Spin motion near snake "resonances"

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Or:

(Almost) everything you wanted to know about snake glitches but nobody bothered to calculate.

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Background knowledge required:

- The Invariant Spin Field (ISF)
- The Amplitude Dependent Spin Tune (ADST)
- The Single Resonance Model.
- The Froissart–Stora formula for polarisation survival when crossing resonances.
- Siberian Snakes.

The ISF $\hat{n}(\vec{z};s)$:

A 3–vector field of unit length obeying the T–BMT equation along particle orbits $(\vec{z}(s);s)$ and fulfilling the periodicity condition

$$\hat{n}(\vec{z};s+C)=\hat{n}(\vec{z};s)$$

where C is the circumference.

Thus

$$\hat{n}(\vec{M}(\vec{z};s);s+C)=\hat{n}(\vec{M}(\vec{z};s);s)=R_{3\times3}(\vec{z};s)\hat{n}(\vec{z};s)$$

where $\vec{M}(\vec{z};s)$ is the new phase space vector after one turn starting at $\vec{z}$ and s and $R_{3\times3}(\vec{z};s)$ is the corresponding spin transfer matrix.

Also have the amplitude dependent spin tune (ADST).

If an ISF exists:

- The scalar product

$$J_{\rm s} = \vec{S} \cdot \hat{n}(\vec{z}; s, \Xi)$$

is invariant along an orbit for fixed parameters Ξ

- The scalar product

$$J_{\rm s} = \vec{S} \cdot \hat{n}(\vec{z}; s, \Xi)$$

can be invariant as parameters Ξ are varied slowly (==> Adiabatic Invariance)
Hoffstaetter, Dumas and Ellison, Phys. Rev. ST Accel. Beams 9, 014001 (2006)

The ISF acts as a template guiding spin motion.

If an ISF exists:

- It specifies the equilibrium spin state on a torus
- Its phase average give the maximum equilibrium polarisation $P_{\text{lim}} = |\langle \hat{n} \rangle|$ or $\vec{P}_{\text{lim}} = \langle \hat{n} \rangle$
- Away from orbital resonances $\langle \hat{n} \rangle$ is also the maximum time averaged polarisation
- At orbital resonance, $\hat{n}$ might not be unique ==> an indefinite number of permissible spin equilibria

Spin–orbit resonances

$$\nu_{\text{spin}} = k + k_1\nu_1 + k_2\nu_2 + k_3\nu_3$$

ν_{spin}: Amplitude dependent spin tune

$\approx$ closed orbit spin tune

$=$ precessions/turn on CO

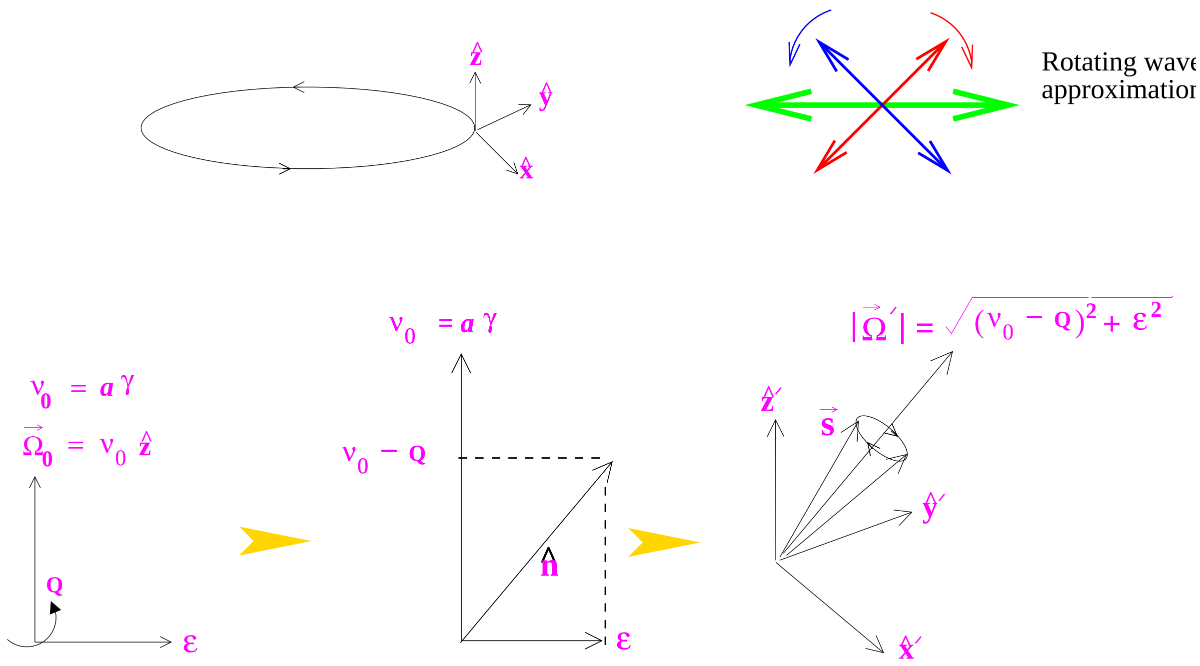
INDEPENDENT OF ORBITAL PHASES!

The parameters of Single Resonance Model (SRM):

- Resonance strength: ϵ
- Vertical betatron tune $Q_2 (= \nu_y)$
- Distance to (parent) resonance: $\delta = a\gamma - Q_2$ with $a = (g-2)/2$
- $\lambda = \sqrt{\delta^2 + \epsilon^2}$

Single resonance model

The elements of the ‘‘single resonance model’’ of spin motion



$$\hat{n}(\phi_2) = \pm\left(\delta\hat{e}_2 + \epsilon(\hat{e}_1\cos\phi_2 + \hat{e}_3\sin\phi_2)\right)/\lambda$$

Acceleration through spin–orbit resonances

In the SRM the loss of polarisation when accelerating initially vertical spins through $\delta = 0$ is given by the Froissart–Stora formula.

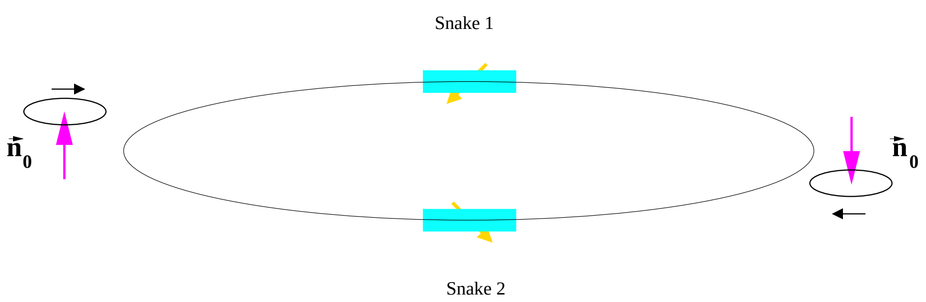
The F–S formula simply describes the dependence of the degree of invariance of $J_{\rm s}$ on the rate of acceleration and the resonance strength!

By accelerating at a low enough rate, the VALUE of the polarisation is preserved but there is full spin flip. E.g. Partial snakes at the AGS.

To avoid resonance crossing (e.g., RHIC):

Siberian snakes for fixing the spin tune at 1/2 on the design orbit

Example with two snakes:



Energy independent spin tune ν_0 = 1/2 on the design orbit:

Snake axes 90 degree apart

For irrational Q_2 the ADST exists and is in the equivalence class of $1/2$

==> low order s–o resonances are avoided

==> polarisation is preserved, e.g. RHIC!

Except close to some special Q_2:

For perfect alignment: $Q_2 = 1/6,\ 5/6,\ 1/10,\ 3/10,\ 7/10,\ 9/10,\ 3/14......$

Pioneering work by:

S.Y. Lee and S. Tepikian, Phys. Rev. Letts., **56** 1653 (1986).

S.Y. Lee, *"Spin Dynamics and Snakes in Synchrotrons"*, World Scientific (1997).

Explanations (?) in terms of "perturbed spin tune" and a perturbative heirarchy of terms in an expansion of the multiturn spin map.

Simulations for RHIC by A. Luccio: BNL Technical Report BNL-52481, (1995).

Observations at RHIC: SPIN2002, SPIN2004, SPIN2006....

What is *really* going on?

The phenomenon is usually called “snake resonance”

BUT exactly at rational tunes there is no common spin precession frequency!! So, e.g.,

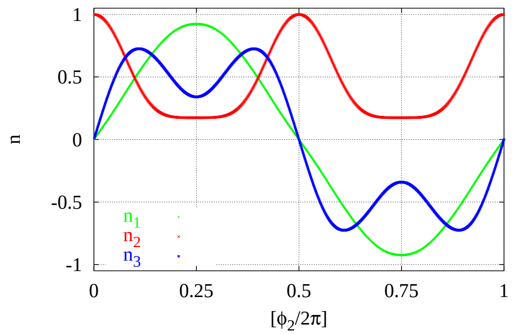
$$\nu_0 = \frac{1}{2} = k_0 + 3Q_2 \quad \text{at} \quad ([Q_2] = \frac{1}{6})$$

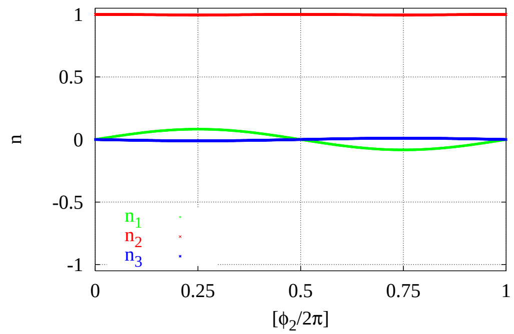
is not s–o resonance condition: at small δ and significant ϵ the 6–turn eigentune depends significantly on the betatron phase.

Look at other cases ==>

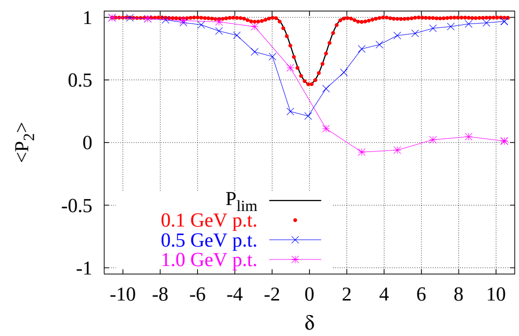
Example: $[Q_2] = \sqrt{5} - 2 = 0.23606797..., \nu_{\rm spin}(\epsilon, \delta) = 1/2$

==> Off s–o resonance, no resonance crossing





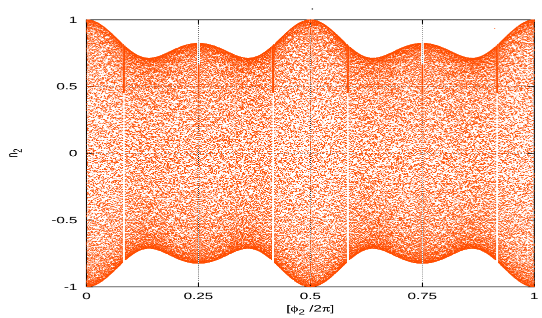
$\delta = 0, \quad \epsilon = 0.4, \quad P_{\rm lim} \approx 0.47 \qquad \delta = 10.6, \quad \epsilon = 0.4, \quad P_{\rm lim} \approx 1.0$

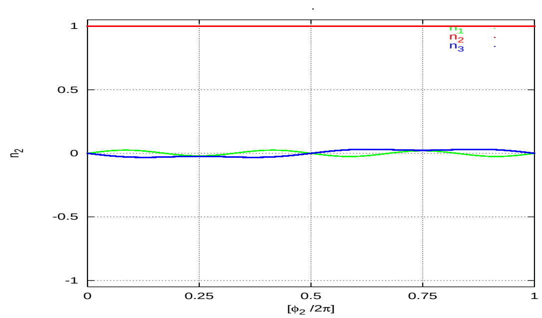


Just before the 0° snake

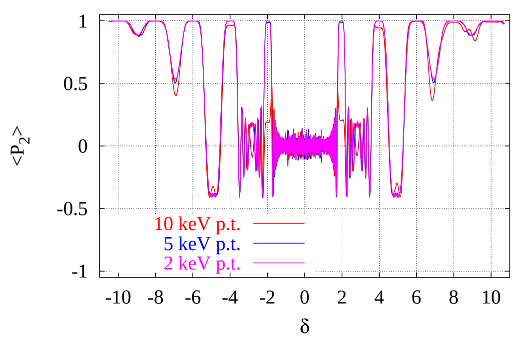
Example: $[Q_2] = 0.16667586.....,\ \nu_{\text{spin}}(\epsilon, \delta) = 1/2$

==> Near to the resonance $\nu_{\text{spin}}(\epsilon, \delta) = [3Q_2]$





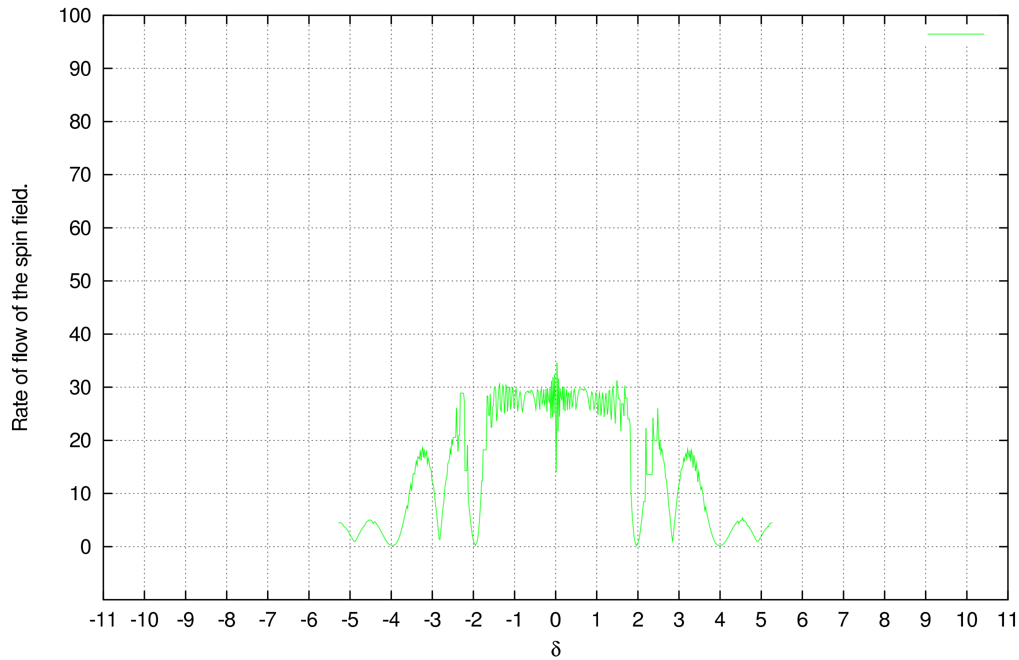
$\delta = 0, \quad \epsilon = 0.4, \quad P_{\text{lim}} \approx 0.0$ $\qquad \delta = 15.6, \quad \epsilon = 0.4, \quad P_{\text{lim}} \approx 1.0$



Just before the 0° snake

The δ–flow: $f_\delta = \langle |\partial \hat{n}/\partial \delta| \rangle$ **for** $[Q_2] = 0.16667586...$

Just before the 0° snake

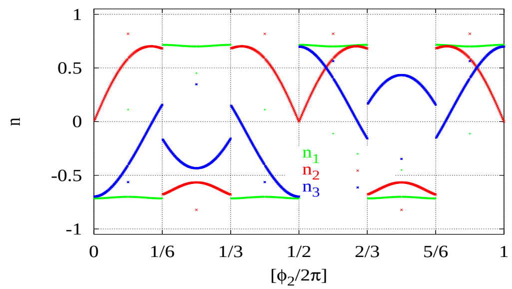


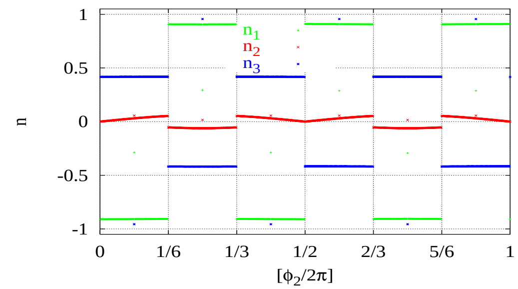
Measures of complexity:
coverage of the unit 2–sphere or the highest (significant) Fourier harmonic.

For the pure SRM: $f_\delta = \frac{\epsilon}{\epsilon^2+\delta^2}$

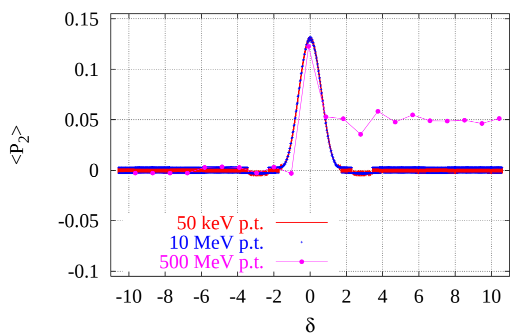
The case: $[Q_2] = 1/6$ is special, $\nu_{\rm spin}(\epsilon, \delta)$ does not exist

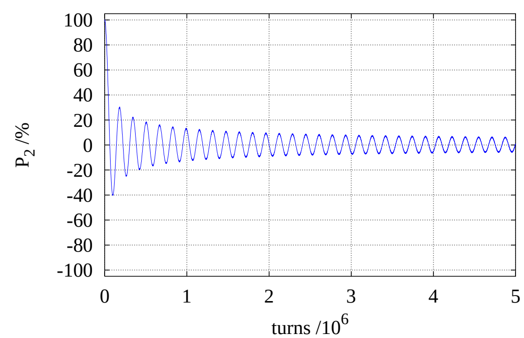
==> Off s–o resonance, no resonance crossing





$\delta = 0, \quad \epsilon = 0.4, \quad P_{\rm lim} \approx 0.13$ $\delta = 10.6, \quad \epsilon = 0.4, \quad P_{\rm lim} \approx 0.0$





$\delta = -10.6$

Just before the 0° snake

Summary I:

Within this model for $[Q_2]$ near and irrational, but not at, $1/6$

- We have a clean picture, in terms of the ISF, for why the polarisation of an initially vertically polarised beam can become depolarised for conventional acceleration rates.
- We see that the polarisation can be preserved at sufficiently low acceleration rates if one begins with the spins either initially aligned along the ISF or vertical while far enough below the parent resonance, i.e., at large enough $|\delta|$.
- Apart from the complexity of the ISF, there is nothing special about these $[Q_2]$.

Summary II:

Within this model for $[Q_2] = 1/6$

- This case is special. The ISF is close to horizontal at large $|\delta|$ and then at large $|\delta|$ the polarisation of an initially vertically polarised beam will be lost even at fixed beam energy since vertical polarisation does not correspond to a permissible equilibrium state and the 6–turn spin phase advances depend of orbital phase.

Comments:

- We have various mathematical indications for why the ISF is so complicated near to, but not at, $[Q_2] = \ 1/6,\ 5/6,\ 1/10,\ 3/10,\ 7/10,\ 9/10,\ 3/14......$, and irreducibly discontinuous at $[Q_2] = 1/6,\ 5/6,\ 1/10,\ 3/10,\ 7/10,\ 9/10,\ 3/14......$ but a physical picture would also be welcome.
- Nevertheless this is a good example of how to use the ISF for insights, guidance, orientation and for systematising the spin motion.
- Extend to larger ϵ
- Look again at real rings