

Betatron Coupling studies PETRA III Optics

Gajendra Kumar Sahoo

Linear Betatron Coupling

$$x'' + k_x x = k_1 y$$

$$y'' + k_y y = k_2 x$$

Sources of Linear Coupling

Skew quadrupoles or rotated normal quadrupoles

Solenoid

Vertical orbit offsets in Sextupoles

Linear coupling between the horizontal and vertical planes of motion is principally induced by rotations of the quadrupoles around the s-axis and vertical misalignments of the sextupoles, each of which produced skew quadrupole fields, with magnitudes which are respectively,

$$k_s = 2k_1 \Delta\theta$$

$$k_s = k_2 \Delta y$$

two linear resonances in Hamiltonian

uncoupled linear motion

$$H = Q_x^0 I_x + Q_y^0 I_y + |\kappa_+| \cos(\phi_x + \phi_y + q_+ + \varphi_0^+) \sqrt{I_x I_y}$$
$$+ |\kappa_-| \cos(\phi_x - \phi_y + q_- + \varphi_0^-) \sqrt{I_x I_y} \quad \text{sum resonance}$$

difference resonance

$$Q_x \pm Q_y + q_\pm = 0$$

resonance driving terms:

$$|\kappa_\pm| = \frac{1}{2\pi} \left| \oint ds k_s(s) \sqrt{\beta_x(s) \beta_y(s)} e^{i(\phi_x(s) \pm \phi_y(s) - (Q_x \pm Q_y - q_\pm) 2\pi s/L)} \right|$$

$k_s(s)$: normalized gradient of skew quadrupole

L : circumference

closest tune approach

$$\rightarrow |\kappa_-|$$

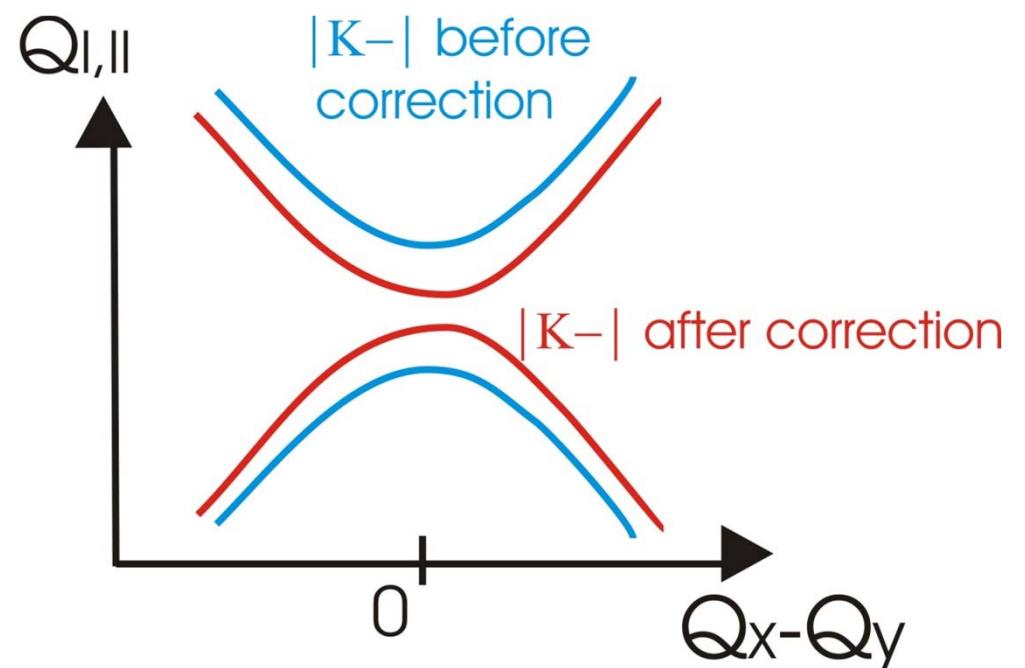
near the difference resonance

$$Q_x - Q_y + q_- \approx 0$$

$$|\kappa_\pm| = \frac{1}{2\pi} \left| \oint ds k_s(s) \sqrt{\beta_x(s)\beta_y(s)} e^{i(\phi_x(s) \pm \phi_y(s) - (Q_x \pm Q_y - q_\pm)2\pi s/L)} \right|$$

tunes can approach each other
only up to distance $|\kappa_-|$

correction strategy;
use two skew quadrupoles
(ideally with $\Delta(\phi_x - \phi_y) \sim \pi/2$) to
minimize $|\kappa_-|$, namely
the distance of closest tune approach



in principle, $|\kappa_+|$ could be compensated by adjusting two skew quadrupoles so as to minimize the stopband width, ideally at locations separated by

$$\Delta(\phi_x + \phi_y) = (n + \frac{1}{2})\pi$$

minimum number of skew quadrupoles for global correction in a ring:

2 for $|\kappa_-|$

2 for $|\kappa_+|$

2 for D_y



6: minimum number for independent correction of 6 global effects and emittance optimization

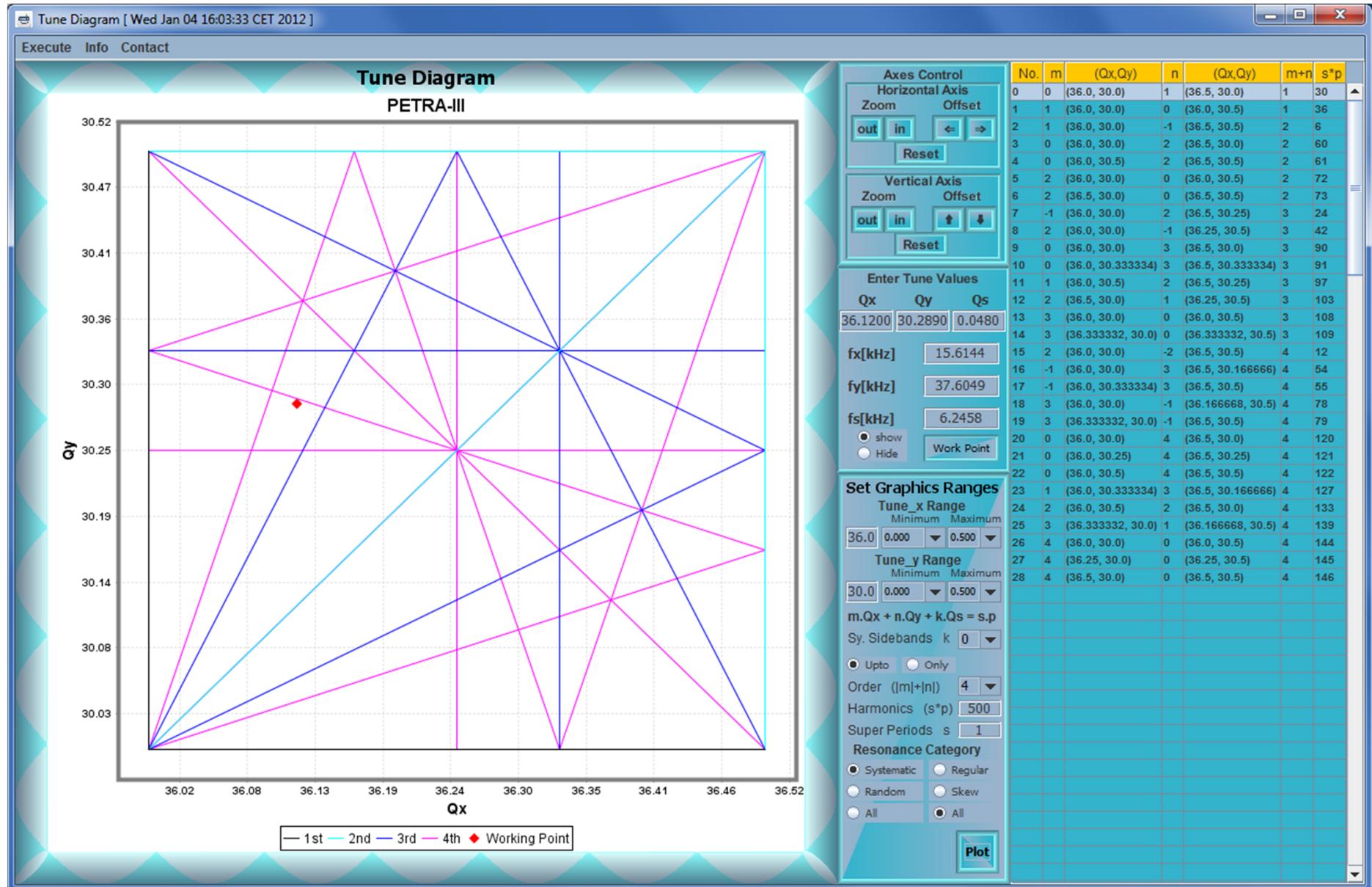
this does not yet correct the local coupling effects, which may also contribute to emittance growth.

Near a difference resonance the eigenvalues tunes of the coupled motion are

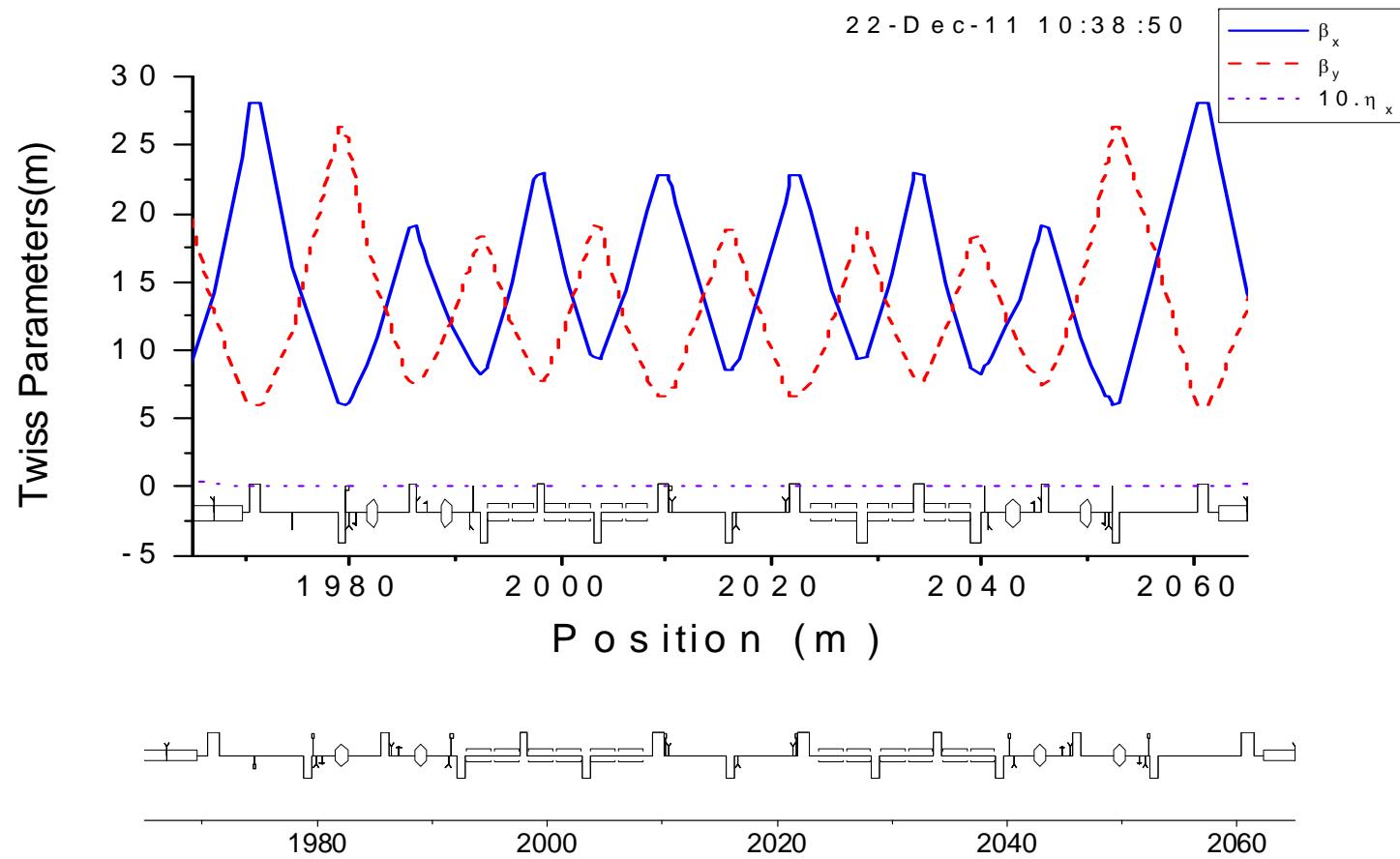
$$\nu_{1,2} = \frac{1}{2} \left(\nu_x + \nu_y + p \pm \frac{1}{4\pi} \sqrt{(\nu_x - \nu_y + p)^2 + 4K_-^2} \right)$$

This resonance can be compensated by using at least two skew quadrupoles to minimize the distance of closest approach. The requiring skew quad strength are,

$$k_s = \frac{4K_-}{L_s \sqrt{\beta_x \beta_y}}$$



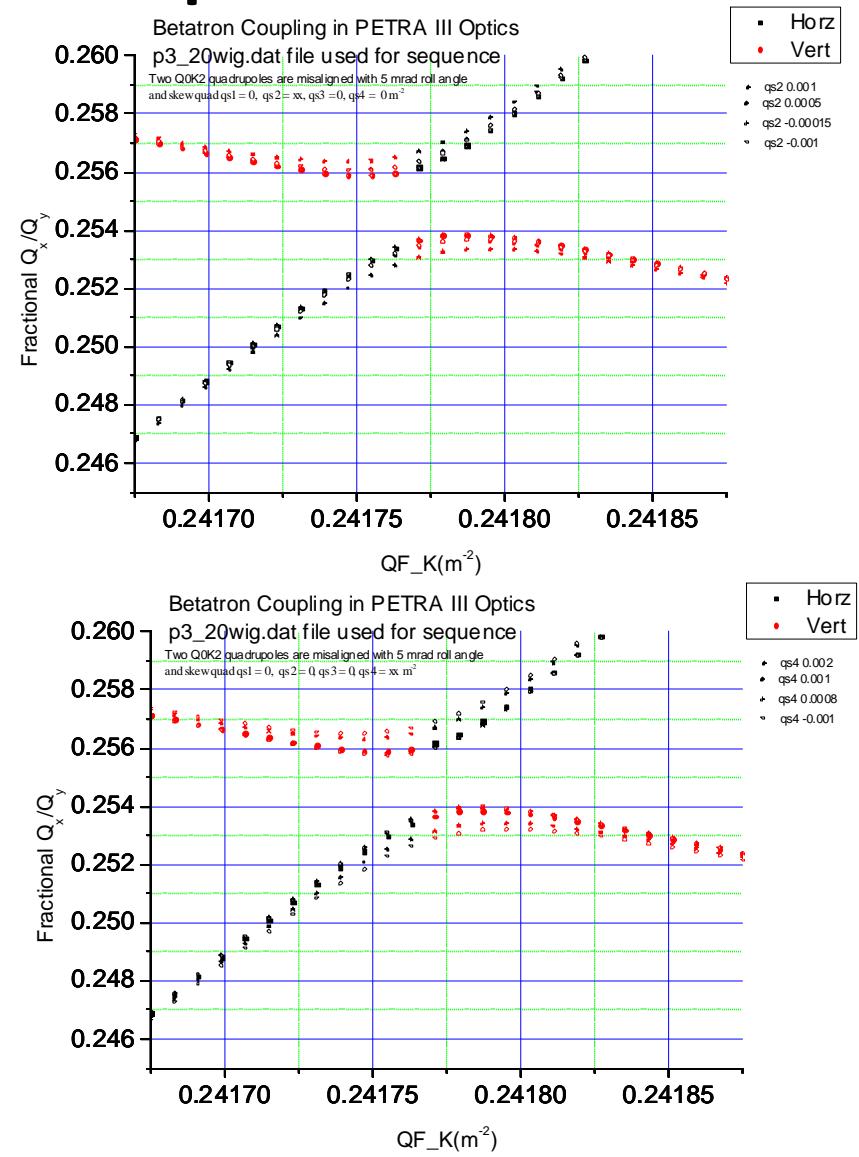
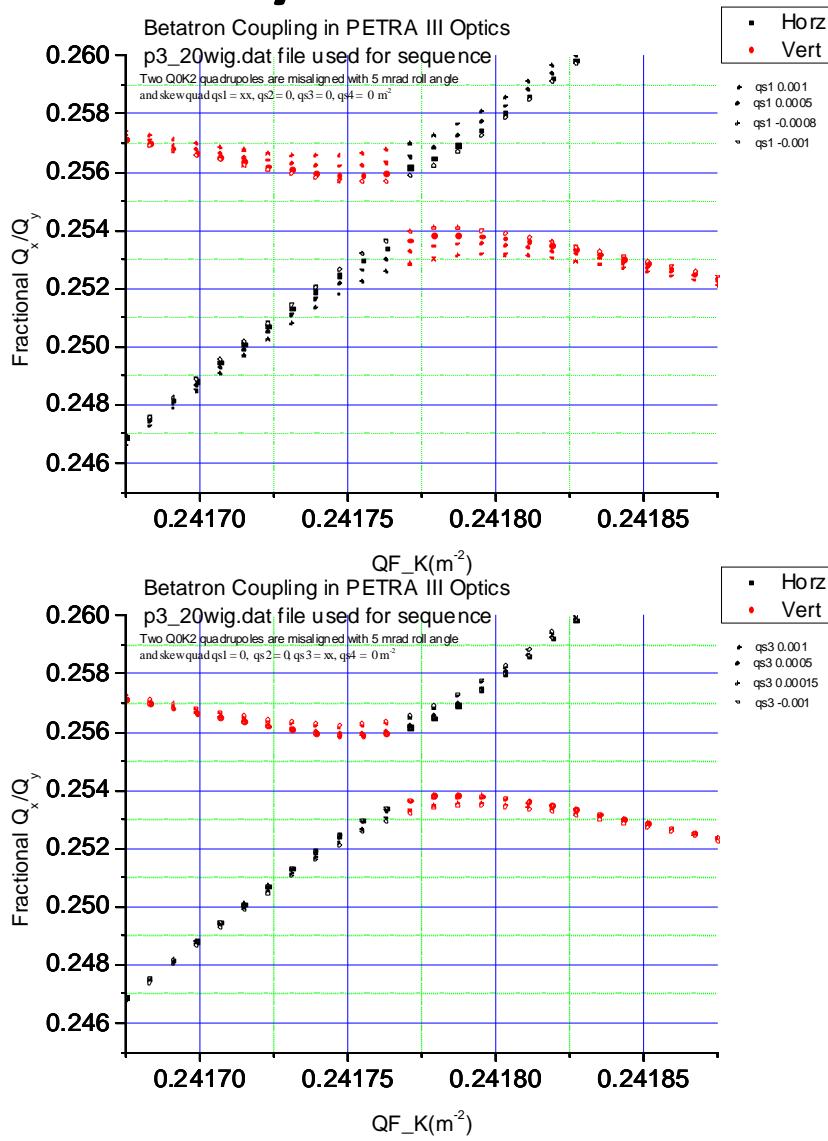
Position of Skew Quads in PETRA III



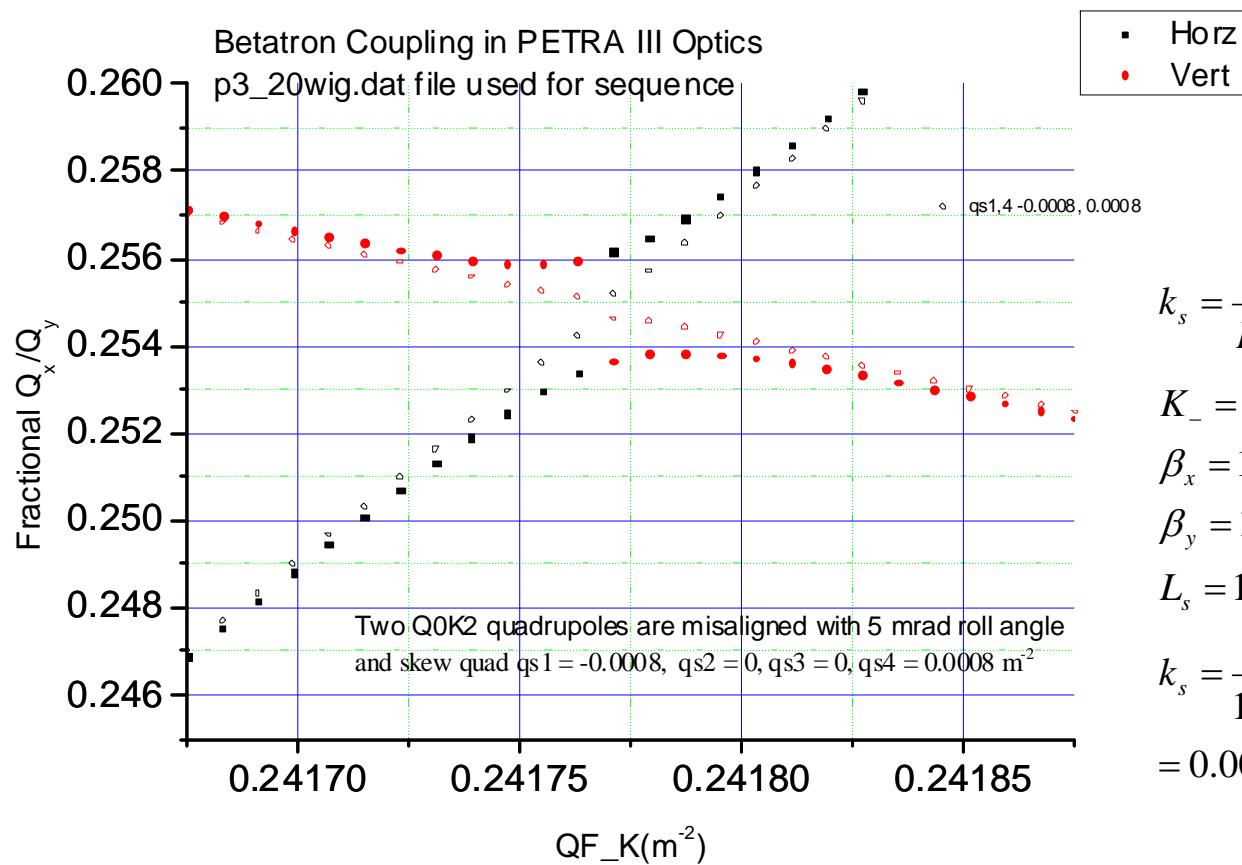
Name	s(m)	$\beta_x(m)$	$\beta_y(m)$	μ_x	μ_y	ϕ_x	ϕ_y
Qs1	1982.641	13.6597	11.21367	31.92606	26.05723	0	0
Qs2	1989.559	13.49109	12.4614	31.98718	26.19062	22.0032	48.0204
Qs3	2043.399	16.61427	9.909611	32.67223	26.92058	246.618	262.7856
Qs4	2050.317	11.18296	13.39165	32.73381	27.05439	22.1688	48.1716

Name	s(m)	$\beta_x(m)$	$\beta_y(m)$	μ_x	μ_y	ϕ_x	ϕ_y
Qs1	1982.641	13.6597	11.21367	31.92606	26.05723	0	0
Qs2	1989.559	13.49109	12.4614	31.98718	26.19062	22.0032	48.0204
Qs3	2043.399	16.61427	9.909611	32.67223	26.92058	246.618	262.7856
Qs4	2050.317	11.18296	13.39165	32.73381	27.05439	22.1688	48.1716

Systematic Roll in quad Q0K2



Systematic Roll in quad Q0K2



$$k_s = \frac{4K_-}{L_s \sqrt{\beta_x \beta_y}}$$

$$K_- = 0.256150 - 0.253657 = 0.002493$$

$$\beta_x = 13.65m$$

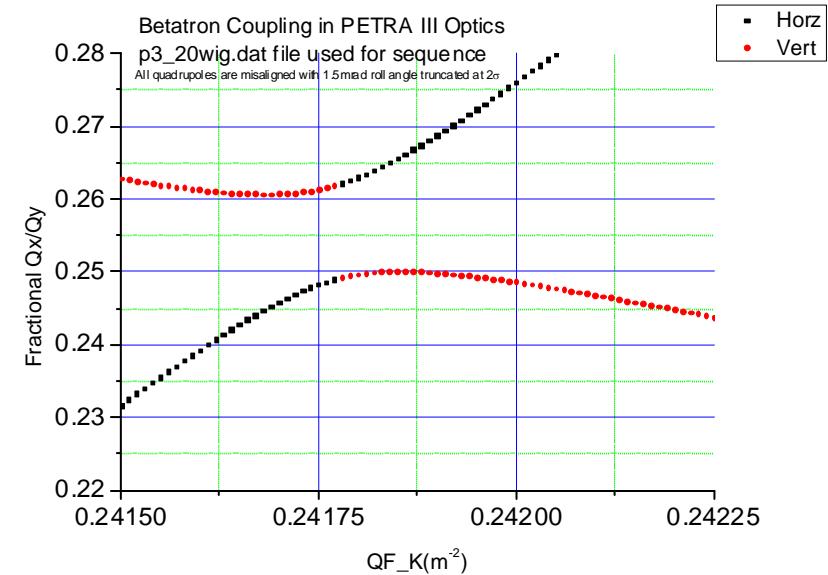
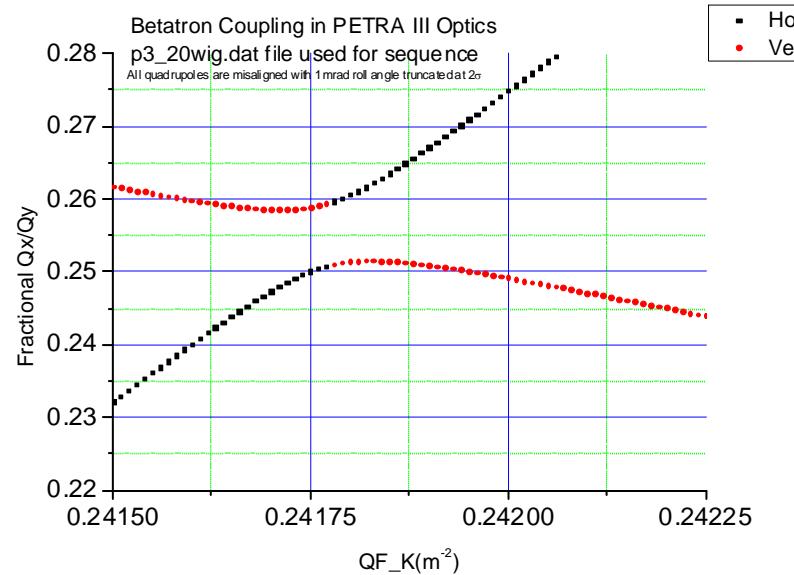
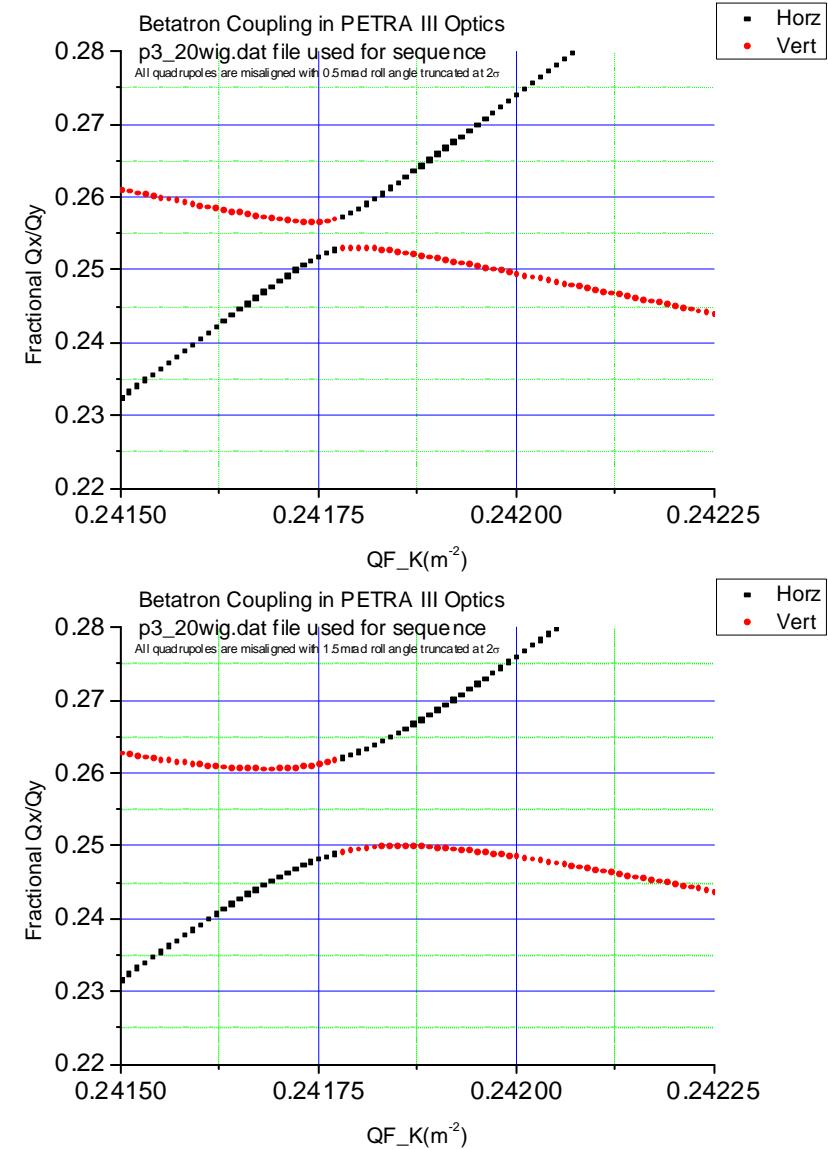
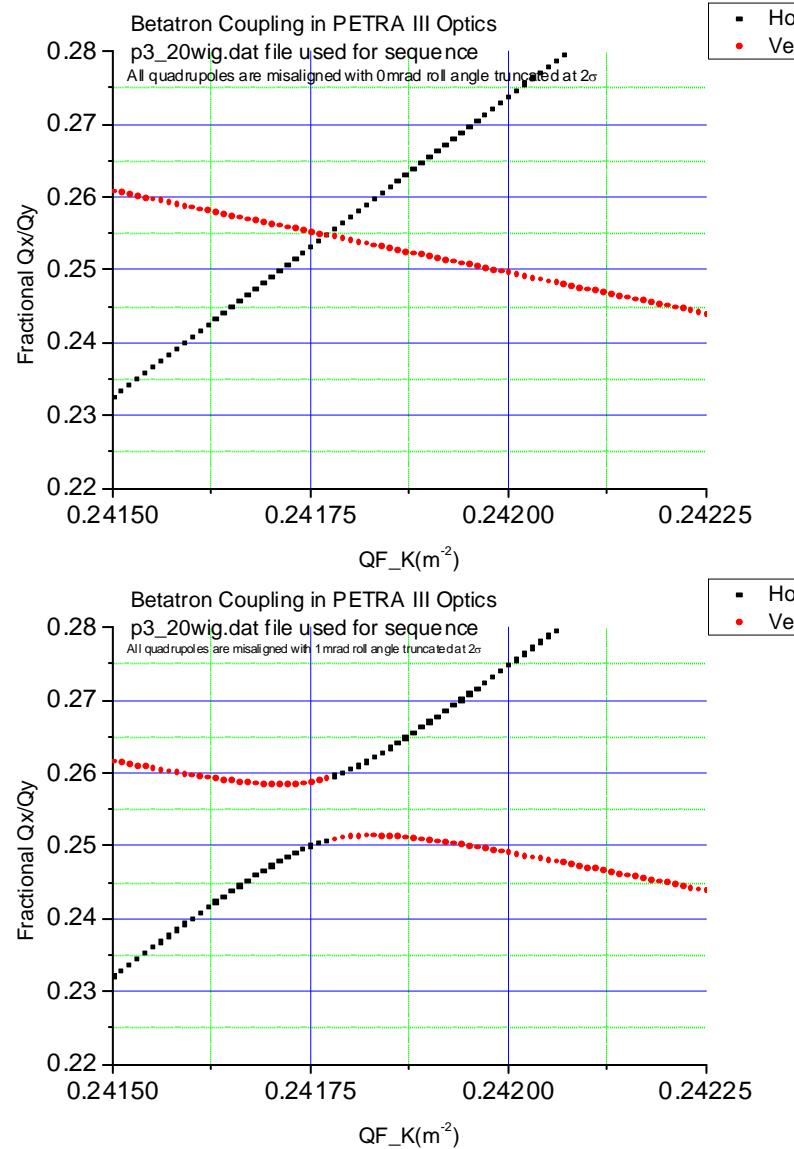
$$\beta_y = 11.21m$$

$$L_s = 1.0426m$$

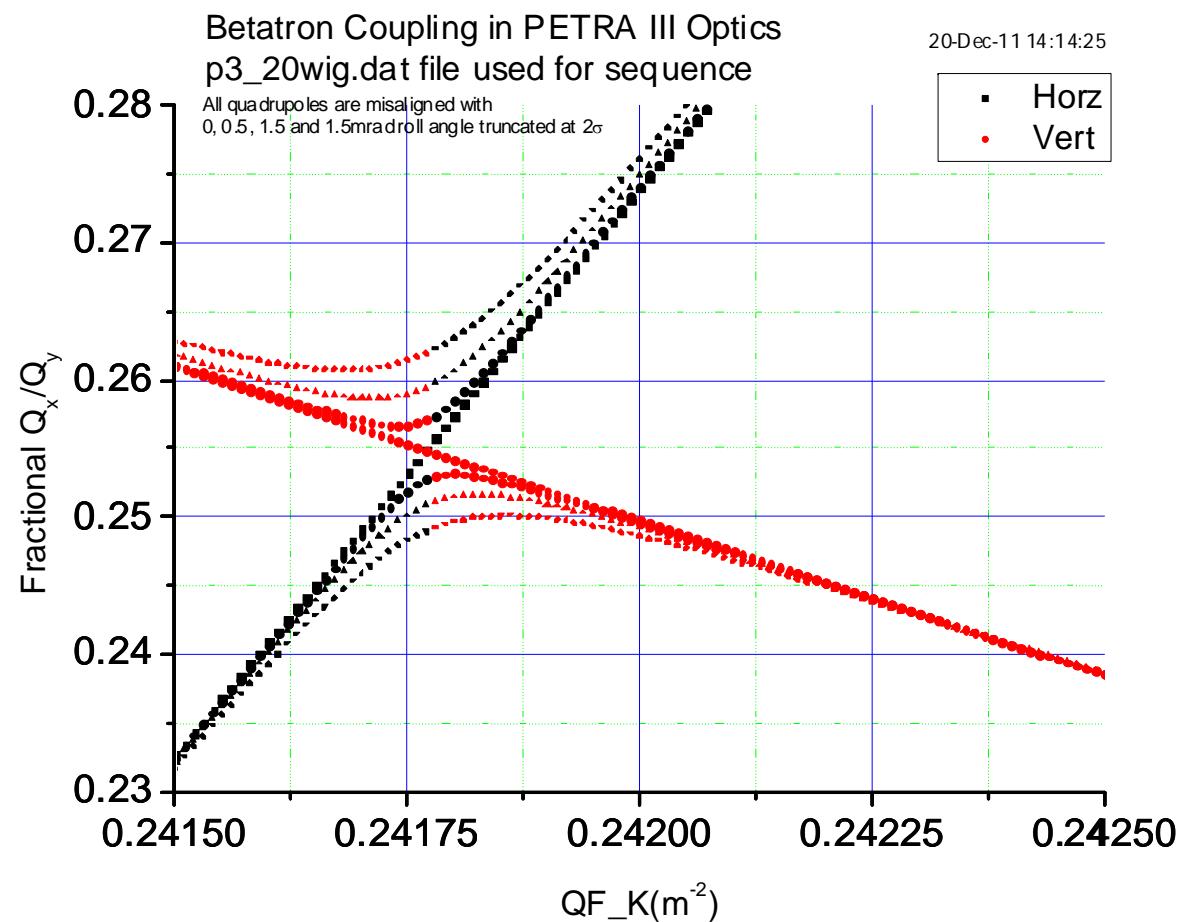
$$k_s = \frac{4 \times 0.002493}{1.0426 \times \sqrt{13.65 \times 11.21}} m^{-2}$$

$$= 0.00077 m^{-2}$$

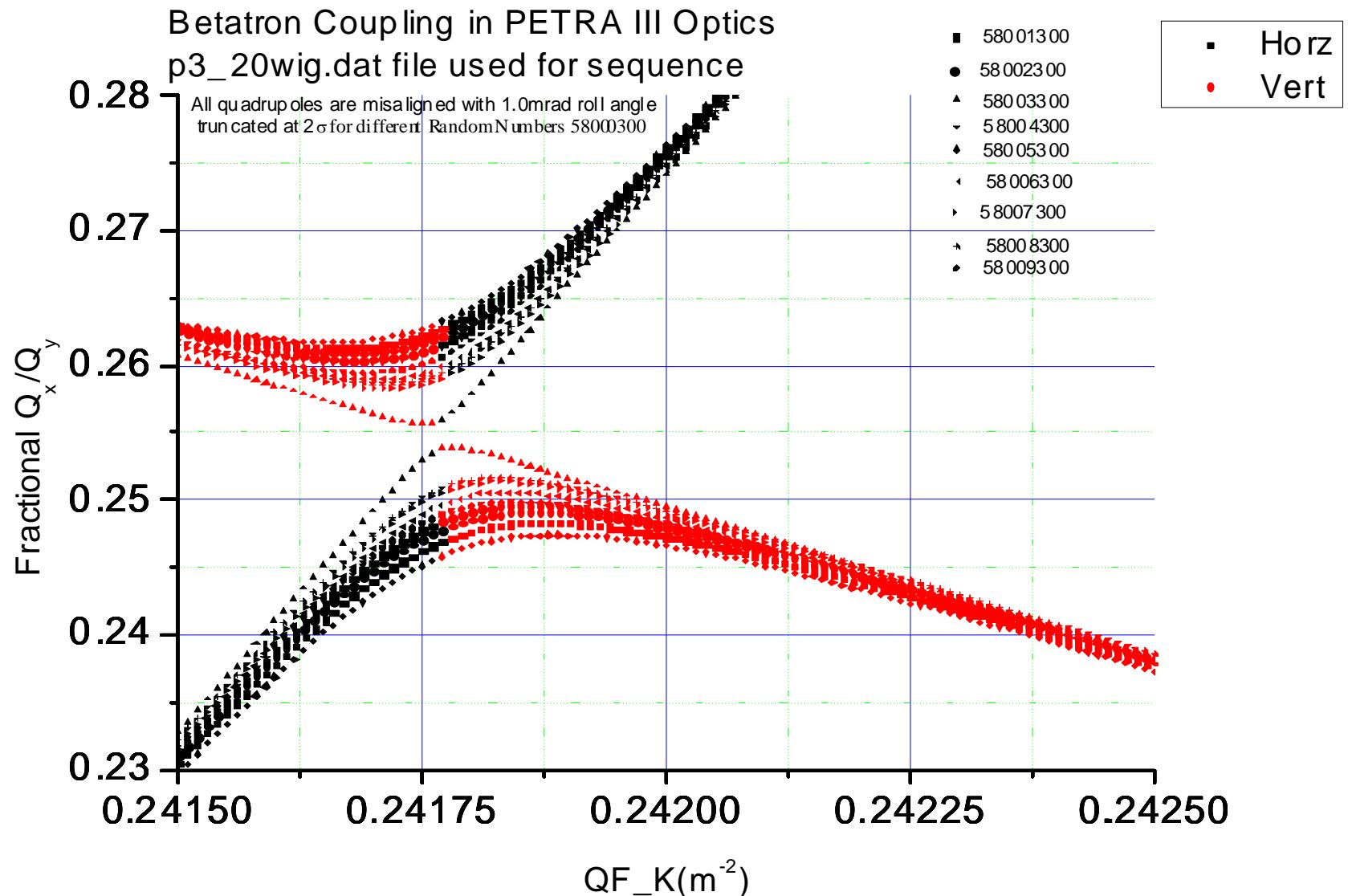
Random Misalignment of Quadrupoles with $\Delta\psi$



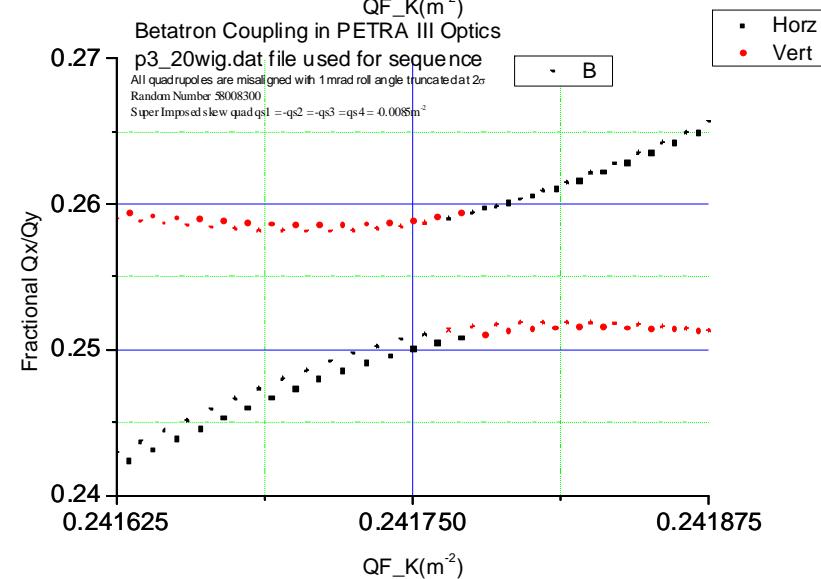
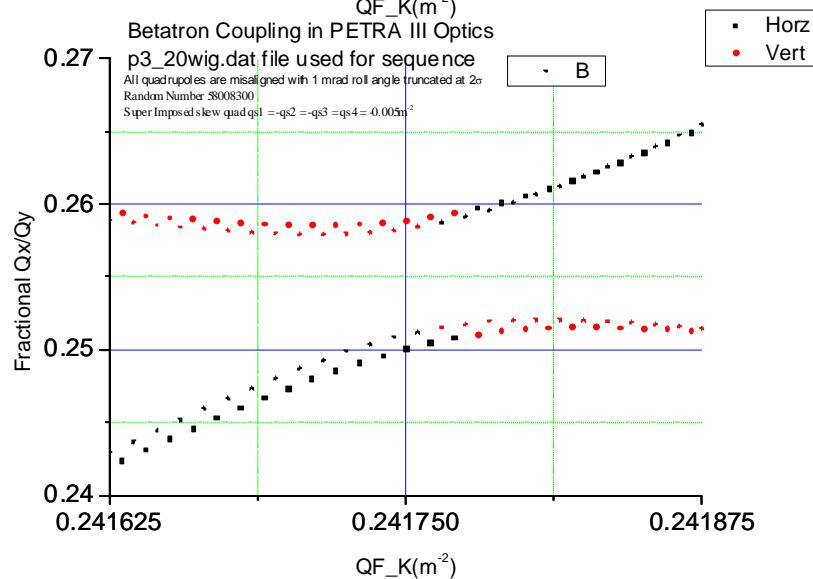
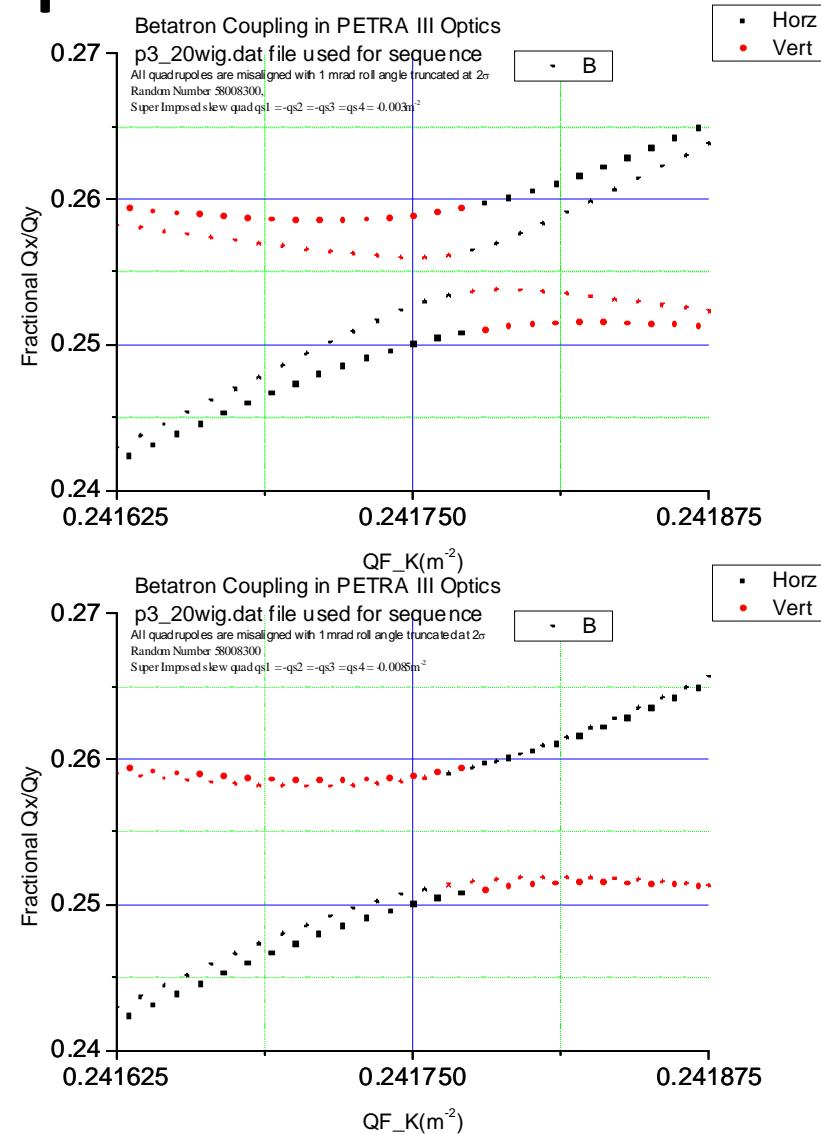
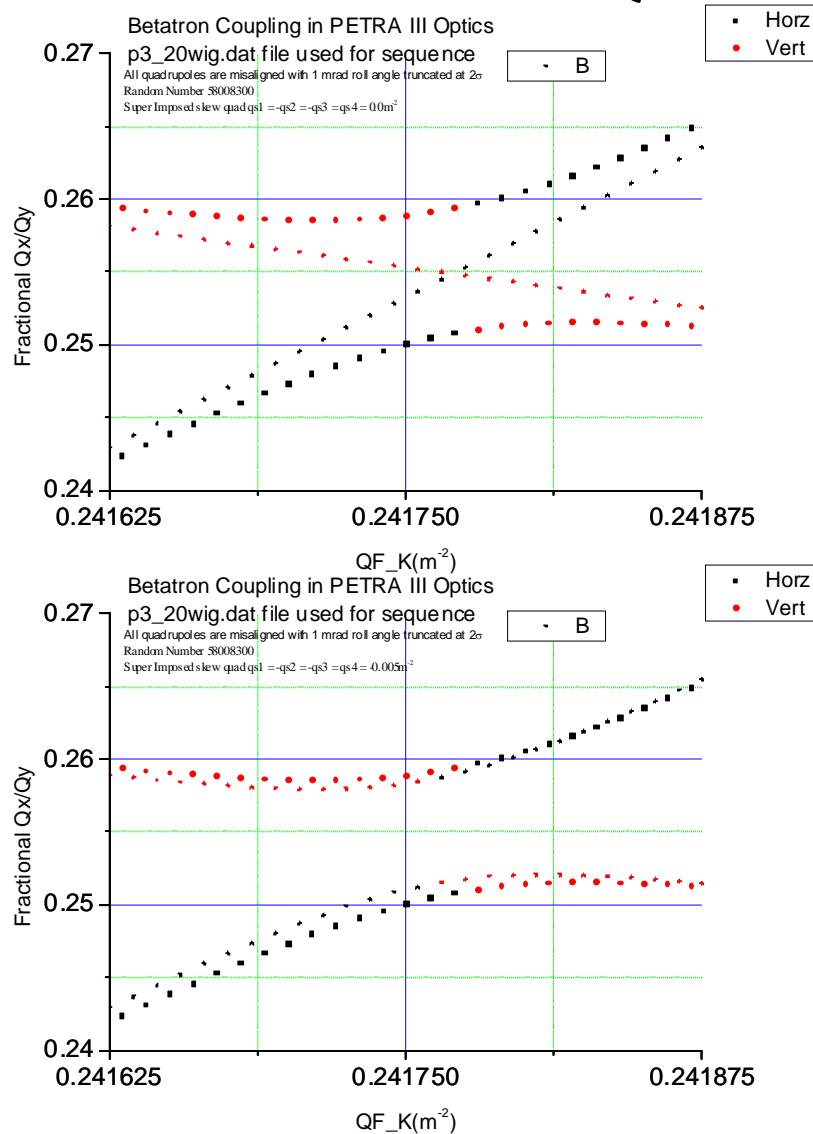
Different roll angles Putting together



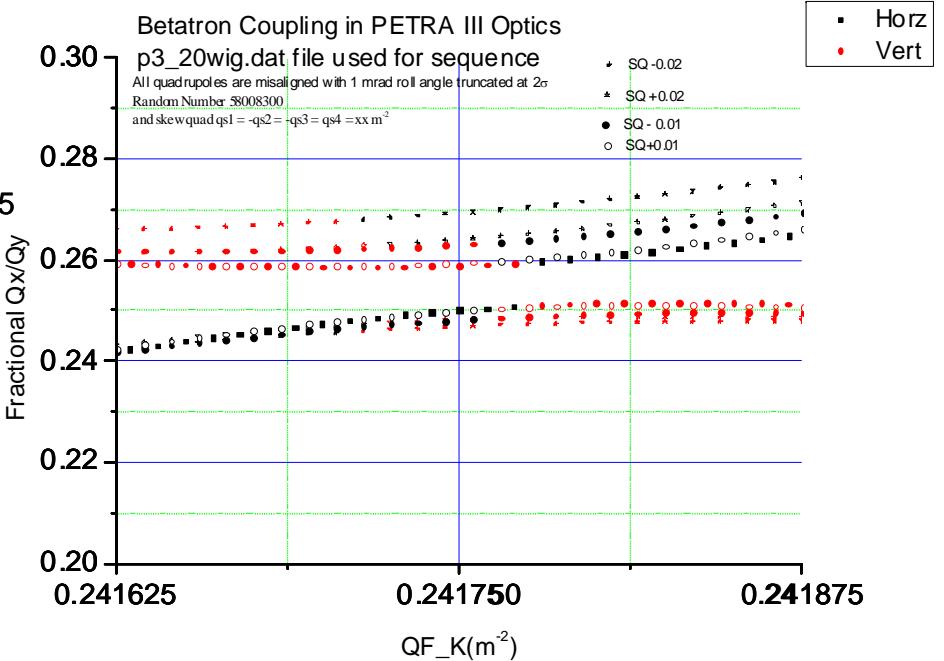
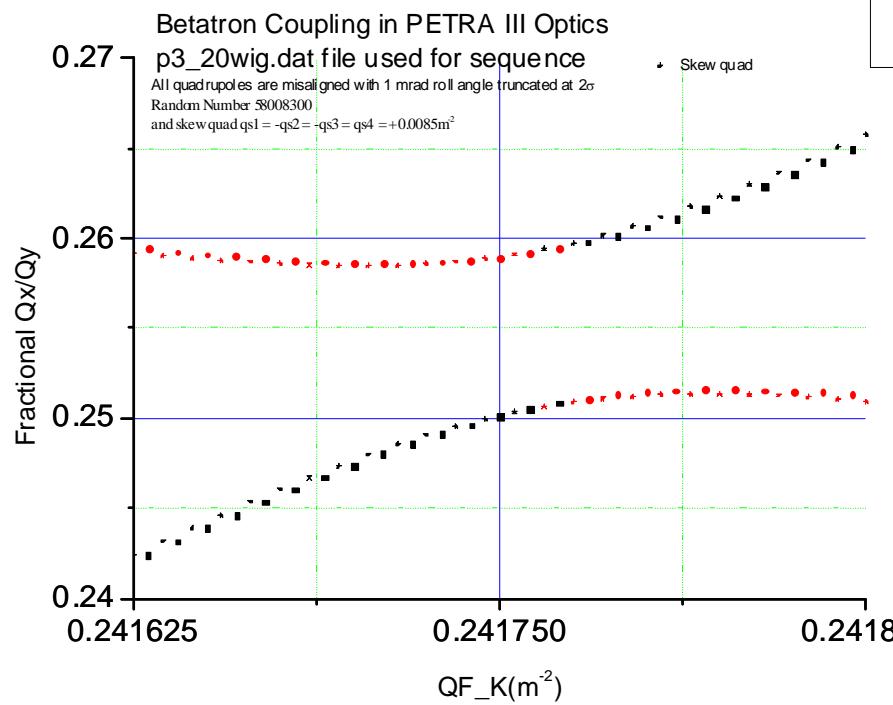
Different Random Seeds



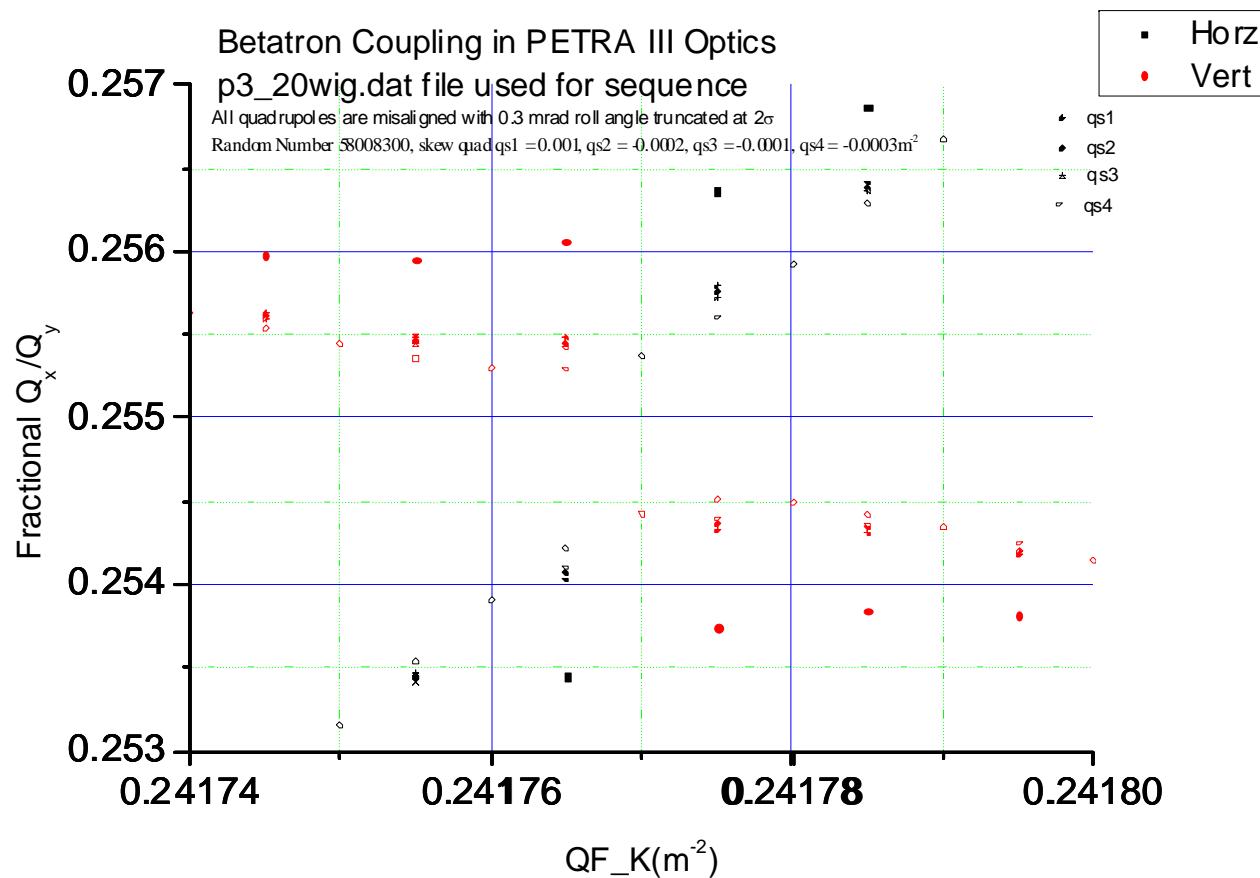
Skew Quadrupoles Used



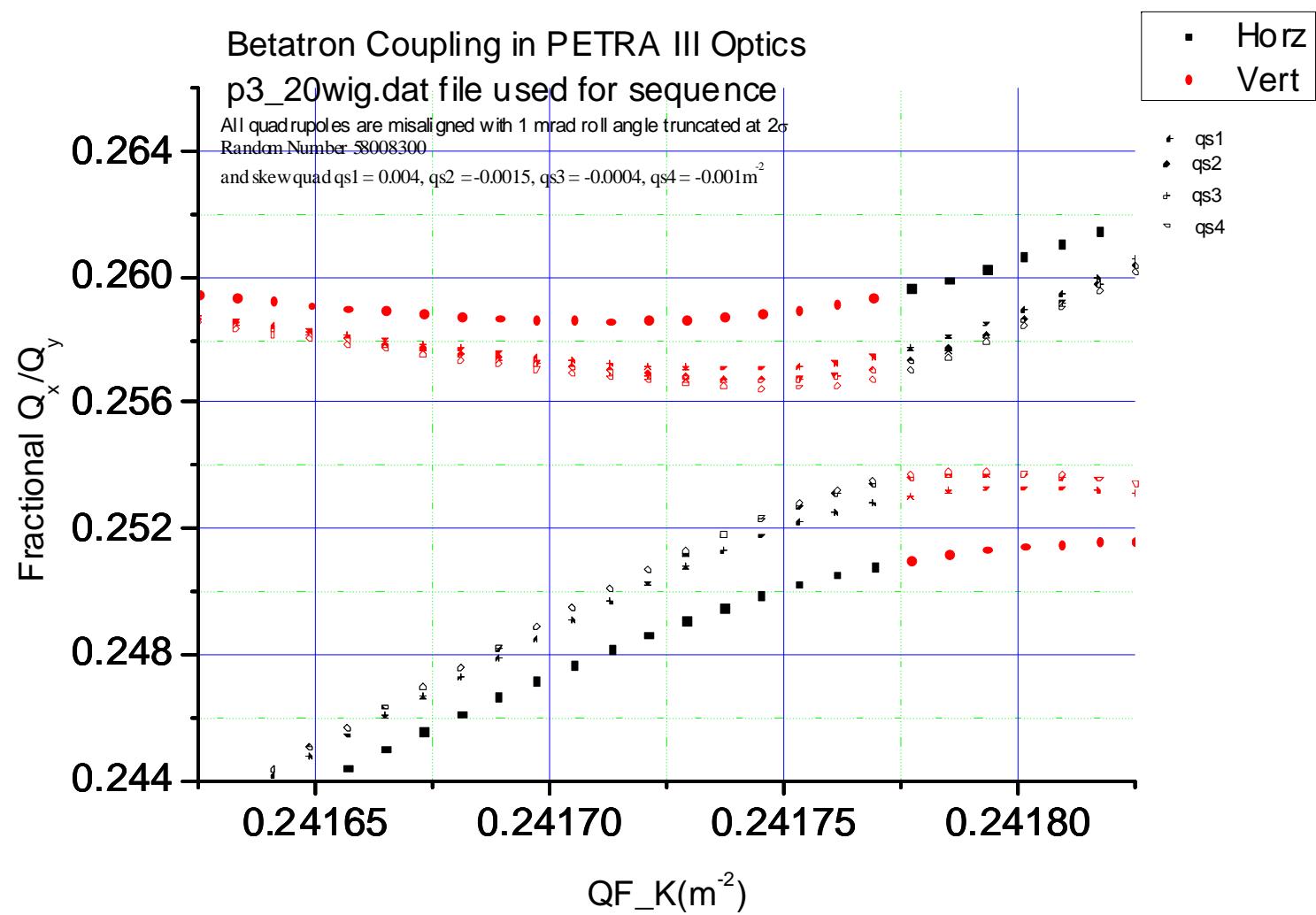
Random roll + skew quad



0.3mrad Random Roll compensation Skew quadrupoles are individually powered



1.0mrad Random Roll 58008300



1.0mrad Random Roll 54008300

