



Final Focus System and Beam Collimation in Linear Collider

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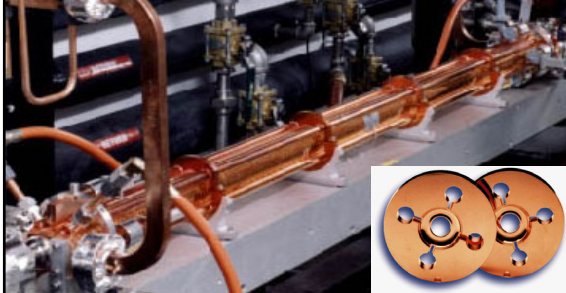
Content

- 1st part of the lecture: stuff that you are expected to learn
- 2nd part: more general overview of BDS developments (mainly NLC)
- Questions at any moment, please !



Linear Colliders - two main challenges

- **Energy** - need to reach at least 500 GeV CM as a start



Normal Conducting (JLC/NLC, CLIC) technology



Super Conducting (TESLA) RF technology

- **Luminosity** - need to reach 10^{34} level

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The Luminosity Challenge

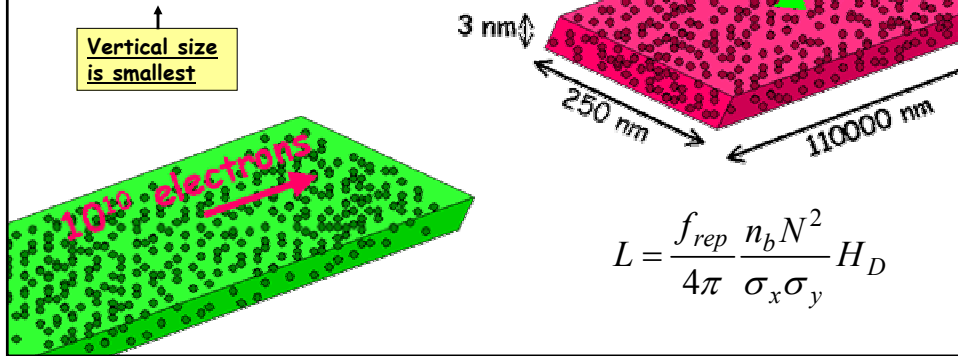
- **Must jump by a Factor of 10000 in Luminosity !!!**
(from what is achieved in the only so far linear collider SLC)
- Many improvements, to ensure this : generation of smaller emittances, their better preservation, ...
- Including better focusing, dealing with beam-beam, and better stability
 - Ensure maximal possible focusing of the beams at IP Lecture 6
 - Optimize IP parameters w.r.to beam-beam effects Lecture 7
 - Ensure that ground motion and vibrations do not produce intolerable misalignments Lecture 8

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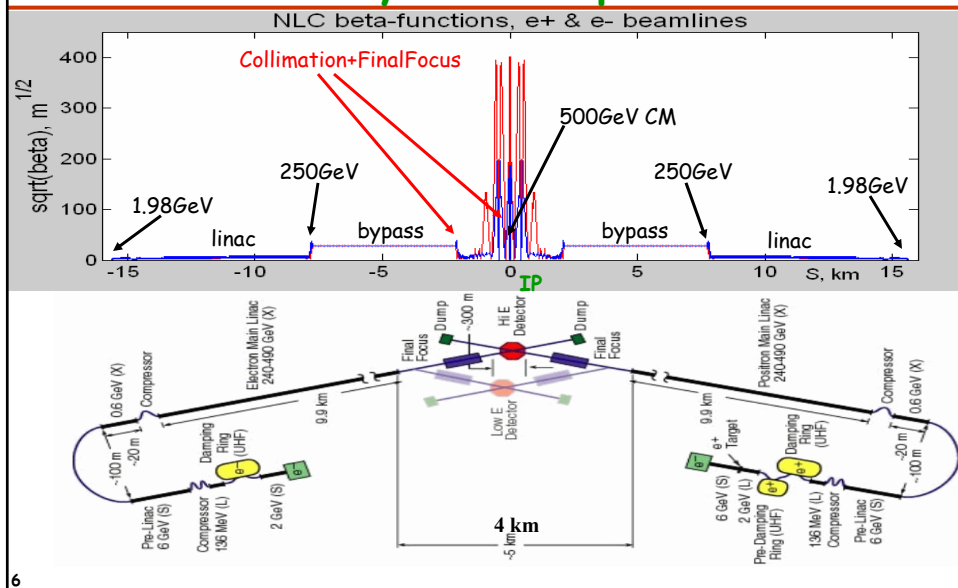


How to get Luminosity

- To increase probability of direct e^+e^- collisions (luminosity) and birth of new particles, beam sizes at IP must be very small
- E.g., NLC beam sizes just before collision (500GeV CM):
 $250 \times 3 \times 110000$ nanometers
 (x y z)



Next Linear Collider layout and optics





TRC table of general parameters of Linear Collider projects

TABLE 2.1: Overall parameters

	TESLA		JLC-C		JLC-X/NLC ^a		CLIC	
	500	800	500	1000	500	1000	500	3000
Center of mass energy [GeV]	500	800	500	1000	500	1000	500	3000
RF frequency of main linac [GHz]	1.3		5.7	5.7/11.4 ^b	11.4		30	
Design luminosity [$10^{33} \text{ cm}^{-2}\text{s}^{-1}$]	34.0	58.0	14.1	25.0	25.0 (20.0)	25.0 (30.0)	21.0	80.0
Linac repetition rate [Hz]	5	4		100	150 (120)	100 (120)	200	100
Number of particles/bunch at IP [10^{10}]	2	1.4		0.75	0.75		0.4	
Number of bunches/pulse	2820	4886		192	192		154	
Bunch separation [nsec]	337	176		1.4	1.4		0.67	
Bunch train length [μsec]	950	860		0.267	0.267		0.102	
Beam power/beam [MW]	11.3	17.5	5.8	11.5	8.7 (6.9)	11.5 (13.8)	4.9	14.8
Unloaded/loaded gradient ^c [MV/m]	23.8 / 23.8 ^d	35 / 35	41.8/31.5	41.8/31.5 / 70/55	65 / 50		172 / 150	
Total two-linac length [km]	30	30	17.1	29.2	13.8	27.6	5.0	28.0
Total beam delivery length [km]	3			3.7	3.7		5.2	
Proposed site length [km]	33			33	32		10.2	
Total site AC power ^e [MW]	140	200	233	300	243 (195)	292 (350)	175	410
Tunnel configuration ^f	Single			Double	Double		Single	

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TRC table of IP parameters of Linear Collider projects

TABLE 2.6: Linear colliders: beam delivery system and interaction point parameters

	TESLA		JLC-C		JLC-X/NLC ^a		CLIC	
	500 GeV	800 GeV	500 GeV	1000 GeV	500 GeV	1000 GeV	500 GeV	3000 GeV
Beam delivery system length ^b [km]	3.2		3.8		3.8		5.2	
Collimation system length ^b [km]	1.4		1.4		1.4		4.1	
Final Focus system length ^b [km]	1.2		1.6		1.6		1.1	
$\gamma\varepsilon_x^* / \gamma\varepsilon_y^*$ [m-rad $\times 10^{-6}$]	10 / 0.03		8 / 0.015		3.6 / 0.04		2.0 / 0.01	
β_x^* / β_y^* [mm]	15 / 0.40		15 / 0.40		8 / 0.20		13 / 0.11	
σ_x^* / σ_y^* before pinch ^c [nm]	554 / 5.0		392 / 2.8		243 / 4.0		219 / 2.1	
σ_z^* [μm]	300		200		110		110	
$\sigma_{\Delta E/E}^*$ [%]	0.14 / 0.04		0.25		0.25		0.25	
Distance between IP and last quad	3.0		4		3.5		4.3	
Crossing Angle at IP [mrad]	0		7		7 (20)		20	
Disruptions D_x / D_y	0.23 / 25.3		0.20 / 28.0		0.29 / 17.5		0.10 / 10.3	
Υ_0	0.05		0.09		0.07		0.28	
δ_B [%]	3.2		4.3		3.4		7.5	
n_γ [number of γ s per e]	1.56		1.51		1.36		1.30	
$N_{\text{pairs}} (p_T^{\text{min}} = 20 \text{ MeV}/c, \Theta_{\text{min}} = 0.2)$	39.4		37.3		10.7		15.0	
$N_{\text{hadron events/crossing}}$	0.248		0.399		0.075		0.103	
$N_{\text{jets}} \times 10^{-2} (p_T^{\text{min}} = 3.2 \text{ GeV}/c)$	0.74		1.90		0.23		2.27	
Geometric Luminosity ^e [$10^{33} \text{ cm}^{-2}\text{s}^{-1}$]	16.4		28.1		8.76		18.5	
H_D	2.11		1.90		1.61		1.42	
Luminosity dilution for tuning [%]	0		5		5		10	
Peak Luminosity ^e [$10^{33} \text{ cm}^{-2}\text{s}^{-1}$]	34.5		53.4		13.6		24.9	
$L_{99\%}$ [%]	66		62		67		58	
$L_{95\%}$ [%]	91		86		90		86	
$L_{90\%}$ [%]	98		95		97		87	

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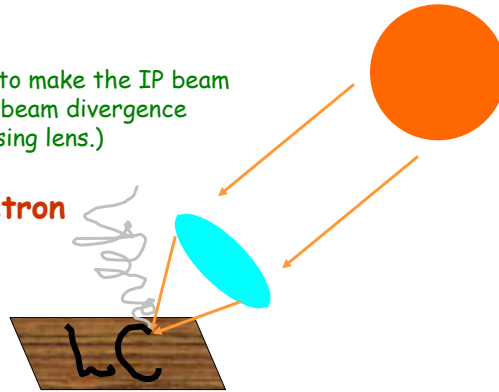


How to focus the beam to a smallest spot?

- Did you ever played with a lens trying to burn a picture on a wood under bright sun ?
- Then you know that one needs a strong and big lens

(The emittance ϵ is constant, so, to make the IP beam size $(\epsilon \beta)^{1/2}$ small, you need large beam divergence at the IP $(\epsilon / \beta)^{1/2}$ i.e. short-focusing lens.)

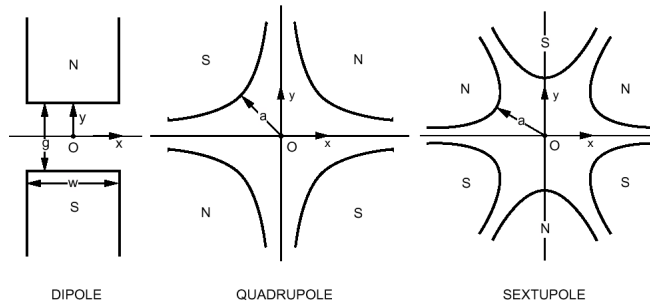
- It is very similar for **electron** or **positron** beams
- But one have to use **magnets**



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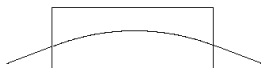


What we use to manipulate with the beam



Etc...

Just bend the trajectory



Focus in one plane,
defocus in another:
 $x' = x' + G x$
 $y' = y' - G y$

Second order
effect:
 $x' = x' + S (x^2 - y^2)$
 $y' = y' - S 2xy$

Here x is transverse coordinate, x' is angle

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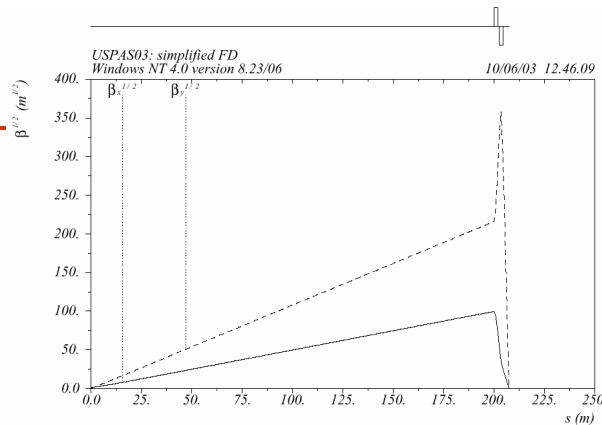
Final telescope

Essential part of final focus is final telescope. It “demagnify” the incoming beam ellipse to a smaller size. Matrix transformation of such telescope is diagonal:

$$R_{X,Y} = \begin{pmatrix} -1/M_{X,Y} & 0 \\ 0 & -M_{X,Y} \end{pmatrix}$$

A minimal number of quadrupoles, to construct a telescope with arbitrary demagnification factors, is four.

If there would be no energy spread in the beam, a telescope could serve as your final focus (or two telescopes chained together).



Shown above is a “telescope-like” optics which consist just from two quads (final doublet). In may have all the properties of a telescope, but demagnification factors cannot be arbitrary. In the example shown the IP beta functions are 15mm for X and 0.1mm for Y. The el-star is 3m.

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Why nonlinear elements

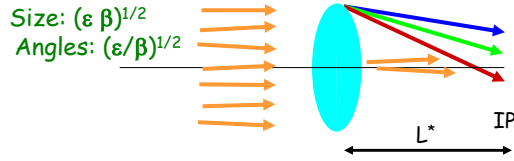
- As sun light contains different colors, **electron beam** has energy spread and get dispersed and distorted => **chromatic aberrations**
- For light, one uses lenses made from different materials to compensate chromatic aberrations
- Chromatic compensation for particle beams is done with **nonlinear magnets**
 - Problem: Nonlinear elements create **geometric aberrations**
- The **task of Final Focus system (FF)** is to focus the beam to required size and compensate aberrations



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How to focus to a smallest size and how big is chromaticity in FF?



Size at IP:
 $L^* (\epsilon/\beta)^{1/2}$
 $+ (\epsilon\beta)^{1/2} \sigma_E$

Beta at IP:
 $L^* (\epsilon/\beta)^{1/2} = (\epsilon\beta^*)^{1/2}$
 $\Rightarrow \beta^* = L^{*2}/\beta$

Chromatic dilution:
 $(\epsilon\beta)^{1/2} \sigma_E / (\epsilon\beta^*)^{1/2}$
 $= \sigma_E L^*/\beta^*$

- The last (final) lens need to be the strongest
 - (two lenses for both x and y => "Final Doublet" or FD)
- FD determines chromaticity of FF
- Chromatic dilution of the beam size is $\Delta\sigma/\sigma \sim \sigma_E L^*/\beta^*$

Typical: σ_E -- energy spread in the beam ~ 0.01
 L^* -- distance from FD to IP ~ 3 m
 β^* -- beta function in IP ~ 0.1 mm

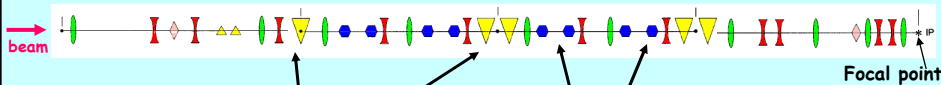
- For typical parameters, $\Delta\sigma/\sigma \sim 300$ too big !
- => Chromaticity of FF need to be compensated

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Example of traditional Final Focus

Sequence of elements in ~100m long Final Focus Test Beam



Dipoles. They bend trajectory, but also disperse the beam so that x depend on energy offset δ

Sextupoles. Their kick will contain energy dependent focusing
 $x' \Rightarrow S(x+\delta)^2 \Rightarrow 2Sx\delta + \dots$
 $y' \Rightarrow -S2(x+\delta)y \Rightarrow -2Sy\delta + \dots$
 that can be used to arrange chromatic correction

Necessity to compensate chromaticity is a major driving factor of FF design

Terms x^2 are geometric aberrations and need to be compensated also

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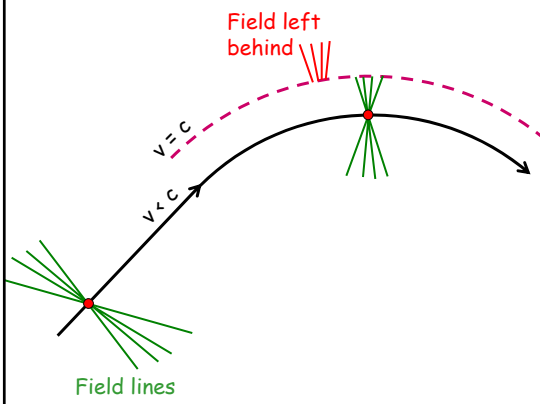
Final Focus Test Beam

Achieved ~70nm vertical beam size

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Synchrotron Radiation in FF magnets

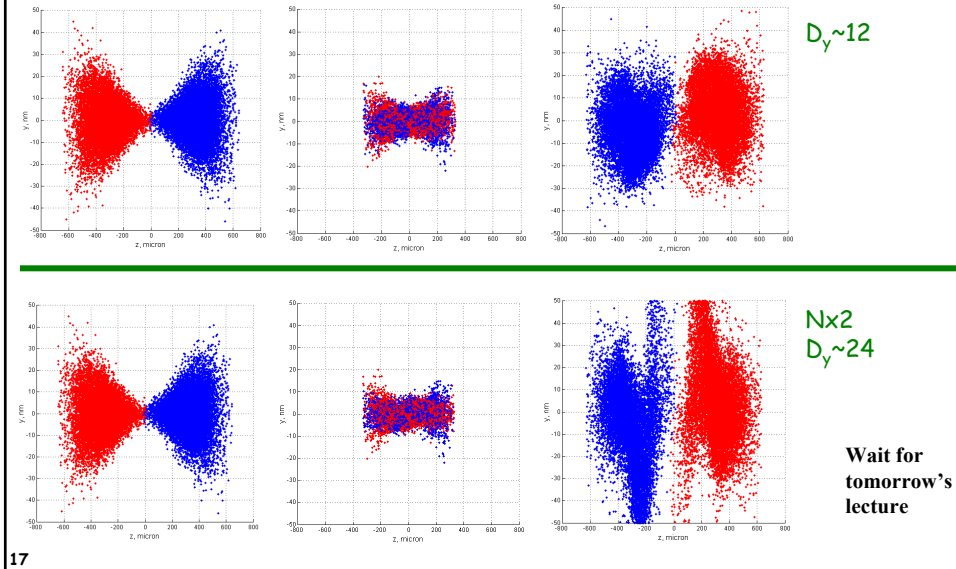


- Bends are needed for compensation of chromaticity
- SR causes increase of energy spread which may perturb compensation of chromaticity
- Bends need to be long and weak, especially at high energy
- SR in FD quads is also harmful (Oide effect) and may limit the achievable beam size

Energy spread caused by SR in bends and quads is also a major driving factor of FF design



Beam-beam (D_y, δ_E, Υ) affect choice of IP parameters and are important for FF design also



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Exercise 1 choice of IP parameters

Consider the CLIC 3TeV CM parameters, but assume that only half of the nominal beam population was achieved. Suggest how the IP beam sizes can be changed to keep the nominal luminosity and to have reasonable beam-beam parameters.

Take into account the following scaling:

$$\text{Lumi} \sim \frac{N^2}{\sigma_x \sigma_y} \quad D_y \sim \frac{N \sigma_z}{\sigma_x \sigma_y} \quad \Upsilon \sim \frac{N}{\sigma_x \sigma_z} \quad \delta_E \sim \frac{N^2}{\sigma_x^2 \sigma_z}$$

Verify your predictions with Beam-Beam simulations using Guinea-Pig program [D.Schulte]

The necessary files are in C:\LC_WORK\ex1 (You may need to read the readme.txt file in this directory and also the file tasks_bb_to_ffs.doc in C:\LC_WORK)

```
$ACCELERATOR:: YOURLC1
```

```
{ energy = 500 ; GeV
  particles = 0.75 ; e10
  sigma_x = 250 ; nm
  sigma_y = 2.0 ; nm
  sigma_z = 100 ; micron
  beta_x = 5.0 ; mm
  beta_y = 0.2 ; mm
  offset_x = 0 ; nm (total offset will be 2*offset_x)
  offset_y = 0 ; nm (-/-)
}
```

```
$PARAMETERS:: LCPARS
```

```
{n_m=20000 ; number of macroparticles
hist_ee_max=1020; max E CM of lumi spectrum
```

Analysing the results. Look into gp.out :

```
lumi_fine (or lumi_ee) -- luminosity [1/m^2]
E_cm and E_cm_var -- CM energy and energy spread
due to beamstrahlung [GeV]
bpm_vx, bpm_vy -- average angular beam
deflection after collision [microrad]
upsmax -- max value of Upsilon parameter
```

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Exercise 2

design a final telescope using MAD

In this case you will design a two-lens telescope to focus a beam with parameters you found in the Exercise 1. You will use MAD program for this. The necessary files are in C:\LC_WORK\ex2

Example of MAD input language is given below with some comments (some lines skipped):

```

MYIPTWSS : BETA0, ENERGY=500, BETX=0.01, ALFX=0.0, DX=0, DPX=0, BETY=0.0001, ALFY=0.0    Specify twiss at IP
LQD0=2.0
KQD0=0.1
QF1: QUAD, L=1, K1=0.01    Define quads. L in meters,     $K1 = \frac{1}{B\rho} \frac{dB_y}{dx}$      $B\rho = \frac{pc}{e}$ 
QD0: QUADRUPOLE, L=LQD0, K1=KQD0    Define the beamline     $B\rho [Gs \cdot cm] = \frac{E [TeV]}{299.792458}$ 
FD_TEL: LINE=(D2,QF1,D1,QD0,D0)    Define the beamline    and the reversed one
REV_FD_TEL: LINE=(-FD_TEL)
USE, REV_FD_TEL    Specify what beamline to use

MATCH, BETA0=MYIPTWSS    Use MAD matching features
VARY, KQD0, STEP=1.E-5, UPPER=0    Tell him what he can vary
VARY, QF1[K1], STEP=1.E-5, LOWER=0
CONSTRAINT, #E, ALFX=0, ALFY=0, BETX< 10, BETY<10    Specify the desired result
LMDIF    Use this particular matching routine
ENDMATCH
VALUE, KQD0    Print matched values to echo file
VALUE, QF1[K1]

TWISS, SAVE, BETA0=MYIPTWSS    Calculate Twiss of the matched beamline
PLOT, TABLE=TWISS, HAXIS=S, VAXIS=BETX,BETY, RANGE=#S/#E    Plot them. (result in mad.metafile.ps)

```

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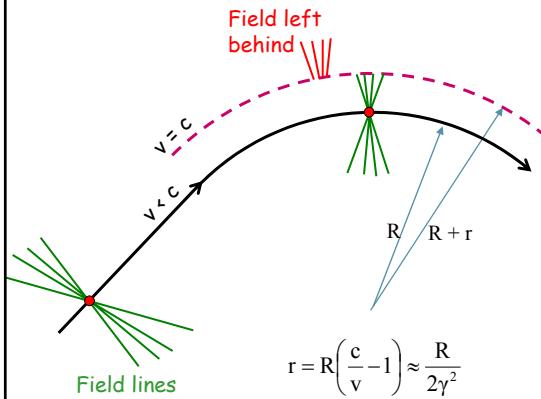
Check point

- What drives, primarily, FF design choices?
- Why the length of NLC FF would be much longer than of the SLC FF or FFTB?
- What defines, among other effects, the choice of beam parameters at the IP?

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Let's estimate SR power



Energy in the field left behind (radiated !):

$$W \approx \int E^2 dV$$

The field $E \approx \frac{e}{r^2}$ the volume $V \approx r^2 dS$

Energy loss per unit length:

$$\frac{dW}{dS} \approx E^2 r^2 \approx \left(\frac{e}{r^2}\right)^2 r^2$$

Substitute $r \approx \frac{R}{2\gamma^2}$ and get an estimate:

$$\boxed{\frac{dW}{dS} \approx \frac{e^2 \gamma^4}{R^2}}$$

Compare with exact formula: $\frac{dW}{dS} = \frac{2}{3} \frac{e^2 \gamma^4}{R^2}$

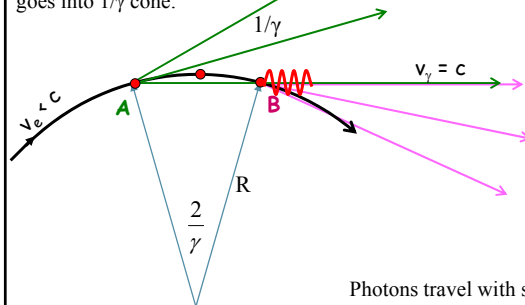
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Let's estimate typical frequency of SR photons

For $\gamma \gg 1$ the emitted photons goes into $1/\gamma$ cone.

During what time Δt the observer will see the photons?



Photons emitted during travel along the $2R/\gamma$ arc will be observed.

Photons travel with speed c , while particles with v .

At point B, separation between photons and particles is

$$dS \approx \frac{2R}{\gamma} \left(1 - \frac{v}{c}\right)$$

Therefore, observer will see photons during $\Delta t \approx \frac{dS}{c} \approx \frac{2R}{c\gamma} (1 - \beta) \approx \frac{R}{c\gamma^3}$

Estimation of characteristic frequency

$$\boxed{\omega_c \approx \frac{1}{\Delta t} \approx \frac{c\gamma^3}{R}}$$

Compare with exact formula: $\omega_c = \frac{3}{2} \frac{c\gamma^3}{R}$

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Let's estimate energy spread growth due to SR

We estimated the rate of energy loss : $\frac{dW}{dS} \approx \frac{e^2 \gamma^4}{R^2}$ And the characteristic frequency $\omega_c \approx \frac{c \gamma^3}{R}$

The photon energy $\epsilon_c = \hbar \omega_c \approx \frac{\gamma^3 \hbar c}{R} = \frac{\gamma^3}{R} \lambda_e mc^2$ where $r_e = \frac{e^2}{mc^2}$ $\alpha = \frac{e^2}{\hbar c}$ $\lambda_e = \frac{r_e}{\alpha}$

Number of photons emitted per unit length $\frac{dN}{dS} \approx \frac{1}{\epsilon_c} \frac{dW}{dS} \approx \frac{\alpha \gamma}{R}$ (per angle θ : $N \approx \alpha \gamma \theta$)

The energy spread $\Delta E/E$ will grow due to statistical fluctuations (\sqrt{N}) of the number of emitted photons :

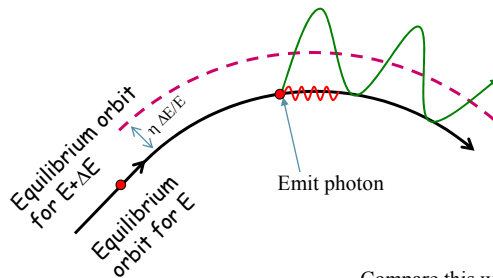
$$\frac{d((\Delta E/E)^2)}{dS} \approx \epsilon_c^2 \frac{dN}{dS} \frac{1}{(\gamma mc^2)^2} \quad \text{Which gives: } \frac{d((\Delta E/E)^2)}{dS} \approx \frac{r_e \lambda_e \gamma^5}{R^3}$$

$$\text{Compare with exact formula: } \frac{d((\Delta E/E)^2)}{dS} = \frac{55}{24\sqrt{3}} \frac{r_e \lambda_e \gamma^5}{R^3}$$

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Let's estimate emittance growth rate due to SR



Dispersion function η shows how equilibrium orbit shifts when energy changes

When a photon is emitted, the particle starts to oscillate around new equilibrium orbit

Amplitude of oscillation is $\Delta x \approx \eta \Delta E/E$

Compare this with betatron beam size: $\sigma_x = (\epsilon_x \beta_x)^{1/2}$

And write emittance growth: $\Delta \epsilon_x \approx \frac{\Delta x^2}{\beta}$

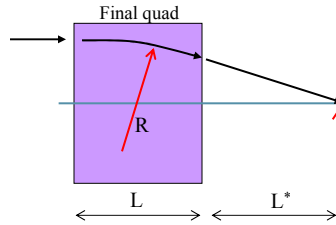
$$\text{Resulting estimation for emittance growth: } \frac{d\epsilon_x}{dS} \approx \frac{\eta^2}{\beta_x} \frac{d((\Delta E/E)^2)}{dS} \approx \frac{\eta^2}{\beta_x} \frac{r_e \lambda_e \gamma^5}{R^3}$$

$$\text{Compare with exact formula (which also takes into account the derivatives): } \frac{d\epsilon_x}{dS} = \frac{(\eta^2 + (\beta_x \eta' - \beta_x' \eta / 2)^2)}{\beta_x} \frac{55}{24\sqrt{3}} \frac{r_e \lambda_e \gamma^5}{R^3} = \mathcal{H}$$

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Let's apply SR formulae to estimate Oide effect (SR in FD)



IP divergence:

$$\theta^* = \sqrt{\varepsilon/\beta^*}$$

IP size:

$$\sigma^* = \sqrt{\varepsilon\beta^*}$$

Energy spread obtained in the quad:

$$\left(\frac{\Delta E}{E}\right)^2 \approx \frac{r_c \lambda_c \gamma^5 L}{R^3}$$

Radius of curvature of the trajectory: $R = L / \theta^*$

Growth of the IP beam size: $\sigma^2 \approx \sigma_0^2 + (L^* \theta^*)^2 \left(\frac{\Delta E}{E}\right)^2$

Which gives $\sigma^2 \approx \varepsilon\beta^* + C_1 \left(\frac{L^*}{L}\right)^2 r_c \lambda_c \gamma^5 \left(\frac{\varepsilon}{\beta^*}\right)^{5/2}$ (where C_1 is ~ 7 (depend on FD params.))

This achieve minimum possible value:

$$\sigma_{\min} \approx 1.35 C_1^{1/7} \left(\frac{L^*}{L}\right)^{2/7} (r_c \lambda_c)^{1/7} (\gamma\varepsilon)^{5/7}$$

When beta* is:

$$\beta_{\text{optimal}} \approx 1.29 C_1^{2/7} \left(\frac{L^*}{L}\right)^{4/7} (r_c \lambda_c)^{2/7} \gamma (\gamma\varepsilon)^{3/7}$$

Note that beam distribution at IP will be non-Gaussian. Usually need to use tracking to estimate impact on luminosity. Note also that optimal β may be smaller than the σ_z (i.e cannot be used).

Task: estimate the minimal vertical size, assuming that horizontal divergence is larger than the vertical.

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Exercise 3 study Oide effect in a FD

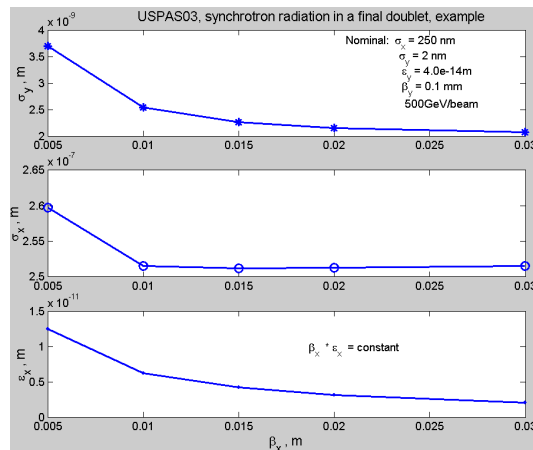
In this example, you will study the Oide effect for your beam parameters and your telescope created in Exercises 1 and 2. Use analytical estimations and verify your results with tracking by DIMAD.

The necessary files are in C:\LC_WORK\lex3

You can do studies similar as shown in <http://www.slac.stanford.edu/~seryi/uspas03/ex3>

In this case, the IP sizes, vertical emittance and vertical beta-function were already chosen, and it was necessary to chose the horizontal beta-function and emittance.

As you can see from the plot, the Oide effect limit the horizontal beta-function to be larger than 15mm, i.e. the horizontal emittance should be smaller than 4e-12 m (i.e. smaller than 3.9e-6m for normalized emittance).



Picture shows the beam sizes obtained by tracking with DIMAD. The sizes are "luminosity equivalent", which deemphasize the importance of tails.

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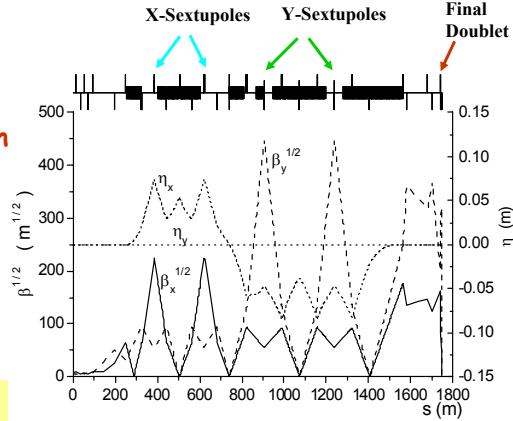
Concepts and problems of traditional FF

- Chromaticity is compensated by sextupoles in dedicated sections
- Geometrical aberrations are canceled by using sextupoles in pairs with $M = -I$

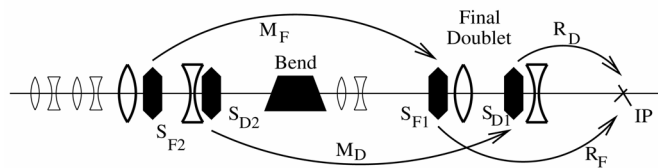
Chromaticity arise at FD but pre-compensated 1000m upstream

Problems:

- Chromaticity not locally compensated
 - Compensation of aberrations is not ideal since $M \neq -I$ for off energy particles
 - Large aberrations for beam tails
 - ...



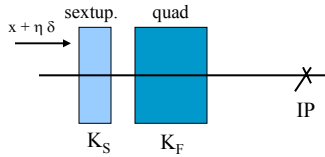
Principles of new FF



- Chromaticity is cancelled locally by two sextupoles interleaved with FD, a bend upstream generates dispersion across FD
- Geometric aberrations of the FD sextupoles are cancelled by two more sextupoles placed in phase with them and upstream of the bend



Chromatic correction in FD



- Straightforward in Y plane
- a bit tricky in X plane:

$$\text{Quad: } \Delta x' = \frac{K_F}{(1+\delta)}(x + \eta\delta) \Rightarrow K_F(-\delta x - \eta\delta^2)$$

chromaticity

Second order dispersion

$$\text{Sextupole: } \Delta x' = \frac{K_S}{2}(x + \eta\delta)^2 \Rightarrow K_S\eta\left(\delta x + \frac{\eta\delta^2}{2}\right)$$

$$\Delta x' = \frac{K_F}{(1+\delta)}(x + \eta\delta) + \frac{K_{\beta\text{-match}}}{(1+\delta)}x \Rightarrow 2K_F(-\delta x - \frac{\eta\delta^2}{2})$$

$$K_{\beta\text{-match}} = K_F \quad K_S = \frac{2K_F}{\eta}$$

If we require $K_S\eta = K_F$ to cancel FD chromaticity, then half of the **second order dispersion** remains.

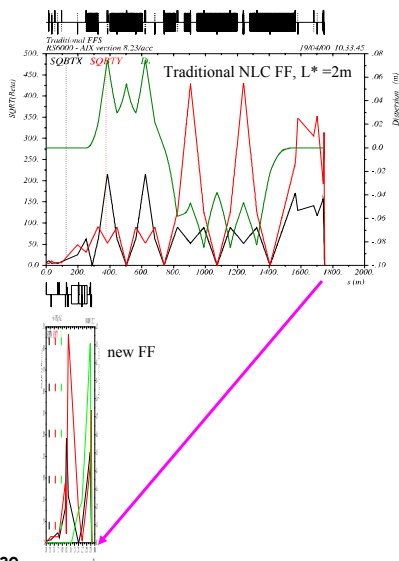
Solution:

The β -matching section produces as much X chromaticity as the FD, so the X sextupoles run twice stronger and cancel the **second order dispersion** as well.

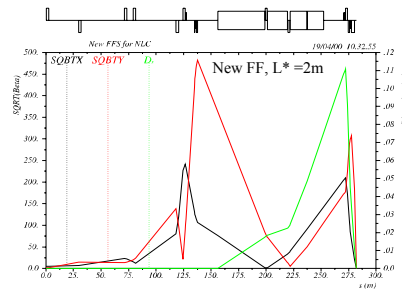
29



Traditional and new FF



A new FF with the same performance as NLC FF can be ~300m long, i.e. 6 times shorter

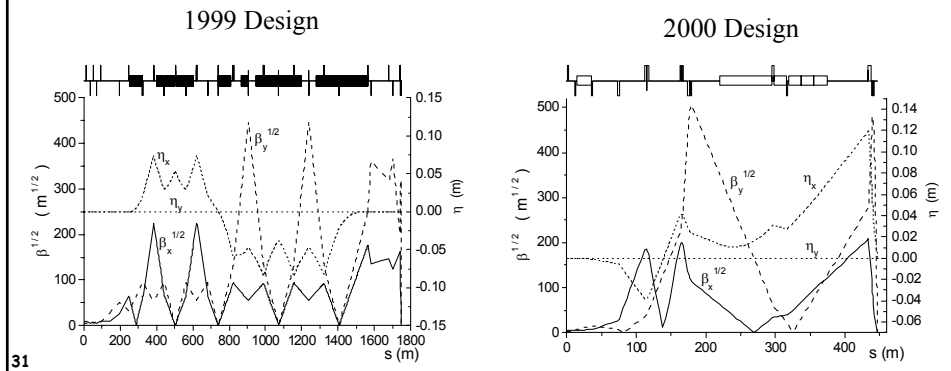


30

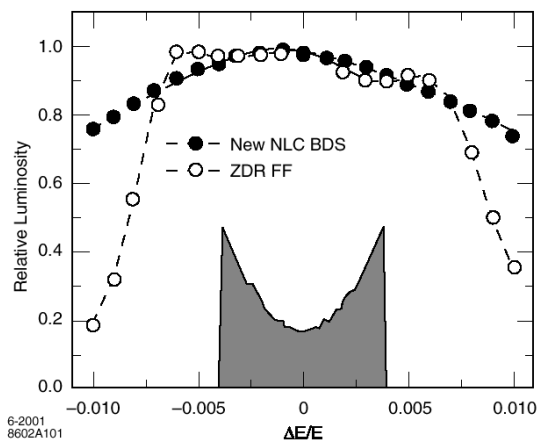


New Final Focus

- One third the length - many fewer components!
- Can operate with 2.5 TeV beams (for 3 ~ 5 TeV cms)
- 4.3 meter L^* (twice 1999 design)



IP bandwidth



Bandwidth is much better for New FF

32



Two more definitions of chromaticity

1st : TRANSPORT

You are familiar now with chromaticity defined as a change of the betatron tunes versus energy. This definition is mostly useful for rings.

In single path beamlines, it is more convenient to use other definitions. Lets consider other two possibilities.

The first one is based on TRANSPORT notations, where the change of the coordinate vector $x_i = \begin{pmatrix} x \\ x' \\ y \\ y' \\ \Delta l \\ \delta \end{pmatrix}$ is driven by the first order transfer matrix R such that $X_i^{\text{out}} = R_{ij} X_j^{\text{in}}$

Can you show that in a FF with zero η and η' at the entrance, the IP η' is equal to R_{26} of the whole system?

The second, third, and so on terms are included in a similar manner:

$$X_i^{\text{out}} = R_{ij} X_j^{\text{in}} + T_{ijk} X_j^{\text{in}} X_k^{\text{in}} + U_{ijkn} X_j^{\text{in}} X_k^{\text{in}} X_n^{\text{in}} + \dots$$

In FF design, we usually call 'chromaticity' the second order elements T_{126} and T_{346} . All other high order terms are just 'aberrations', purely chromatic (as T_{166} , which is second order dispersion), or chromo-geometric (as U_{32446}).

33



Several useful formulae

TRANSPORT \leftrightarrow Twiss

1) If you know the Twiss functions at point 1 and 2, the transfer matrix between them is given by

$$\begin{pmatrix} x_2 \\ x_2' \end{pmatrix} = \mathbf{M}_{21} \cdot \begin{pmatrix} x_1 \\ x_1' \end{pmatrix} \quad (\text{see DR notes, page 12}).$$

$$\mathbf{M}_{21} = \begin{pmatrix} \sqrt{\beta_2} & 0 \\ -\frac{\alpha_2}{\sqrt{\beta_2}} & \frac{1}{\sqrt{\beta_2}} \end{pmatrix} \cdot \begin{pmatrix} \cos(\Delta\phi) & -\sin(\Delta\phi) \\ \sin(\Delta\phi) & \cos(\Delta\phi) \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{\beta_1}} & 0 \\ \frac{\alpha_1}{\sqrt{\beta_1}} & \sqrt{\beta_1} \end{pmatrix}$$

2) If you know the transfer matrix between two points, the Twiss functions transform in this way:

$$\beta = R_{11}(R_{11}\beta_0 - R_{12}\alpha_0) - R_{12} \left(R_{11}\alpha_0 - R_{12} \frac{(1 + \alpha_0^2)}{\beta_0} \right) \quad \eta = R_{11}\eta_0 + R_{12}\eta_0' + R_{16}$$

$$\alpha = R_{21}(R_{12}\alpha_0 - R_{11}\beta_0) + R_{22} \left(R_{11}\alpha_0 - R_{12} \frac{(1 + \alpha_0^2)}{\beta_0} \right) \quad \eta' = R_{21}\eta_0 + R_{22}\eta_0' + R_{26}$$

$$\Phi = \Phi_0 + \text{arctg} \left(\frac{R_{12}}{R_{11}\beta_0 - R_{12}\alpha_0} \right)$$

And similar for the other plane

34



Two more definitions of chromaticity

2nd : W functions

Let's assume that betatron motion without energy offset is described by twiss functions α_1 and β_1 and with energy offset δ by functions α_2 and β_2

Let's define chromatic function W (for each plane) as $W = (iA + B)/2$ where $i = \sqrt{-1}$

And where: $B = \frac{\beta_2 - \beta_1}{\delta(\beta_2 \cdot \beta_1)^{1/2}} \approx \frac{\Delta\beta}{\delta\beta}$ and $A = \frac{\alpha_2\beta_1 - \alpha_1\beta_2}{\delta(\beta_2 \cdot \beta_1)^{1/2}} \approx \frac{\Delta\alpha}{\delta} - \frac{\alpha}{\beta} \frac{\Delta\beta}{\delta}$

Using familiar formulae $\frac{d\beta}{ds} = -2\alpha$ and $\frac{d\alpha}{ds} = K \cdot \beta - \frac{(1 + \alpha^2)}{\beta}$ where $K = \frac{e}{pc} \frac{dB_y}{dx}$

And introducing $\Delta K = \frac{K(\delta - K(0))}{\delta} \approx -K$ we obtain the equation for W evolution:

Can you show this?

$$\frac{dW}{ds} = \frac{2i}{\beta} W + \frac{i}{2} \beta \Delta K$$

knowing that the betatron phase is $\frac{d\Phi}{ds} = \frac{1}{\beta}$

can see that if $\Delta K=0$, then W rotates with double betatron frequency and stays constant in amplitude. In quadrupoles or sextupoles, only imaginary part changes.

Show that if in a final defocusing lens $\alpha=0$, then it gives $\Delta W=L^*/(2\beta^*)$

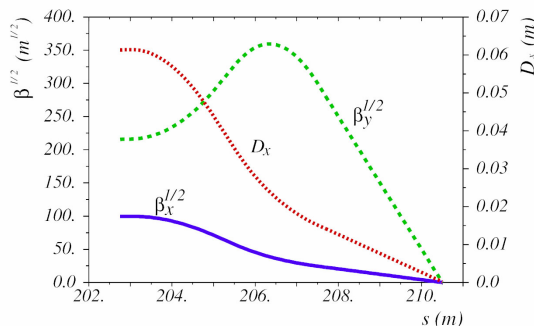
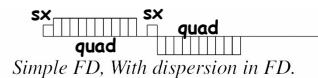
Show that if T_{346} is zeroed at the IP, the W_y is also zero. Use approximation $\Delta R_{34} = T_{346} \delta$, use DR notes, page 12, to obtain $R_{34} = (\beta\beta_0)^{1/2} \sin(\Delta\Phi)$, and the twiss equation for $d\alpha/d\Phi$.

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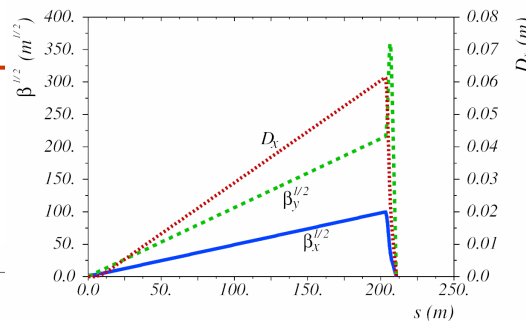


Let's consider chromatic correction in more details

Assume that FF is represented by a final telescope. Example show telescope-like optics which consists just from two quadrupoles. A short bend in the beginning creates dispersion' at the IP.



Simple final telescope. With dispersion in FD.



The left picture show the final doublet region in details. (The FD quads are split in ten pieces for convenience). Two sextupoles are inserted in the final doublet for chromatic correction.

The $\eta' = 0.005$ in this case and $L^* = 3.5m$ (Note that $\eta_{max} \sim 4 L^* \eta'$)

You will design similar telescope and study its chromatic properties in Ex.2-4.

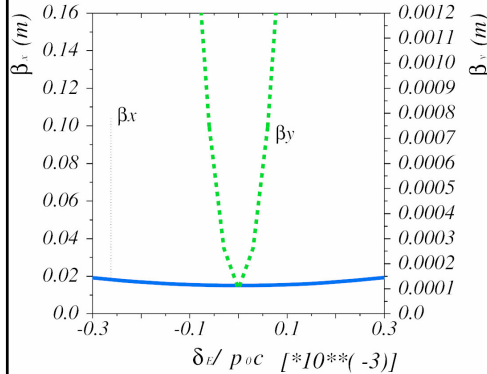
36



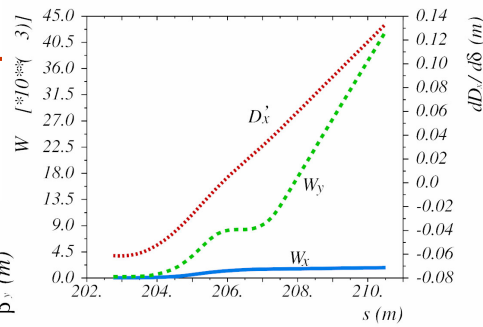
Chromatic properties before any correction

The chromatic functions W and the second order dispersion for this telescope are shown on the right picture.

Simple FD, Before chromatic correction.



Simple, Chromatic study. With dispersion in FD.



The bandwidth, calculated by program MAD, is shown in the left picture. Note that the vertical bandwidth is approximately $1/W_y$.

Note that MAD does not include the second order dispersion (or the first order) into calculation of the horizontal bandwidth. The present $d\eta/d\delta$ at the IP is 0.14m and if the energy spread is 0.3%, it would increase the beam size by about 1micron.

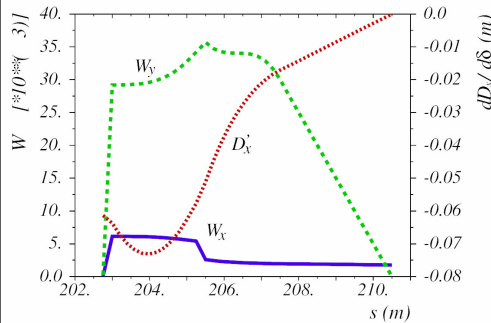
37



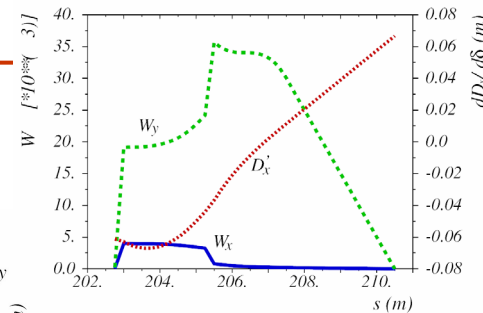
Chromatic properties after correction

As you see, we have 3 chromatic functions that we want to zero at the IP. As discussed above, with two sextupoles in the FD we can cancel only two of them simultaneously.

Zero Y chromaticity and DDX, but not X chromaticity



Zero X and Y chromaticity, but not DDX



In the picture above the W_x and W_y are zeroed. Note that the remaining η' is about half of the original value, as we discussed.

The left picture show the case when W_y and η' are zeroed at the IP.

You will study such corrections and the bandwidth of a similar telescope in Exercise 4.

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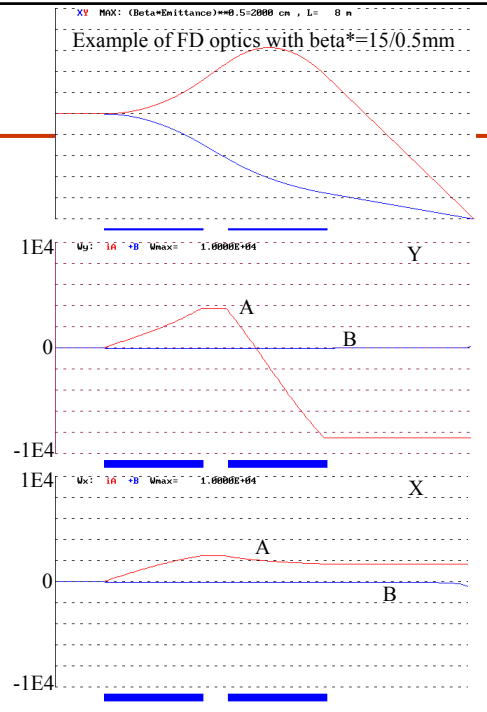


Chromatic properties of FD

The modulus of W chromatic functions plotted by MAD, shown on the previous page, apparently do not correspond to the expected behavior. For example, the modulus of W should not change along the el-star drift, but it does.

We are trying to figure out the reason for such behavior.

Pictures shown on this page represent another FD example, where the components of W (functions A and B) behaves as expected.



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Exercise 4 chromatic correction in a FD

In this case you will study chromatic correction in a two-lens telescope you designed in the previous Exercises. You will use MAD. The necessary files are in C:\LC_WORK\ex4

In addition to the beamline definition you already have, you will now include a bend and sextupoles. You will make matching of bend strength to have desired h' at IP and match sextupoles to correct either x and y chromaticities or y chromaticity and second order dispersion. Some new MAD definitions are given below.

```

SF1: SEXTUPOLE, L=0.25, K2=1.0e-10
SD0: SEXTUPOLE, L=0.25, K2=1.0e-10
BND0: SBEND, L=3, ANGLE=0
FD_TEL: LINE=(BND0,D2,SF1,QF1,D1,SD0,QD0,D0,IP)
USE, FD_TEL
MATCH, BETA0=MYTWSS0
VARY, ABND0, STEP=1.E-5
CONSTRAINT, #E, DX=0, DPX=DPXIP
LMDIF
ENDMATCH
MATCH, BETA0=MYTWSS0
VARY, SF1[K2], STEP=1.E-5
VARY, SD0[K2], STEP=1.E-5
TMATRIX, #S/#E, TM(1,2,6)=0, TM(3,4,6)=0
LMDIF
ENDMATCH
TWISS, CHROM, SAVE, BETA0=MYTWSS0
PLOT, TABLE=TWISS, HAXIS=S, VAXIS1=WX,WY, VAXIS2=DDX

```

Specify bend and sextupoles

Use MAD to match η' at IP
(bend must be short to also have $\eta \sim 0$!)

Match sextupoles

Request chromaticity to be zero

Calculate Twiss and chromatic functions

Plot chromatic functions and 2nd order dispersion

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Let's estimate required length of the bends in FF

We know now that there should be nonzero horizontal chromaticity W_x upstream of FD (and created upstream of the bend). SR in the bend will create energy spread, and this chromaticity will be 'spoiled'. Let's estimate the required length of the bend, taking this effect into account.

Parameters: length of bend L_B , assume total length of the telescope is $2 * L_B$, the el-star L^* , IP dispersion' is η'

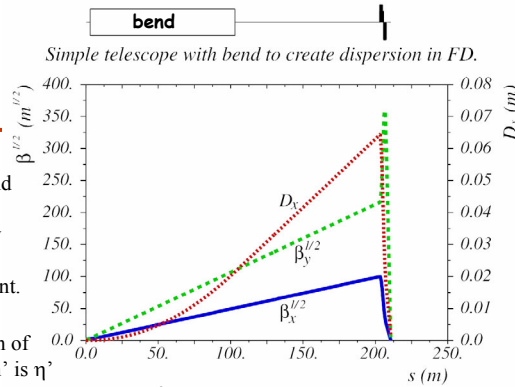
Dispersion at the FD, created by bend, is approximately $\eta_{\max} \approx \frac{3}{2} \frac{L_B^2}{R}$

Recall that we typically have $\eta_{\max} \sim 4 L^* \eta'$, therefore, the bending radius is $R \approx \frac{3}{8} \frac{L_B^2}{L^* \eta'}$

The SR generated energy spread is then $\left(\frac{\Delta E}{E}\right)^2 \approx 19 \frac{r_e \lambda_e \gamma^5 L^{*3} \eta'^3}{L_B^5}$ And the beam size growth $\frac{\Delta \sigma^2}{\sigma^2} \approx W_x^2 \left(\frac{\Delta E}{E}\right)^2$

Example: 650 GeV/beam, $L^*=3.5\text{m}$, $\eta'=0.005$, $W_x=2E3$, and requesting $\Delta\sigma/\sigma < 1\%$ $\Rightarrow L_B > 110\text{m}$

Energy scaling. Usually $\eta' \sim 1/\gamma^{1/2}$ then the required L_B scales as $\gamma^{7/10}$



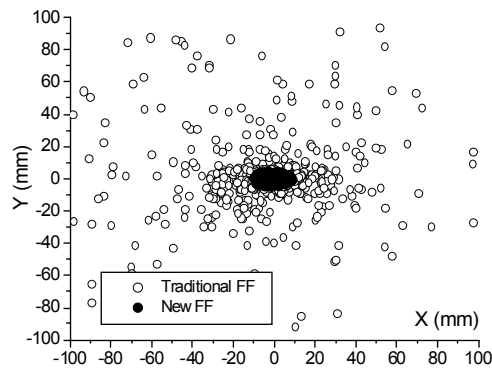
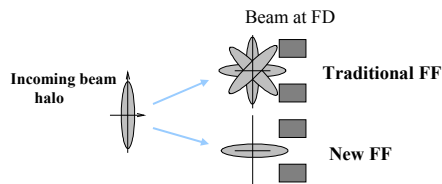
Can you show this ?

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Aberrations for beam halo

- Traditional FF generate beam tails due to aberrations and it does not preserve betatron phase of halo particles
- New FF is virtually aberration free and it does not mix phases particles



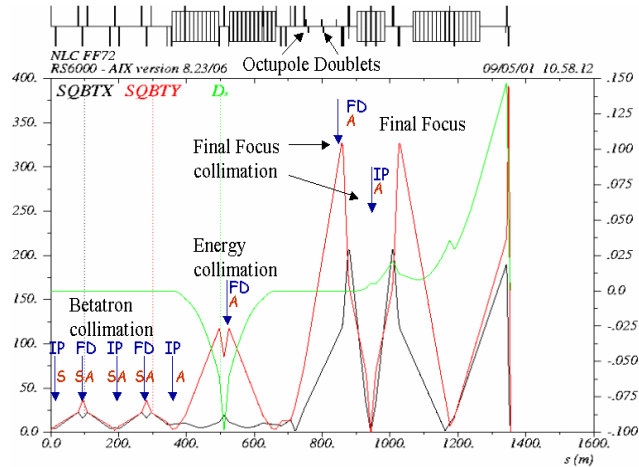
Halo beam at the FD entrance. Incoming beam is ~ 100 times larger than nominal beam

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Overview of a complete NLC BDS

- Compact system with local chromaticity corrections
- Collimation system has been built in the Final Focus system
- Two octupole doublets are placed in NLC FF for active folding of beam tails



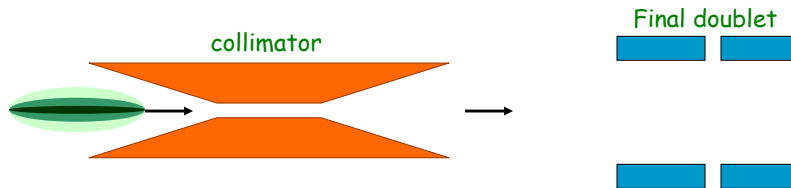
- NLC Beam Delivery System Optics
shown not the latest version but very close to it

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Why collimation?

- Would like to scrape out the beam halo well before the IP, to prevent halo particle hitting FD or detector and blinding the detector



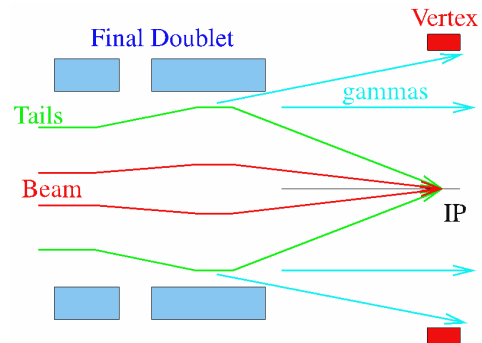
- Issues with collimators:
 - Survivability - may consider rotating renewable collimators
 - Wakes (effect on the beam core) - small gaps (sub mm) may be an issue
- There are solutions that we believe will work

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Beam halo & background

- Major source of detector background:
 - particles in the beam tail which hit FD and/or emit photons that hit vertex detector
- Tails can come
 - From FF, due to aberrations
 - From linac, etc.
- Tails must be collimated, amount of collimation usually determined by
 - Ratio : FD bore / beam size at FD
- Most tough in x-plane => collimation at just ~10 sigmas (collimation depth)

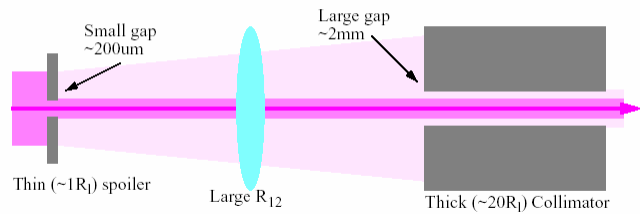


45



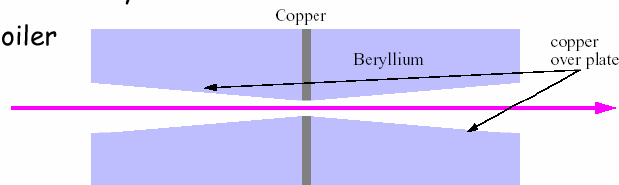
Consumable / renewable spoilers

Spoiler / Absorber Scheme



Tapered low resistivity surface for wakefields

Thin hi-Z spoiler



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1988 CuBe COMPOSITE SPOILER CONCEPT 8/17/99

depth	absorption
50 um	21 r.l.
500 um	30 r.l.
1000 um	39 r.l.

3.0mm
R.100mm
0.032
10.0cm
Cu PLATE
Ba
Cu

Rotating "Wheel" Collimator

Rotation
Rotation
Damaged Area

Rotating wheel option

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1988 Halo collimation in NLC BDS

0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
X, mm

0.008
0.006
0.004
0.002
0
-0.002
-0.004
-0.006
-0.008
Y, mm

Integral of particle loss

0.001
0.0001
1e-05
1e-06
1e-07
1e-08
1e-09
1e-10
1e-11
1e-12

Path length, m

-2000 -1500 -1000 -500 0

NLC, octupoles ON
NLC, octupoles OFF

Assumed halo sizes. Halo population is 0.001 of the main beam.

Assuming 0.001 halo, beam losses along the beamline behave nicely, and SR photon losses occur only on dedicated masks

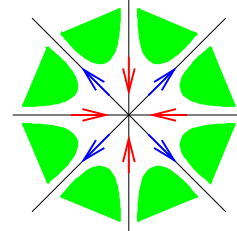
Smallest gaps are +/-0.6mm with tail folding Octupoles and +/-0.2mm without them.

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Nonlinear handling of beam tails in NLC BDS

- Can we ameliorate the incoming beam tails to relax the required collimation depth?
- One wants to **focus beam tails** but not to change the core of the beam
 - use **nonlinear** elements
- **Several nonlinear elements** needs to be **combined** to provide **focusing in all directions**
 - (analogy with **strong focusing by FODO**)
- **Octupole Doublets (OD)** can be used for **nonlinear tail folding** in NLC FF



Single octupole focus in planes and defocus on diagonals.

An octupole doublet can focus in all directions !

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Strong focusing by octupoles

- **Two octupoles of different sign separated by drift** provide **focusing in all directions for parallel beam**:

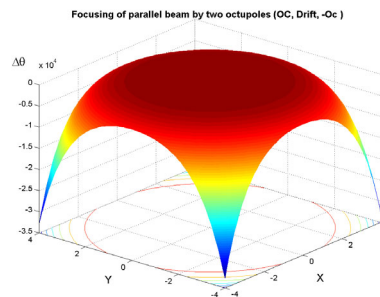
$$\Delta\theta = \alpha r^3 e^{-i3\varphi} - \left(\alpha r^3 e^{i3\varphi} (1 + \alpha r^2 L e^{-i4\varphi})^3 \right)^*$$

$$x + iy = r e^{i\varphi}$$

$$\Delta\theta \approx -3\alpha^2 r^5 e^{i\varphi} - 3\alpha^3 r^7 L^2 e^{i5\varphi}$$

Focusing in all directions

Next nonlinear term focusing - defocusing depends on φ



Effect of octupole doublet (Oc,Drift,-Oc) on parallel beam, $\Delta\theta(x,y)$.

- For this to work, the beam should have **small angles**, i.e. it should be parallel or **diverging**

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Schematic of folding with Octupole or OD

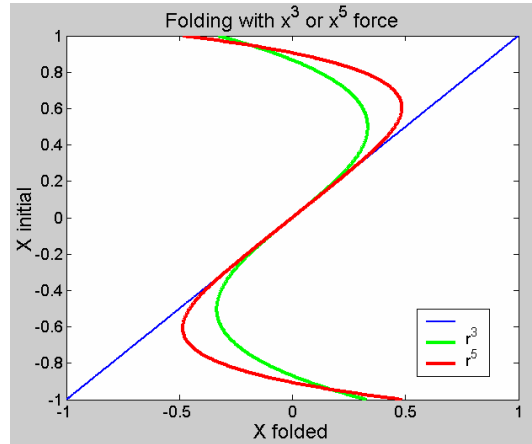
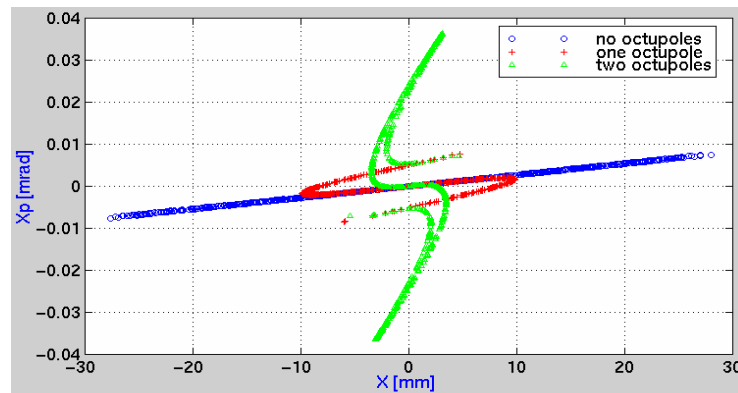


Illustration of folding of the horizontal phase space.
Octupole like force give factor of 3 (but distort diagonal planes)
OD-like force give factor of 2 (OK for all planes)
"Chebyshev Arrangement" of strength.

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Schematic of double folding (with two doublets)



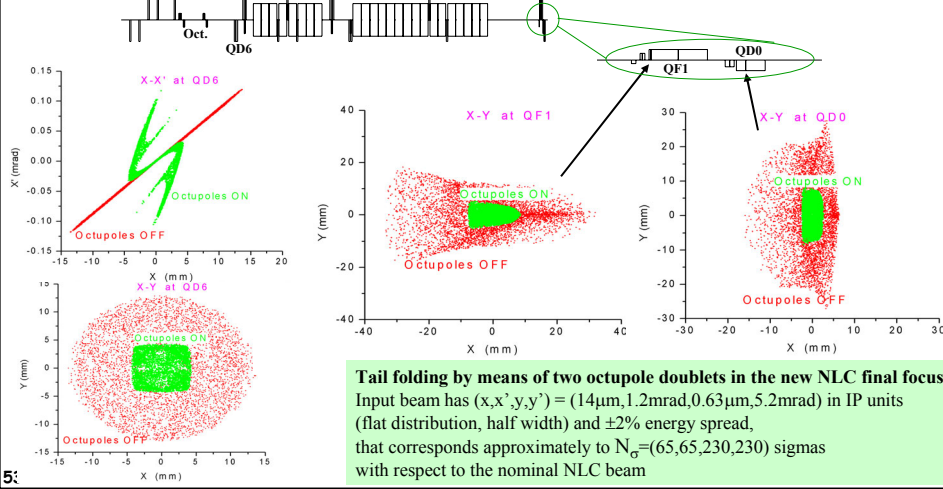
Folding of the horizontal phase space distribution at the entrance of the Final Doublet with one or two octupoles in a "Chebyshev Arrangement".

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Tail folding in new NLC FF

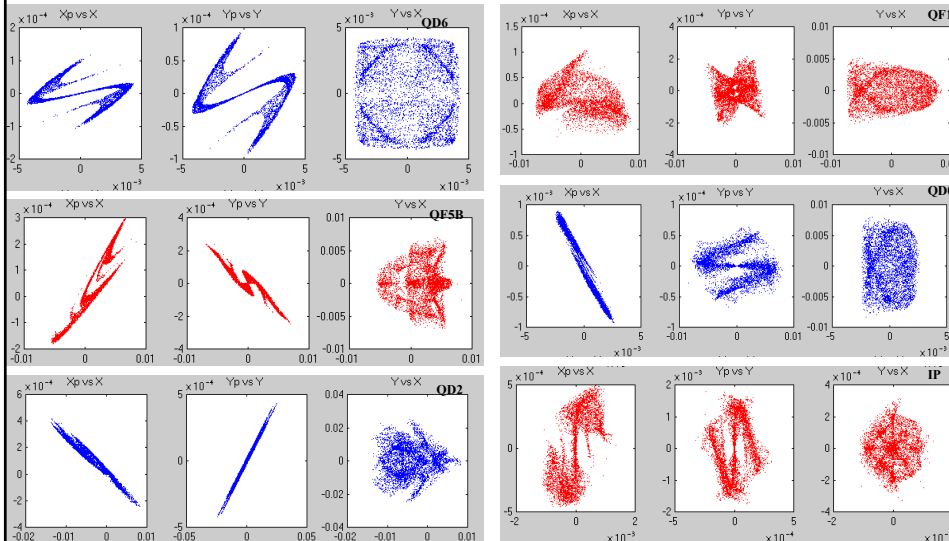
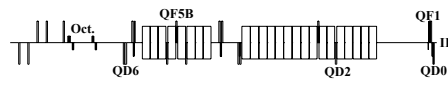
- Two octupole doublets give tail folding by ~ 4 times in terms of beam size in FD
- This can lead to relaxing collimation requirements by \sim a factor of 4



5:



Tail folding or Origami Zoo

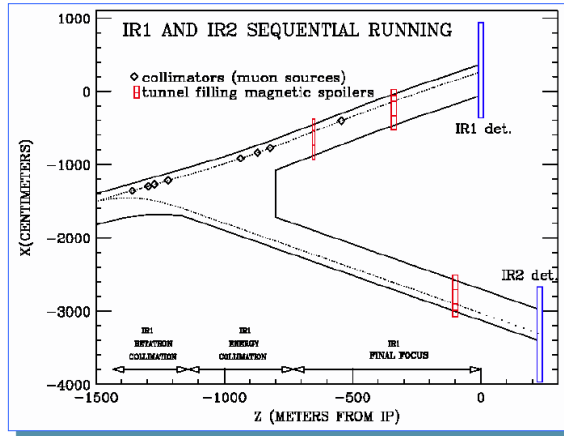




Dealing with muons in NLC BDS

Assuming 0.001 of the beam is collimated, two tunnel-filling spoilers are needed to keep the number of muon/pulse train hitting detector below 10

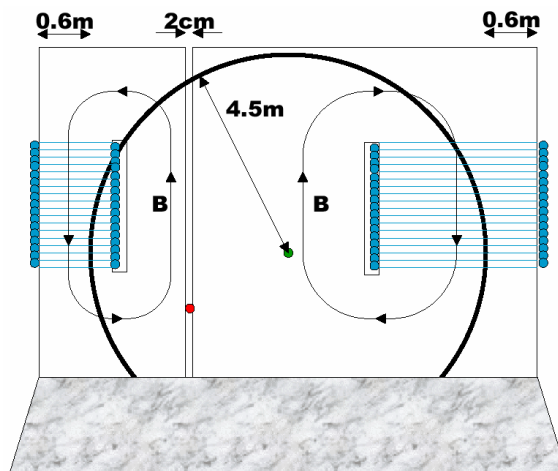
Good performance achieved for both Octupoles OFF and ON



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9 & 18 m Toroid Spoiler Walls

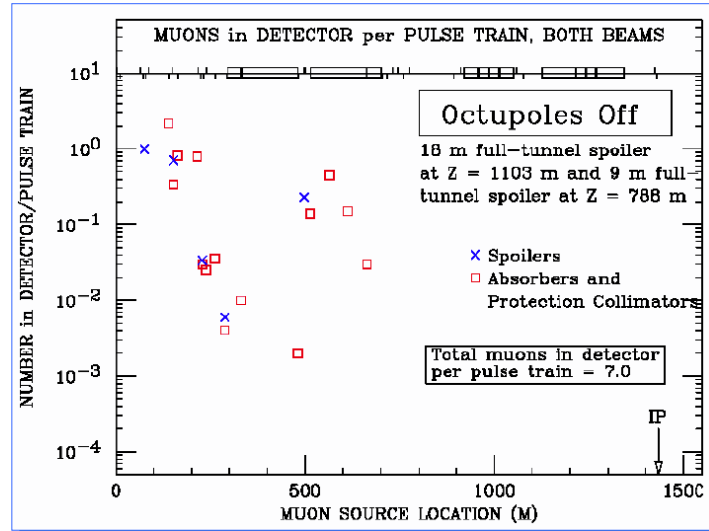


Long magnetized steel walls are needed to spray the muons out of the tunnel

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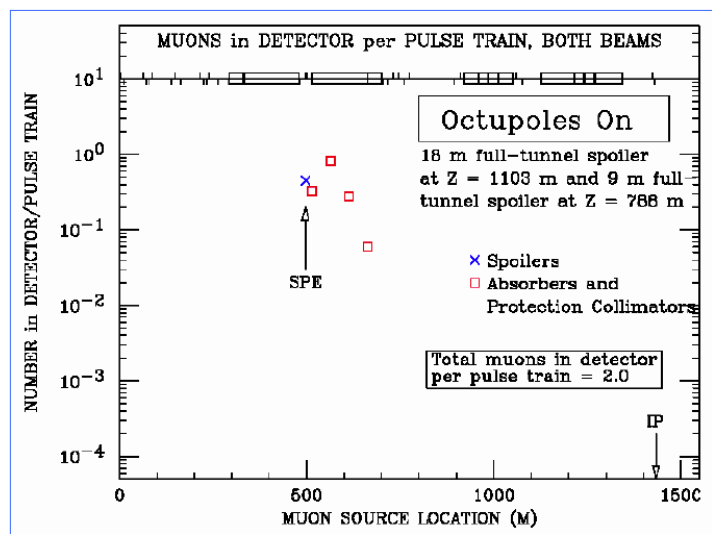
Muons (Oct. OFF)



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Muons (Oct. ON)



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Beam Delivery Systems of LC projects (2002 status)

NLC and CLIC use new FF with local chromaticity compensation

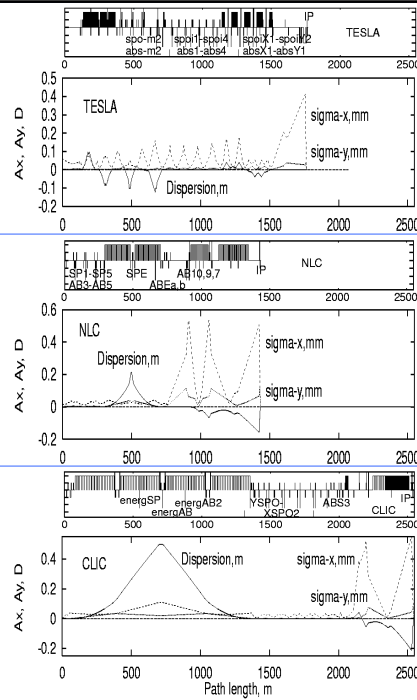
TESLA - traditional FF design

JLC/NLC and CLIC have crossing angle

TESLA - no crossing angle: more complications for setting the collimation system

NLC: Betatron coll. => Energy coll.

TESLA and CLIC: Energy coll. => Betatron coll.



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May/03 NLC IR layout

1st and 2nd IR configuration and optics

Crossing angle:
IP1: 20 mrad
IP2: 30 mrad

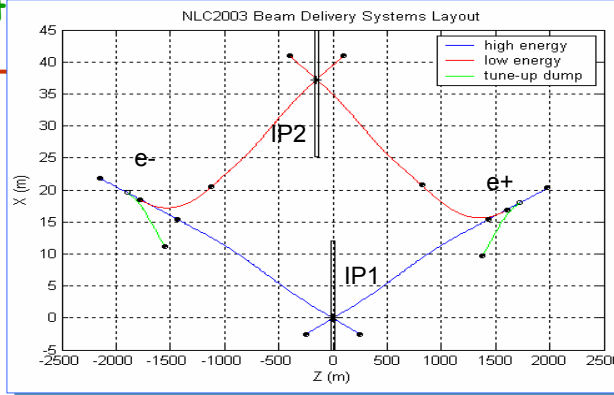
dPath(1st IR - 2nd IR) = 299.79 m
(which is DR perimeter) for timing system

1st IR BDS: "full length" (1434 m)
TRC era version

2nd IR BDS: "2/3 length" (968 m)
4/28/03 version

Bends in optics as shown optimized for 250GeV/beam

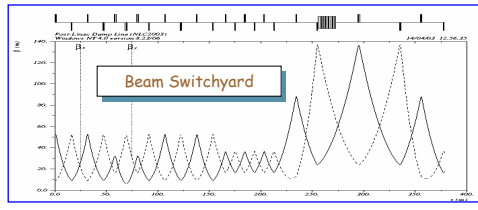
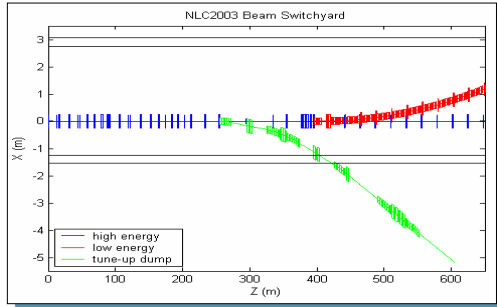
Less than 30% emittance growth in 2nd IR big bend at 1.3TeV CM



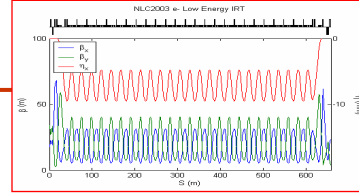
60



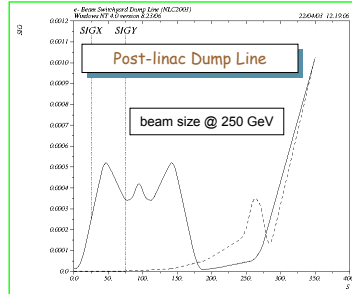
NLC IR May/03 layout details



Low Energy Interaction Region Transport



IP2 crossing angle = 30 mrad
 $\Delta\epsilon/\epsilon$ from ISR <30% for 650 GeV beam
 Combined function fodo 23 cells ($L_{cell} = 23$ m)



full bunch train; nominal charge, $\epsilon_x, \sigma_z, \sigma_y$; full machine rate (120 Hz)
 13 MW for 750 GeV beam ($\sigma_x = 0.5$ mm); $\pm 20\%$ energy acceptance
 8 cm bore (diameter); $L = 350$ m, $\Delta X = 5$ m, $\Delta Y = -1$ m
 separate enclosure (vault) for dump



May/03 config

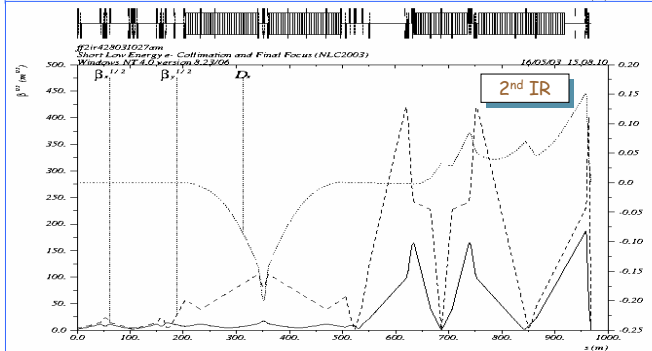
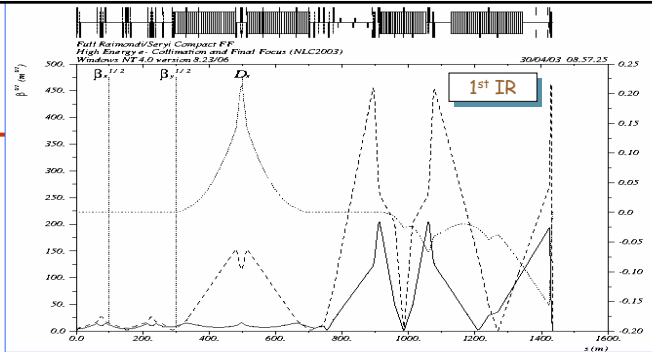
Collimation / Final Focus Optics

High energy:
 "full length" (1434 m)
 compact system (TRC
 report version)

Low energy:
 "2/3 length" (968 m)
 compact system
 (4/28/03 version)

Bends in optics as
 shown optimized for
 250GeV/beam

Note that both BDS
 have bending in
 E-Collimation opposite
 to bending in FF, to
 nearly cancel the total
 bend angle.
 (Either one fits in a
 straight tunnel).





BDS layout change in upgrade to 1TeV CM (example for "standard" BDS)

For upgrade :
Reduce bending angle in FF
twice, and increase
bending angle in E-
Collimation by ~15%.

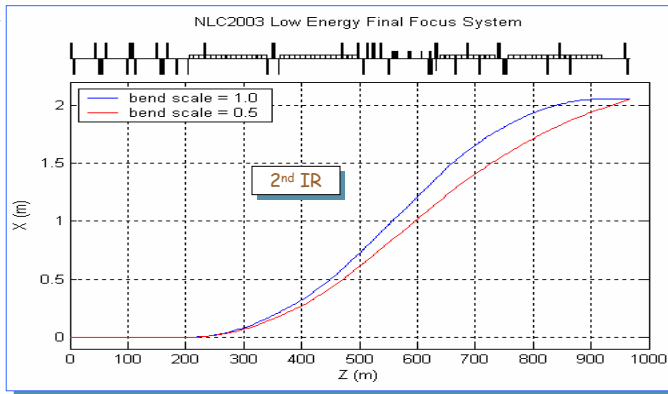
Location of IP is fixed.

With proper rescaling of
SX, OC, DEC fields
aberration cancellation is
preserved

BDS magnets need to be
moved by ~20cm.

Outgoing angle change by
~1.6 mrad (\Rightarrow the
extraction line also need
to be adjusted)

Bends in 1TeV optics are
optimized for
650GeV/beam



$$\Delta x^* = 0, \Delta z^* = 583.3 \mu\text{m}, \Delta\theta^* = 1.6767 \text{ mrad}$$

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June/03 NLC layout 2nd IR with "one-way" bending BDS

The Big Bend goes from 23 cells to 10 cells
for <30% emittance growth @ 650 GeV/beam

All "stretches" in high E beamlines are
removed, making the two high energy BDS
systems mirror symmetric about IP1 once again

We get 125 m of "extra" space in the short
low energy e- beamline

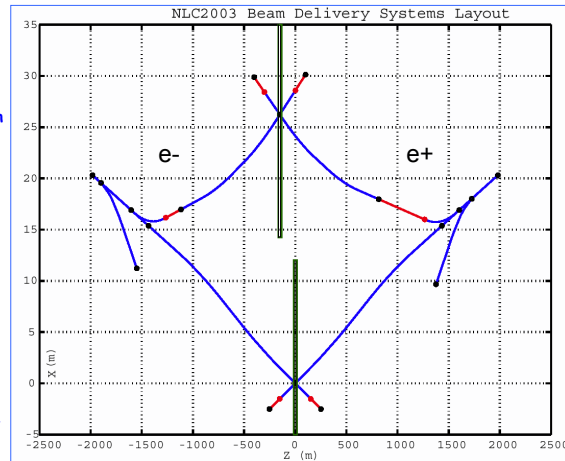
The IP2 crossing angle at 30 mrad and 1 DR
turn path-length difference between the low
energy BDS systems

The overall "Z-length" of the entire
BDS is now determined by the high energy
systems

We can make the e- low energy BDS system
longer by these extra 125 m

We can make the e+ low energy BDS longer by
450 m, which makes it equal to high E system

the "Z-length" of the BDS now 3962.7 m, so
the NLC site got shorter by 172.5 m



Based on: 1st IR: TRC version of NLC BDS (~1400m) ;
2nd IR: May 2003 version of one-way BDS (~970m)

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BDS layout change in upgrade to 1TeV CM (example for one-way BDS for 2nd IR)

Upgrade is done in the same way as for standard BDS:

Reduce bending angle in FF twice, and increase bending angle in E-Collimation by ~15%.

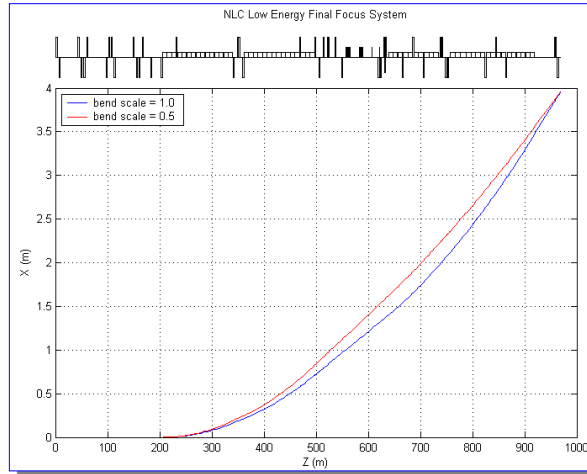
Location of IP is fixed.

With proper rescaling of SX, OC, DEC fields aberration cancellation is preserved

BDS magnets need to be moved by ~20cm.

Outgoing angle change by ~1.6 mrad (\Rightarrow the extraction line also need to be adjusted)

Bends in 1TeV optics are optimized for 650GeV/beam

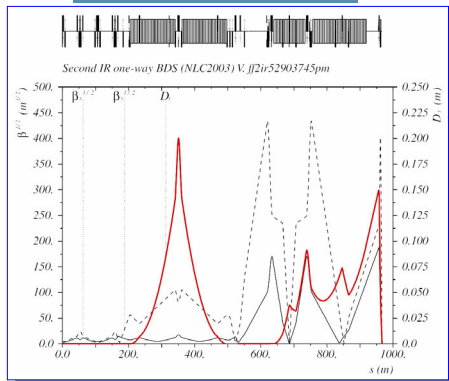


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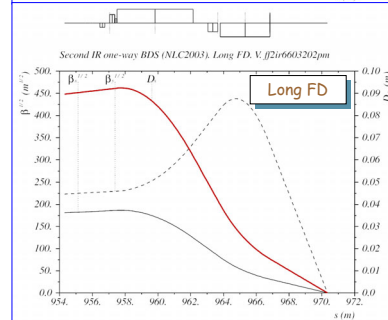
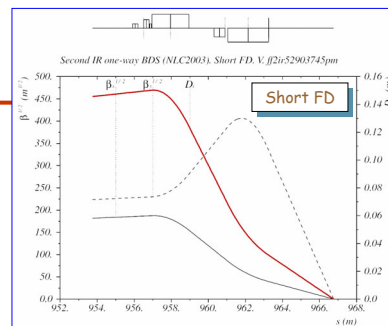


2nd IR BDS optics

2nd IR BDS for 250GeV/beam



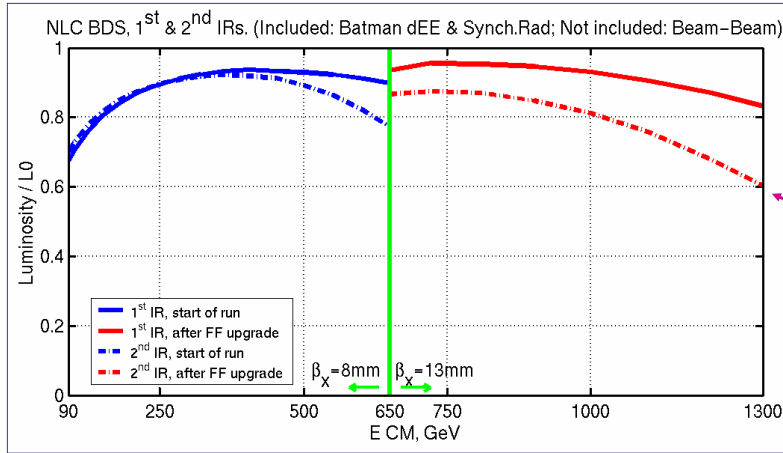
250GeV/beam: ff2ir52903745pm one-way bending BDS
500GeV/beam: ff2ir6603202pm (less bending in FF and long FD)



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BDS performance (June layout) 1st and 2nd IR



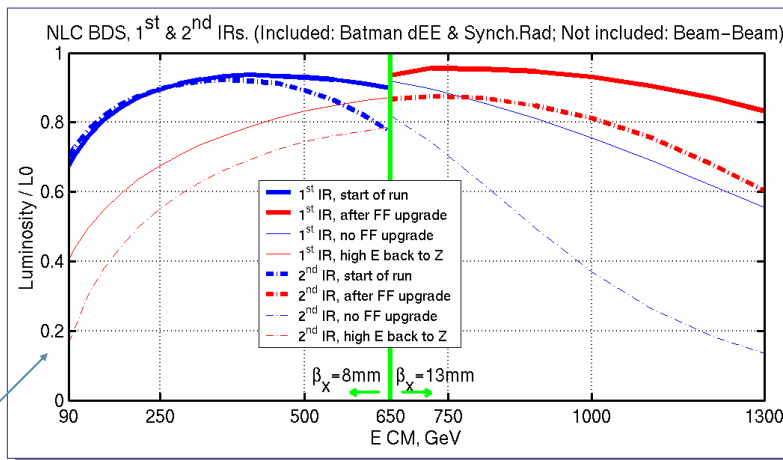
**Performance of NLC BDS (optics only: include aberration and synch.radiation).
Effect such as beam beam or collimator wakes (!) are not taken into account.**

Based on: 1st IR: ff112, ff112ffd (long FD), ~1400m; 2nd IR: ff2ir52903745pm (one way FF), ff2ir6603202pm (one way FF, long FD), ~970m. Same nominal s
67 Upgrade= reduce by ~50% the bend angles in FF and increase by ~15% in energy collimation (IP location is fixed but beamline relocated).

The 2nd IR BDS can be lengthened and performance will improve.



BDS performance more details



When luminosity loss is high, it can be partly regained by increasing β^*

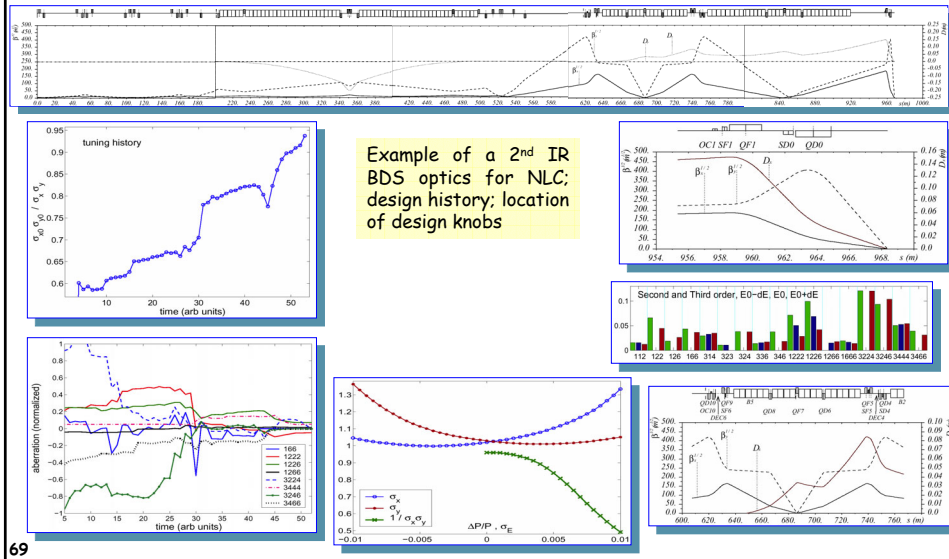
**Thin curves show performance if upgrade (=layout change) was not made,
or if one goes back from 1TeV to Z pole.**

Luminosity loss scales as $dL/L \sim \gamma^{1.75} / \mathcal{L}^{2.5}$. That means that though the required length scales only as $\mathcal{L} \sim \gamma^{0.7}$, the luminosity loss can be significant when the length is decreased.

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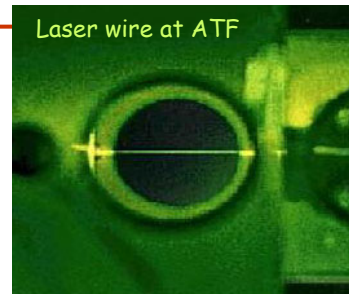


BDS design methods & examples



In a practical situation ...

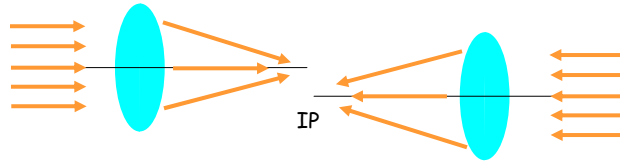
- While **designing** the FF, one has a **total control**
- When the system is built, one has just limited number of observable parameters (measured orbit position, beam size measured in several locations)
- The system, however, may initially have **errors** (errors of strength of the elements, transverse misalignments) and **initial aberrations** may be large
- **Tuning** of FF has been done so far by tedious optimization of "knobs" (strength, position of group of elements) chosen to affect some particular aberrations
- Experience in SLC FF and FFTB, and simulations with new FF give confidence that this is possible



Laser wire will be a tool for tuning and diagnostic of FF



Stability - tolerance to FD motion



- Displacement of FD by dY cause displacement of the beam at IP by the same amount
- Therefore, stability of FD need to be maintained with a fraction of nanometer accuracy
- How would we detect such small offsets of FD or beams?
 - Using Beam- beam deflection ! (Tuesday lecture)
- How misalignments and ground motion influence beam offset?
 - => Wednesday lecture on LC stability

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Maybe YOU will solve this?

- Overcome Oide limit and chromaticity by use of other methods of focusing - plasma focusing, focusing by additional low energy and dense beam - or something else?
- Collimate the halo by something "invisible" for the beam core... Photons?

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Join the LC work!