The physics of particle accelerators and storage rings

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Plan

Generalities:

- Introduction
- Why accelerators?
- Examples
- .....

Key concepts:

- Transverse optics: quads etc
- Cavities
- Synchrotron radiation
- 6×6 symplectic dynamics, resonances...

Characteristics:

- “Goodness”
- Contrasts: rings/linacs
- ...

Spin polarisation!!!!!

- Why polarised beams?
- Where to find information

All kinds of good things can be found at:

www.desy.de/~mpybar

Why accelerators and storage rings?

Understand the structure of matter and the fundamental forces of nature by colliding electrons/positrons, protons/antiprotons with each other

or

electrons/positrons, protons/antiprotons, photons, muons, neutrinos, pions, kaons....

with protons or nuclei in external targets

===> observe the scattering products.

Kinematics

Energy in the centre of momentum:

$$\begin{aligned}\frac{E_{cm}^2}{c^2} &= (p_1^\mu + p_2^\mu)_{cm}(p_{\mu 1} + p_{\mu 2})_{cm} \\ &= (p_1^\mu + p_2^\mu)(p_{\mu 1} + p_{\mu 2}) \\ &= s \qquad \text{(conventional symbol)} \\ &= \frac{(E_1 + E_2)^2}{c^2} - (\vec{p}_1 + \vec{p}_2)^2 \\ &= 2(\frac{E_1 E_2}{c^2} + p_1 p_2) + m_1^2 + m_2^2\end{aligned}$$

Beam into target:
$E_1 >> m_1,\ m_2,\ \ p_2 = 0,\ \ E_2 = m_2 :\Longrightarrow E_{cm} \approx \sqrt{2E_1 m_2}$

Colliding beams (with storage rings or linear accelerators):
$E_1 >> m_1,\ \ E_2 >> m_2 :$ $\Longrightarrow E_{cm} \approx 2\sqrt{E_1 E_2}$

At HERA:
$E_e = 27.5\text{GeV}$,$E_p = 920\text{GeV}$,$m_e = 0.51\text{MeV}$,$m_p = 938\text{MeV}$:
$\sqrt{s} \approx 318$ GeV.

At LHC at CERN:
$E_{p1} = 7\text{TeV}$, $E_{p2} = 7\text{TeV}$, : $\sqrt{s} = 14$ TeV

Basic techniques: $\vec{F} = e\,(\vec{E} + \vec{v} \times \vec{B})$

Symbolic! Much over simplified!!!

Basic linear acc (linac).

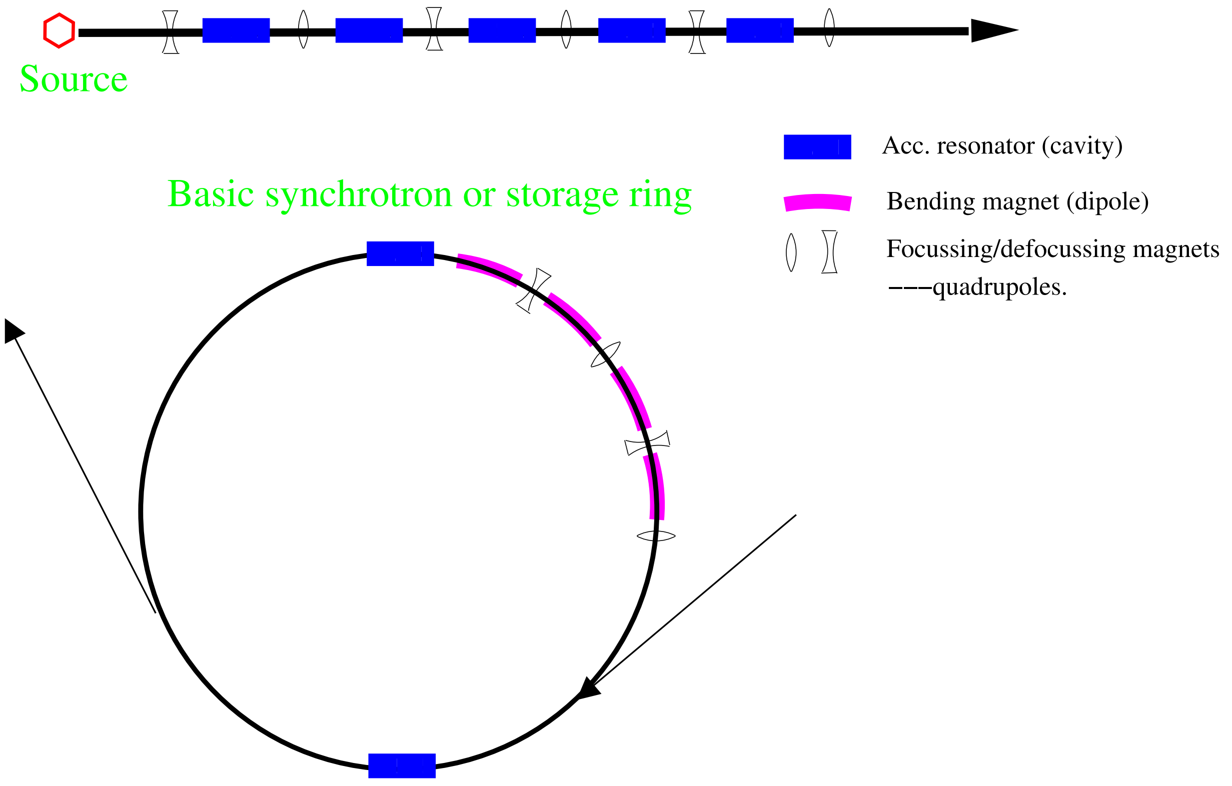


Table 1: **The highest energy machines past, present, future**

Name	type	location	Energy
LEP (†)	ring e^+–e^-	CERN	105 + 105 GeV
LHC (*)	rings p–p	CERN	7 + 7 TeV
Tevatron	ring p–$\bar{p}$	Fermilab	900 + 900 GeV
HERA	rings $e^\pm$–p	DESY	27 + 920 GeV
RHIC	rings p–p,A–A	Brookhaven	250 + 250 GeV
SLC(†)	linac e^+–e^-	SLAC	46 + 46 GeV
TESLA(?)	linac e^+–e^-	DESY	250 + 250 GeV

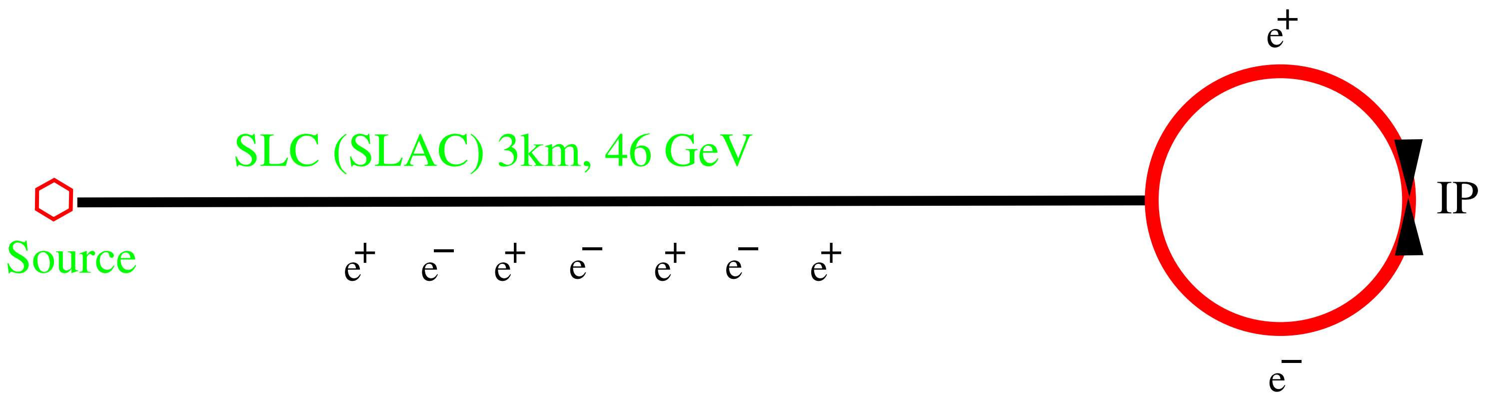
With e^+–e^- and p–$\bar{p}$: 1 vacuum pipe

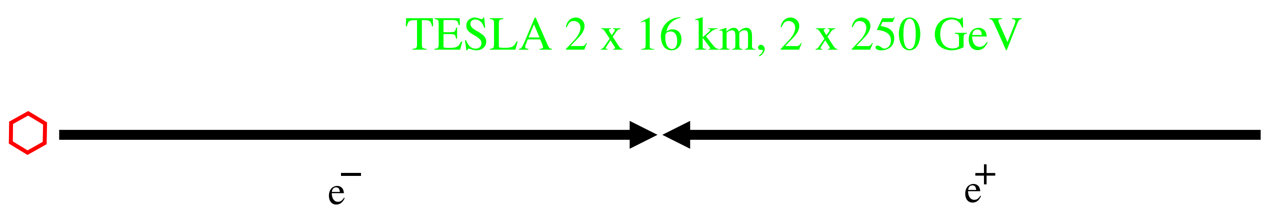
With p–p : 2 interleaved vacuum pipes

Lots of all kinds of low energy machines!!!!

Typical linacs

Symbolic! Much over simplified!!!





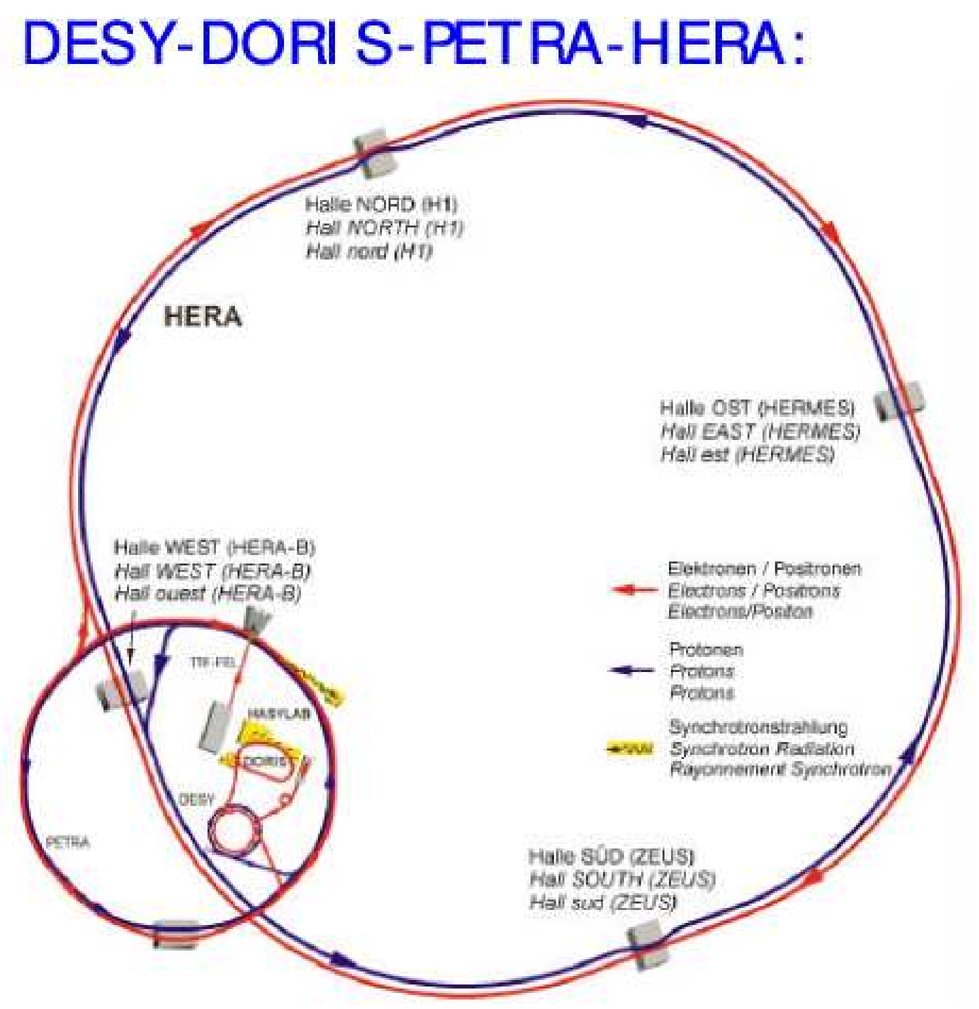
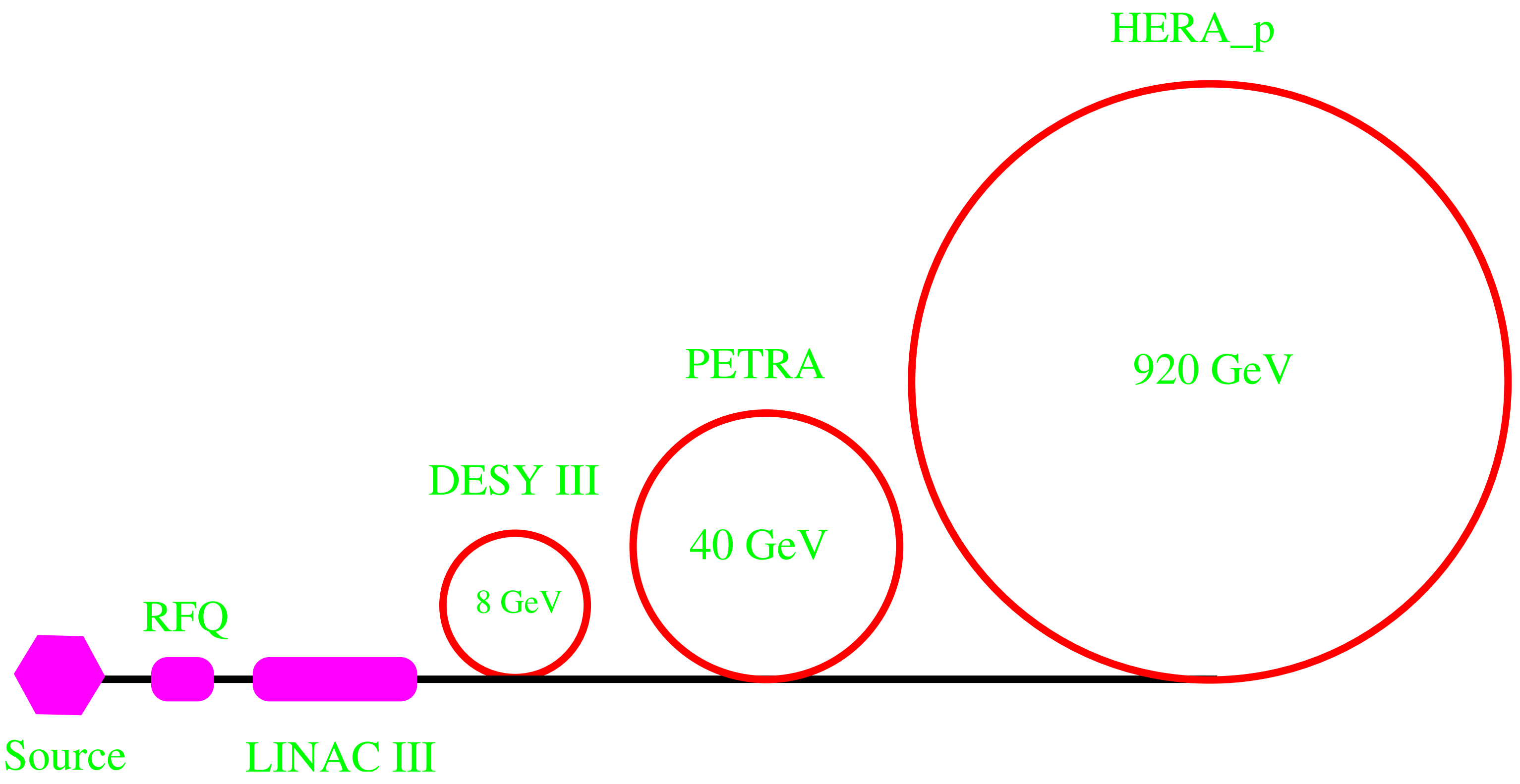


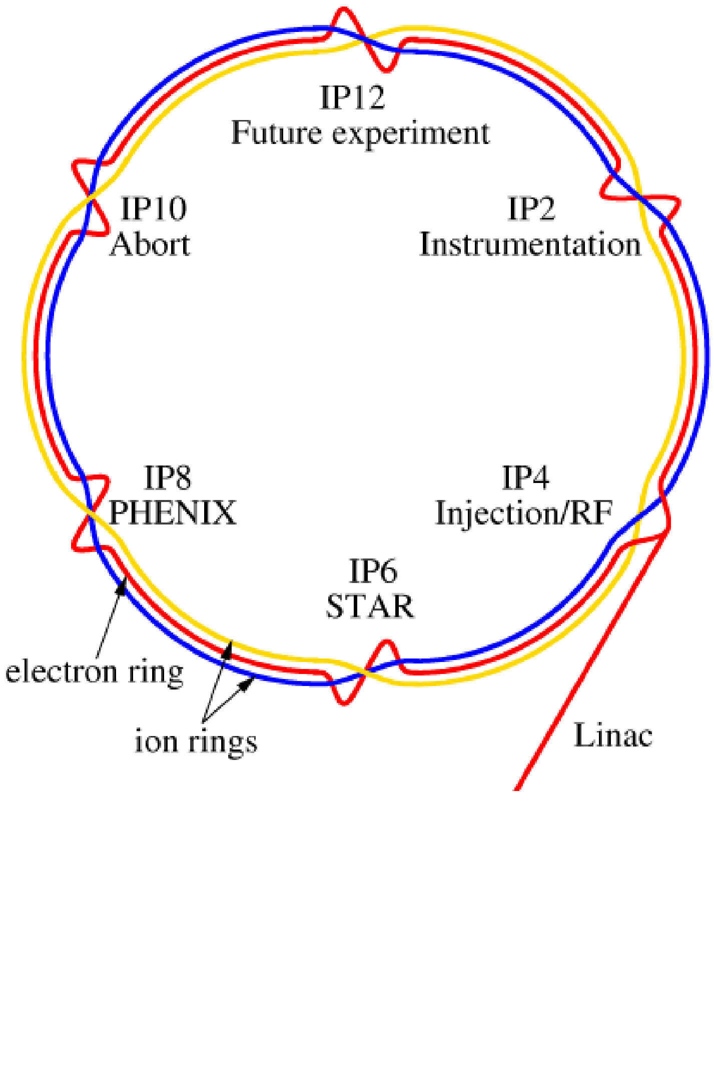
Figure 1: The DESY rings. Apologies for the low quality.

The HERA Proton Chain

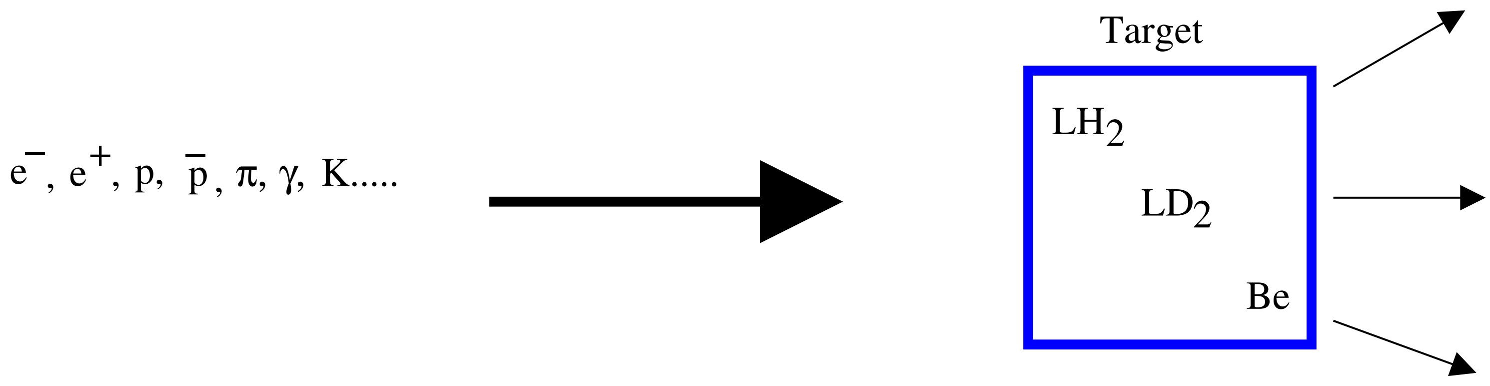


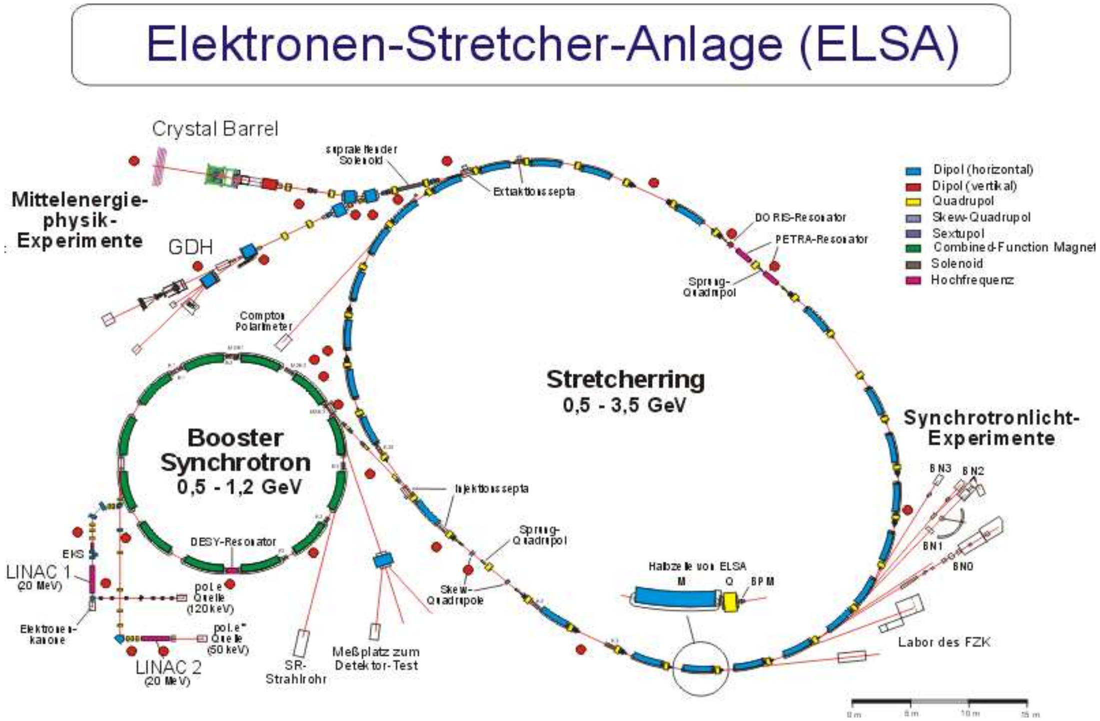
eRHIC: 10 GeV electron ring addition to the 250 + 250 GeV protons of RHIC

Ben–Zvi et al., NIM A463 (2001) and EIC white paper. 2001



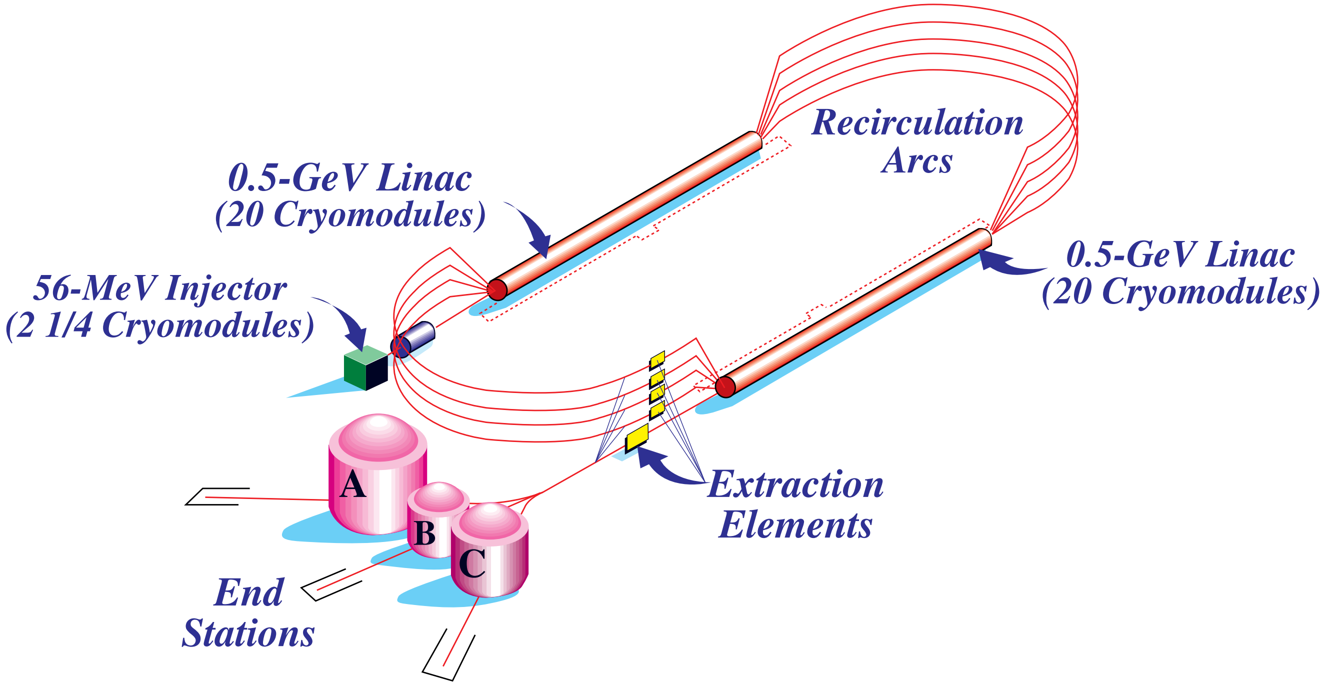
External beam into a target





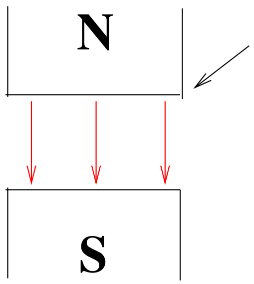
From http://www.physik.uni-bonn.de/physik/elsa99de.html

CEBAF at TJNAL: Up to 5.73 GeV, up to 80 % polarisation.



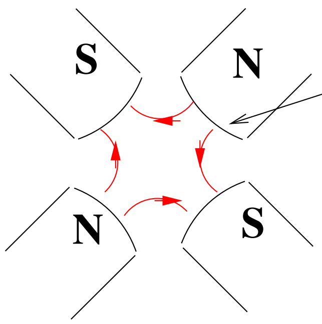
Optics, dynamics, transport maps.....

Basic components



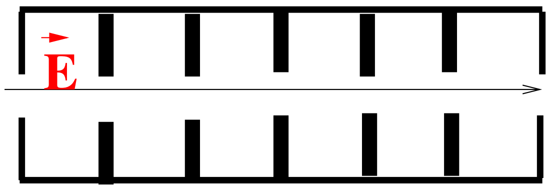
Parallel iron poles.

Bending magnet (dipole): constant vertical field B_z



Hyperbolic iron poles.

Quadrupole: $B_x = g\,z, \quad B_z = g\,x$



R.F. resonator (accelerating cavity)

Most compact form: the helical Siberian Snakes of V. Ptitsin and Yu. Shatunov.
4 modules, super cond. 4 Tesla, each 2.4 m.

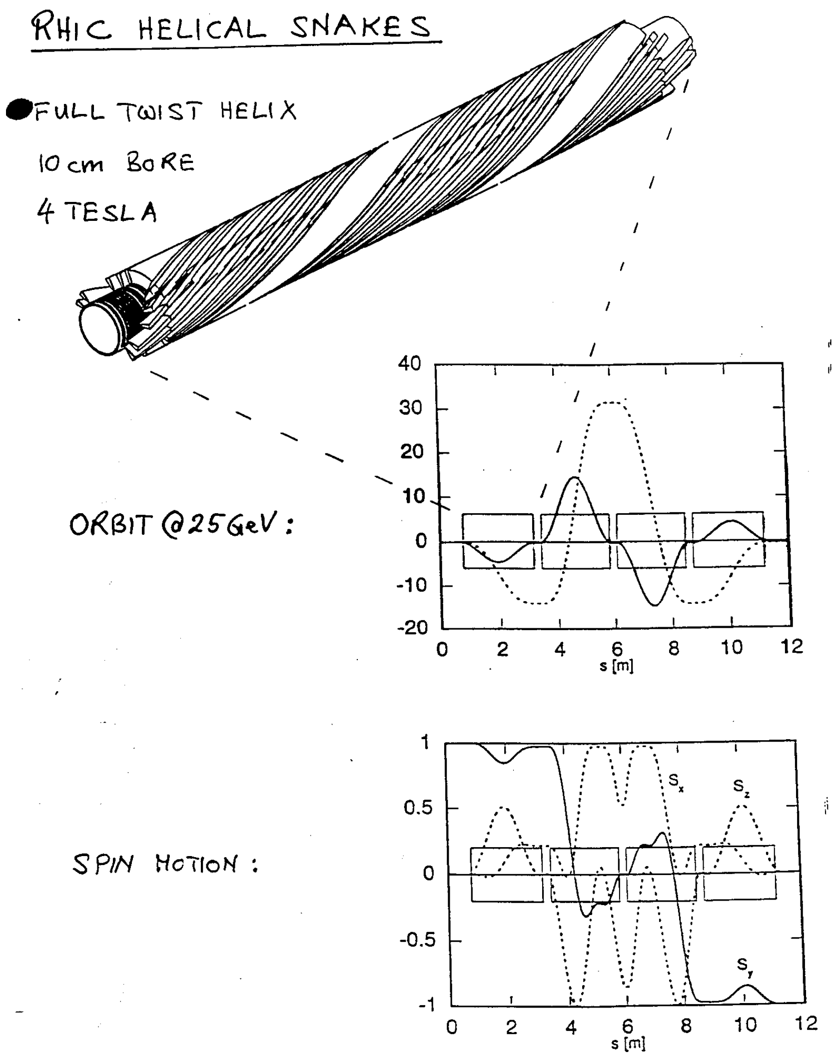
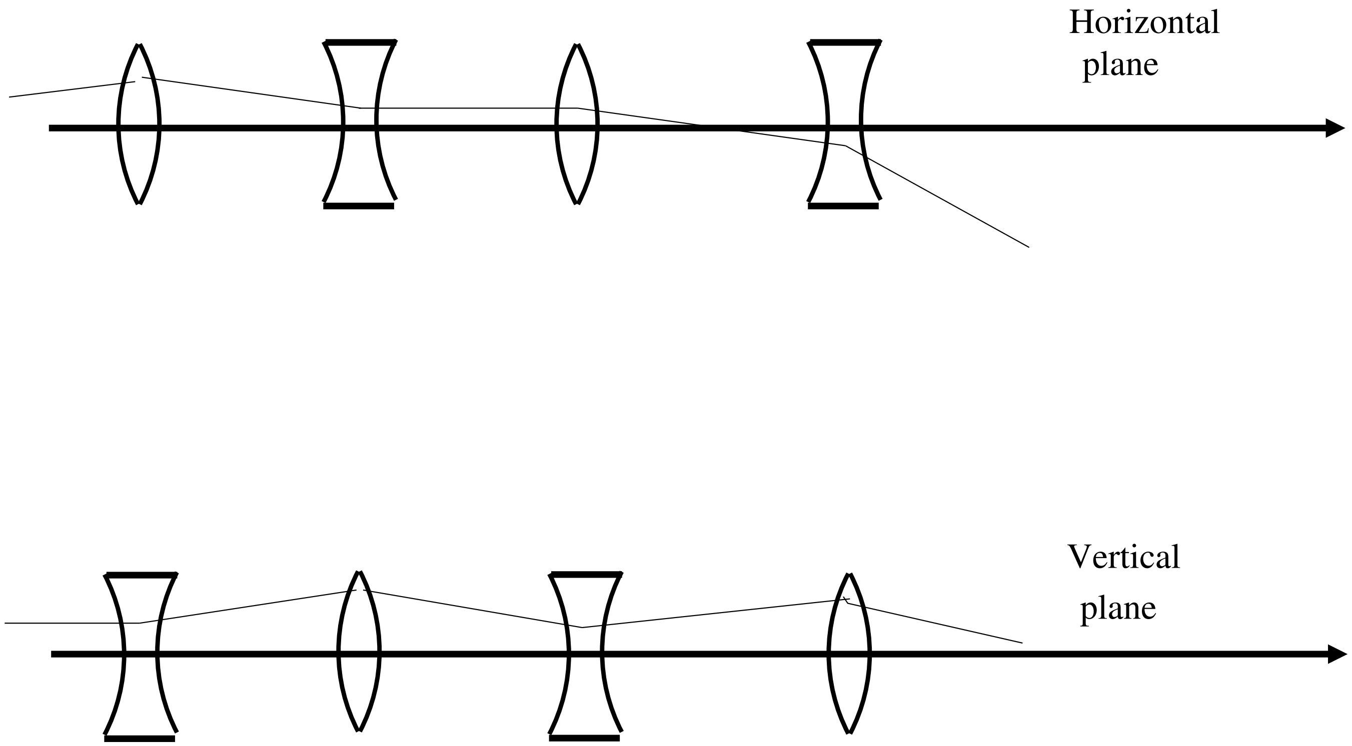


Figure 2: Rotators can be formed by choosing other combinations of the 4 modules Sketch from T. Roser, May 99 DESY Workshop.

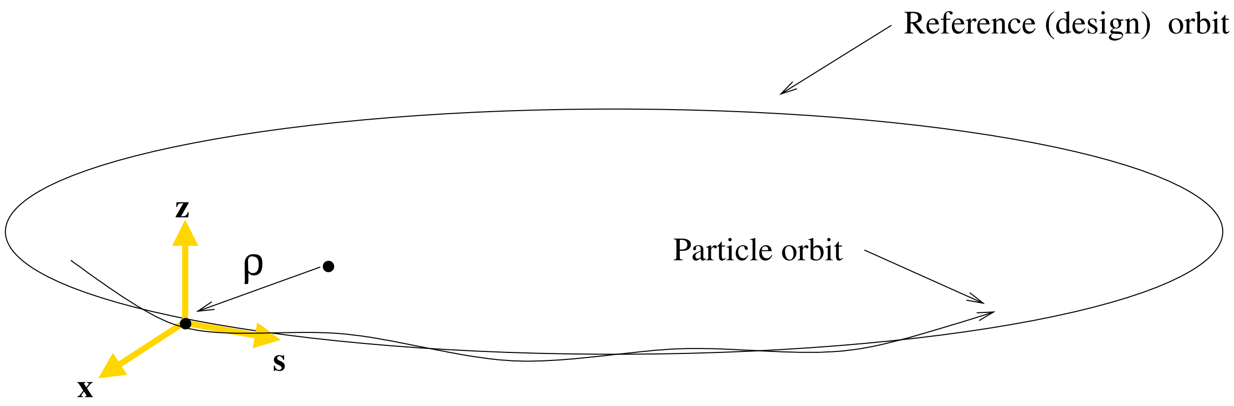
A strong focussing channel

To confine the particles and transport the beam over long distances:

Alternate focusing and defocusing magnets: quadrupoles (symbolic)



Attach a local curvilinear coordinate system to each point on the design orbit



Equations of motion in dipoles and quadrupoles.....

- Work in the curvilinear coordinate system defined by the design orbit
  Longitudinal coordinate: "s", circumference: C
- Exploit: $\vec{F} = e(\vec{E} + \vec{v} \times \vec{B})$
- Reduce the equations of particle motion to **geometry** ===>

- Dipoles: $K_x(s) = \frac{1}{\rho(s)} = \frac{e}{p_0} B_z(s)$, defines the B_z in a dipole if ρ and p_0 given.
- Quadrupoles: $B_z = x\frac{\partial B_z}{\partial x} = gx,$ $\quad B_x = z\frac{\partial B_x}{\partial z} = gz,$ $\quad k(s) = \frac{e}{p_0} g(s)$
- Equations of motion in quadrupoles and dipoles (hor. bend) relative to the local curvilinear coordinates in **paraxial approx.** :

$$x'' + (K_x^2(s) - k(s))x = K_x(s)\frac{\Delta p}{p_0}$$
$$z'' + k(s)z = 0$$

i.e. **linear equations of motion.**

Solutions in quadrupoles.....

Horizontal motion with constant $k(s)$ in a quadrupole of length l: just SHM!

$$\begin{pmatrix} x \\ x' \end{pmatrix}' = \begin{pmatrix} 0 & 1 \\ k & 0 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$

For $k > 0$: horizontally defocussing.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} \cosh(\sqrt{|k|}l) & \frac{1}{\sqrt{|k|}} \sinh(\sqrt{|k|}l) \\ \sqrt{|k|} \sinh(\sqrt{|k|}l) & \cosh(\sqrt{|k|}l) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

For $k < 0$: horizontally focussing.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} \cos(\sqrt{|k|}l) & \frac{1}{\sqrt{|k|}} \sin(\sqrt{|k|}l) \\ -\sqrt{|k|} \sin(\sqrt{|k|}l) & \cos(\sqrt{|k|}l) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

For a drift of length l, i.e. no magnet $k = 0$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

In quadrupoles: for z motion: JUST CHANGE THE SIGN OF k.

Solution for strings of quadrupoles.....

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = M_{2\times 2,n} \quad M_{2\times 2,n-1} \ldots M_{2\times 2,1} \begin{pmatrix} x \\ x' \end{pmatrix}_{in}$$

Solution for whole RING.....

The gradient parameter: $k(s+C) = k(s)$: ONE TURN PERIODICITY!

===> Floquet methods, Hill's eqn. recall Bloch wave functions in crystals!

Get the 1–turn map: $M_{2\times 2}(s+C) = M_{2\times 2}(s)$

$$\text{Det } M_{2\times 2} = 1$$

Eigenstates of motion, stability analysis.....

$$M_{2\times 2}(s)v(s) = \lambda v(s),$$

===> two eigenvectors: $v_-(s) = v_+^*(s)$ $\qquad \lambda = e^{\pm 2\pi i Q_x}$ —- independent of s !!

Eigen–analysis cont.....

Can always write:

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = A_x v_+(s) + A_x^* v_-(s)$$

for some constant amplitudes A_x determined by the initial conditions.

To transport some x, x' turn–by–turn just operate repeatedly with $M_{2\times 2}(s)$:

$$M_{2\times 2}^N(s) \begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = A_x \lambda^N v_+(s) + A_x^* \lambda^{*N} v_-(s)$$

===> unstable motion unless $|\lambda| = 1$!!!!

Eigen–analysis more.....

If stable:

$$v_+(s) = \frac{1}{2\sqrt{\beta_x(s)}} \begin{pmatrix} \beta_x(s) \\ -(\alpha_x(s)+i) \end{pmatrix} e^{-i\phi_x(s)} \qquad \alpha_x(s) = -\beta_x(s)/2$$

$$\beta_x(s+C) = \beta_x(s)$$

$\beta_x(s)$ fulfils a NONLINEAR diff. eqn. ===> has fixed scale! The scaling is in $|A_x|$.

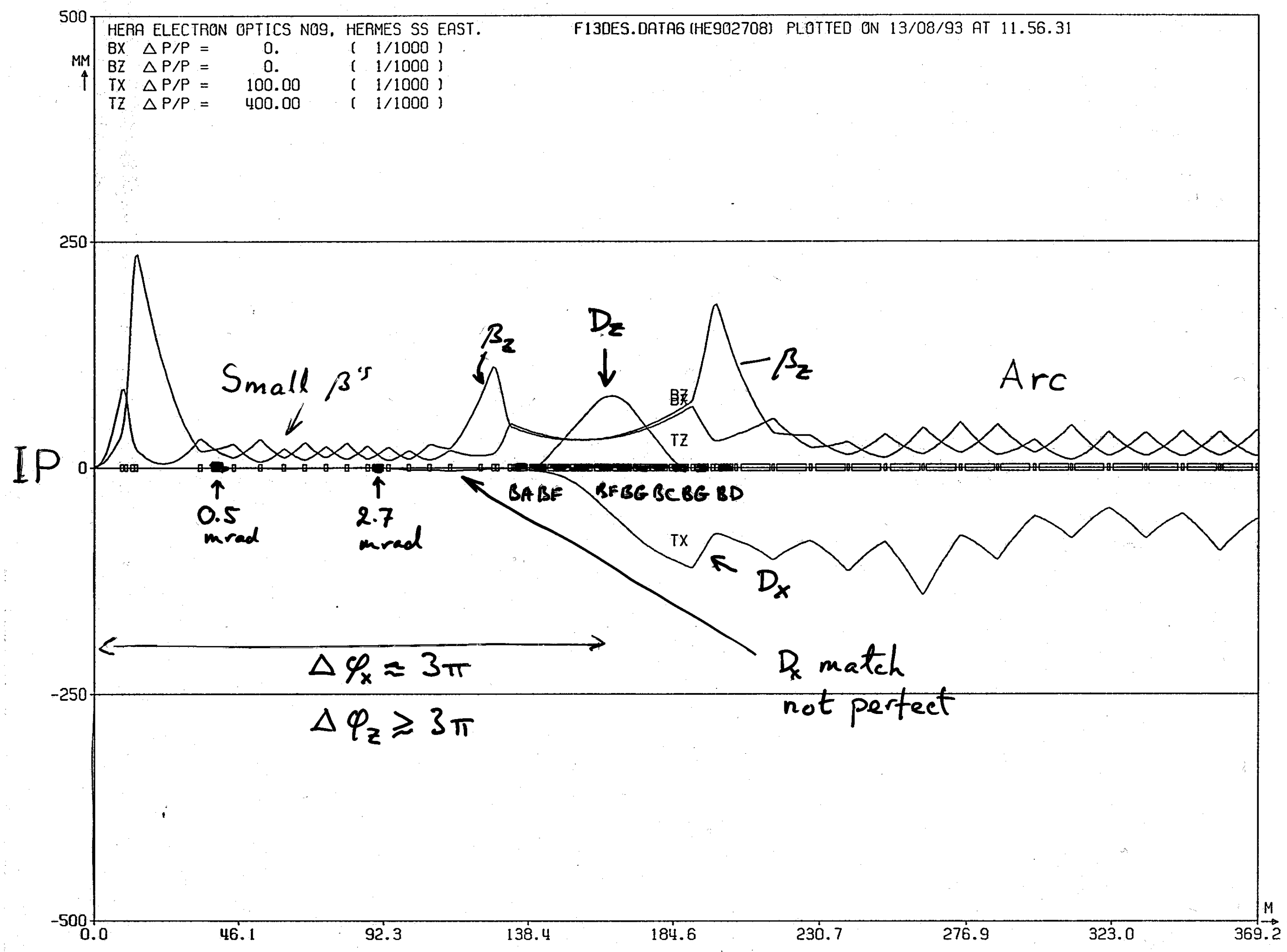
Then $x(s) = |A_x|\sqrt{\beta_x(s)}\cos(\phi_x(s)+\phi_{x0})$, $\qquad x'(s) = |A_x|\sqrt{\beta_x(s)}(............)$

Vertical plane: $z(s) = |A_z|\sqrt{\beta_z(s)}\cos(\phi_z(s)+\phi_{z0})$

The beta functions determine the BEAM ENVELOPE.

This is the COURANT–SNYDER formalism

Electron beta functions near an interaction point

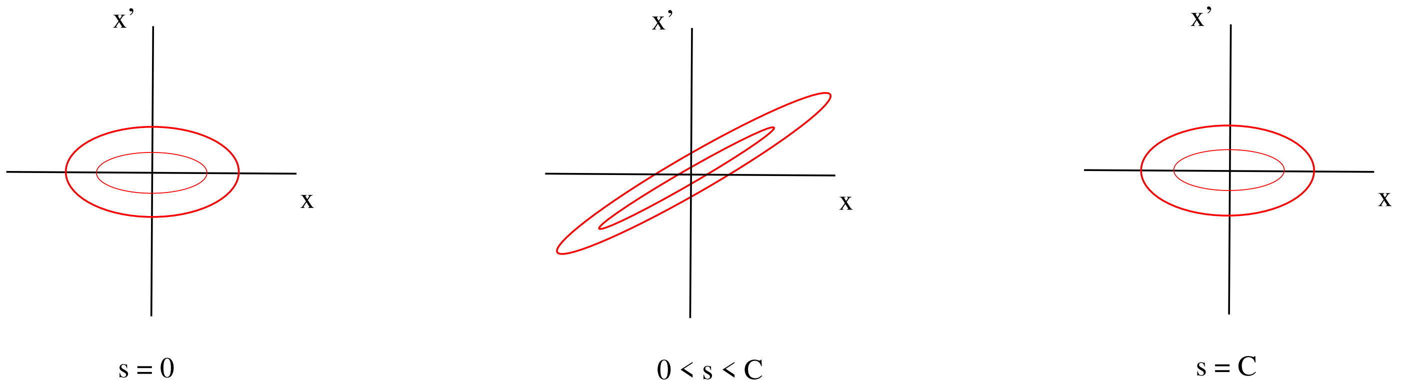


Liouville's theorem/Poincare invariant

In a system described by a Hamiltonian:

The particle density along an orbit is conserved

The volume of phase space is conserved: $\text{Det}\, M_{2x2} = 1$



Shape and tilt depend on β and α

$|A_x|$ is a constant (integral) of motion and the scale factor

The “tune”: the number of transverse oscillations/turn

$$\lambda = e^{\pm 2\pi i Q_x}$$

For stable motion Q_x is the characteristic frequency of “betatron” motion.

$$\text{Also } \Delta\phi_x = \int \frac{ds}{\beta_x} ==> Q_x = \frac{1}{2\pi} \oint \frac{ds}{\beta_x}$$

$$\text{and} \quad Q_z = \frac{1}{2\pi} \oint \frac{ds}{\beta_z}$$

Synchrotron radiation (electrons).....

Accelerating particles emit e.m. radiation ==> radio technology.

Electrons confined or deflected by magnetic fields emit e.m. radiation:

e.g. in HERA dipole magnets at HERA at 27.5 GeV: $E_c \approx 70$ keV!

$$\text{Power} \propto \frac{1}{m_e^4}\frac{E^4}{\rho^2} \quad — \quad m_e <<< m_p.$$

Synchrotron radiation emission has profound effects on electron dynamics.

So far we have neglected radiation.

A stored electron beam must be continually "accelerated"!

Acceleration, energy replacement and longitudinal motion

- A particle following a closed curve in a static electric field gains no net energy!
- ===> Use e.m. fields in cavity resonators with fields in synchronism with the circulating particles.

  - e.g. for HERA electrons $f_c \approx 47.3$ kHz
  - cavities run at ≈ 500 MHz
  - and replace ≈ 80 MeV per electron per turn (sync. rad. loss):
  - ===> megavolts and megawatts

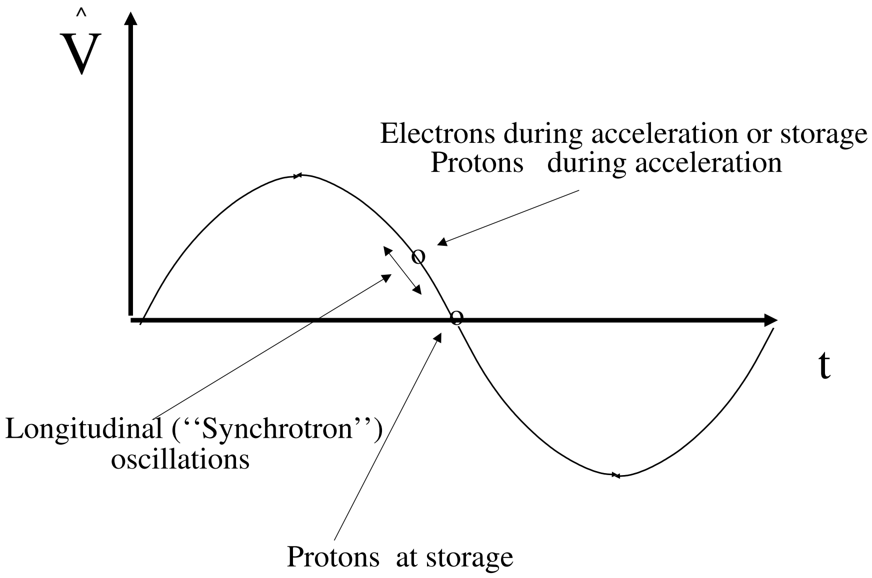
- The r.f. field leads to longitudinal bunching ===> electrons are always bunched even in storage mode

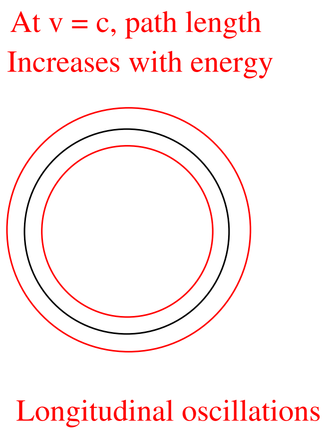
  It's good to keep the protons bunched ==> higher luminosity

- The r.f. field leads to longitudinal oscillations: tune = Q_s

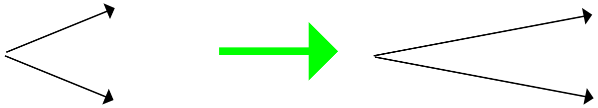
Radio frequency resonators (rf cavities)

to accelerate and/or give longitudinal confinement−−bunching





Note: non−−zero voltage for electrons at storage to replace energy lost by synchrotron radiation.



i.e. Transverse damping!

if accelerating and/or for electrons.

Equations of motion in dipoles and quadrupoles again.....

Paraxial approx. :

$$x'' + (K_x^2(s) - k(s))x = K_x(s)\frac{\Delta p}{p_0}$$
$$z'' + k(s)z = 0$$

===> **transverse–longitudinal coupling!**

Transverse – longitudinal coupling...

We have 3 tunes Q_x, Q_z, Q_s — ad hoc.

Combine all three motions into one formalism with variables:

e.g. for (ultra–relativistic) electrons: $x, x', z, z', \Delta l, \eta = \Delta E/E_0$.

For the linear approx. use 6×6 transport matrices:

For flat ring.

$$\mathbf{M_{6\times 6}} \quad = \quad \begin{pmatrix} \star & \star & 0 & 0 & \star & \star \\ \star & \star & 0 & 0 & \star & \star \\ 0 & 0 & \star & \star & 0 & 0 \\ 0 & 0 & \star & \star & 0 & 0 \\ \star & \star & 0 & 0 & \star & \star \\ \star & \star & 0 & 0 & \star & \star \end{pmatrix} \; ; \quad \vec{v}_x = \begin{pmatrix} \star \\ \star \\ 0 \\ 0 \\ \star \\ \star \end{pmatrix} \; ; \quad \vec{v}_y = \begin{pmatrix} 0 \\ 0 \\ \star \\ \star \\ 0 \\ 0 \end{pmatrix} \; ; \quad \vec{v}_z = \begin{pmatrix} \star \\ \star \\ 0 \\ 0 \\ \star \\ \star \end{pmatrix}$$

Symplectic unit matrix:

$$\mathbf{S} \ = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

"Symplecticity" — the geometry of Hamiltonian mechanics

$$MSM^T = S:$$

Compare orthogonality of 3×3 rotation matrices:— the geometry of 3–space.

$$R_{3\times3}R^T_{3\times3} = I_{3\times3}$$

Effect of radiation on electron dynamics...

The radiated energy must be replaced by cavities and etc... ===> damping i.e. collapse of phase space volume.

But single photon emission (recoils) is a source of noise.

==> Need a STOCHASTIC DIFFERENTIAL EQUATION!

$$\begin{pmatrix} x \\ x' \\ z \\ z' \\ \Delta l \\ \eta \end{pmatrix}' = (\underbrace{\mathbf{A_{6\times6}}}_{\text{Radiationless}} + \underbrace{\delta\mathbf{A_{6\times6}}}_{\text{Damping}}) \begin{pmatrix} x \\ x' \\ z \\ z' \\ \Delta l \\ \eta \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \delta(s) \end{pmatrix}}_{\text{Single photon noise}}$$

Can only predict STATISTICAL quantities: e.g. $\langle|A_x|^2\rangle$, $\langle|A_z|^2\rangle$, $\langle|A_s|^2\rangle$ ===> calculate beam sizes.

Damping + noise ===> equilibrium (Gaussian) phase space density distribution.

Key electron–proton difference in storage ring...

Proton motion is mosly deterministic: protons have a very long memory and don't enjoy being perturbed!

The size of a proton beam depends on its history.

Electron beams are much more forgiving: a perturbed electron beam in HERA re-establishes equilibrium on the "damping time scale" of 10's of milliseconds!

The size of a stored electron beam depends on the state of the ring.

Beyond linear approximations...

Begin with a Hamiltonian, with time as the independent variable:

$$\mathcal{H} = c\sqrt{(\vec{p} - \vec{A})^2 + m^2c^2} + e\phi$$

Do a canonical transformations to get a new Hamiltonian w.r.t. the curvilinear machine coordinates:

$$\tilde{\mathcal{H}}(x, p_x, z, p_z, s, p_s; t)$$

Exchange t and s to make s the independent variable:

$$\hat{\mathcal{H}} = -p_s(x, p_x, z, p_z, t, -\tilde{\mathcal{H}}; s)$$

The final Hamiltonian is 1–turn periodic:

$$\hat{\mathcal{H}}(s + C) = \hat{\mathcal{H}}(s)$$

Now use Hamilton's eqns. e.g. $\frac{dx}{ds} = \frac{\partial \hat{\mathcal{H}}}{\partial p_x}$ etc etc... to get the full non–linear dynamics SYSTEMATICALLY and beyond paraxial approx.

For example Taylor expansions in the small quantities x, p_x......., perturbation methods around the linear theory,

Instabilities: the importance of "tunes"..

If the ring has imperfections and nonlinear fields and if the nonlinearities implied by the full orbit geometry are included, then the particle motion is unstable if the unperturbed motions are mutually coherent (in resonance):

$$k_0 + k_1 Q_1 + k_2 Q_2 + k_3 Q_3 \ = \ 0$$

where the k_i($k = 0, 1, 2, 3$) are integers and at least one is non–zero.

The optics must be carefully set up to avoid dangerous tunes.

It is usually difficult to get precise analytical estimates of beam behaviour ==> massive amounts of purely numerical tracking by various good, efficient approximations using the best computers available.

For example, which tunes will work if all the details beyond linear approx. are included. What tune "windows"?

For example, the beam–beam interaction is totally non-linear!!!! Will the beams survive?

Need stable beam for many hours or days. Particles travel 2.6 10^{13} metres/day !!!

“Goodness”

- Scattering rate ===> Luminosity
- Energy in the centre of mass
- Spin polarisation!!!!

"Luminosity"

The counting rate for a particular process is expressed in terms of the *luminosity* $\mathcal{L}$, which describes the geometry and characteristics of the incident beams, by

$$\mathcal{R} = \sigma \, \mathcal{L} \, \mathcal{A}$$

- σ: total cross section for the process
- $\mathcal{A}$: the corresponding acceptance of the detector

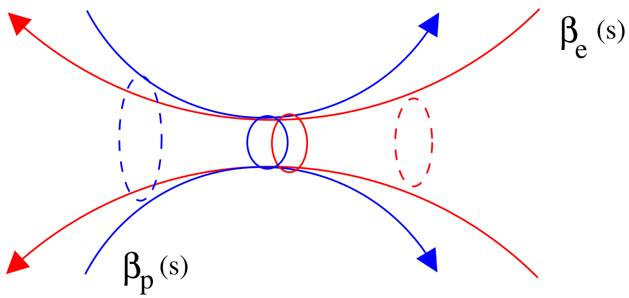
For example the luminosity of HERA can be written as

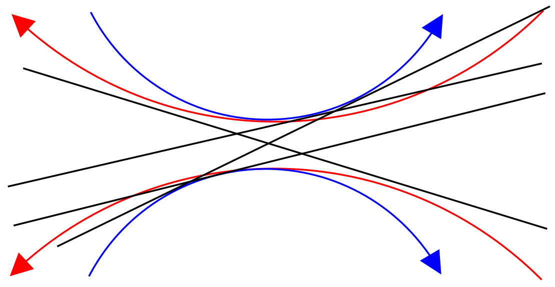
$$\mathcal{L} = \frac{N_e N_p N_{b,col} \, f_{rev}}{2\pi \sqrt{\sigma_{xe}^2 + \sigma_{xp}^2} \sqrt{\sigma_{ze}^2 + \sigma_{zp}^2}}$$

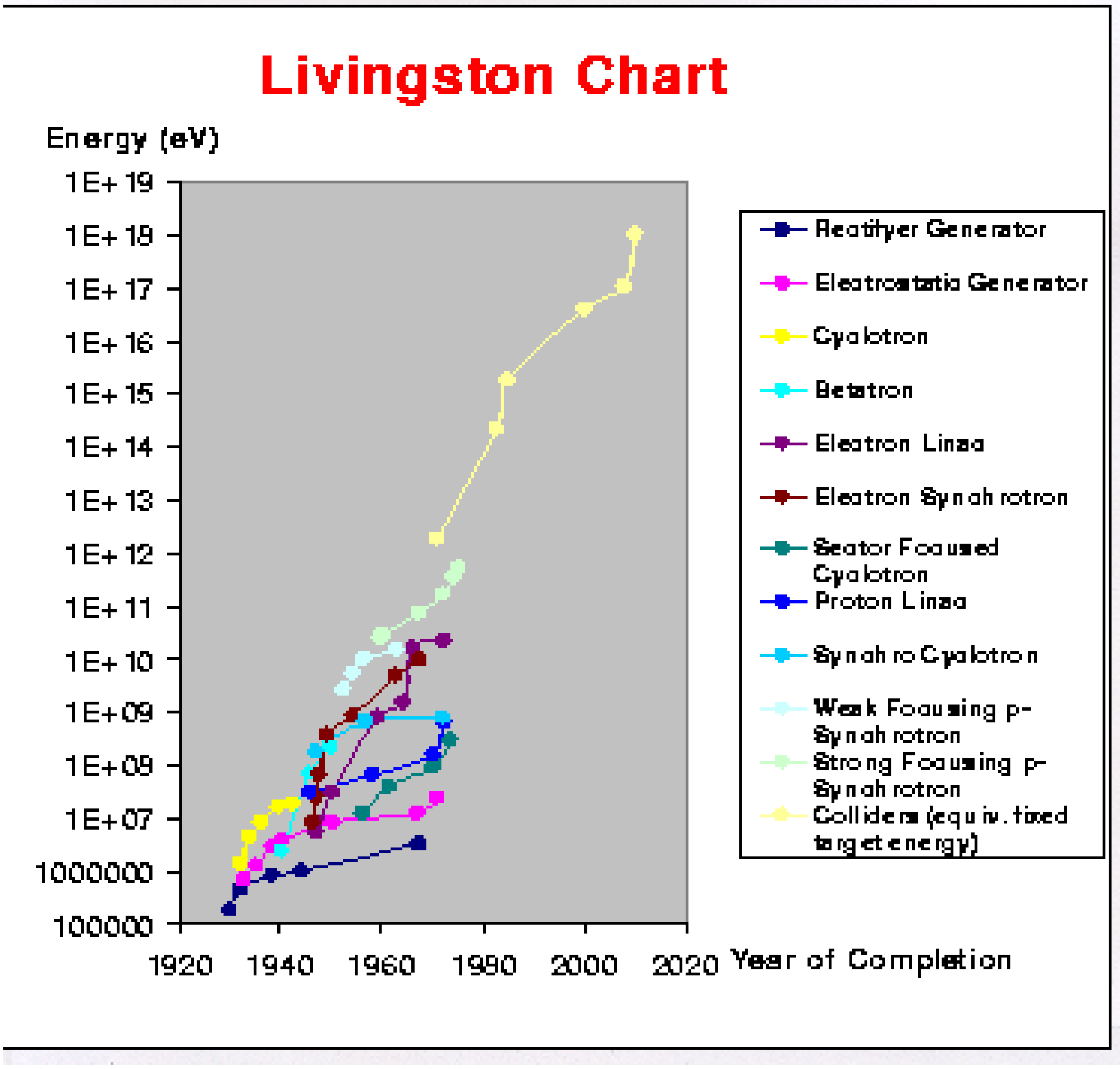
- N_e: the number of leptons per bunch
- N_p: the number of protons per bunch
- $N_{b,col}$:the number of colliding bunches per beam
- f_{rev}: the revolution frequency (47.3 kHz in HERA)
- $\sigma_{x,z;e,p}$: the rms beam sizes at interaction points (IP).

Beam geometry at the interaction point: ‘‘minibeta principle’’

The beta functions define the beam envelope






Livingston Chart
Energy (eV)
1E+19
1E+18
1E+17
1E+16
1E+15
1E+14
1E+13
1E+12
1E+11
1E+10
1E+09
1E+08
1E+07
1000000
100000
1920
1940
1960
1980
2000
2020
Year of Completion
Rectifyer Generator
Electrostatic Generator
Cyclotron
Betatron
Electron Linac
Electron Synchrotron
Sector Focused Cyclotron
Proton Linac
Synchro Cyclotron
Weak Focusing p-Synchrotron
Strong Focusing p-Synchrotron
Colliders (equiv. fixed target energy)

Some limitations: energy and luminosity

Proton rings:

- Maximum current (current per bunch, number of bunches)
  - Wake fields ==> instabilities
  - Space charge forces
  - Beam–beam forces
  - Antiprotons: the supply.
  - etc.......
- Maximum energy: Maximum circumference available, maximum fields.

Electrons/positrons rings:

- Maximum current (current per bunch, number of bunches)
  - Wake fields ==> instabilities.
  - Space charge forces
  - Beam–beam forces
  - Replenishing radiated energy (synchrotron radiation).
  - etc.......
- Maximum energy ==>: Maximum circumference available: synchrotron radiation energy/turn: $\propto \frac{E^4}{\rho}$.
  E.g. an electron in HERA at 27.5 GeV radiates 80 MeV per turn.
  ===> Accelerating power, cooling of beam pipe, radiation in experiments.

Limitations: energy and luminosity

Linacs:

- Available length <===> available voltage/metre: super conducting cavities?
- Wake fields ==> instabilities
- etc.......

Rings versus linacs..

Rings:

- Re-use the bunches as they circulate.
- Re-use the cavities for acceleration.
- If magnetic fields are not the limit, limited land area needed.
- BUT for electrons: energy limited by synchrotron radiation losses and replenishment.
- etc...

Linacs:

- Continually replenish bunches.
- Energy normally lost, continually accelerating new bunches
- Need enormous Volts/metre in the cavities to limit the length.
- etc....

Miscellaneous stuff not mentioned so far.....

- Vacuum: need typically 10^{-12} atmospheres.
- HERA: 180 bunches of each type $(e^{\pm}, p)$
- HERA: about 3 10^{10} particles per bunch.
- HERA: electron bunches 2 cm long and very roughly!
  1 mm wide, 0.1 mm high
- HERA: proton bunches, very roughly!:
  20cm long, 1mm wide 1 mm high
- HERA: $\gamma_e \approx 60000$, $\gamma_p \approx 900$
- HERA: luminosity $\approx 1.5\ 10^{31}\mathrm{cm}^{-2}\mathrm{sec}^{-1}$
- HERA: the quadrupoles must be positioned to ≈ 0.3 mm so that the periodic orbit is OK.
- Software: many standard packages:
  e.g. **MAD** = Methodical Accelerator Design.
  for all of optics and long term numerical
  tracking.
  e.g. **Sixtrack** for long term numerical
  symplectic tracking: don't want numerical
  noise or bad approxs causing artificial change of
  phase space volume ==> beam loss.
  SPRINT for spin–orbit tracking.

HERA is very special.....

The only (high energy) $e^{\pm}$–p collider ever.

The only high energy $e^{\pm}$ ring to have LONGITUDINAL SPIN POLARISATION.

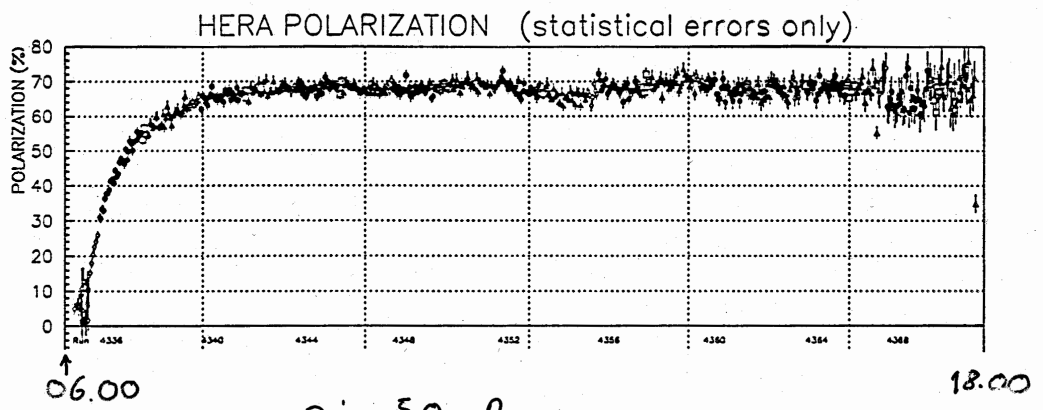
===>A unique chance to learn about it...

Why do we want spin polarised beams?

- Because it's fun!!!! and it's a chance to keep me happy by combining quantum mechanics with large scale machine physics.
  And also
  - Because it's useful for understanding the angular momentum distribution inside nucleons: The "spin crisis"===> where is the angular momentum? What constraint is put on the topology of quantum fields?
  - To get more information about the spin structure of the weak interaction. New weak bosons?
  - etc

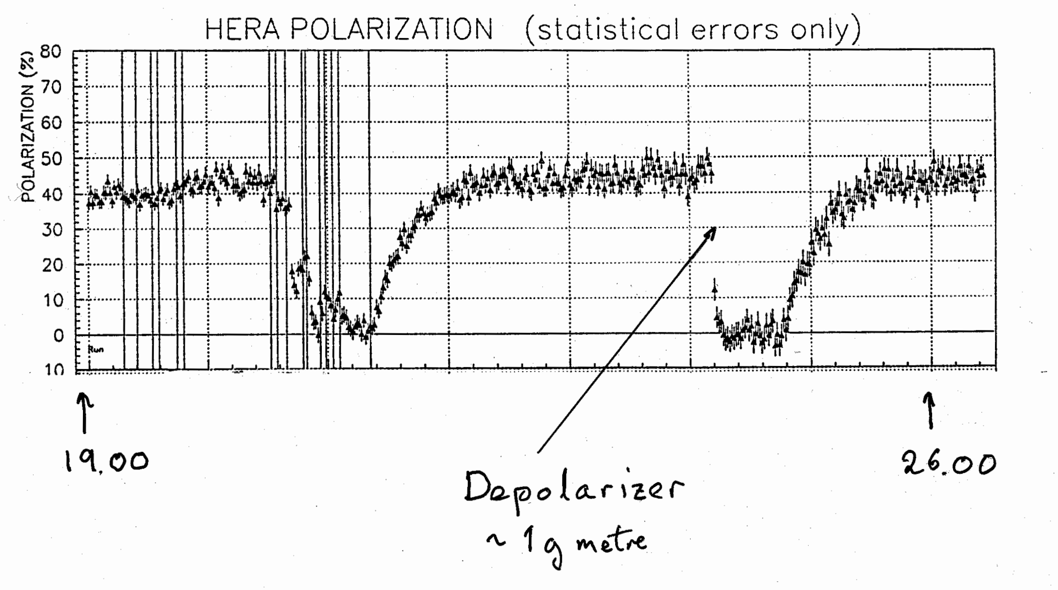
If one gets it right, electron polarisation survives spin gymnastics at 47 kHz in a 6 km ring!!!

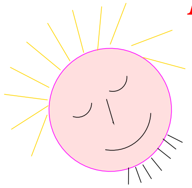
July '95 : Hera Polarimeter Group.



p : 50 mA

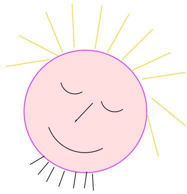
e^+ : 30 mA → 10 mA





Electron and proton spin polarization

in storage rings

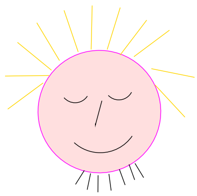


For a journey through the world of electron and proton spin polarizatio

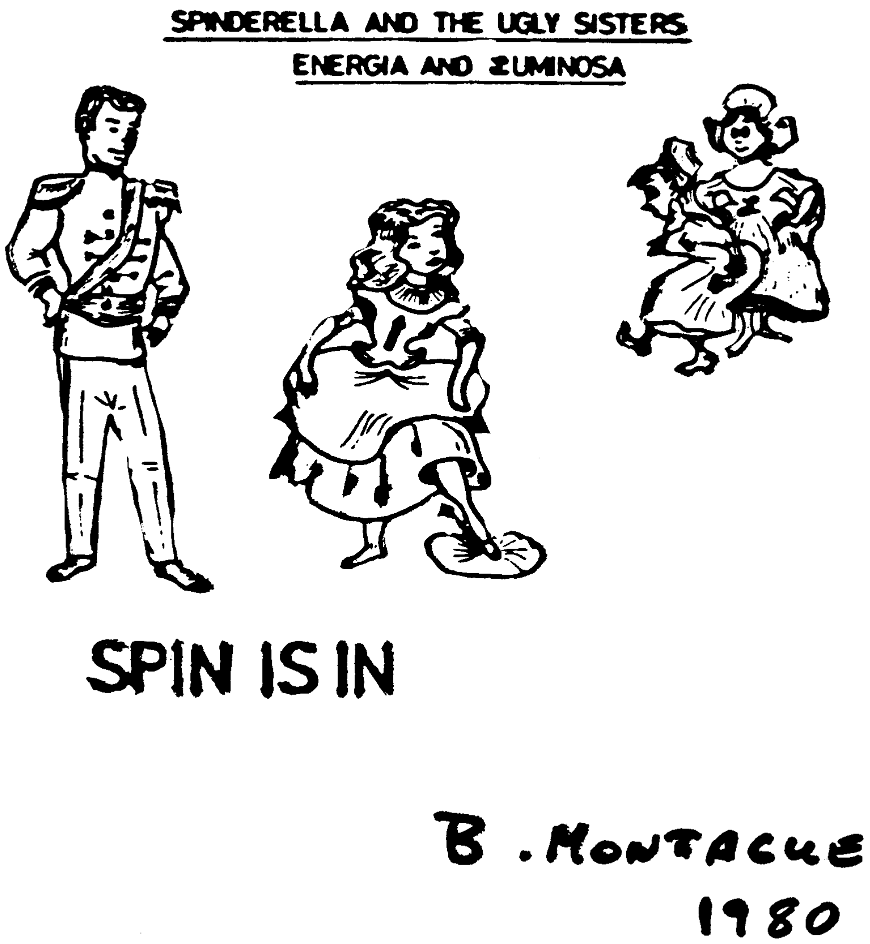
with tutorials and pictures of rotators:

see e.g. ‘‘Recent talks’’ and ‘‘Radiative polarization....’’ under:

www.desy.de/~mpybar



But:.....


SPINDERELLA AND THE UGLY SISTERS
ENERGIA AND LUMINOSA
SPIN IS IN
B. MONTAGUE
1980

Books on introductory accelerator physics:

K. Wille
The Physics of Particle Accelerators
Oxford University Press.

H. Wiedemann
Particle Accelerator Physics Part I and II:
Basic Principles and Linear Beam Dynamics/Nonlinear and Higher-order Beam Dynamics
Springer.