An Introduction to the Physics and Technology of e+e- Linear Colliders

Lecture 9: a) Beam Based Alignment

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USPAS, Santa Barbara, 16th-27th June, 2003

Emittance tuning in the LET

- LET = Low Emittance Transport
  - Bunch compressor (DR→Main Linac)
  - Main Linac
  - Beam Delivery System (BDS), inc. FFS
- DR produces tiny vertical emittances ($\gamma\varepsilon_y \sim$ 20nm)
- LET must preserve this emittance!
  - strong wakefields (structure misalignment)
  - dispersion effects (quadrupole misalignment)
- Tolerances too tight to be achieved by surveyor during installation

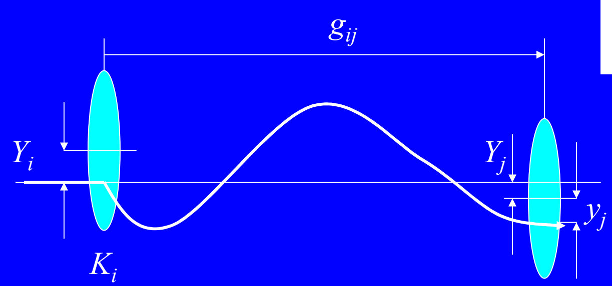
$\Rightarrow$ Need beam-based alignment

mma!

Basics (linear optics)

thin-lens quad approximation: $\Delta y' = -KY$

$$g_{ij} = \left.\frac{\partial y_i}{\partial y'_j}\right|_{y'_j=0} = R_{34}(i,j)$$



$$y_j = \left(-\sum_{i=1}^{j} g_{ij} K_i Y_i \right) - Y_j$$

linear system: just superimpose oscillations caused by quad kicks.

Introduce matrix notation

Original Equation $\quad y_j = \left(-\sum_{i=1}^{j} g_{ij} K_i Y_i \right) - Y_j$

Defining *Response Matrix* **Q**: $\quad \mathbf{Q} = \mathbf{G} \cdot \mathbf{diag(K)} + \mathbf{I}$

Hence beam offset becomes $\quad \mathbf{y} = -\mathbf{Q} \cdot \mathbf{Y}$

G is lower diagonal: $\quad \mathbf{G} = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ g_{21} & 0 & 0 & 0 & \cdots \\ g_{31} & g_{32} & 0 & 0 & \cdots \\ g_{41} & g_{42} & g_{43} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$

Dispersive Emittance Growth

Consider effects of finite energy spread in beam δ_{RMS}

chromatic response matrix: $\mathbf{Q}(\delta) = \mathbf{G}(\delta)\cdot\mathbf{diag}\left(\frac{\mathbf{K}}{1+\delta}\right)+\mathbf{I}$

lattice chromaticity

dispersive kicks

$$\mathbf{G}(\delta) = \mathbf{G}(0) + \left.\frac{\partial \mathbf{G}}{\partial \delta}\right|_{\delta=0}$$

$$R_{34}(\delta) = R_{34}(0) + T_{346}\delta$$

dispersive orbit: $\boldsymbol{\eta}_y \approx \frac{\Delta\mathbf{y}(\delta)}{\delta} = -\left[\mathbf{Q}(\delta)-\mathbf{Q}(0)\right]\cdot\mathbf{Y}$

What do we measure?

BPM readings contain additional errors:

$\mathbf{b}_{\mathrm{offset}}$ static offsets of monitors wrt quad centres

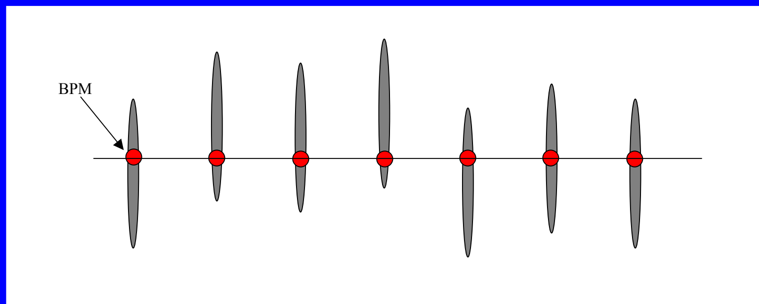
$\mathbf{b}_{\mathrm{noise}}$ one-shot measurement noise (resolution σ_{RES})

$$\mathbf{y}_{\mathrm{BPM}} = -\mathbf{Q}\cdot\mathbf{Y} + \underbrace{\mathbf{b}_{\mathrm{offset}}}_{\text{fixed from shot to shot}} + \underbrace{\mathbf{b}_{\mathrm{noise}}}_{\text{random (can be averaged to zero)}} + \mathbf{R}\cdot\mathbf{y}_0 \qquad \mathbf{y}_0 = \begin{pmatrix} y_0 \\ y_0' \end{pmatrix}$$

launch condition

In principle: all BBA algorithms deal with $\mathbf{b}_{\mathrm{offset}}$

Scenario 1: Quad offsets, but BPMs aligned



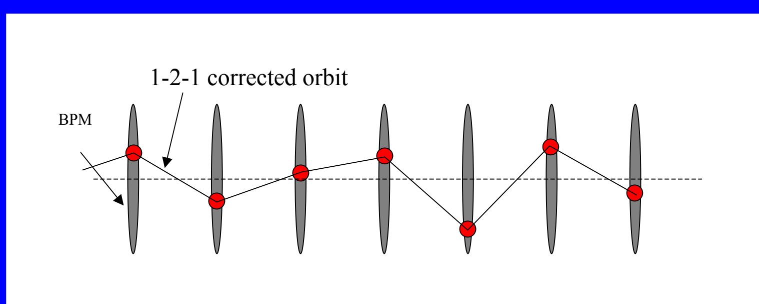
Assuming:

- a BPM adjacent to each quad
- a 'steerer' at each quad

steerer { quad mover / dipole corrector }

simply apply one to one steering to orbit

Scenario 2: Quads aligned, BPMs offset



one-to-one correction BAD!

Resulting orbit not <u>Dispersion Free</u> ⇒ <u>emittance growth</u>

Need to find a steering algorithm which effectively puts BPMs on (some) reference line

real world scenario: some mix of scenarios 1 and 2

BBA

- Dispersion Free Steering (DFS)
  - Find a set of steerer settings which minimise the dispersive orbit
  - in practise, find solution that minimises difference orbit when 'energy' is changed
  - Energy change:
    - true energy change (adjust linac phase)
    - scale quadrupole strengths
- Ballistic Alignment
  - Turn off accelerator components in a given section, and use 'ballistic beam' to define reference line
  - measured BPM orbit immediately gives $\mathbf{b}_{\text{offset}}$ wrt to this line

DFS

Problem:

$$\Delta\mathbf{y} = -\left[\mathbf{Q}(\frac{\Delta E}{E}) - \mathbf{Q}(0)\right]\left(\frac{\Delta E}{E}\right)\cdot\mathbf{Y}$$
$$\equiv \mathbf{M}\left(\frac{\Delta E}{E}\right)\cdot\mathbf{Y}$$

Note: taking difference orbit $\Delta\mathbf{y}$ removes $\mathbf{b}_{\text{offset}}$

Solution (trivial): $\mathbf{Y} = \mathbf{M}^{-1}\cdot\Delta\mathbf{y}$

Unfortunately, not that easy because of noise sources:

$$\Delta\mathbf{y} = \mathbf{M}\cdot\mathbf{Y} + \mathbf{b}_{\text{noise}} + \mathbf{R}\cdot\mathbf{y}_0$$

DFS example

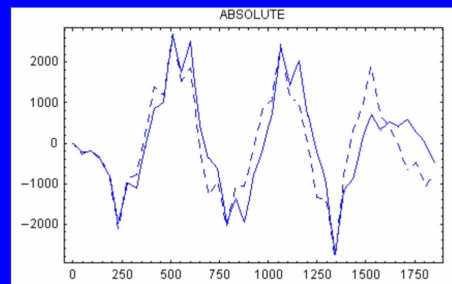
300µm random quadrupole errors

20% *ΔE/E*

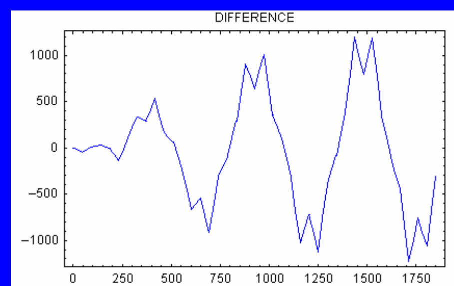
No BPM noise

No beam jitter

µm



µm

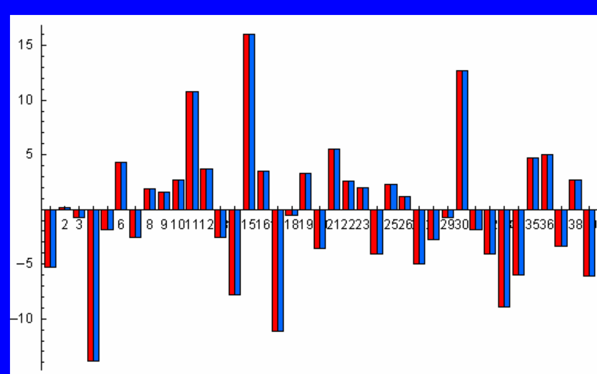


DFS example

Simple solve

$\mathbf{Y} = \mathbf{M}^{-1} \cdot \Delta\mathbf{y}$

In the absence of errors, works exactly



Resulting orbit is flat

⇒Dispersion Free

(perfect BBA)

original quad errors

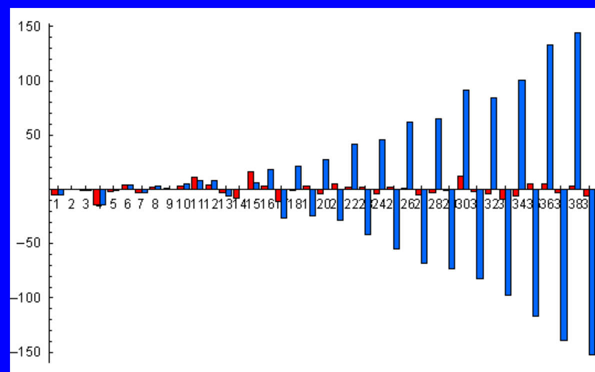
fitter quad errors

Now add 1µm random BPM noise to measured difference orbit

DFS example

Simple solve

$$\mathbf{Y} = \mathbf{M}^{-1} \cdot \Delta\mathbf{y}$$



original quad errors

fitter quad errors

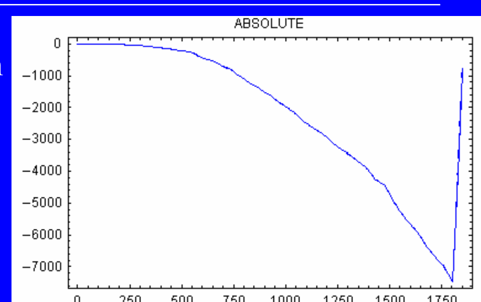
Fit is ill-conditioned!

DFS example

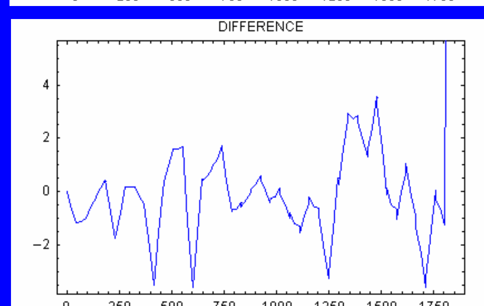
Solution is still Dispersion Free

but several mm off axis!

µm



µm



DFS: Problems

- Fit is ill-conditioned
  - with BPM noise DF orbits have very large unrealistic amplitudes.
  - Need to constrain the absolute orbit

minimise $$\frac{\mathbf{\Delta y}\cdot\mathbf{\Delta y}^{\mathrm{T}}}{2\sigma_{\mathrm{res}}^{2}}+\frac{\mathbf{y}\cdot\mathbf{y}^{\mathrm{T}}}{\sigma_{\mathrm{res}}^{2}+\sigma_{\mathrm{offset}}^{2}}$$

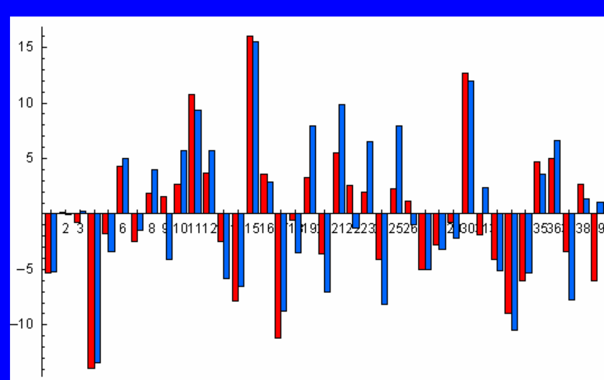
- Sensitive to initial launch conditions (steering, beam jitter) $\mathbf{R}\cdot\mathbf{y}_0$
  - need to be fitted out or averaged away

DFS example

Minimise

$$\frac{\mathbf{\Delta y}\cdot\mathbf{\Delta y}^{\mathrm{T}}}{2\sigma_{\mathrm{res}}^{2}}+\frac{\mathbf{y}\cdot\mathbf{y}^{\mathrm{T}}}{\sigma_{\mathrm{res}}^{2}+\sigma_{\mathrm{offset}}^{2}}$$

absolute orbit now constrained



remember

$\sigma_{\mathrm{res}} = 1\mu\mathrm{m}$

$\sigma_{\mathrm{offset}} = 300\mu\mathrm{m}$

original quad errors

fitter quad errors

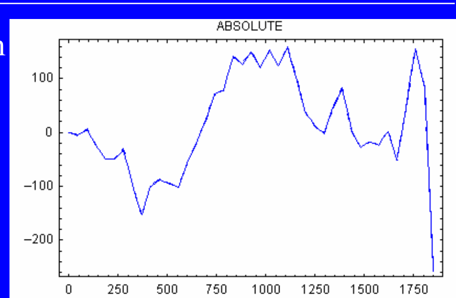
DFS example

Solutions much better behaved!

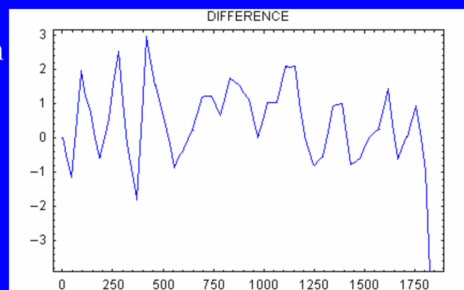
! Wakefields !

Orbit *not quite* Dispersion Free, but very close

µm



µm



DFS practicalities

- Need to align linac in sections (bins), generally overlapping.
- Changing energy by 20%
  - quad scaling: only measures dispersive kicks from quads. Other sources ignored (not measured)
  - Changing energy upstream of section using RF better, but beware of RF steering (see initial launch)
  - dealing with energy mismatched beam may cause problems in practise (apertures)
- Initial launch conditions still a problem
  - coherent β-oscillation looks like dispersion to algorithm.
  - can be random jitter, or RF steering when energy is changed.
  - need good resolution BPMs to fit out the initial conditions.
- Sensitive to model errors (**M**)

Ballistic Alignment

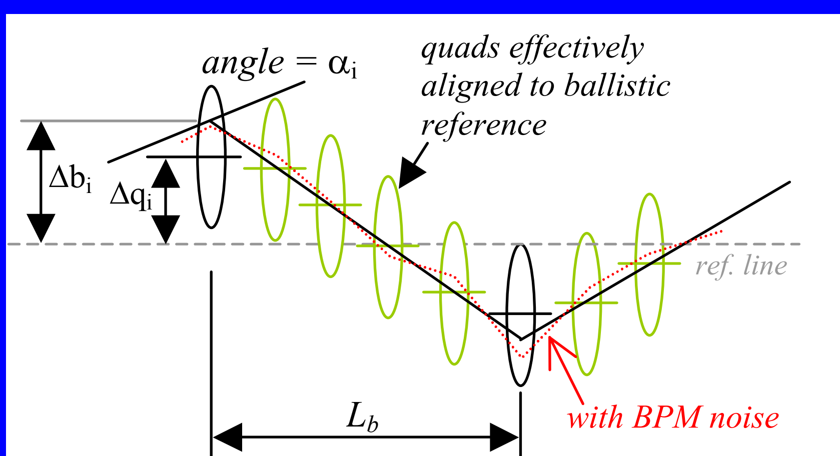
- Turn of all components in section to be aligned [magnets, and RF]
- use ‘ballistic beam’ to define straight reference line (BPM offsets)

$$y_{\mathrm{BPM},i} = y_0 + s_i y_0' + b_{\mathrm{offset},i} + b_{\mathrm{noise},i}$$

- Linearly adjust BPM readings to arbitrarily zero last BPM
- restore components, steer beam to adjusted ballistic line

62

Ballistic Alignment



62

Ballistic Alignment: Problems

- Controlling the downstream beam during the ballistic measurement
  - large beta-beat
  - large coherent oscillation
- Need to maintain energy match
  - scale downstream lattice while RF in ballistic section is off
- use feedback to keep downstream orbit under control

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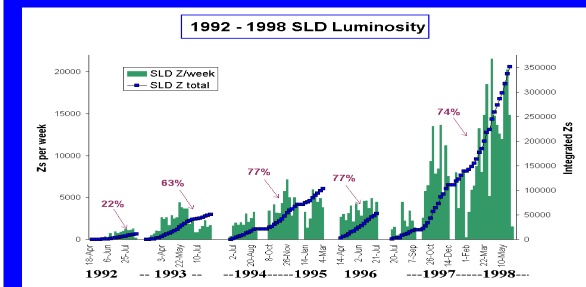
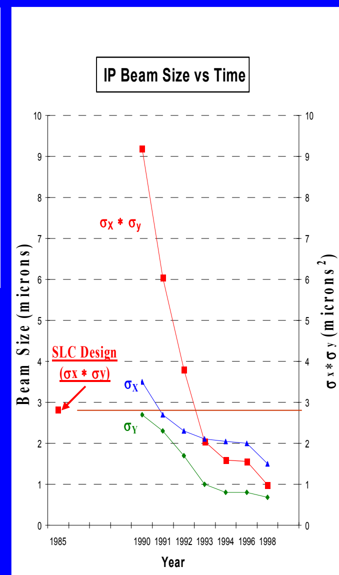
Lecture 9: b) Lessons learnt from SLC

Nick Walker (DESY)

USPAS, Santa Barbara, 16th-27th June, 2003

Lessons from the SLC





New Territory in Accelerator Design and Operation

- **Sophisticated on-line modeling of non-linear physics.**
- **Correction techniques expanded from first-order (trajectory) to include second-order (emittance), and from hands-on by operators to fully automated control.**
- **Slow and fast feedback theory and practice.**

D. Burke, SLAC

The SLC

	1980	1998	
	Design	Achieved	Units
Beam charge	7.2e10	4.2e10	$e^{\pm}$/bunch
Rep. rate	180	120	Hz
DR ε_x	3.0e-5	3.0e-5	m rad
DR ε_y	3.0e-5	3.0e-6	m rad
FF ε_x	4.2e-5	5.5e-5	m rad
FF ε_y	4.2e-5	1.0e-5	m rad
IP σ_x	1.65	1.4	µm
IP σ_y	1.65	0.7	µm
Pinch factor	220%	220%	Hd
Luminosity	6e30	3e30	$cm^{-2}sec^{-1}$

note: SLC was a single bunch machine (n_b = 1)

taken from *SLC – The End Game* by R. Assmann *et al*, proc. EPAC 2000

SLC: lessons learnt

- Control of wakefields in linac
  - orbit correction, closed (tuning) bumps
  - the need for continuous emittance measurement (automatic wire scanner profile monitors)
- Orbit and energy feedback systems
  - many MANY feedback systems implemented over the life time of the machine
  - operator ‘tweaking’ replaced by feedback loop
- Final focus optics and tuning
  - efficient algorithms for tuning (focusing) the beam size at the IP
  - removal (tuning) of optical aberrations using orthogonal knobs.
  - improvements in optics design
- many many more!

The SLC was an 10 year accelerator R&D project that also did some physics ☺

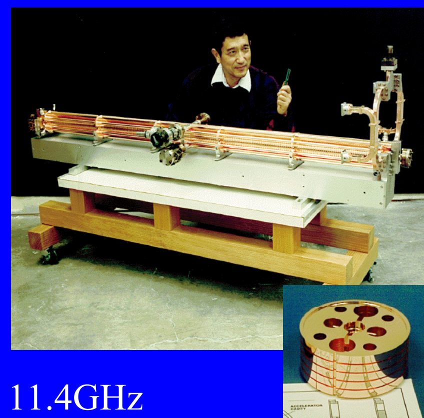
The Alternatives

2003 E_{cm}=500 GeV

		TESLA	JLC-C	JLC-X/NLC	CLIC
f	GHz	1.3	5.7	11.4	30.0
L	$\times 10^{33}$ $cm^{-2}s^{-1}$	34	14	20	21
P_{beam}	MW	11.3	5.8	6.9	4.9
P_{AC}	MW	140	233	195	175
$\gamma\varepsilon_y$	$\times 10^{-8}$m	3	4	4	1
σ_y*	nm	5	4	3	1.2

Examples of LINAC technology

9 cell superconducting Niobium cavity for TESLA (1.3GHz)



11.4GHz
structure for NLCTA

(note older 1.8m structure)

Competing Technologies: swings and roundabouts

RF frequency

CLIC (30GHz)

NLC (11.4GHz)

SLC (3GHz)

TESLA (1.3GHz SC)

- higher gradient = short linac ☺
- higher r_s = better efficiency ☺
- High rep. rate = GM suppression ☺
- smaller structure dimensions = high wakefields ☹
- Generation of high pulse peak RF power ☹

- long pulse low peak power ☺
- large structure dimensions = low WF ☺
- very long pulse train = feedback within train ☺
- SC gives high efficiency ☺
- Gradient limited <40 MV/m = longer linac ☹
- low rep. rate bad for GM suppression (ε_y dilution) ☹
- very large unconventional DR ☹☹

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Lecture 9: c) Summary

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The *Luminosity* Issue

$$L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}} \sqrt{\frac{\delta_{BS}}{\varepsilon_{n,y}}} H_D \qquad \beta_y \approx \sigma_z$$

- high RF-beam conversion efficiency η_{RF}
- high RF power P_{RF}
- small normalised vertical emittance $\varepsilon_{n,y}$
- strong focusing at IP (small β_y and hence small σ_z)
- could also allow higher beamstrahlung δ_{BS} if willing to live with the consequences
- Valid for low beamstrahlung regime (Υ<1)

High Beamstrahlung Regime

low beamstrahlung regime $\Upsilon<<1$: $L \propto P_{\text{beam}} \frac{\sqrt{\delta_{\text{BS}}}}{\sqrt{\varepsilon_{y,n}}}$

high beamstrahlung regime $\Upsilon>>1$: $L \propto P_{\text{beam}} \frac{\delta_{\text{BS}}^{3/2}}{\sigma_z \sqrt{\varepsilon_{y,n}}}$

with $\beta_y^* \approx \sigma_z$

Pinch Enhancement

$$L \propto \frac{\eta_{RF} P_{RF}}{E_{cm}} \sqrt{\frac{\delta_{BS}}{\varepsilon_{n,y}}} H_D$$

$$H_D = H_D(D_y)$$

$$D_y = \frac{\sigma_z}{f_{beam}} \approx \frac{2N_b r_e \sigma_z}{\gamma \sigma_y (\sigma_x + \sigma_y)} \approx \frac{2N_b r_e \sigma_z}{\gamma \sigma_y \sigma_x}$$

Trying to push hard on D_y to achieve larger H_D leads to single-bunch kink instability ➔ detrimental to luminosity

The Linear Accelerator (LINAC)

- Gradient given by *shunt impedance*: $E_z(z) = \sqrt{P_{RF}(z) r_l}$
  - P_{RF} RF power /unit length
  - r_l shunt impedance /unit length
- The cavity Q defines the *fill time:*
  - v_g = *group velocity,*
  - l_s = *structure length*

$$t_{fill} = \begin{cases} 2Q/\omega & \text{SW} \\ 2\tau Q/\omega = l_s/v_g & \text{TW} \end{cases}$$

- For TW, τ is the structure attenuation constant: $P_{RF,out} = P_{RF,in} e^{-2\tau}$
- RF power lost along structure (TW):

$$\frac{dP_{RF}}{dz} = -\frac{E_z^2}{r_l} - i_b E_z$$

power lost to structure　　beam loading

η_{RF}

would like R_S to be as high as possible

$R_s \propto \sqrt{\omega}$

The Linear Accelerator (LINAC)

For constant gradient structures:

$$V_u = \sqrt{r_l P_0 L \left(1 - e^{-2\tau}\right)}$$

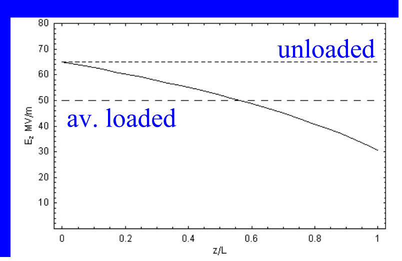
unloaded structure voltage

$$V_l = V_u - \frac{1}{2} r_l L i_{\text{beam}} \left(1 - \frac{2\tau e^{-2\tau}}{1 - e^{-2\tau}}\right)$$

loaded structure voltage (steady state)

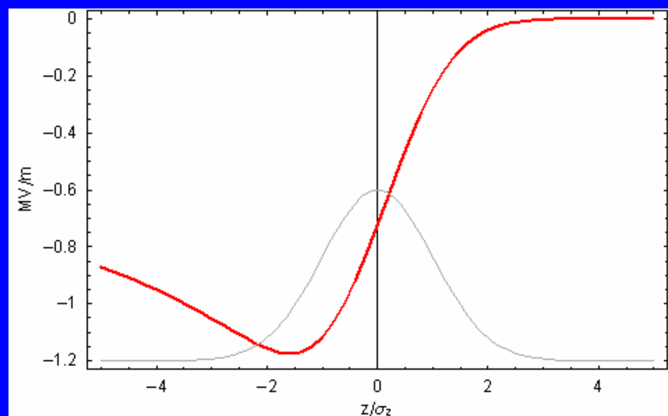
$$i_{\text{opt}} = \sqrt{\frac{P_0}{r_l L} \frac{\left(1 - e^{-2\tau}\right)^{3/2}}{1 - (1 + 2\tau) e^{-2\tau}}}$$

optimum current (100% loading)



The Linear Accelerator (LINAC)

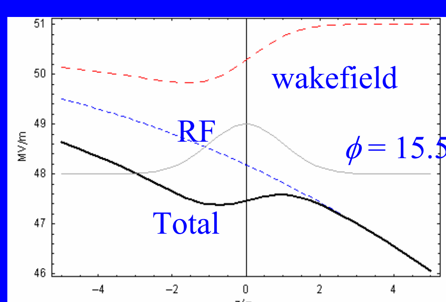
Single bunch beam loading: the Longitudinal wakefield

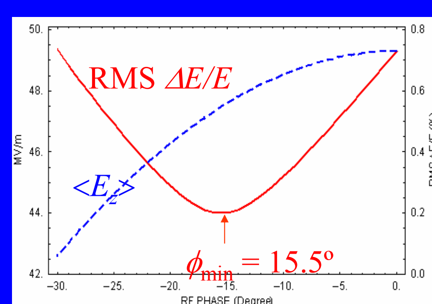


NLC X-band structure: $\langle \Delta E_z \rangle_{bunch} \approx 700\,\mathrm{kV/m}$

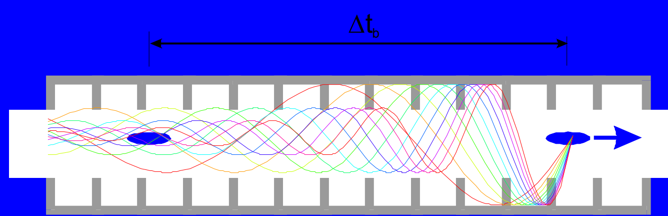
The Linear Accelerator (LINAC)

Single bunch beam loading Compensation using RF phase





Transverse Wakes: The Emittance Killer!



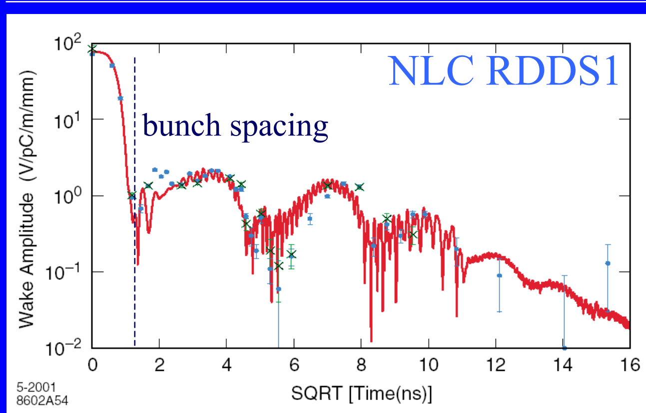
$$V(\omega,t) = I(\omega,t)Z(\omega,t)$$

Bunch current also generates transverse deflecting modes when bunches are not on cavity axis

Fields build up resonantly: latter bunches are kicked transversely

$\Rightarrow$ multi- and single-bunch beam breakup (MBBU, SBBU)

Damped & Detuned Structures



$$\Delta t \approx \frac{2Q_{\mathrm{HOM}}}{\Delta\omega}$$

Slight random detuning between cells causes HOMs to decohere.

Will recohere later: needs to be damped (HOM dampers)

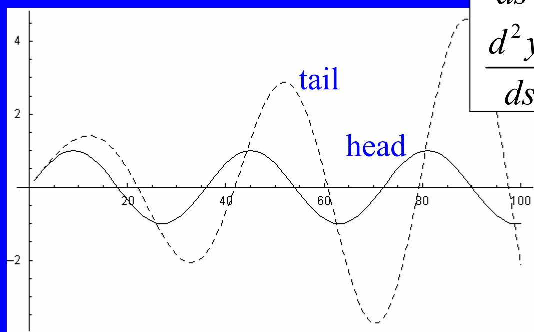
Single bunch wakefields

Effect of coherent betatron oscillation

- head *resonantly* drives the tail

$$\frac{d^2 y_h}{ds} + k^2 y_y = 0 \qquad \text{head}$$

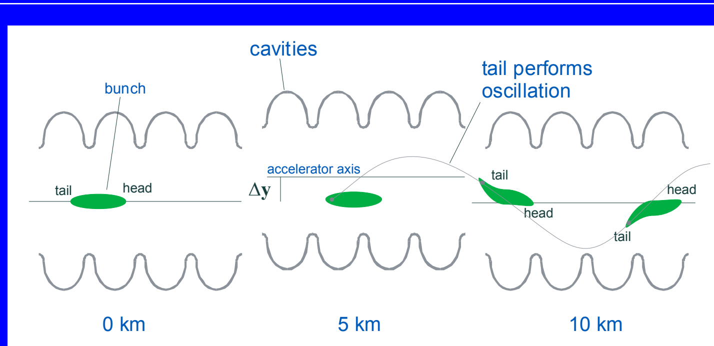
$$\frac{d^2 y_t}{ds} + k^2 y_t = -w_{wf}\, y_h \qquad \text{tail}$$



Cancel using BNS damping:

$$\delta_{\mathrm{BNS}} \approx \frac{1}{16}\frac{W'_{\perp} q \sigma_z}{E}\frac{L^2_{\mathrm{cell}}}{\sin^2(\pi \nu_\beta)}$$

Wakefields (alignment tolerances)



$$\delta Y_{\mathrm{RMS}} \propto \frac{1}{N W_{\perp}}\sqrt{\frac{E_z}{\beta}}$$

$$\propto \frac{f^{-3}}{N}\sqrt{\frac{E_z}{\beta}}$$

higher frequency = stronger wakefields

-higher gradients

-stronger focusing (smaller β)

-smaller bunch charge

Damping Rings

initial emittance
(~0.01m for e^+)

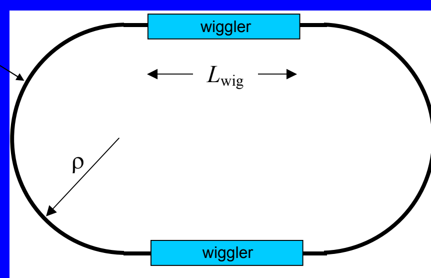
$$\varepsilon_f = \varepsilon_{eq} + (\varepsilon_i - \varepsilon_{eq})e^{-2T/\tau_D}$$

final emittance

equilibrium emittance

damping time

$$\Delta E_{\text{arc}} = C_\gamma \frac{E^4}{\rho}$$



$$\tau_D = \frac{2E}{P_\gamma} \le \frac{n_{\text{train}}}{8 f_{\text{rep}}}$$

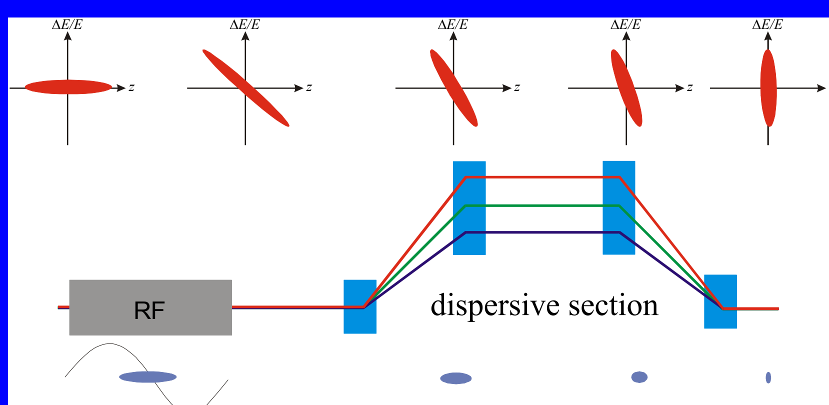
$$C = n_{train} n_b \Delta t_b c$$

$$\Delta E_{\text{wig}} \approx 1.27 \times 10^{-6} \left\langle B^2 \right\rangle (\text{T}) E^2 (\text{GeV}) L_{\text{wig}} (\text{m})$$

Bunch Compression

- bunch length from ring ~ few mm
- required at IP 100-300 μm

long. phase space



The linear bunch compressor

see lecture 6

initial (uncorrelated) momentum spread:	δ_u
initial bunch length	$\sigma_{z,0}$
compression ration	$F_c=\sigma_{z,0}/\sigma_z$
beam energy	E
RF induced (correlated) momentum spread:	δ_c
RF voltage	V_{RF}
RF wavelength	$\lambda_{RF}=2\pi/k_{RF}$
longitudinal dispersion:	R_{56}

conservation of longitudinal emittance

$$F_c=\frac{\sqrt{\delta_c^2+\delta_u^2}}{\delta_u}\Leftrightarrow\delta_c=\delta_u\sqrt{F_c^2-1}$$

RF cavity

$$\delta_c\approx\frac{k_{RF}V_{RF}\sigma_{z,0}}{E}\Leftrightarrow V_{RF}=\frac{E\delta_c}{k_{RF}\sigma_{z,0}}=\frac{E}{k_{RF}}\left(\frac{\delta_u}{\sigma_{z,0}}\right)\sqrt{F_c^2-1}$$

The linear bunch compressor

chicane (dispersive section)

$$R_{56}=-\frac{\langle\delta z\rangle}{\delta^2}=-\frac{\delta_c\sigma_{z,0}}{F^2\delta_u^2}=\frac{k_{RF}V_{RF}}{E}\left(\frac{\sigma_{z,0}}{\delta_u}\right)^2\frac{1}{F^2}$$

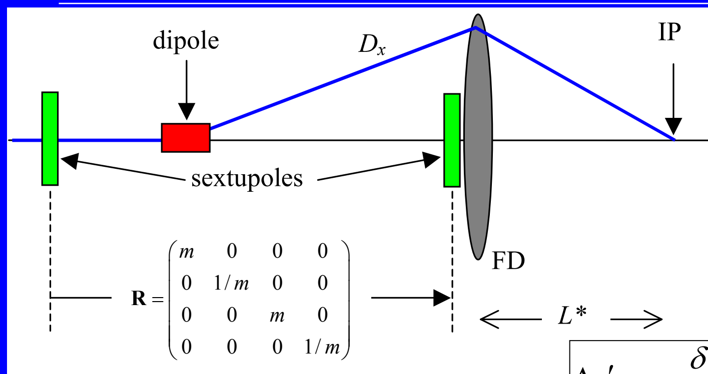
$$R_{56}\begin{cases}<0 & \text{wiggler}\\ >0 & \text{arc}\end{cases}$$

Two stage compression used to reduce final energy spread

E_0 —— COMP #1 (F_1) —— LINAC (ΔE) —— COMP #2 (F_2) ——→

$$\begin{aligned}\delta_f&=(F_1\delta_i)\cdot\left(\frac{E_0}{E_0+\Delta E}\right)\cdot F_2\\&=F_T\delta_i\left(\frac{E_0}{E_0+\Delta E}\right)\end{aligned}$$

Final Focusing



$$\Delta y'_{\mathrm{FD}} = -\frac{\delta y}{L^*}$$

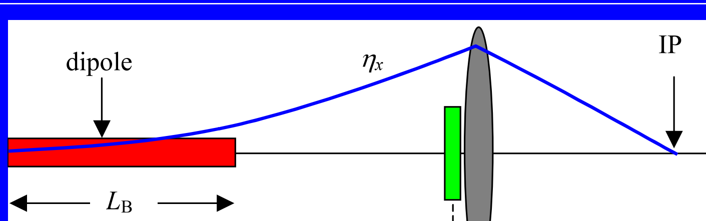
$$\Delta x'_{\mathrm{FD}} = -\frac{\delta(x+\eta\delta)}{L^*}$$

$$\Delta y'_{\mathrm{SX}} = S\left(x+\eta\delta\right)y$$

$$\Delta x'_{\mathrm{SX}} = -S\left[(x+\eta\delta)^2 - y^2\right]$$

chromatic correction $S = \dfrac{1}{L^*\eta}$

Synchrotron Radiation effects



$$\left(\frac{\Delta \mathrm{E}}{\mathrm{E}}\right)^2 \approx 19 \frac{r_e \lambda_e \gamma^5 L^{*3} \eta'^3}{L_B^5}$$

$$\frac{\Delta\sigma_y^{*2}}{\sigma_y^{*2}} \approx W_y^2 \left(\frac{\Delta \mathrm{E}}{\mathrm{E}}\right)^2$$

FD chromaticity + dipole SR sets limits on minimum bend length

Final Focusing: Fundamental limits

Already mentioned that $\beta_y \geq \sigma_z$

At high-energies, additional limits set by so-called *Oide Effect*: synchrotron radiation in the final focusing quadrupoles leads to a beamsize growth at the IP

minimum beam size: $\sigma \approx 1.83\left(r_e \lambda\!\!\!^{-}_e F\right)^{1/7} \varepsilon_n^{5/7}$ *independent of E!*

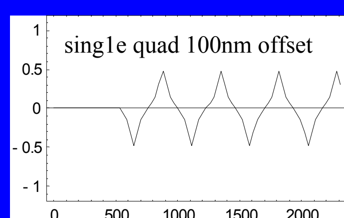
occurs when $\beta \approx 2.39\left(r_e \lambda\!\!\!^{-}_e F\right)^{2/7} \gamma \varepsilon_n^{3/7}$

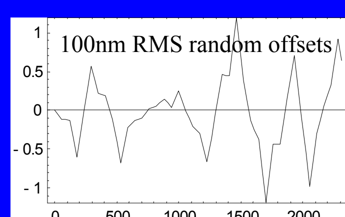
F is a function of the focusing optics: typically $F \sim 7$ (minimum value ~0.1)

LINAC quadrupole stability

$$y^* = \sum_{i=1}^{N_Q} k_{Q,i} \Delta Y_i g_i = k_Q \sum_{i=1}^{N_Q} \Delta Y_i g_i$$

$$g_i = \sqrt{\frac{\gamma_i}{\gamma^*}} \sqrt{\beta_i \beta^*} \sin(\Delta\phi_i)$$





for uncorrelated offsets

$$\left\langle y^{*2} \right\rangle = \frac{\beta^* \left\langle \Delta Y^2 \right\rangle}{\gamma^*} \sum_{i=1}^{N_Q} \gamma_i k_{Q,i}^2 \beta_i \sin^2(\Delta\phi_{ij})$$

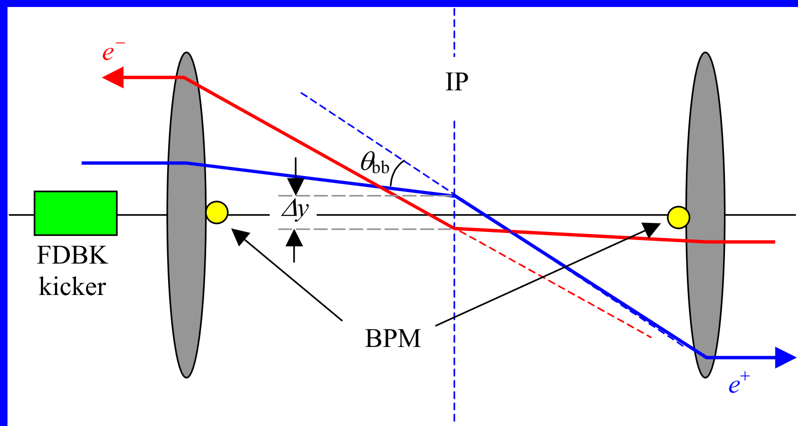
Dividing by $\sigma_y^{*2} = \beta^* \varepsilon_{y,n} / \gamma^*$ and taking average values:

$$\frac{\left\langle y_j^2 \right\rangle}{\sigma_y^{*2}} \approx \frac{N_Q k_Q^2 \bar{\beta} \bar{\gamma}}{2\varepsilon_{y,n}} \sigma_{\Delta Y}^2 \leq 0.3^2$$

take $N_Q = 400$, $\varepsilon_y \sim 6\times10^{-14}$ m, $\beta \sim 100$ m, $k_1 \sim 0.03$ m^{-1} $\Rightarrow$ ~25 nm

see lecture 8

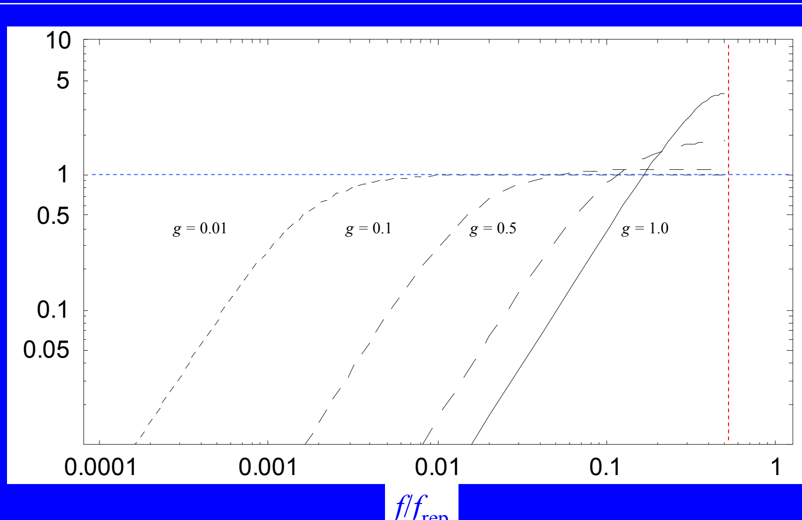
Beam-Beam orbit feedback



use strong beam-beam kick to keep beams colliding

Generally, orbit control (feedback) will be used extensively in LC

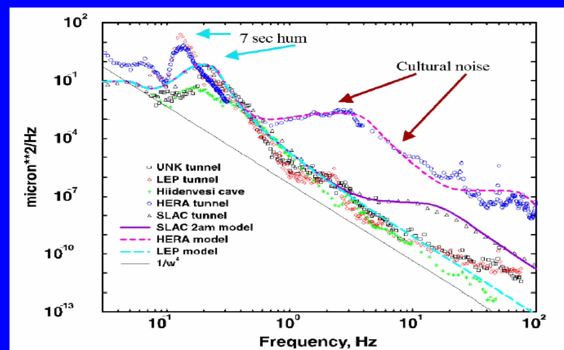
Beam based feedback: bandwidth



Good rule of thumb: attenuate noise with $f<f_{rep}/20$

Ground motion spectra

Both frequency spectrum and *spatial* correlation important for LC performance



2D power spectrum

$$P(\omega, k)$$

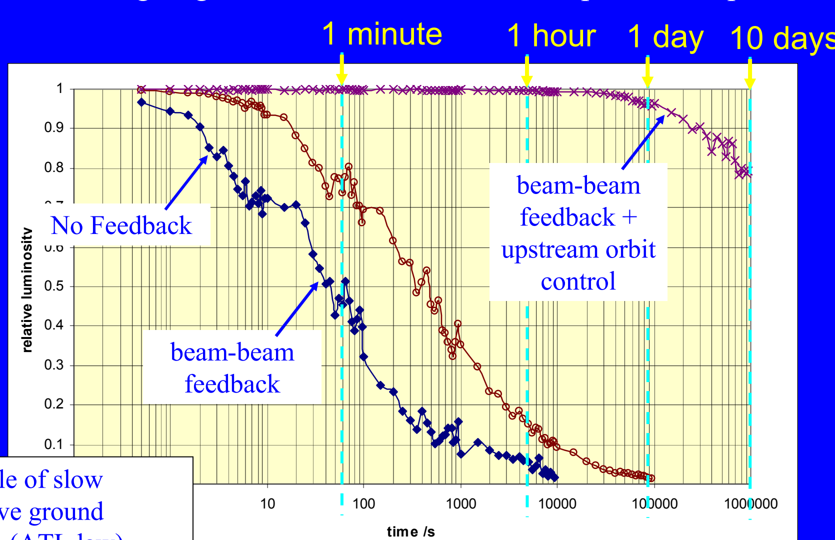
measurable relative power spectrum

$$\rho(\omega, L) = \frac{1}{\pi}\int_{-\infty}^{+\infty} P(\omega,k)\left[1-\cos(kL)\right]dk$$

Long Term Stability

see lecture 8

understanding of ground motion and vibration spectrum important



example of slow diffusive ground motion (ATL law)

A Final Word

- Technology decision due 2004
- Start of construction 2007+
- First physics 2012++
- There is *still* much to do!

WE NEED YOUR HELP

for

the Next Big Thing

hope you enjoyed the course

Nick, Andy, Andrei and PT