Pre-existing betatron motion and spin flipping with RF fields in storage rings

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For students of the recent history of these matters:

Every cloud has a silver lining.

– English proverb

Physics now stands on three legs – has three branches

- Experimental physics
- Theoretical physics
- Computational physics

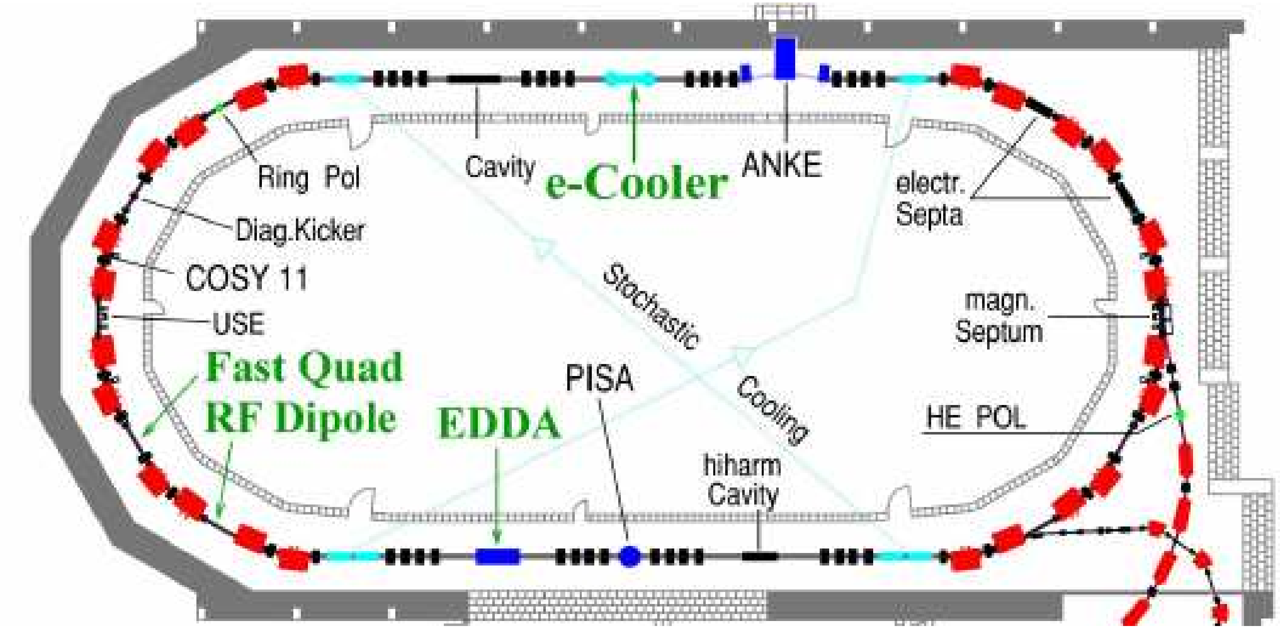
– FZ Juelich: publicity material about the powerful computers.

Essential, pre-requisite tools and knowledge:

- The **T-BMT equation** for spin motion:
  $d\vec{S}/ds = \vec{\Omega}(z,s) \times \vec{S}$; $\vec{\Omega}(z,s) = \vec{\Omega}_0(s) + \vec{\omega}(z,s)$; $z = (x,\ p_x,\ y,\ p_y, \sigma, \delta)$; $\theta = 2\pi s/C$
- The 1-turn periodic coordinate system on the CO: $\hat{l}(s), \hat{n}_0(s), \hat{m}(s)$
- The **invariant spin field** (ISF):
  $\hat{n}(z,s)$ or $\hat{n}(z,\theta)$; $\hat{n}(z, s+C) = \hat{n}(z,s)$
- The **spin tune** on the closed orbit ν_0: $= a\gamma_0$ in a simple perfectly flat ring.
- The **uniform invariant frame field** (u-IFF): $\hat{u}_1(z,s)$, $\hat{n}(z,s)$, $\hat{u}_2(z,s)$
- The **equivalence class of amplitude dependent spin tunes** (ADST).
- The **naive single resonance model** (SRM) for the ISF and ADST: a single Fourier harmonic of $\vec{\omega}(z,s) \cdot (\hat{l}(s) - i\hat{m}(s))$ has dominant control of spin motion
  $\Longrightarrow$ the spin tune ν_0 on the closed orbit matches an orbital tune:
  $\nu_0 = \kappa = k_0 \pm Q_{\mathrm{I}}$, $k_0 \pm Q_{\mathrm{II}}$, $k_0 \pm Q_{\mathrm{III}}$ or $\nu_0 = \kappa = k_0 \pm Q_{\mathrm{rf}}$ for an integer k_0.
- The well known notation for the SRM. For example:
  $\delta = \nu_0 - \kappa$, the resonance strength ϵ_κ
- The **naive Froissart–Stora formula** for survival of vertical polarisation $\big(1960!\big)$ when δ crosses 0:

$$\frac{S_y^{\mathrm{final}}}{|\vec{S}|} = 2\exp(-\Xi) - 1 = 2\exp\left(-\frac{\pi|\epsilon_\kappa^{\mathrm{v}}|^2}{2\alpha_{\mathrm{FS}}}\right) - 1\,, \quad \alpha_{\mathrm{FS}} = \frac{d\delta}{d\theta}$$

COSY


Ring Pol
Cavity
e-Cooler
ANKE
electr.
Septa
Diag.Kicker
COSY 11
USE
Stochastic
Cooling
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Septum
Fast Quad
RF Dipole
EDDA
PISA
HE POL
hiharm
Cavity

The SLIM/SLICK formalism

Linearised orbital and spin motion for first order analytical estimates of radiative depolarisation in electron storage rings, e.g., HERA, eRHIC, ELIC, ENC@FAIR, SuperB, LHeC.....

Attach an orthonormal 1-turn periodic coordinate system $\hat{l}(s), \hat{n}_0(s), \hat{m}(s)$ to the closed orbit.

$\hat{n}_0(s)$ obeys the BMT equation on the closed orbit – "the stable spin direction".

$$\vec{S} \approx \hat{n}_0(s) + \alpha\hat{m}(s) + \beta\hat{l}(s)$$

α, β: 2 small spin tilt angles — have subtracted out the big rotations!

$$\hat{\mathbf{M}}_{8\times 8} = \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix}$$

acting on $\vec{z} = (x,\ p_x,\ y,\ p_y,\ \sigma,\ \delta)$ and α, β

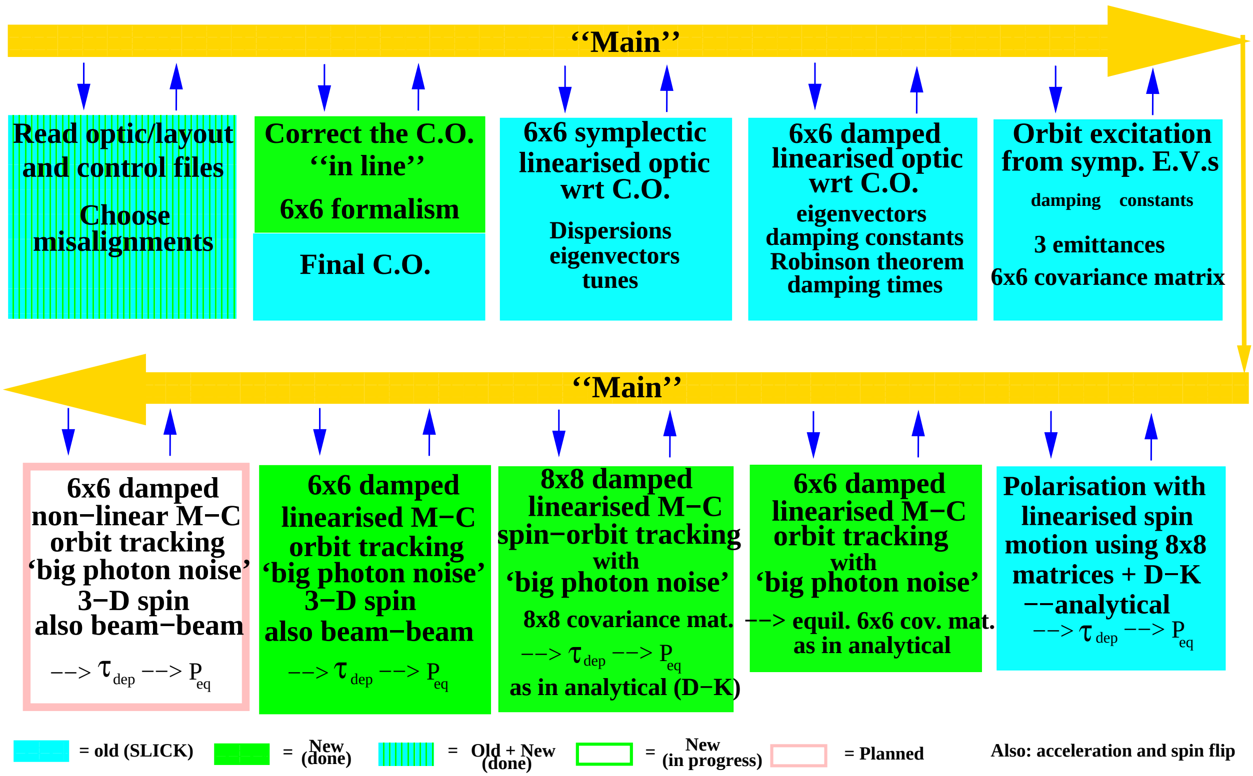
$\mathbf{G_{2\times 6}}$ represents the linearised solution of the T-BMT equation for α, β .

This is the **SLIM formalism**: originally A.W. Chao 1981 – working at DESY.

Theory and codes developed further by H. Mais and G. Ripken, D. Barber.

D. Barber: also a thick-lens version, SLICK and with Monte-Carlo extensions, SLICKTRACK.

The structure of SLICKTRACK



Using $G_{2\times 6}$ to get $\epsilon^{\mathrm{tot}}, \epsilon^{\mathrm{rfd}}$ etc

See for example:

- G.H. Hoffstaetter, *"High Energy Polarised Proton Beams: a Modern View"*, Springer Tract in Modern Physics, Vol 218 (2006).
- M. Vogt, PhD Thesis, University of Hamburg, Germany, DESY-THESIS-2000-054 (2000): http://www-library.desy.de/preparch/desy/thesis/desy-thesis-00-054.pdf
- D.P. Barber and G. Ripken in *"Handbook of Accelerator Physics and Engineering"*, Eds: A.W. Chao and M. Tigner, World Scientific, 3rd printing (2006).

Just need the 1-turn $G_{2\times 6}$ for free synchro-betatron motion and with the RFD, a trivial extension for the forced motion + a term for the RFD itself.

For example, for free synchro-betatron motion:

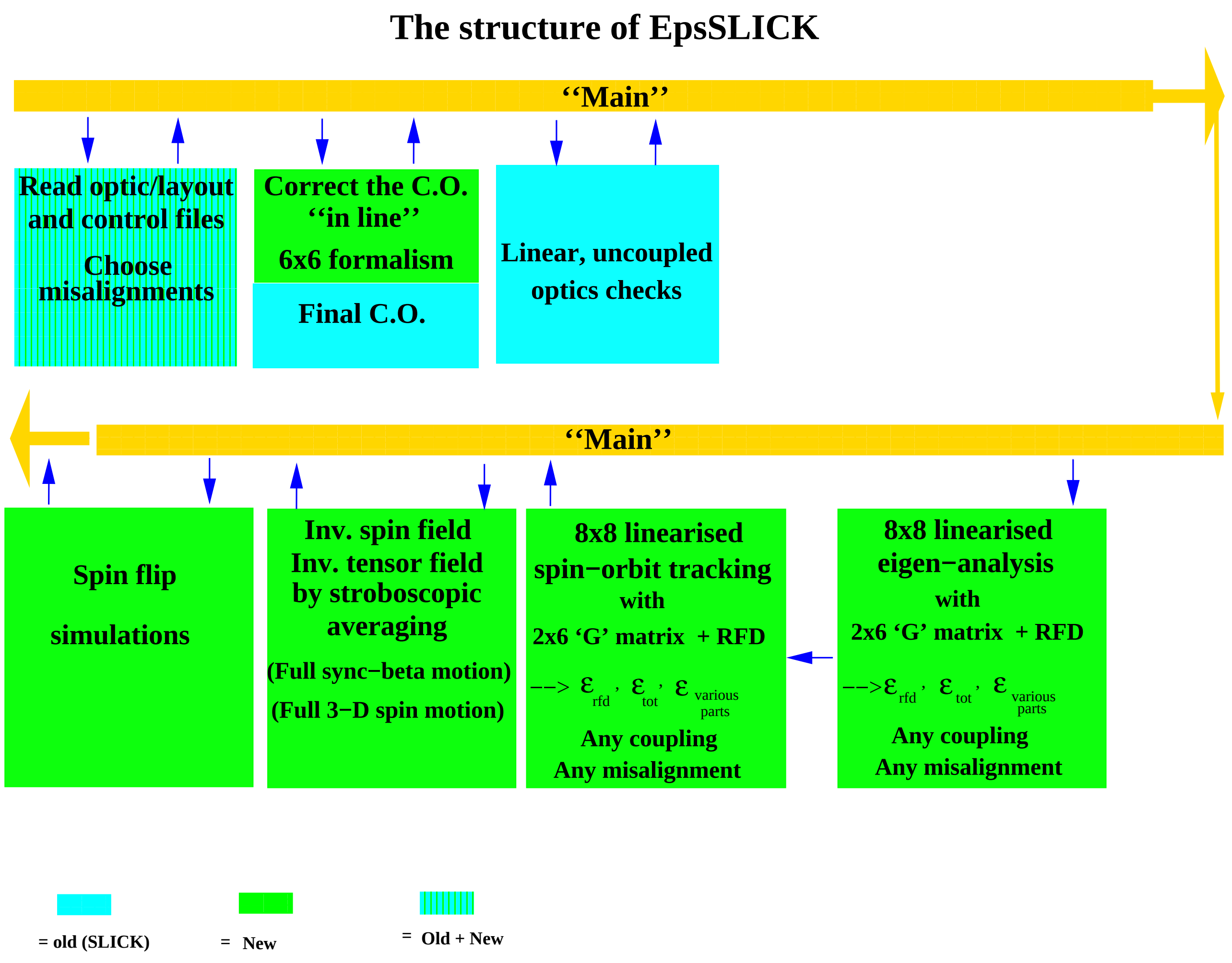
$$\epsilon_{\kappa}^{\mathrm{sb}} = \frac{1}{2\pi}(1 \quad i)\,\bar{\bar{G}}(C,0)\sqrt{2J_k}e^{-i\,\phi_k}\,\check{v}_k(0)$$

Or do long term tracking and averaging to get the Fourier integral.

Any ring geometry, any misalignment, any linear coupling.
Full 6-D orbital motion!

No need for obscene contortions!

Getting ϵ by tracking or eigenanalysis: including the (inhomogeneous) rf dipole with the matrix $G_{2\times 6}$

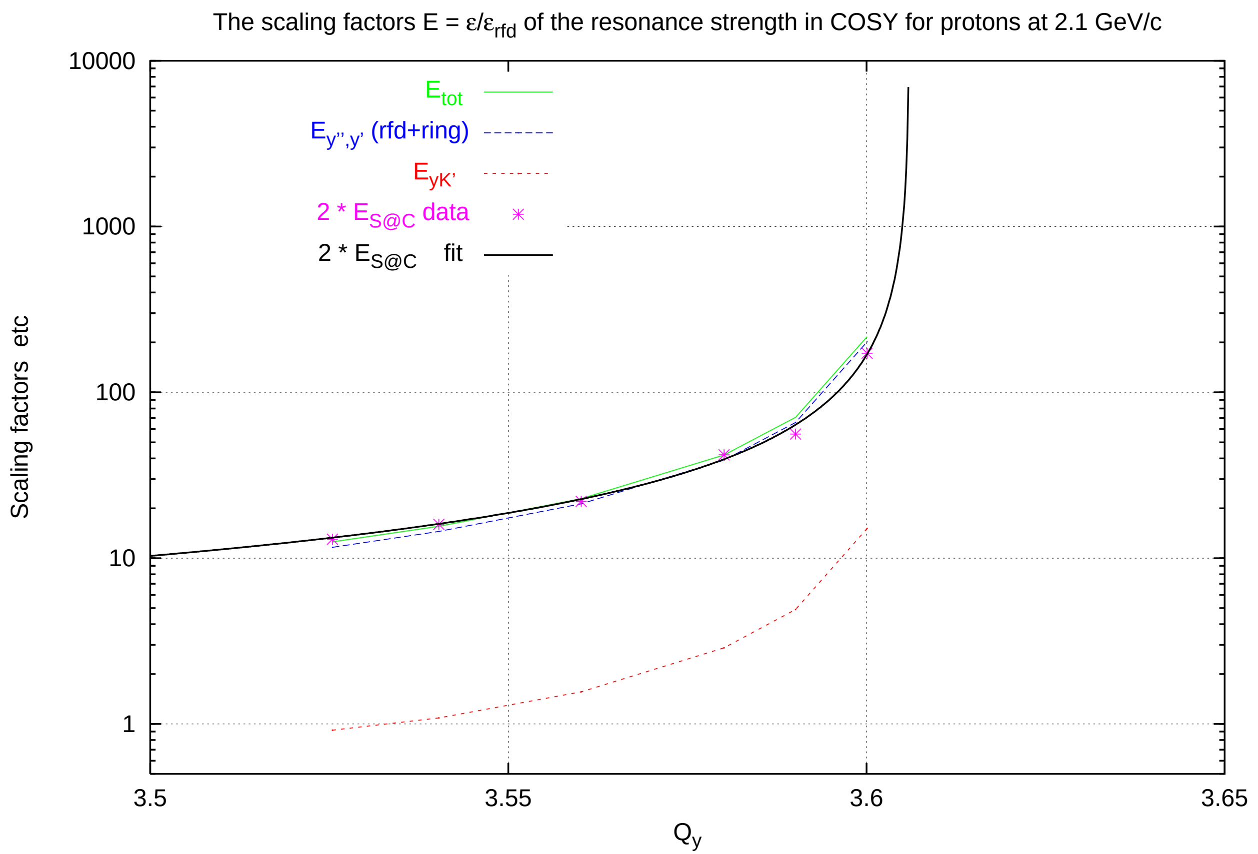


Diagnostics!

Switch spin-orbit coupling off/on to see what does what.

For example:
Check that $\epsilon^{\rm rfd}$ comes out correctly.
Study contributions from y'', y', y.
Getting the relative signs right.

Protons



The values for $E_{y''}(\text{rfd} + \text{ring})$ confirm earlier preliminary results (2006!) from A. Lehrach.

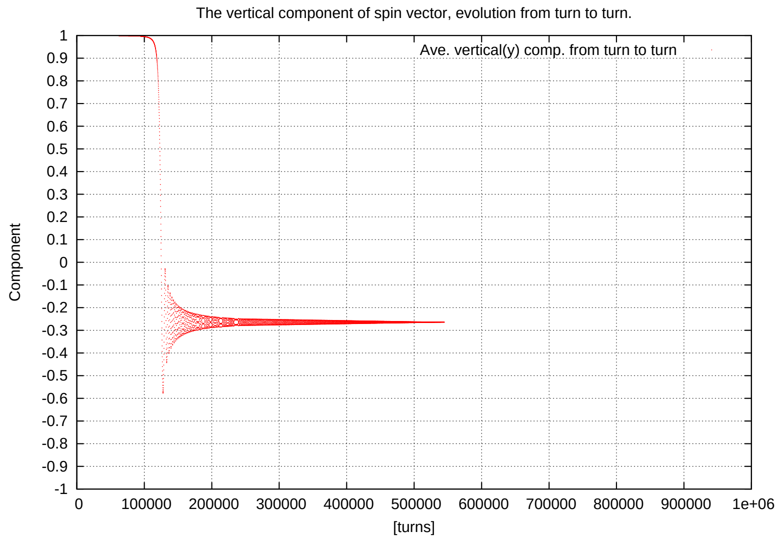
The values for $E_{y'',y'}(\text{rfd} + \text{ring})$ are confirmed by M. Vogt with the code SPRINT.

Nothing unexpected – just well known effects. Subject closed.

Spin flip simulations:

Extend the 8x8 formalism of basic SLICK to handle full 3-D spin motion. A perfect formalism for such problems.

± 4 kHz $\equiv \pm 57|\epsilon_{\kappa}^{\mathrm{rf}}|$. Spins initially parallel to ISF....i.e vertical.
Run down the RF amplitude at the end. $\Xi = 1 \Longrightarrow S_{n_0}^{\mathrm{final}} = -0.264$.



Now look properly

beyond the naive SRM:

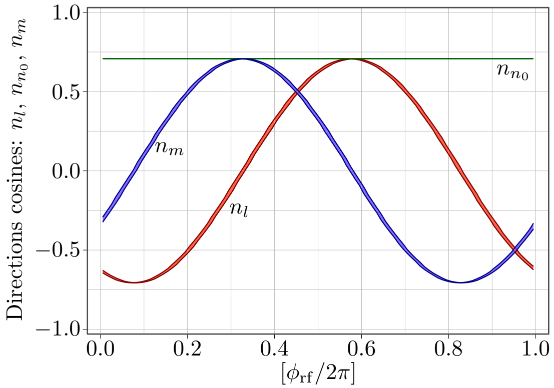
$$\hat{n}^{\rm rf}(\phi_{\rm rf}) = (\mathrm{sgn}\,\delta^{\rm rf})\,\frac{\left(\delta^{\rm rf}\,\hat{n}_0(s) + |\epsilon_\kappa^{\rm rf}|\,(\hat{l}(s)\cos\breve{\phi}_{\rm rf} + \hat{m}(s)\sin\breve{\phi}_{\rm rf})\right)}{\Lambda^{\rm rf}}\,,$$

where $\delta^{\rm rf} = \nu_0 - \kappa^{\rm rf}$, $\Lambda^{\rm rf} = \sqrt{(\delta^{\rm rf})^2 + |\epsilon_\kappa^{\rm rf}|^2}$ and $\breve{\phi}_{\rm rf} = \kappa^{\rm rf}\theta - \phi_\epsilon - \pi$ where ϕ_ϵ is the phase of $\epsilon_\kappa^{\rm rf}$.

$|\epsilon_\kappa^{\rm rf}| = 47.2\times 10^{-6}, \quad \kappa^{\rm rf} = 5 - Q_{\rm rf}, \quad Q_{\rm rf} \approx 0.605$

The $\hat{l}(s)$ and $\hat{m}(s)$ are chosen so that spins on the CO precess at the uniform rate ν_0 in the $(\hat{l}(s),\ \hat{n}_0(s),\ \hat{m}(s))$ frame — NOT using $(\hat{x},\ \hat{y},\ \hat{s}) \Longrightarrow$ any ring geometry.

Protons, just before the RFD. RFD + forced solution. $\delta^{\rm rf} = \pm|\epsilon_\kappa^{\rm rf}|$



Include large-amplitude pre-existing vertical betatron motion.
— off spin-orbit resonance.

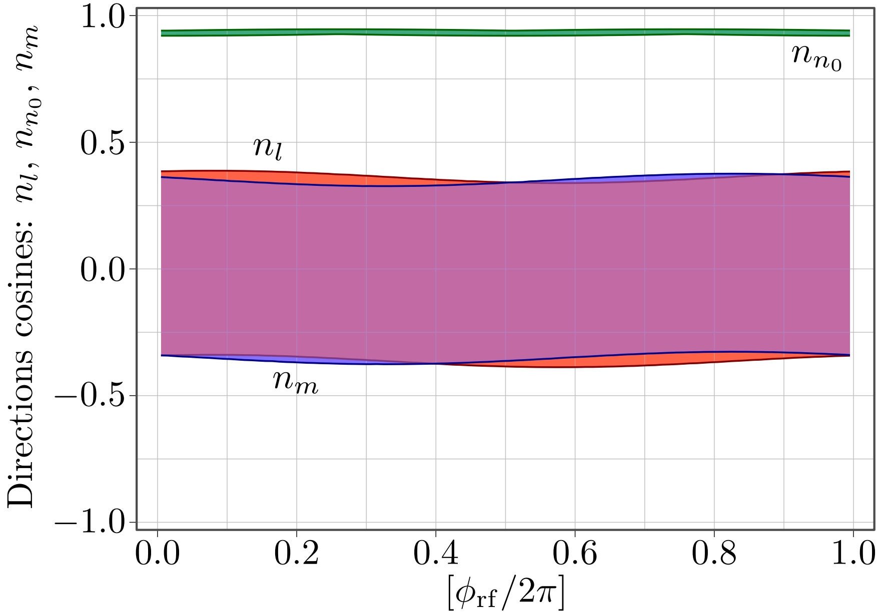
- No misalignments
- 2.1 GeV/c $\Longrightarrow \nu_0 = a\gamma_0 = 4.395$
- $Q_y = 3.580, \quad Q_x = 3.543$
- $\kappa^{\mathrm{v}} = 8 - Q_y \Longrightarrow \delta^{\mathrm{v}} = \nu_0 + Q_y - 8 = -0.02490...$
- Before cooling, the $3-\sigma$ vertical emittance for protons at 2.1 GeV/c is $2\pi - 4\pi$ mm.mrad.

  For 4π mm.mrad: $\beta_y^{\max} \approx 32$ metre $\Longrightarrow y_{\max} \approx 11$ mm.
- For this simulation, sit ON the 18π mm.mrad ellipse and imagine that the motion is still linear.

  Then $y_{\max} \approx 24$ mm:

  Relax! it's only a simulation!

Then:


Directions cosines: n_l, n_{n_0}, n_m
1.0
0.5
0.0
−0.5
−1.0
0.0
0.2
0.4
0.6
0.8
1.0
$[\phi_{\rm rf}/2\pi]$
n_{n_0}
n_l
n_m

What's happening?

The "ground state" has shifted!!! and so has the spin tune – and this is not just naive interference.

The uniform Invariant Frame Field

- $\hat{u}_1(z,s)$, $\hat{n}(z,s)$, $\hat{u}_2(z,s)$ with $\hat{u}_1(z,s+C)=\hat{u}_1(z,s)$ and $\hat{u}_2(z,s+C)=\hat{u}_2(z,s)$
- – a local coordinate system attached to each (z,s) to appraise spin motion as particles flow through phase space. No History. Generalises $\hat{l}(s), \hat{n}_0(s), \hat{m}(s)$
- $\hat{u}_1(z,s)$ and $\hat{u}_2(z,s)$ are chosen so that spins precess at a uniform rate.

The proper definition of spin tune and spin-orbit resonance

See "Quasiperiodic spin–orbit motion and spin tunes in storage rings"
Barber, Ellison and Heinemann, PRST-AB, **7**, 124002 (2004).

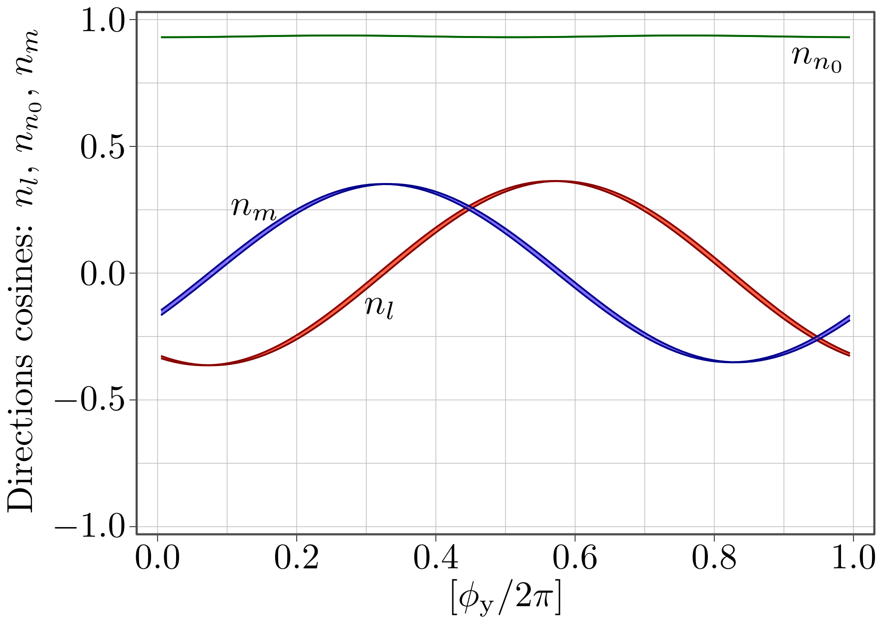
- An ADST $\nu(J)$ describes the rate of uniform spin precession around $\hat{n}$ in a u-IFF. $(\hat{u}_1(z,s),\ \hat{n}(z,s),\ \hat{u}_2(z,s))$. where J is the list of 3 amplitudes or, if an external RF field is applied, 4 amplitudes.
- The transform $\hat{u}_1 \pm i\hat{u}_2 \Longrightarrow (\hat{u}_1 \pm i\hat{u}_2)\exp\{(j_0 + j \cdot Q)\theta\}$ with $(j_0, j) \in \mathbb{Z} \times \mathbb{Z}^3$ causes the change $\nu(J) \Longrightarrow \pm\nu(J) + j_0 + j \cdot Q$
- This motivates the definition of an equivalence relation where ν_1 and ν_2 in $[0, 1)$ are said to be equivalent - and we write $\nu_1 \sim \nu_2$ - iff there exist $(\varepsilon, j_0, j) \in \{-1, 1\} \times \mathbb{Z} \times \mathbb{Z}^3$ such that $\nu_2 = \varepsilon\nu_1 + j_0 + j \cdot Q$.
- $\Longrightarrow$ The ADST is an equivalence class! – countably infinite in size.
- The system is on spin-orbit resonance if the equivalence class contains 0, i.e. if some $\nu(J) = j_0 + j \cdot Q$.
- The condition $\nu_0 = j_0 + j \cdot Q$ is NOT a spin-orbit resonance condition but the system will be near resonance if some $\nu(J) \approx \nu_0$.
- Systems tend to avoid exact resonance! – see the SRM
- The ADST does **not exist** on orbital resonance.

The preferred member of the equivalence class

- The equivalence class is countably infinite
- The member $\nu(J)$ which $\to \pm\nu_0$ as its u-IFF reduces smoothly to $(\hat{l}(s),\ \hat{n}_0(s),\ \hat{m}(s))$ – if such a member exists.
- $\Longrightarrow$ these $(\hat{u}_1(z,s),\ \hat{u}_2(z,s))$ are the generalisation of $(\hat{l}(s),\ \hat{m}(s))$
- The new reference spin tune is the preferred member of the ADST of the **new ground state**!!

The spin motion for the pre-existing vertical betatron motion?

Just before the RFD



This looks like SRM – expected from experience.

The SRM for vertical betatron motion

$$\hat{n}^{\rm v}(s,\phi_\kappa) = (\mathrm{sgn}\,\delta^{\rm v})\frac{\Big(\delta^{\rm v}\hat{n}_0(s) + |\epsilon_\kappa^{\rm v}|\,(\hat{l}(s)\cos\breve{\phi}_\kappa + \hat{m}(s)\sin\breve{\phi}_\kappa)\Big)}{\Lambda^{\rm v}}$$

where $\Lambda^{\rm v} = \sqrt{(\delta^{\rm v})^2 + |\epsilon_\kappa^{\rm v}|^2}$ with $\delta^{\rm v} = \nu_0 - \kappa^{\rm v}$ and $\breve{\phi}_\kappa = \theta\kappa^{\rm v} - \phi_\epsilon + \pi$ where ϕ_ϵ is the phase of $\epsilon_\kappa^{\rm v}$.

- With $\delta^{\rm v} = -0.02490$ and $\hat{n}_{n_0}^{\rm v} = 0.934$, $|\epsilon_\kappa^{\rm v}|$ should be 0.009582.
- This agrees exactly with the value from the Fourier integral
- $\hat{n}_{n_0}^{\rm v} = 1/\sqrt{2}$ when $\delta^{\rm v} = \pm|\epsilon_\kappa^{\rm v}|$ etc etc.

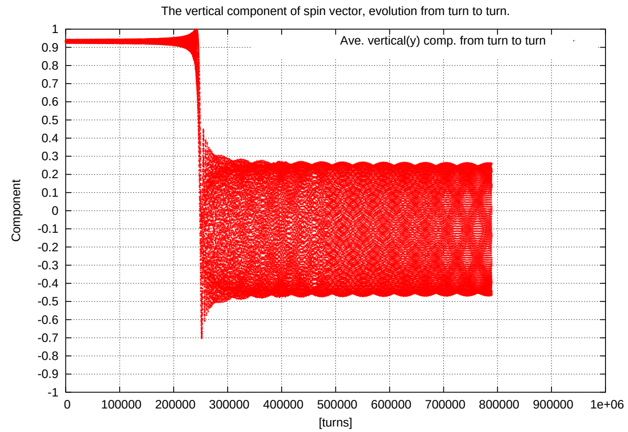
$\Longrightarrow$ spin motion for the pre-existing vertical betatron motion is well described by an SRM — expected!

The preferred member of the ADST for the SRM

The preferred member of the ADST for the SRM is

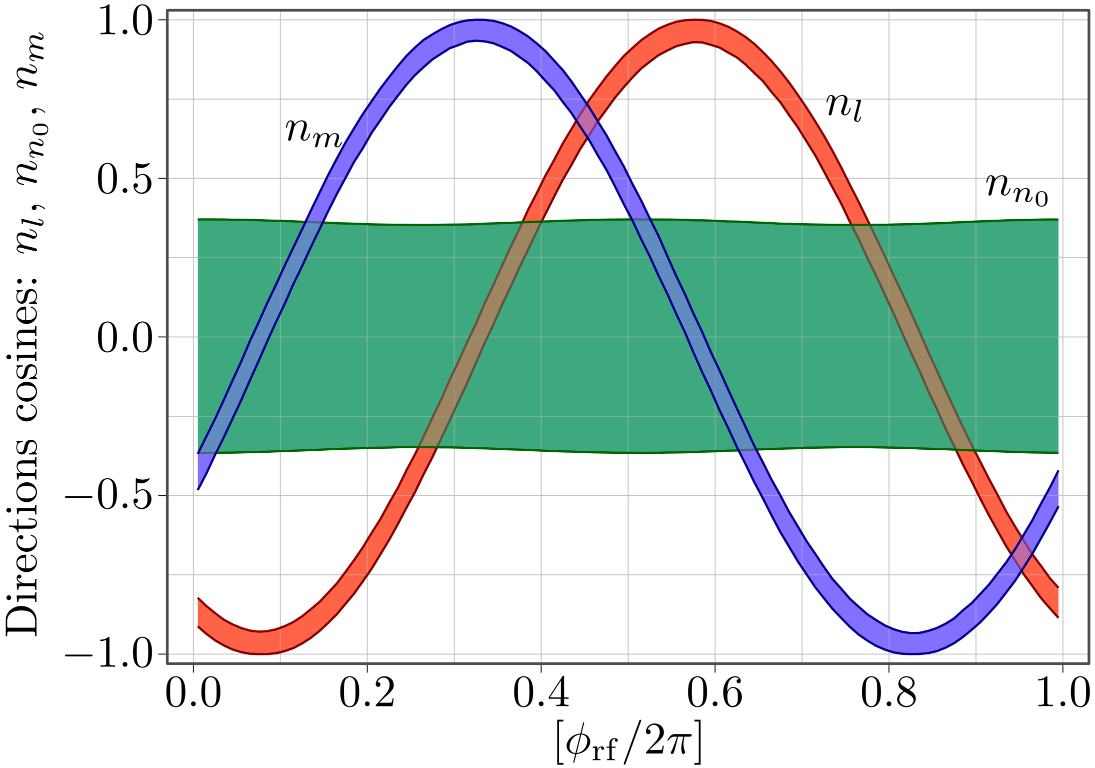
$\nu(J_y) = -\Lambda^{\rm v} + \kappa^{\rm v} \Longrightarrow \nu(J_y) - \nu_0 = -\Lambda^{\rm v} - \delta^{\rm v} = -0.001779...$ The shift is negative!

- The full $\nu_0 = 4.39498$ at 2.1 GeV/c
- The shift represents about 2.6 kHz in the original scan range of ± 4 kHz!
  $\Longrightarrow$ Chao's "matrix formalism" would not be too relevant.
- **Spectral analysis** of spin motion with **Theorem 9.1.c in BEH (2004)**:
  Lines at $j_\nu \nu(J_y) + j_0 + j_y Q_y : j_\nu \in \{0, 1, -1\}$ The line with $[\nu(J_y)] = 0.393201....$ is prominent
- The $Q_{\rm rf}$ at which spins flip in a Froissart-Stora scan is shifted **upwards** by 0.001779

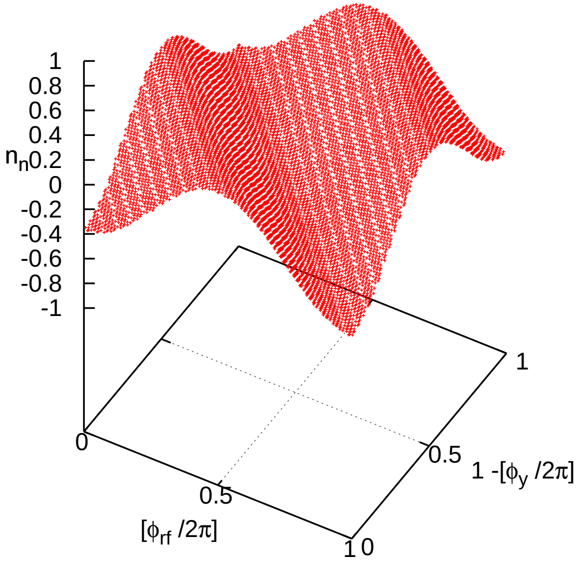


$\Xi = 1$ with $\vec{S}$ initially parallel to $\hat{n}^{\rm v}$: final average -0.11 instead of -0.264.

Everything is consistent, and with the RFD running such that $\tilde{\delta}^{\rm rf} := \nu(J_y) - \kappa^{\rm rf} = 0$ we get



The $\hat{n}_0$ component of the full ISF

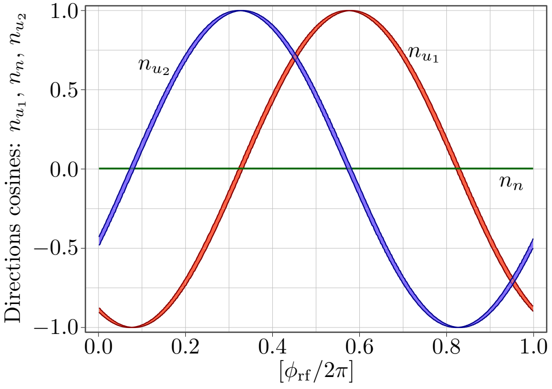


A **surface** so that the ISF is a single valued function of the phases $\phi_{\rm rf}$ and ϕ_y as required.

A very simple form, being very well parametrised by the function

$$\hat{n}_{n_0}(\phi_{\rm rf}, \phi_y) = (|\epsilon_\kappa^{\rm v}|/\Lambda^{\rm v})\ \cos\{[(\phi_{\rm rf} - \phi_y)/2\pi]\}$$

Now view within the u-IFF of the pre-existing vertical betatron motion



An SRM within an SRM!

$$\hat{n}^{\rm full}(\phi_{\rm v}, \phi_{\rm rf}) = ({\rm sgn}\, \tilde{\delta}^{\rm rf}) \frac{\left(\tilde{\delta}^{\rm rf}\hat{n}^{\rm v} + |\tilde{\epsilon}^{\rm rf}_{\kappa}|(\hat{u}^{\rm v}_1 \cos\breve{\phi}_{\rm rf} + \hat{u}^{\rm v}_2 \sin\breve{\phi}_{\rm rf})\right)}{\tilde{\Lambda}^{\rm rf}} \,,$$

$\tilde{\epsilon}^{\rm rf}_{\kappa}$ is the resonance strength in the u-IFF, and $\tilde{\Lambda}^{\rm rf} = \sqrt{(\tilde{\delta}^{\rm rf})^2 + |\tilde{\epsilon}^{\rm rf}_{\kappa}|^2}$ and $\tilde{\delta}^{\rm rf} = \nu(J) - \kappa^{\rm rf}$

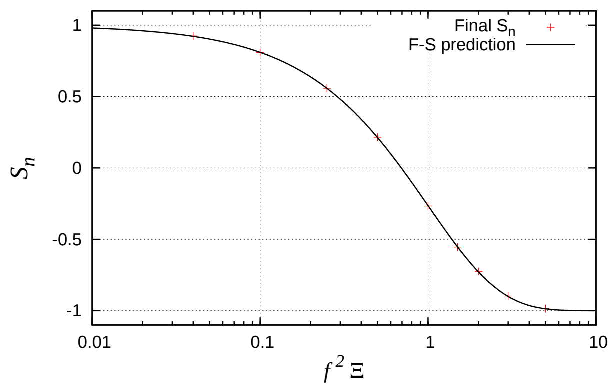
What is $|\tilde{\epsilon}^{\rm rf}_{\kappa}|$???

We naively expect:
$|\tilde{\epsilon}^{\rm rf}_{\kappa}| \leq |\epsilon^{\rm rf}_{\kappa}|$ since $\hat{n}^{\rm v}$ is tilted from the vertical while the perturbing fields are horizontal.

Three tests

- With $|\tilde{\delta}^{\rm rf}| = |\tilde{\epsilon}^{\rm rf}_\kappa| \equiv f|\epsilon^{\rm rf}_\kappa| = 0.901\,|\epsilon^{\rm rf}_\kappa| \longrightarrow \hat{n}^{\rm full}\cdot\hat{n}^{\rm v} = 1/\sqrt{2}$ etc etc
- The Froissart-Stora formula describes the level of adiabatic invariance of $\vec{S}\cdot\hat{n}$

  A scan with $\vec{S}$ initially parallel to $\hat{n}^{\rm v}$ and observing the final $\hat{n}^{\rm full}\cdot\hat{n}^{\rm v}$ i.e. a F-S scan within the u-IFF. For $f = 0.901$



  A scan with $f = 1.0$ gives a bad fit.

- With the RFD **included** at various $Q_{\rm rf}$ the "preferred member of the ADST" in the **u-IFF** (instead of in $(\hat{l},\ \hat{n}_0,\ \hat{m})$ frame) should be:

$$\nu^{\rm full} = ({\rm sgn}\,\tilde{\delta}^{\rm rf})\tilde{\Lambda}^{\rm rf} + \kappa^{\rm rf} \quad = \quad ({\rm sgn}\,\tilde{\delta}^{\rm rf})\tilde{\Lambda}^{\rm rf} + \nu(J_y) - \tilde{\delta}^{\rm rf}\,.$$

  Spectral analysis of spin motion including the RFD:

  With **Theorem 9.1.c in BEH (2004)**, the observed spectra require that $|\tilde{\epsilon}^{\rm rf}_\kappa| = 0.901\,|\epsilon^{\rm rf}_\kappa|$

Now some fun with spinors

The Pauli matrices and spinors (NOT wavefunctions!):

$$\sigma_l = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_{n_0} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma_m = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \vec{S} = \Psi^\dagger \vec{\sigma} \Psi$$

With two dominant harmonics in $\vec{\omega}(z,s) \cdot (\hat{l}(s) - i\hat{m}(s))$:

$$\frac{d\Psi}{d\theta} = -\frac{i}{2} \begin{pmatrix} -\nu_0 & -i(\epsilon_\kappa^{\rm v})^* \exp(i\kappa^{\rm v}\theta) - i(\epsilon_\kappa^{\rm rf})^* \exp(i\kappa^{\rm rf}\theta) \\ i\epsilon_\kappa^{\rm v} \exp(-i\kappa^{\rm v}\theta) + i\epsilon_\kappa^{\rm rf} \exp(-i\kappa^{\rm rf}\theta) & \nu_0 \end{pmatrix} \Psi$$

Then with a sequence of unitary transformations into the u-IFF (Maple):

$$\frac{d\Psi_{\rm uiff}}{d\theta} = -\frac{i}{2}(\mathfrak{M}_{\rm res} + \mathfrak{D} + \mathfrak{N})\Psi_{\rm uiff} \,,$$

$$\mathfrak{M}_{\rm res} = \begin{pmatrix} -\nu(J_y) & -i(\mathcal{E}_\kappa^{\rm rf})^* \exp(i\kappa^{\rm rf}\theta) \\ i\mathcal{E}_\kappa^{\rm rf} \exp(-i\kappa^{\rm rf}\theta) & \nu(J_y) \end{pmatrix},$$

with $(\mathfrak{D}, \mathfrak{N})$ oscillating but far off resonance and with $(\mathfrak{D}, \mathfrak{N}) \to 0$ as the component $\hat{n}^{\rm v}_{n_0} \to 1$

$$|\mathcal{E}_\kappa^{\rm rf}| = \frac{(1 + \hat{n}^{\rm v}_{n_0})}{2} |\epsilon_\kappa^{\rm rf}|(\tilde{\delta}=0) \;=\; 0.967 \times |\epsilon_\kappa^{\rm rf}|(\tilde{\delta}=0) \text{ for } \hat{n}^{\rm v}_{n_0} = 0.934$$

BUT, $|\epsilon_\kappa^{\rm rf}|(\tilde{\delta}=0) = 0.930 \times |\epsilon_\kappa^{\rm rf}|(\delta=0) \implies |\tilde{\epsilon}_\kappa^{\rm rf}| = |\mathcal{E}_\kappa^{\rm rf}| = 0.899 \times |\epsilon_\kappa^{\rm rf}|(\delta=0) \implies f = 0.899$!!!

...and some serious stuff with COSY

- If spin motion for pre-existing vertical betatron motion is described by an SRM and $|\epsilon_\kappa^{\rm v}|$ is known, we know the preferred member of the ADST.
- At fixed $|\epsilon_\kappa^{\rm v}|$: $|\nu(J_y) - \nu_0| = |-\Lambda^{\rm v} - \delta^{\rm v}|$ **increases** as $|\delta^{\rm v}|$ **decreases**.
- Measurements of resonance strengths for protons were made at $Q_y = 3.6 \Longrightarrow \delta^{\rm v} = -0.0050$.
- Then if $|\epsilon_\kappa^{\rm v}|$ is insensitive to Q_y, $\nu(J_y) - \nu_0 = -0.00158$ ON the ellipse with 3.6 mm.mrad. $\Longrightarrow y^{\rm max} \approx 10.7$ mm.
- Two parameters: the preferred member of the ADST for the pre-existing vertical betatron motion $\nu(J)$ and the attenuation f.

Suggestion

- Run at $Q_y = 3.6$ and 2.1 GeV/c with polarised protons.
- Cool the beam to get a $3-\sigma$ vertical emittance of ≈ 0.3 mm.mrad.
- Use a vertical kicker to put the beam onto ellipses in the range $0 - 3.6$ mm.mrad.
- Use the field of the RF solenoid as a **probe** to measure $\nu(J_y) - \nu_0$ by measuring the RF tune needed to get zero time averaged vertical polarisation in each case and check the dependence of $\nu(J_y) - \nu_0$ on J_y against the expectation.
- Or run at $Q_y = 3.58$ and $a\gamma_0 = 4.4148....$

Why?

(1) Because it's never been done.

(2) Because the literature on spin tune shifts contains too much mumbo-jumbo.

(3) Because COSY is perfectly placed and has a very experienced crew.

(4) Because people wanting to measure beam energies very precisely might want to have a better understanding of the systematics. See (2).

(5) Etc.....

Questions:

- Is it feasible to put the beam on the ellipse? See DESY 09-15 – in preparation.
- Can the energy spread be kept low enough? Orbit length?
- How long does the beam stay on the ellipse (non-linear motion, scattering in the polarimeter, IBS, space charge, image forces....)?
- How can the phase space distribution be measured for checking that the particles are on/off the ellipse?
- If it's only a short time, is it feasible to set up an injection/measurement routine that can deliver enough precision?
- Etc.

$\Longrightarrow$ Homework for the COSY crew.

In simple rings the shifts might be bigger at higher energies – we need large $|\epsilon_{\kappa}^{\mathrm{v}}|$

Summary

- Going beyond the common perception that non-spin-resonant background vertical betatron motion is not too interesting.
- With appropriate computational tools:
  - Stroboscopic averaging to get the ISF in general situations without models,
  - Efficient spin-orbit tracking
  - Spectral analysis
- and clearly defined mathematical concepts with theorems (rigour). E.g. the proper definition of the ADST based on conventional mathematics!

$\Longrightarrow$ **Efficient analysis.**

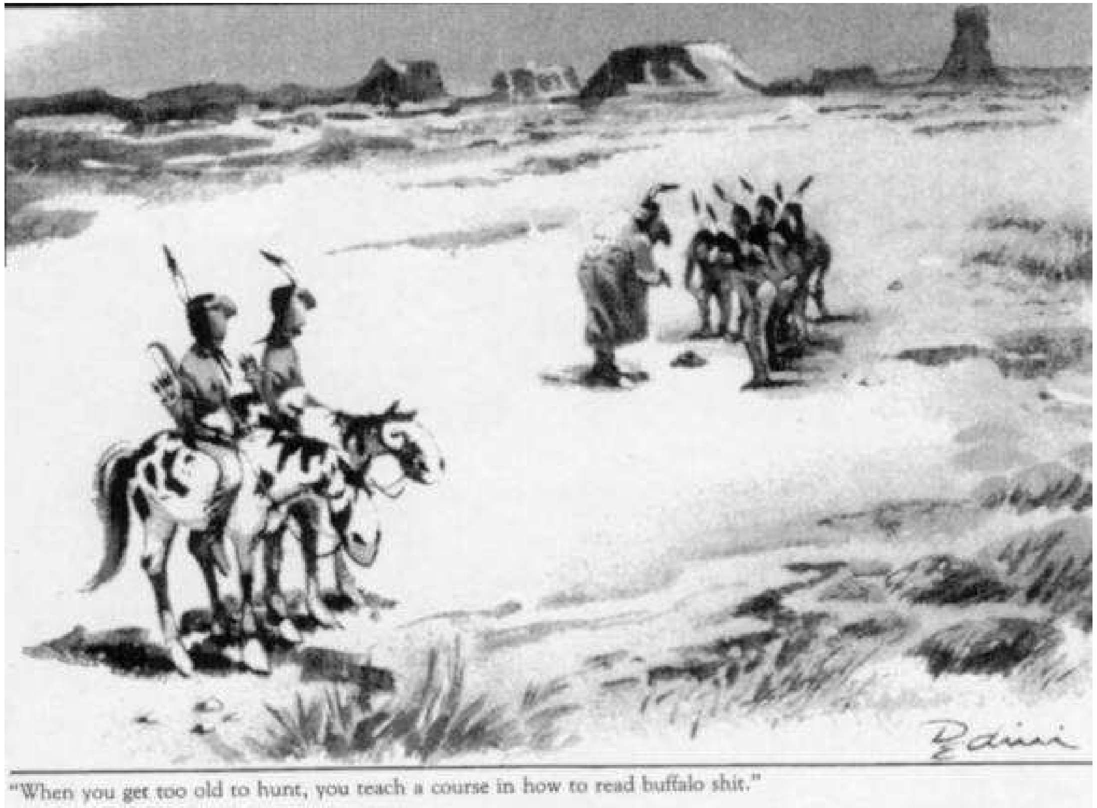
- The resonance strength for the RFD **does** depend on the coordinate system

AND

$\Longrightarrow$ A suggestion for a test using COSY, a perfect facility with its experts.

$\Longrightarrow$ A chance to clean up this business of spin tune shifts.
Confusions, ignorance and laziness about such shifts in the literature.

More information in DESY 09-15 – in preparation.



When you get too old to hunt, you teach a course on how to read buffalo shit.

(Eldon Dedini: The Tracker Magazine 1985.)