USPAS Linear Colliders class – exercises for Lecture 3
PT, 13-june-2003

**1. Consider a 10-to-250 GeV/beam main linac consisting of accelerator structures operating at 11.424 GHz (4 x the SLAC frequency) with the same value of t (0.57), the same phase advance per cell (120º), and the same ratio of magnets to structures (1 quad per 4 structures, 90º betatron phase/FODO cell).**

**a. Estimate the Q, shunt impedance per unit length $\eta$, and filling time $t_f$ for such a structure.**

The most straightforward method for this estimation is to use the scaling laws on page 38: since $\eta$ scales with $?^{1/2}$, then this structure should have values twice as high as the SLAC structure, or 104-120 MO/m. Similarly, Q is proportional to $?^{-1/2}$ and should thus be half as large as the SLAC structure, or 6500. The filling time, 2Qt/?, is therefore smaller than the SLAC structure by a factor of $4^{3/2}$, or about 107 nsec.

**b. If the structure is to have the same number of cells as the SLAC structure (86), what is the structure length? Compute the profile of group velocity, a, and b for the structure. What input power level is needed to attain a gradient of 60 MeV/meter?**

The wavelength of 11.424 GHz is 2.63 cm; 1/3 of this is the cell length, or 8.75 mm. A structure with 86 cells yields 75.3 cm.

Since the structure is one-quarter as long as the SLAC structure but has a fill time one-eighth as long, the group velocity must be twice as high, or 4% c at the upstream end and 1.3% c at the downstream end.

If we assume that the disc thickness is exactly ¼ as great as in the SLAC structure, then the group velocity expression in Equation 97 shows that, in order to have 2x the group velocity in this structure, the quantity $(a^3/b^3)$ should be twice as large as in the SLAC structure. Similarly, the phase velocity expression can be re-written as:

$$\frac{v_{ph}}{c} = \frac{z_{01}}{k_z b} + B(h/a, k_z d)\frac{a^3}{b^3} = 1.$$

For the SLAC structure at the upstream end, $a^3/b^3 = 3.10\times10^{-2}$ and $z_{01}/k_z b=0.964$; therefore, B = 1.168. Since $a^3/b^3$ doubles for this structure, $z_{01}/k_z b$ must be 0.928 and thus $b=z_{01}/0.928k_z=1.08$ cm (remember that $k_z$ for this structure = 4 x $k_z$ for the SLAC structure). This yields an upstream a of 4.27 mm. Similarly, at the downstream end of the structure, b=1.04 cm and a = 3.06 mm. Note that b is close to ¼ as large as in the SLAC structure, while a is closer to 1/3.2 as large.

For a gradient of 60 MeV/meter, we can use Equation 116 to find a requisite input power of 35.6 MW. Note that we have not included the reduction in shunt impedance due to the relatively large aperture in this calculation, but it's not a huge effect (order of 10%).

**c. For the canonical LC bunch (200 um RMS length, 1.6 nC), what is the mean phase offset needed for single-bunch energy spread compensation?**

We can take as a reasonable average value for a the numerical average of the upstream and downstream values, or 3.67 mm. An approximate phase offset is given by Equation 151, yielding f = 0.209 radians (or 12 degrees). More accurately, $s_z$ = 561 microns so exp(-?) = -0.48 and Equation 150 yields sin f = 0.0901 so f = 5.2 degrees.

Note that, although the short-range loading is more severe in the high-frequency structure, the average phase needed for energy spread compensation is smaller. This is partially because the wavelength is shorter and partially because the gradient is larger (60 MeV/m rather than 20 MeV/m).

**d. What is the RMS energy spread (in MeV) needed to achieve autophasing?**

The transverse wake slope, $W'_{\perp}$, is given by $2Zc/pa^4 = 3.97\times10^{20}$ V/C/m$^3$. $L_{cell}$ is about 8 structures, or 6 meters. Equation 172 therefore tells us that the required energy spread is about 531 MeV. At the 10 GeV end of the linac this is 5% of the beam energy, while at the 250 GeV end it's about 0.2%.

**e. What is the bunch spacing required to achieve 20% beam loading?**

The full-loading current is given by Equation 139 to be 1.16 amperes. One-fifth of this current is 231 milliamperes. Note that at 1.6 nC/bunch, the bunch spacing is about 1 per 7 nanoseconds.

**f. What is the overall energy efficiency if a train repetition rate of 120 Hz is desired?**

Since the average current required is 42 microamperes, a beam duty cycle of $1.8\times10^{-4}$ will yield a beam current of 231 milliamperes. A repetition rate of 120 Hz implies a cycle time of 8.33 msec, and $1.8\times10^{-4}$ of this time is 1.5 microseconds of beam time. The maximum efficiency for t=0.57 is $?_{max}$ = 0.73. Folding in the optimum current, the beam current, the beam time, and the fill time, the efficiency comes out to roughly 25%.

**2. Consider a 10-to-250 GeV/beam main linac consisting of superconducting accelerator structures operating at 1.428 GHz (1/2 the SLAC frequency). The structures operate in pi-mode, with 10 cells per structure (1.05 meter length) and a wall Q of $10^{10}$ at an accelerating gradient of 20 MeV/meter. Assume that the cell geometry is the standard "tuna can" (or "pillbox") geometry discussed for travelling-wave structures, and that the disc thickness is infinitessimal.**

**a. How much RF power is required to maintain 20 MeV/meter, given the Q of $10^{10}$?**

The R/Q for an RF structure with pi phase advance per cell is found, via eqn 64, to be 200 O. Since Q=$10^{10}$, the shunt impedance of 1 cavity is 2x$10^{12}$ O, which implies that the power needed to maintain 2.1 MV (20 MeV/m * 0.105 m cell length) in 1 cell is 2.2 watts. For 10 cells, then, a total of about 22 watts is needed.

**b. At 20 MeV/meter, what beam current will be required to absorb an amount of energy equal to the wall losses?**

In this case (21 MV)(I) = 22 watts, so a beam current of about 1 microampere will absorb 22 watts out of each structure.

**c. Suppose that a beam duty cycle of 1% is desired (ie, beam is present in the linac for 1% of the time). This implies that a beam current of 4.2 mA is required. How much RF power will the beam absorb from the structure?**

A beam current of 4.2 mA absorbs 88 kW out of the structure.

**d. Given the solution to c., what coupler coefficient is required to ensure that when the beam is present, the net power directed back towards the RF power source will go to zero?**

If the steady-state condition described above is achieved, then the power going backwards up the input line is zero (all incoming power goes into the beam or the cavity walls). If the beam and the RF power are simultaneously switched off, then the cavity will begin to radiate power up the input coupler, and that power is given by

$$P_{out} = P_c ß_c.$$

Since the emitted *field* does not change instantaneously, then the sudden appearance of radiated power is due to the disappearance of the input field, which was originally canceling the output field by superposition. We can thus deduce that the power from the RF source (when it was on) must have been equal to the power coming out of the structure (now that the source is off), or 88 kW. So 88 kW = $P_c ß_c$, therefore $ß_c$ = 88 kW / 22 W = 4000.

**e. What is the loaded Q of the structure? The characteristic time $t_c$?**

The loaded Q of the structure = $10^{10}/4001 = 2.5\text{x}10^6$. The characteristic time = $2Q_L/?$ = 557 µsec.

**f. How much time is required for the RF source to fill the cavity to the desired 20 MeV/meter gradient? What is the net energy efficiency if the linac is pulsed once per second? Five times per second? Ten times per second?**

We can use Equation 130,

$$V(t) = \left(1 - e^{-t/t_c}\right)\sqrt{\frac{R}{Q} w t_c P_{in} \frac{2b_c}{1+b_c}} \ ,$$

where the factor inside the square root equals 4.19 MV (remember that our R/Q is for a single cell, so the power we use in the equation is 8.8 kW and the resulting voltage is also single-cell). The structure voltage, then, is V(t) = 41.9 MV($1\text{-}e^{-t/tc}$). Thus we find that the exponential factor $e^{-t/tc}$ must be 0.501 and t = 0.691 $t_c$ = 385 microseconds.

For a linac pulse rate of 1 Hz and a beam duty cycle of 1%, the bunch train length is 10 msec and the efficiency = (10 msec)/(10 msec + 385 µsec) = 96%. For 5 Hz linac pulse rate the train is 2 msec so the efficiency = 83.8%. For 10 Hz the efficiency is 72%. Interestingly, if we wanted to go to 120 Hz, the beam train would be 83 microseconds long for 1% beam duty cycle and the efficiency would decrease to 17.7%, comparable to the efficiency calculated for a normal-conducting linac with 20% beam loading.