec{\sigma}_1}{m_1^2} + \frac{\vec{\sigma}_2}{m_2^2}\right)\left[\frac{\vec{r}}{r}\,\vec{q}\,\right] - \frac{i\sigma}{2}\left(\frac{1}{m_1}\frac{\vec{\alpha}_1\vec{r}}{r} - \frac{1}{m_2}\frac{\vec{\alpha}_1\vec{r}}{r}\right)$$

$$\vec{q} = \vec{p}_1 - \vec{p}_2 \qquad \vec{q}_T = \vec{q} - \frac{\vec{r}(\vec{q}\,\vec{r})}{r^2}$$

$$\Sigma_{\mu\nu}\sigma_{\mu\nu} = 2\,\sigma_{0i}\,\Sigma_{0i} + \Sigma_{ik}\,\sigma_{ik}$$

$$\Sigma_{ik} = i\varepsilon_{kil}\begin{pmatrix}\sigma_l & 0\\ 0 & \sigma_l\end{pmatrix} \qquad \Sigma_{0i} = \begin{pmatrix}0 & \sigma_i\\ \sigma_i & 0\end{pmatrix}$$

The potential contains nonhermitian part!

Requires <u>nonunitary</u> F.-W. transformation

$$U = e^{S} \qquad S = \frac{i}{2\omega_1}\vec{\alpha}_1\vec{q} - \frac{i}{2\omega_2}\vec{\alpha}_2\vec{q}$$

$$\tilde{V} = \sigma r - \frac{\sigma}{6}\left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2} - \frac{1}{\omega_1\omega_2}\right)\left(\vec{q}_T^{\,2}\, r\right) \leftarrow \text{velocity correction}$$

$$+ \frac{1}{8}\left(\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}\right)\Delta\, \sigma r \qquad \leftarrow \text{Darwin term}$$

$$- \frac{\sigma}{2r}\left[\frac{1}{\omega_1^2}\vec{S}_1(\vec{r}\times\vec{p}_1) - \frac{1}{\omega_2^2}\vec{S}_2(\vec{r}\times\vec{p}_2)\right] \leftarrow \text{spin-orbit}$$

Spin-orbit looks like scalar, but it not scalar

Problem: Supply $\tilde{V}$ with Fermi-Breit reduction of one gluon exchange potential, compare the spin-orbit force.

What is wrong?

$\Rightarrow$ Backward motion was neglected unconsistently with spin content

Can we do it consistently?

$\Rightarrow$ No, we cannot beyond $1/m^2$

* There was an extra symmetry in the theory at the classical level (supersymmetry) which is respected after backward motion neglection at the classical level, and is not respected at the quantum level

1+1 QCD in $N_c \to \infty$ limit

't Hooft model

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + g\,[A_\mu, A_\nu]$$

axial gauge $A_- = 0$ $\Rightarrow$ effectively Abelian

$N_c \to \infty$ limit $\Rightarrow$ only planar graphs

Gluon propagator is known (free propagator in the axial gauge)

Graphs:

self-energy 

$q\bar{q}$ Green function 

Eigenvalue problem is given by the equation

$$\left(\frac{m^2 - 2\gamma}{x} + \frac{m^2 - 2\gamma}{1-x} - M^2 \right) \phi(x) = 2\gamma \fint_0^1 \frac{dy}{(x-y)^2}\, \phi(y)$$

$0 \leq x \leq 1$, $\gamma = \dfrac{g^2 N}{4\pi}$

Axial gauge $A_1 = 0$

$\gamma_0 = \sigma_3$

$\gamma_1 = i\sigma_2$

$k_\mu = (k_0, k)$

$\mathcal{D}_{00}(k) = -\frac{P}{k^2}$

Self-energy:  =  rainbow procedure

$S_0^{-1}(q_0, q) - S^{-1}(q_0, q) = \Sigma(q) =$

$$= \frac{i g N^2}{2(2\pi)^2} \int dp_0 \, dp \, \gamma_0 S(p_0, p) \gamma_0 \frac{1}{(q-p)^2}$$

Parametrize $\Sigma(q) = E(q)\cos\theta(q) - m + \gamma_1 \{E(q)\sin\theta(q) - q\}$

$$\Sigma(p) = E(p)\cos\theta(p) - m + \gamma_1 \{E(p)\sin\theta(p) - p\} =$$

$$= \frac{\sigma}{2} \int \frac{dk}{(p-k)^2} \{\gamma \sin\theta(k) + \cos\theta(k)\}$$

To solve one needs F.-W. transformation

$$T(p) = \exp\left(-\tfrac{1}{2}\theta(p)\gamma\right)$$

$$p\cos\theta(p) - m\sin\theta(p) = \frac{\gamma}{2} {-\!\!\!\!\!\!\int} \frac{dk}{(p-k)^2}\sin\big(\theta(p)-\theta(k)\big)$$

$$E(p) = m\cos\theta(p) + p\sin\theta(p) +$$

$$+ \frac{\gamma}{2} {-\!\!\!\!\!\!\int} \frac{dk}{(p-k)^2}\cos\big(\theta(p)-\theta(k)\big)$$

with the result for quark Green function

$$S(p) = \frac{u(p)\bar{u}(p)}{p_0 - E(p) + i\epsilon} + \frac{v(-p)\bar{v}(-p)}{p_0 + E(p) - i\epsilon}$$

where the spinors u and v are

$$u(p) = T(p)\begin{pmatrix}1\\0\end{pmatrix} \quad , \quad v(-p) = T(p)\begin{pmatrix}0\\1\end{pmatrix}$$

The <u>Bogolubov - Valatin angle</u> which transforms from bare (free) quarks to dressed quarks is the <u>Foldy angle</u> which decouples upper/lower components.

Bound state problem:

$$\Gamma(P_\mu, p_{1\mu}) = \frac{iG}{2\pi} \int \frac{dk_0\, dk}{(p_1-k)^2} S(p_{1\mu}) \gamma_0 \Gamma(P_\mu, k_\mu) \gamma_0 S(p_{1\mu} - P_\mu)$$

Instantaneous interaction:

$$\phi(P_\mu, p) = \int dp_0\, \Gamma(P_\mu, p_\mu)$$

F.-W. transformed wave function

$$\tilde{\phi}(P_\mu, p) = T^+(p)\, \phi(P_\mu, p)\, T(p-P)$$

$$\tilde{\phi} = \frac{1+\gamma_0}{2} \gamma_5 \phi_+ + \frac{1-\gamma_0}{2} \gamma_5 \phi_-$$

$$\left(E(p) + E(P-p) - P_0 \right) \phi_+(p) =$$

$$= G \int \frac{dk}{(p-k)^2} \left\{ C(p,k,P) \phi_+ - S(p,k,P) \phi_- \right\}$$

$$\left(E(p) + E(P-p) - P_0 \right) \phi_-(p) =$$

$$= G \int \frac{dk}{(p-k)^2} \left\{ C(p,k,P) \phi_- - S(p,k,P) \phi_+ \right\}$$

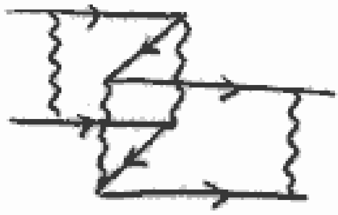
Bars-Green equation

⑤

$$C(p,k,P) = \cos\left(\tfrac{1}{2}(\theta(p)-\theta(k))\right)\cos\left(\tfrac{1}{2}\theta(P-p)-\tfrac{1}{2}\theta(P-k)\right)$$

$$S(p,k,P) = \sin\left(\tfrac{1}{2}(\theta(p)-\theta(k))\right)\sin\left(\tfrac{1}{2}\theta(P-p)-\tfrac{1}{2}\theta(P-k)\right)$$

* * *



Backward motion is present

Problems:

i) Show that $\theta(p) \to \pm\frac{\pi}{2}$ as $p \to \pm\infty$

ii) Letting $P \to \infty$ demonstrate that the spectrum is equivalent to 't Hooft equation one

iii) Find the heavy quark limit of Bars-Green equation, and compare with Fermi-Breit reduction of one gluon exchange potential in 1+1

χSB in 1+1

$m = 0$

$$p\cos\theta(p) = \frac{\sigma}{2} \int \frac{dk}{(p-k)^2} \sin(\theta(p) - \theta(k))$$

$$E(p) = p\sin\theta + \frac{\sigma}{2} \int \frac{dk}{(p-k)^2} \cos(\theta(p) - \theta(k))$$

obvious solution $\quad \theta(p) = \frac{\pi}{2} \quad p>0$

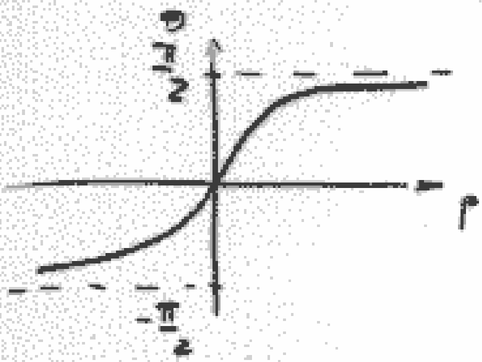
$$-\frac{\pi}{2} \quad p<0$$

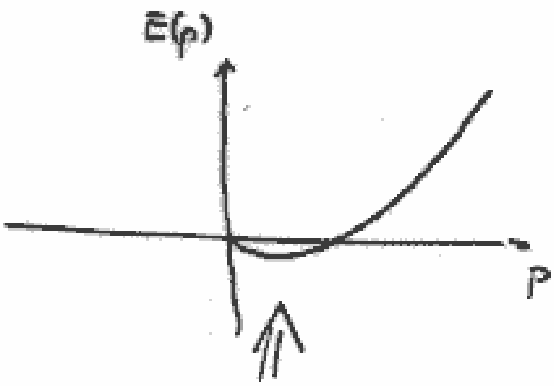
$$E(p) = |p| - \frac{\sigma}{|p|}$$

$$\langle \bar\psi\psi \rangle = -i\,\mathrm{Sp}\, S(x,x) = -\frac{N}{\pi} \int_0^\infty dp \cos\theta(p)$$

$\langle \bar\psi\psi \rangle$ is 0 for this solution

another solution exists, with $\cos\theta \neq 0$, $\langle \bar\psi\psi \rangle \neq 0$





General properties:

i) For $M > 0$

ϕ_- is small for large n and for large m

$(m \gg \sqrt{\sigma})$

ii) For large n

$$\cancel{2E(p)\phi_+ \approx \frac{\sigma}{2} \int \frac{dk}{(p-k)^2}}$$

$$\left(2E(p) - M\right)\phi_+ = \frac{\sigma}{2} \int \frac{dk}{(p-k)^2}\, \phi_+(k)$$

$$E(p) \to |p|$$

in the coordinate space $\frac{\sigma}{2} \int \frac{dk}{(p-k)^2}\, \phi_+(k) \Rightarrow$

$$\sigma z\, \phi_+(z)$$

Goldstone particle:

in the c.m.f.

$$\left(2E(p) - M\right)\phi_+(p) = \int \frac{dk}{(p-k)^2}\left(C(p,k)\,\phi_+(k) - S(p,k)\,\phi_-(k)\right)$$

$$\left(2E(p) + M\right)\phi_-(p) = \int \frac{dk}{(p-k)^2}\left(C(p,k)\,\phi_-(k) - S(p,k)\,\phi_+(k)\right)$$

There exists the solution with $M=0$

Problem: Find this solution, show that there is no such solution for chirally invariant $\theta(p)$.