Electron/positron polarization in rings

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.

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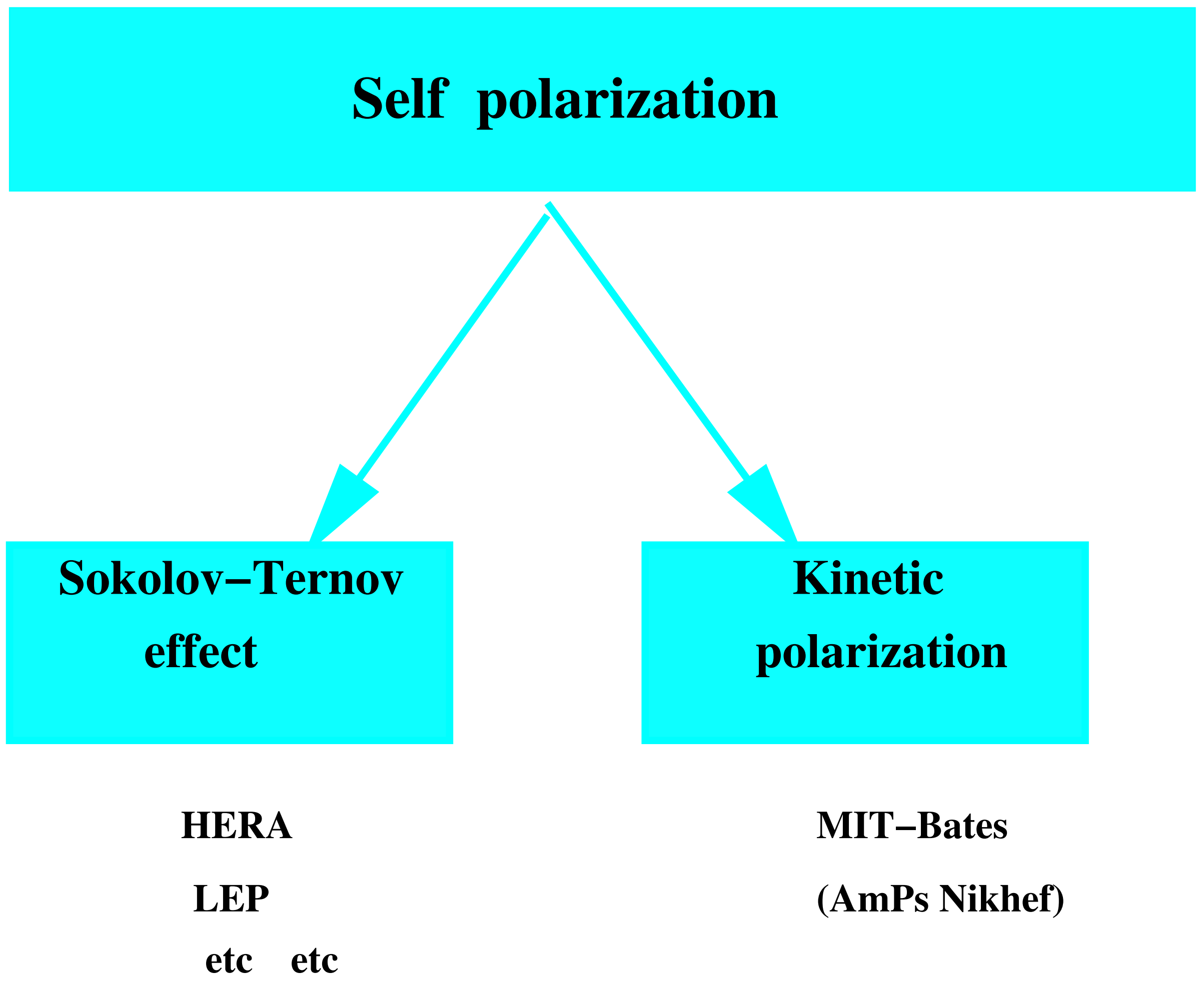
Hamburg Germany

6 July 2001


Self polarization
Sokolov–Ternov effect
Kinetic polarization
HERA
LEP
etc etc
MIT–Bates
(AmPs Nikhef)

Self polarization / depolarization.

- Electrons in storage rings can become spin POLARIZED due to emission of synchrotron radiation:
  Sokolov–Ternov effect (1964).

- The polarization is perpendicular to the machine plane.

- The maximum value is $P_{st} = 92.4\%$.

- Sync. radn. also excites orbit motion. So sync.radn. **also** causes spin diffusion i.e. DEPOLARIZATION!!!!! Baier + Orlov (1966), BINP, Chao, Yokoya, DESY

- For longitudinal polarization the polarization vector must be rotated into the longitudinal direction before an IP and back to the vertical afterwards ===> spin rotators.

- Depolarization can be very strong if the polarization vector is horizontal in parts of the ring or if vertical dispersion generates vertical emittance.

Cont./

- So with rotators the depolarization can be very dangerous.
- However this source of depolarization can in principle be combatted by a special choice/adjustment of the **optic** called ‘spin matching’.
- But in principle nasty higher order effects remain.......
- In spite of these problems longitudinal polarization **has** been achieved at high energy, namely at HERA. Spin matching works.
- Use the HERA experience and that at LEP for future high energy rings.

Sokolov–Ternov effect: flat ring, no sols.

- $$W_{\uparrow\downarrow/\downarrow\uparrow} \;=\; \frac{5\sqrt{3}}{16}\frac{e^2\gamma^5\hbar}{m_e^2c^2|\rho|^3}\left(1 \pm \frac{8}{5\sqrt{3}}\right)$$
- $P_{st} = \frac{8}{5\sqrt{3}}$
- $\frac{\text{flip power}}{\text{non–flip power}} \approx (\frac{E_c}{E})^2$ –very small!
- Time constant: $\tau_{st}^{-1} = \frac{5\sqrt{3}}{8}\frac{e^2\gamma^5\hbar}{m_e^2c^2|\rho|^3}$ –small!

Homogeneous fields, Dirac wave functions, simple pert. theory.

But in real rings: inhomogeneous fields, various directions ====>

The T-BMT equation.

$$\frac{d\vec{S}}{ds} = \vec{\Omega}(\gamma, \vec{v}, \vec{B}, \vec{E}) \times \vec{S}$$

In transverse magnetic fields:

$$\delta\theta_{spin} = a\gamma \cdot \delta\theta_{orbit}$$

$a = (g-2)/2$ where g is the electron g factor.

At 27.5 GeV $a\gamma = 62.5$ (HERA)
At 185. GeV $a\gamma = 420$ (VLLC)

Spin motions

- Protons: largely deterministic — unless IBS.
- Electrons/positrons:
  If a photon causes a spin flip, what are the other $\approx 10^{10}$ photons doing? ===>

  Stochastic/damped orbital motion due to synchrotron radiation
  + inhomogeneous fields
  + T–BMT
  ===> spin diffusion i.e. depolarization!!!

  Balance of poln. and depoln. ===>

$$P_\infty \approx P_{BK} \frac{1}{1 \; + \; (\frac{\tau_{dep}}{\tau_{BK}})^{-1}} \qquad (P_{ST} \to P_{BK})$$

$$\tau_{dep}^{-1} \;\propto\; \gamma^{2N}\; \tau_{st}^{-1} \quad \text{(polynomial in } \gamma\text{)}$$

===> Trouble at high energy!

Periodic solution $\hat{n}_0$ on closed orbit.

The real unit eigenvector of:

$$R_{3\times 3}(s+C,s)\vec{S} = \vec{S}$$

$\hat{n}_0$ is 1–turn periodic: $\hat{n}_0(s+C) = \hat{n}_0(s)$

$\hat{n}_0$: direction of measured equilibrium radiative polarization.

The **value** of the polarization is the same at all azimuths — time scales.

$$P_{BK} \propto \oint \frac{\hat{B}\cdot\hat{n}_0 ds}{|\rho|^3}$$

If $\hat{n}_0$ is horizontal $P_{BK} = 0$

If $\hat{n}_0$ is evenly up/down $P_{BK} = 0$

Spin–orbit resonances

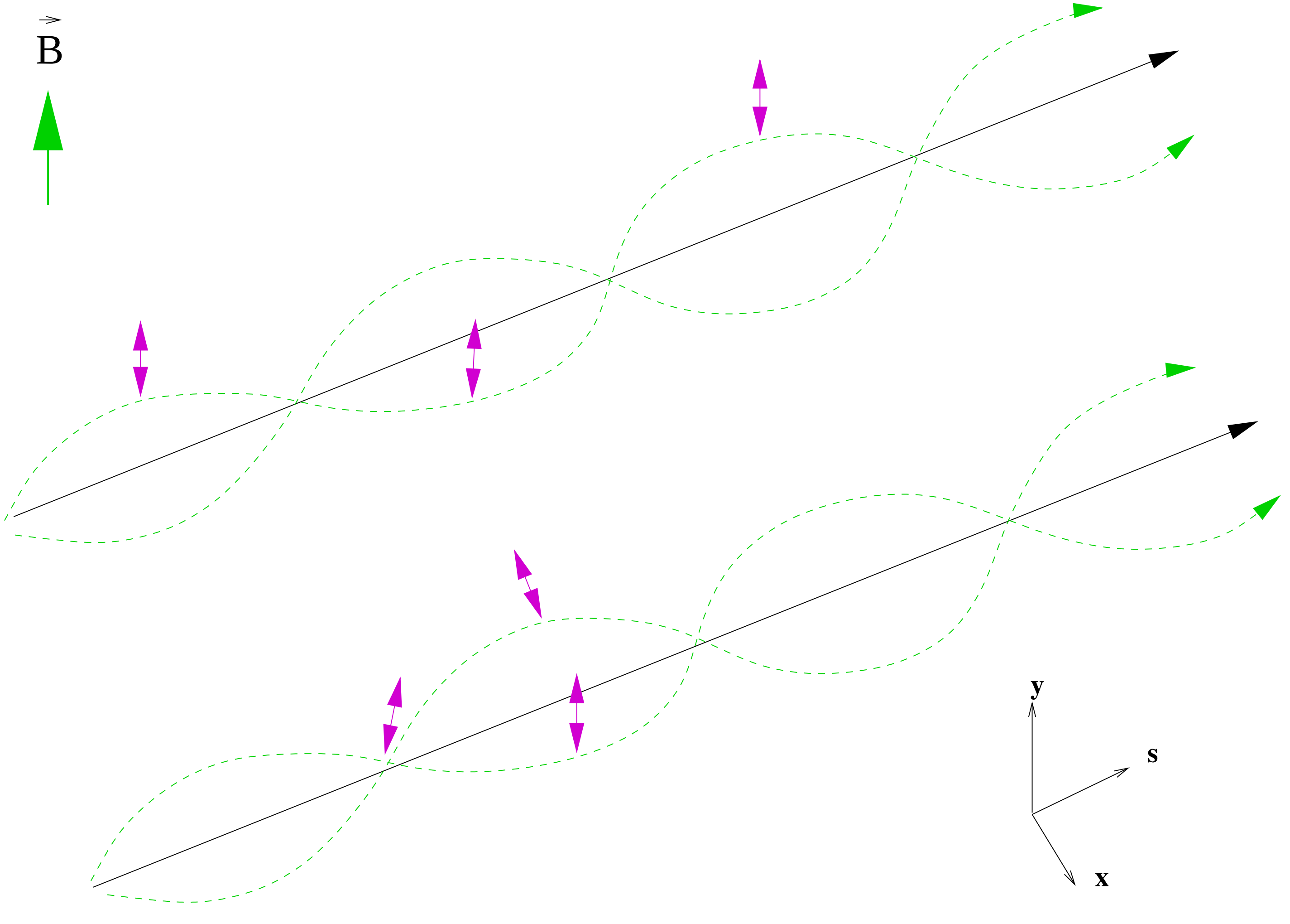
$$\nu_{\rm spin} \;=\; k + k_I\nu_I + k_{II}\nu_{II} + k_{III}\nu_{III}$$

$\nu_{\rm spin}$: amplitude dependent spin tune ≈ closed orbit spin tune = precessions /turn on CO

- Orbit “drives spins” ===> Resonant enhancement of spin diffusion.
- Resonance order: $|k_I| + |k_{II}| + |k_{III}|$
- First order: $|k_I| + |k_{II}| + |k_{III}| = 1$ e.g. SLIM like formalisms.
- Strongest beyond first order:
  synchrotron sidebands of first order parent betatron or synchrotron resonances

$$\nu_{\rm spin} \;=\; k + k_i\nu_i + k_{III}\nu_{III}, \quad i = I, II \text{ or } III$$

EXAMPLE: Depolarization in arcs if $\vec{n}_0$ (s) not vertical



Main sources of depolarization

- Vertical misalignments ===> tilted $\hat{n}_0$ in arcs ===>
  In lowest order:

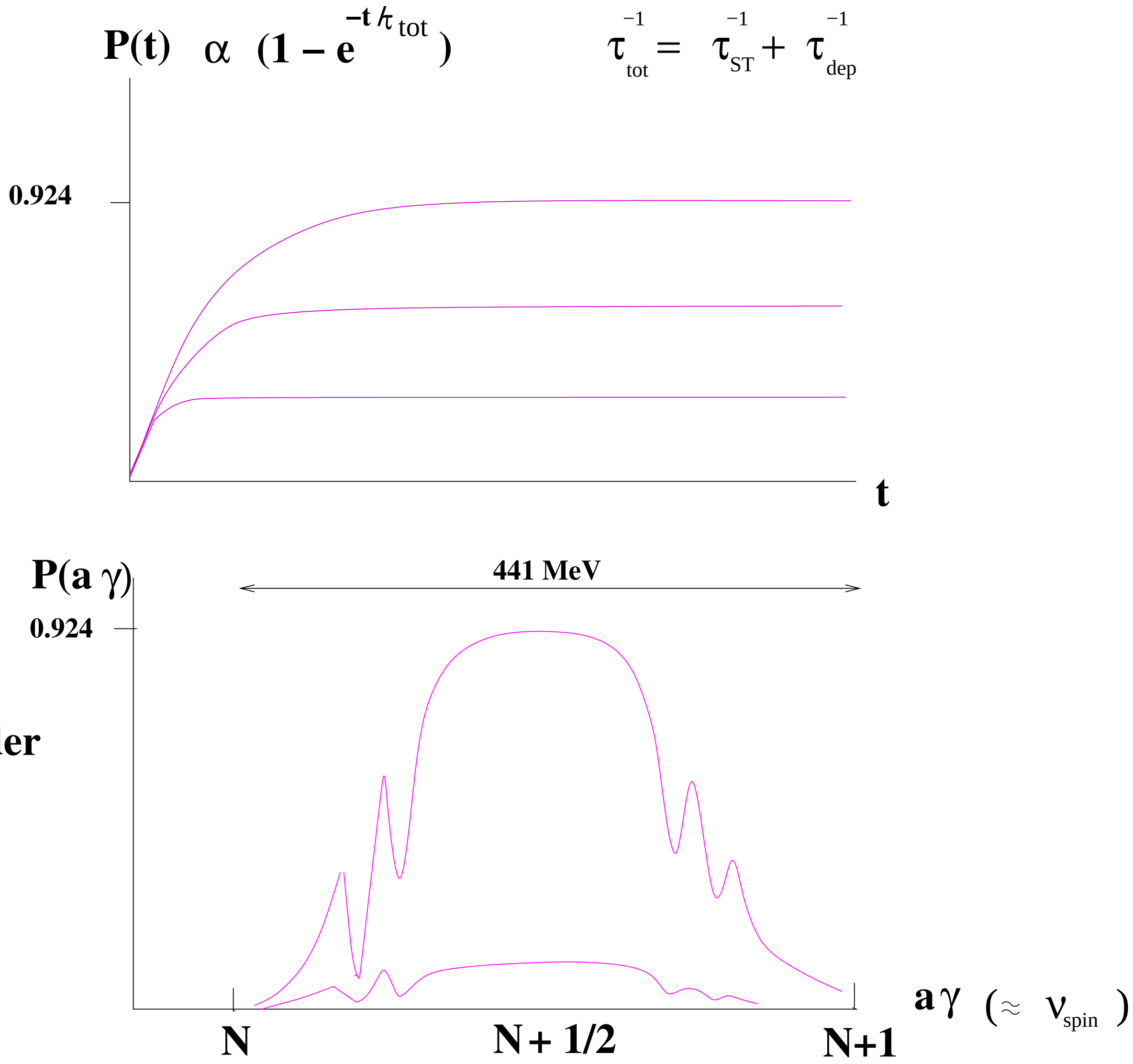
$$P \approx \frac{P_{BK}}{1 + (A\ E^2)(B\ E^2)}$$

  – $A\ E^2$: $(\text{tilt})^2$ of $\hat{n}_0$

  – $B\ E^2$: $(a\gamma)^2$ coupling of spin to sync–beta motion.

  Potential very strong effect:
  at HERA a tilt of 50 mrad ===> very small polarization ===> keep $\hat{n}_0$ very close to vertical in the arcs.

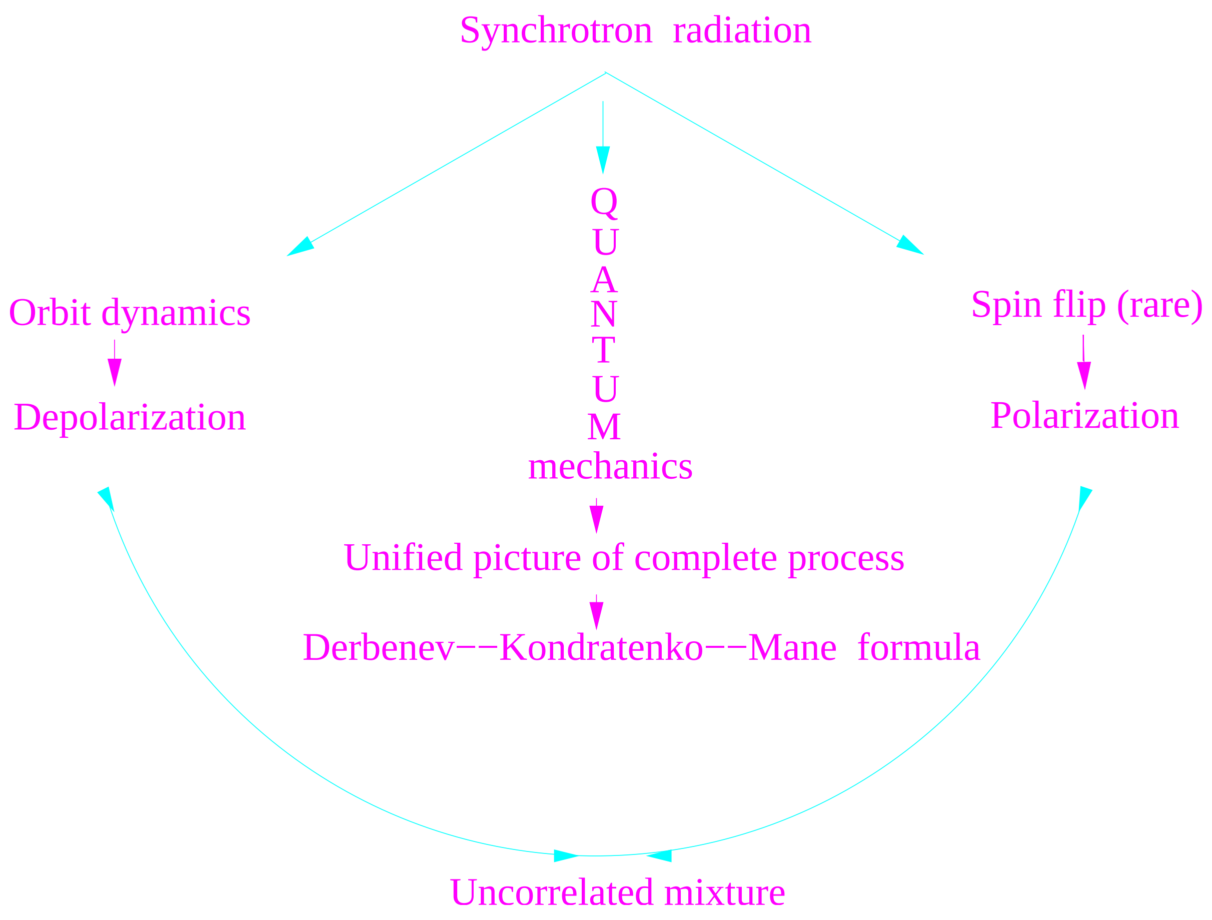
- Regions of horizontal $\hat{n}_0$ between rotator pairs: i.e. maximum tilt ===> trouble.



SLIM formalism:

$\longrightarrow$ **only first order resonances**

Self Polarization of electrons/positrons



Particle transport in the presence of damping and diffusion.

Fokker–Planck equation:

$$\frac{\partial W_{\rm orb}}{\partial s} = \mathcal{L}_{\rm FP,orb}\, W_{\rm orb}$$

where with synchrotron photon emission modelled as additive noise the orbital Fokker–Planck operator can be decomposed into the form:

$$\mathcal{L}_{\rm FP,orb} = \underbrace{\mathcal{L}_{\rm ham}}_{\mathcal{L}_{\rm ham}\to\text{Liouville}} + \underbrace{\mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2}_{\text{damping and noise}} .$$

Without the S–T terms, the corresponding form for the

Polarization Density $\vec{\mathcal{P}}$:

$$\boxed{\frac{\partial \vec{\mathcal{P}}}{\partial s} = \mathcal{L}_{\mathrm{FP,orb}} \vec{\mathcal{P}} + \vec{\Omega}(\vec{u};s) \times \vec{\mathcal{P}}}$$

Barber + Heinemann 1990's: NIM-A463, A469 2001

$$\vec{P}(s) = \int d^6u \; \vec{\mathcal{P}}(\vec{u};s).$$

This equation:

- can be derived in a classical picture,
- is homogeneous in $\vec{\mathcal{P}}$ i.e. it's "universal",
- is valid far from spin–orbit equilibrium,
- contains the whole of depolarization!

After including the S–T terms, this becomes (Derbenev + Kondratenko, Barber + Heinemann):

$$\underbrace{\frac{\partial \vec{\mathcal{P}}}{\partial s} = \mathcal{L}_{\mathrm{ham}} \vec{\mathcal{P}} + \vec{\Omega}(\vec{u};s) \times \vec{\mathcal{P}}}_{\equiv \text{Damping and noise free part}} \underbrace{+ \mathcal{L}_0\, \vec{\mathcal{P}} + \mathcal{L}_1\, \vec{\mathcal{P}} + \mathcal{L}_2\, \vec{\mathcal{P}} + \underbrace{\frac{1}{\tau_0(\vec{u};s)} \left[\vec{\mathcal{P}} - \frac{2}{9} \hat{v}(\vec{\mathcal{P}} \cdot \hat{v}) + \frac{8\hat{b}(\vec{u})}{5\sqrt{3}} W_{\mathrm{orb}} \right]}_{\text{ST in BKS form}} + \underbrace{\text{X–terms}}_{\text{Kinetic pol.}}}_{\text{SMALL}}$$

$\Downarrow$

$\equiv$ T-BMT equation (BIG)

$\Downarrow$

Stationary state

$\Downarrow$

$\hat{n}$-axis (Invariant spin field) $\rightarrow$ DETERMINES DIRECTI ON

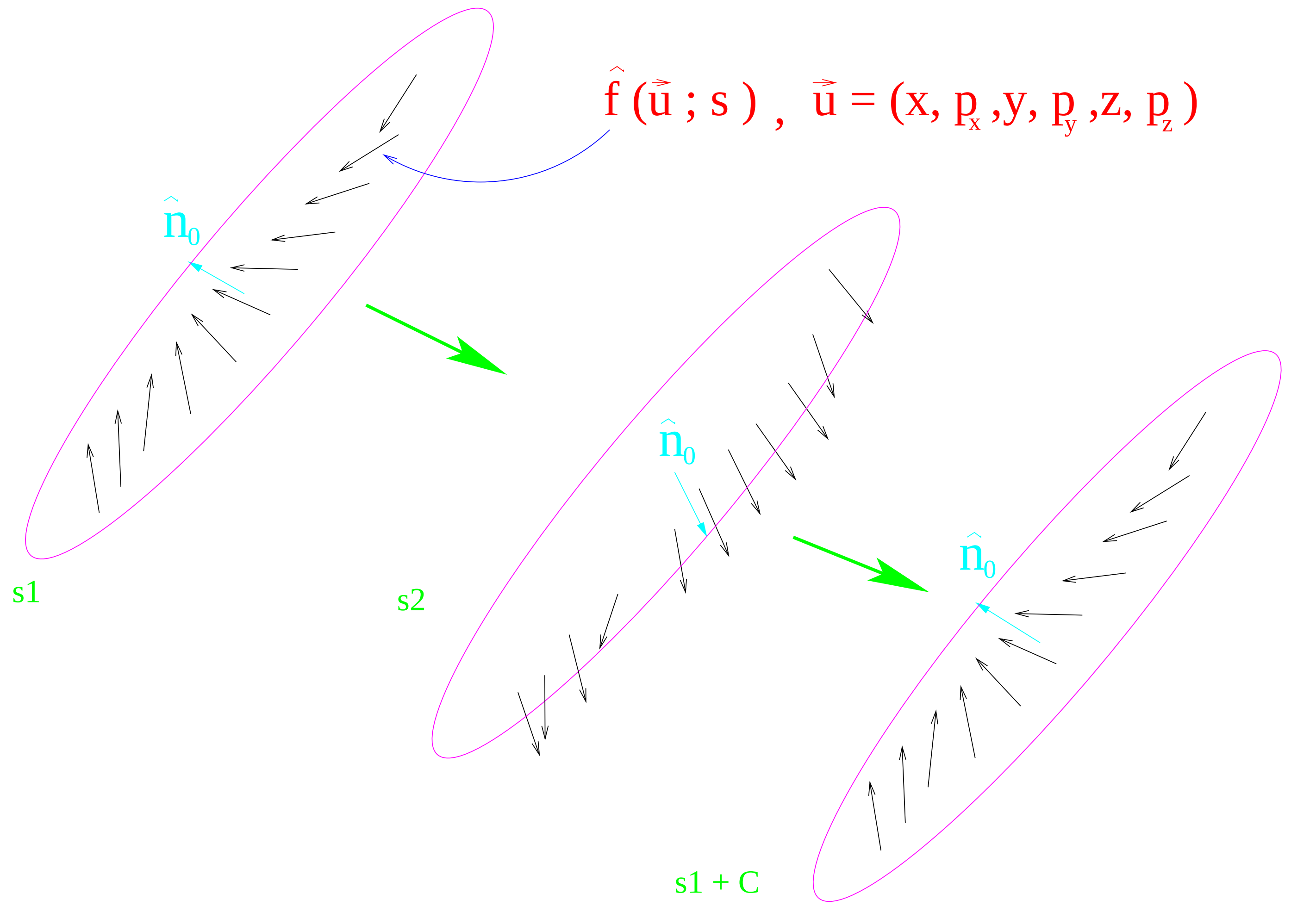
$\Downarrow$

Rate of polarisation loss $\propto$ Functional of $\hat{n}$, $\partial_{\vec{u}}\, \hat{n}$, $\partial^2_{\vec{u}}\, \hat{n} \ldots\ldots$ (e.g. DK formula).

$\Longrightarrow$ large near spin orbit resonances — since $\hat{n}$ is then very sensitive to $\vec{u}$.

The invariant spin field (n–axis, Derbenev–Kondratenko vector)

A pre−established s−periodic unit vector field at each phase space point



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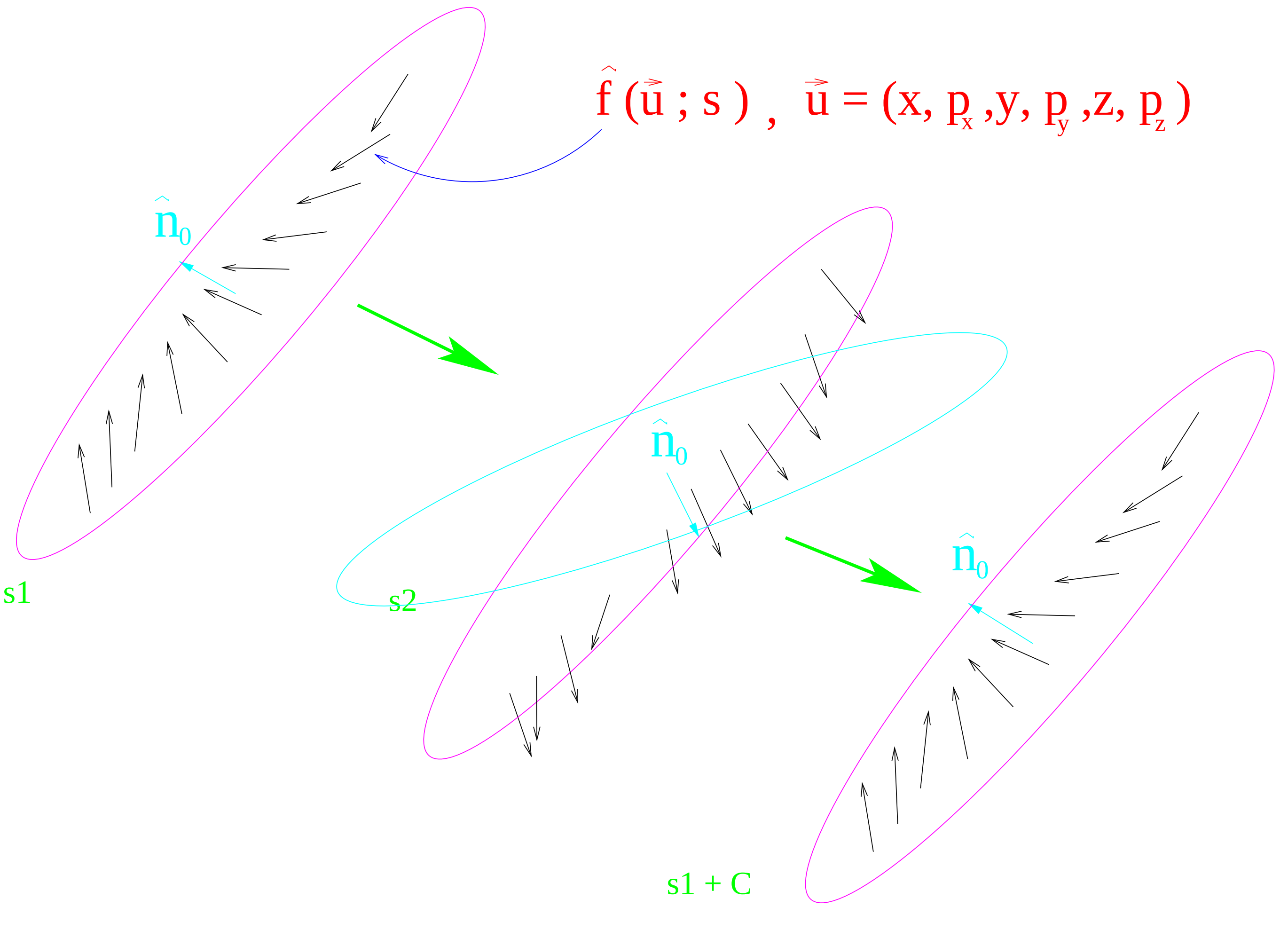
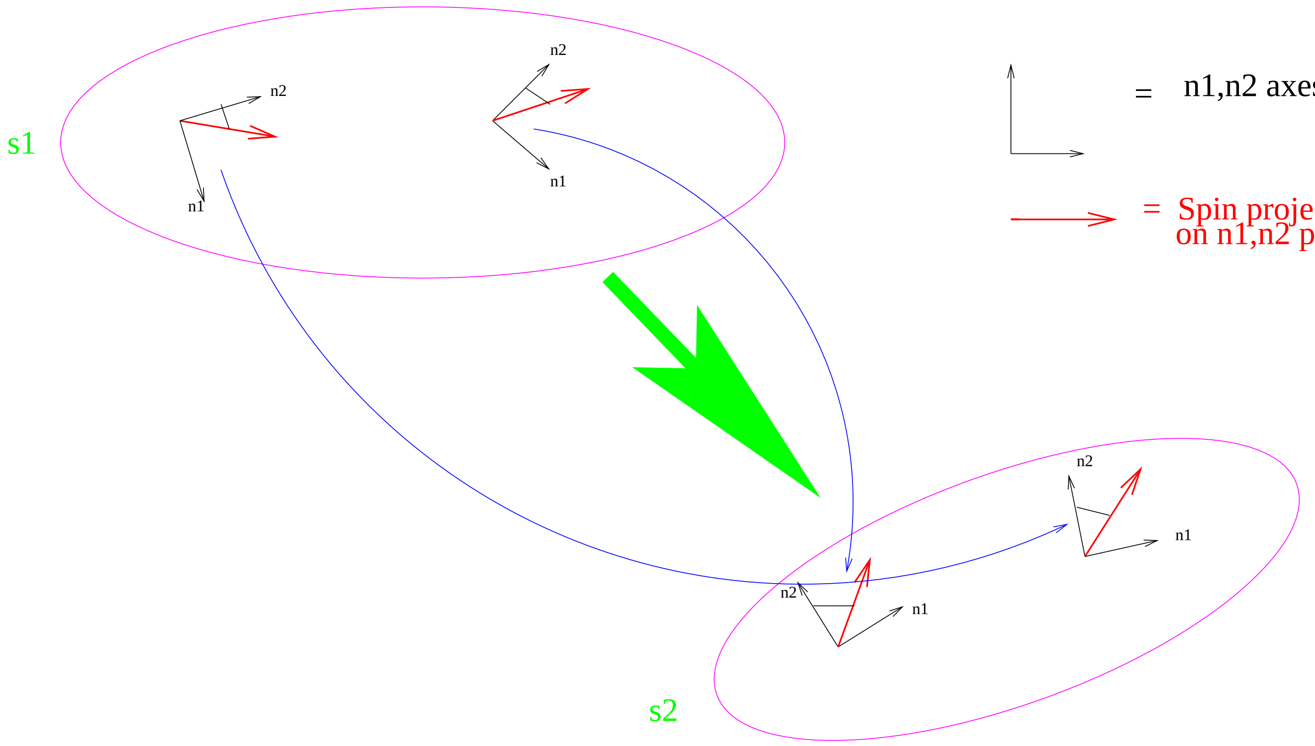


Attaching coordinate axes to each phase space point



Spin precession rate w.r.t. n1, n2 is the same at all phase space points with same J_x, J_y, J_z.

→ Amplitude dependent spin tune! $\nu_{spin}(J)$

- Measured polarization:

$$\vec{P}_{meas} = P\ \hat{n}_0$$

- In detail:

$$\vec{P}_{eq}(\vec{u};s) = P_{_{DK}}\hat{n}(\vec{u};s)$$

- ======>

$$\vec{P}_{meas}(s) = <\vec{P}_{eq}(\vec{u};s)>_{_s} = P_{_{DK}} <\hat{n}(\vec{u};s)>_{_s} \approx P_{_{DK}}\hat{n}_0(s)$$

since at normal energies

$$|\hat{n}(\vec{u};s) - \hat{n}_0(s)| \approx \text{ a few tens of milliradians at the most!}$$

The Derbenev–Kondratenko–Mane Formula.

$$P_{\text{eq,DK}} = -\frac{8}{5\sqrt{3}} \frac{\langle \frac{1}{|\rho|^3} \hat{b} \cdot \left[\hat{n} - \frac{\partial \hat{n}}{\partial \delta}\right] \rangle}{\langle \frac{1}{|\rho|^3} \{1 - \frac{2}{9}(\hat{n}\cdot\hat{v})^2 + \frac{11}{18}\left|\frac{\partial \hat{n}}{\partial \delta}\right|^2\} \rangle}$$

$$\vec{P}_{\text{ens,DK}}(s) \; = \; P_{\text{eq,DK}} \, \langle \hat{n} \rangle_s \; \approx \; P_{\text{eq,DK}} \, \hat{n}_0$$

$\hat{n}$: Semiclassical quantization axis: $\hat{n}(\vec{u}, s) = \hat{n}(\vec{u}, s + C)$.

$\hat{b} = \bar{v} \times \dot{\bar{v}} / \mid \bar{v} \times \dot{\bar{v}} \mid$

δ: fractional energy deviation (6th canonical coordinate).

$\langle \quad \rangle$: ring and ensemble average, $\langle \quad \rangle_s$: ensemble average.

Tears of frustration

e.g. e-Print Archive: physics/9901038

- The S–T effect is **NOT** due to $\Delta\vec{\mu}\cdot\vec{B}$: $E_\gamma \approx$ tens of keV but S–G energy is a fraction of eV. P_{ST} does not vanish at $g = 0$ etc etc.
- The invariant spin field $\hat{n}$ is **NOT a "spin closed solution"**: it is analogous to the invariant torus of orbital motion.
- High order spin–orbit resonances (including synchrotron sideband resonances!) are in general **NOT** due to orbital coupling or non–linear fields, but are simply due to the non–commutative behaviour of rotations. High order resonances occur for perfectly linear orbital motion and skew quads primarily just introduce more 1st order resonances and shift the resonant spin tune as the orbital eigen–tunes shift.
- The off–closed–orbit (amplitude dependent) spin tune does **NOT** depend on the orbital phases:
  "tunes" depending on phases are meaningless.
  It is **NOT** derived from a simple eigenproblem.
- The value of P_{eq} is **NOT** due to the opening angle $|\hat{n}(\vec{u};s) - \hat{n}_0(s)|$ — in a simple picture it is due to the balance of Sokolov–Ternov effect and **diffusion**
- To paraphrase or misquote Pauli: "Some of the things in the literature are not even just wrong, they are pathological."

Linear spin matching

$$\hat{\mathrm{M}}_{8\times 8} = \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix}$$

acting on $\vec{u} = (x,\ x',\ y,\ y',\ l,\ \delta)$ and α, β

$$\hat{n}(\vec{u}; s) \approx \hat{n}_0(s) + \alpha(\vec{u}; s)\hat{m}(s) + \beta(\vec{u}; s)\hat{l}(s)$$

α, β: 2 small spin tilt angles.

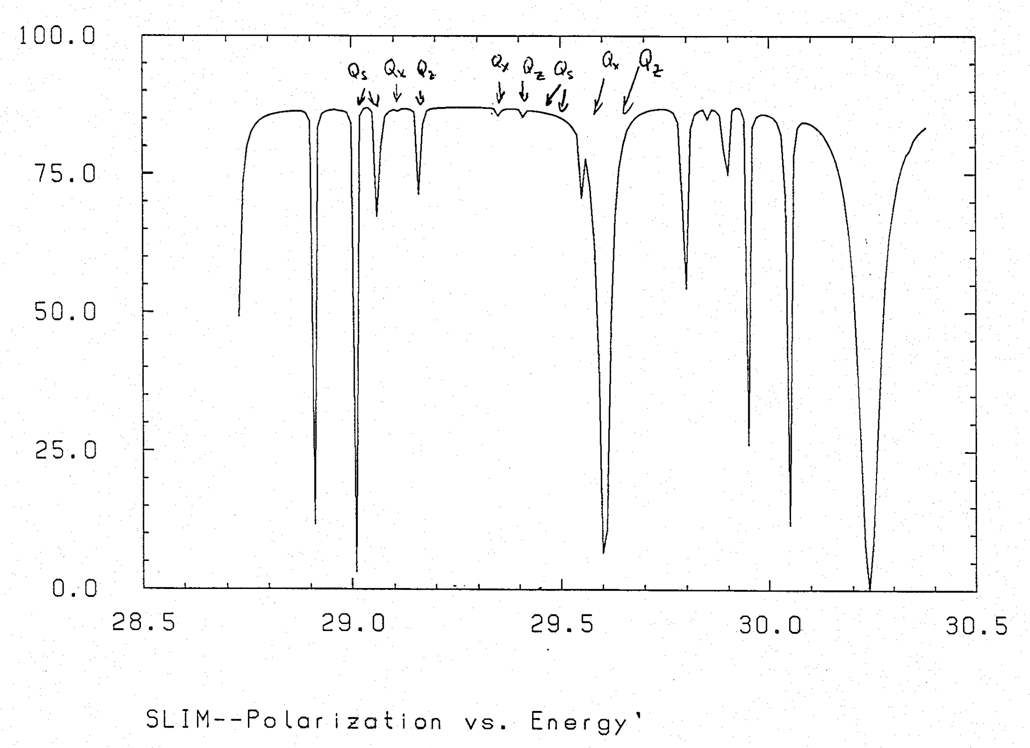
To minimize depolarization:
minimize $\mathbf{G_{2\times 6}}$ for appropriate stretches of ring
===> lots of independent quadrupole circuits.

The matrix approach to linear spin matching: minimize $\mathbf{G_{2\times 6}}$

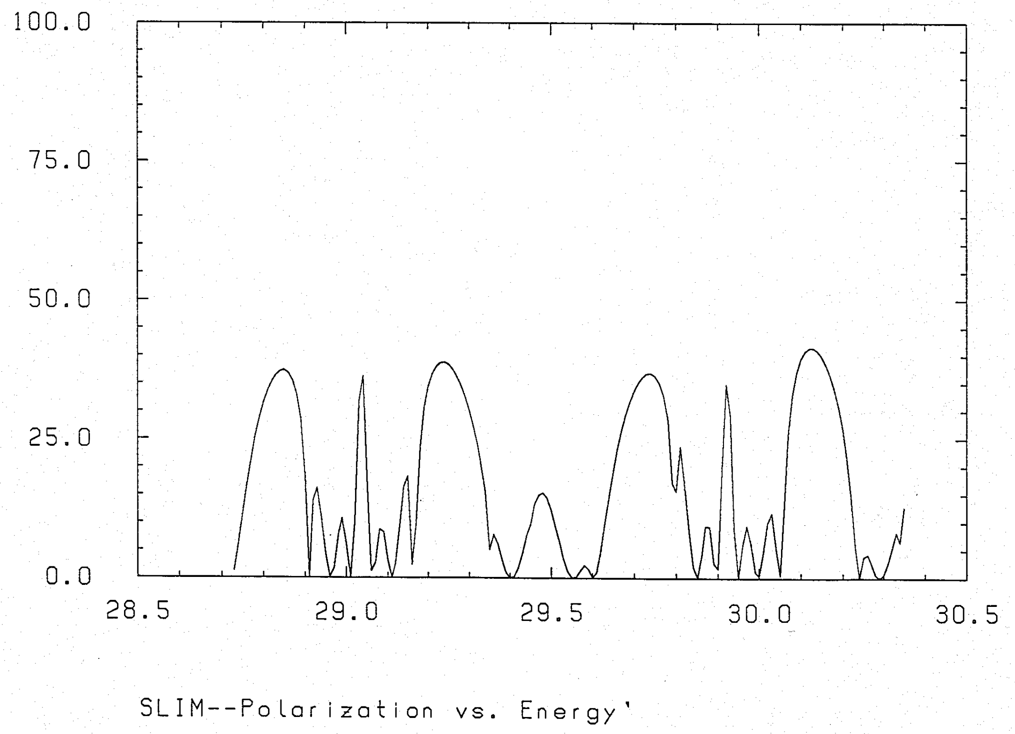
Advantages:

- Direct connection to quantities appearing in SLIM (SLICK).
- Necessary for coupled systems (skew quads, solenoids).
- For a big ring:
  Evaluation (numerical) of integrals in a thick lens optimization program is too slow ===> analytic integration? ===> integrals already contained in $\mathbf{G_{2\times 6}}$.
- "Locality": once $\mathbf{G_{2\times 6}}$ is zero for a section of the ring it remains zero no matter what changes are made to the optics outside.
- Provides a systematic basis for investigation of the algebraic properties using e.g. REDUCE, MATHEMATICA, MAPLE.
- The interpretation is usually transparent, e.g. arbitrary string of quads and drifts.

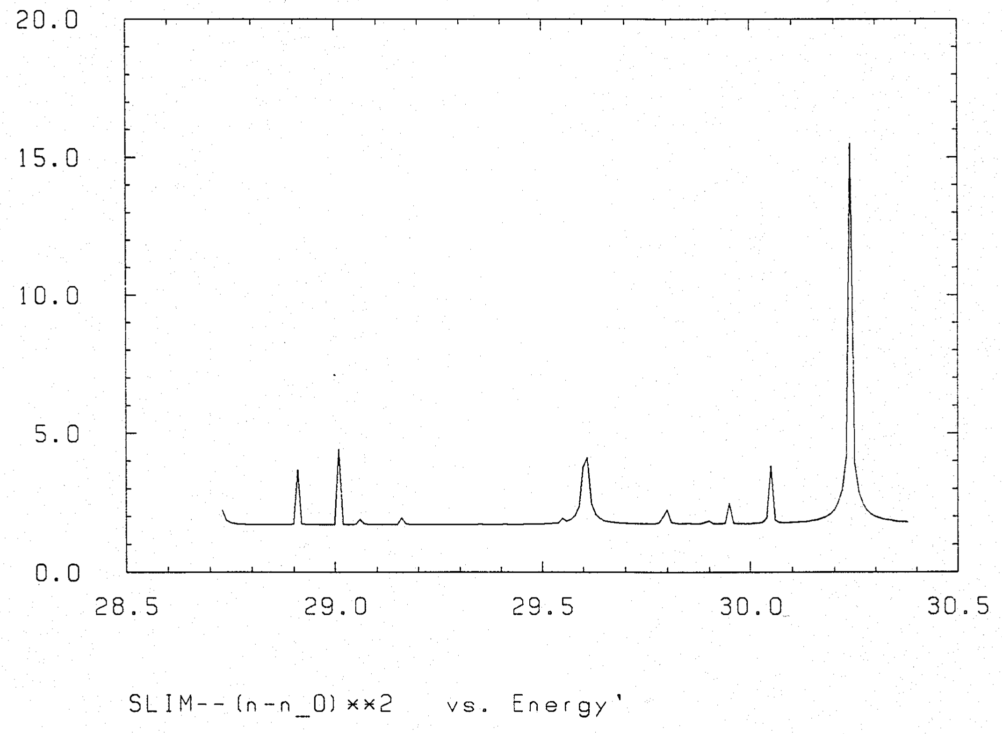
HERA: SLIM polarisation:
3 rotator pairs, spin matched at 29.23 GeV, no distortions

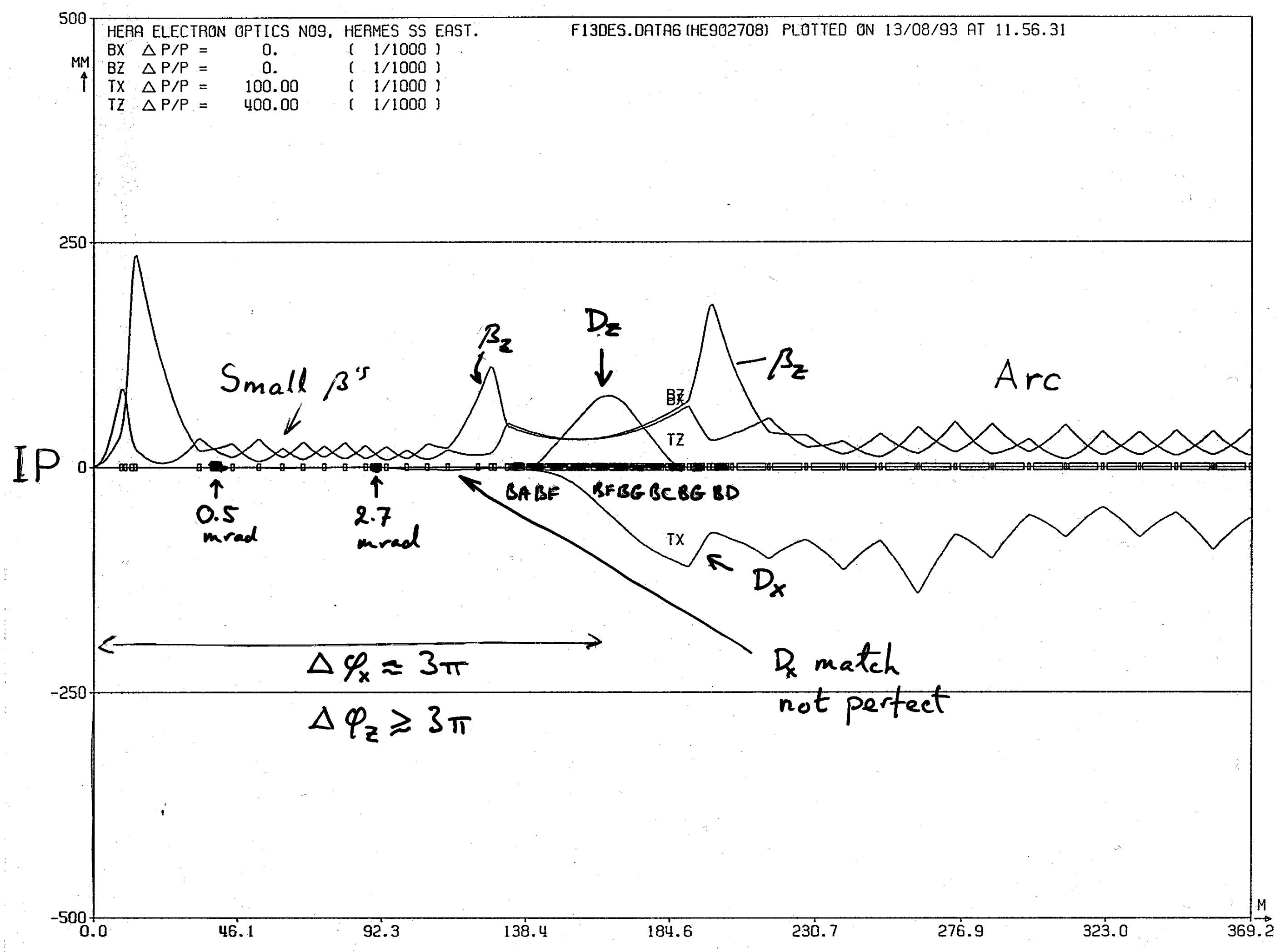


HERA: SLIM polarisation:
3 rotator pairs, spin match broken, no distortions



HERA: SLIM r.m.s angle between $\vec{n}$ and $\vec{n}_0$,
3 rotator pairs, spin matched at 29.23 GeV, no distortions




HERA ELECTRON OPTICS N09, HERMES SS EAST.
F13DES.DATA6 (HE902708) PLOTTED ON 13/08/93 AT 11.56.31
BX △P/P = 0. (1/1000)
BZ △P/P = 0. (1/1000)
TX △P/P = 100.00 (1/1000)
TZ △P/P = 400.00 (1/1000)
MM
500
250
0
-250
-500
0.0
46.1
92.3
138.4
184.6
230.7
276.9
323.0
369.2
M
IP
Small β's
β_z
D_z
β_z
Arc
BZ
BX
TZ
TX
0.5 mrad
2.7 mrad
BA BF
BF BG BC BG BD
D_x
D_x match not perfect
$\Delta\varphi_x \approx 3\pi$
$\Delta\varphi_z \gtrsim 3\pi$

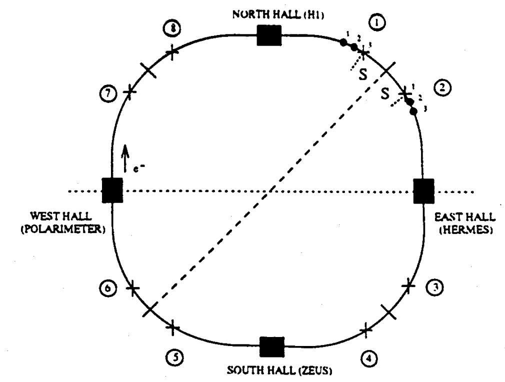


Fig. 2. A schematic drawing of the HERA ring showing the arrangement of the eight closed orbit bumps (numbered 1 through 8). The positions of the three coils (see Fig. 1) of bumps 1 and 2 are also shown. The distance S is the distance from the center of the arc to the nearest coil in the bump. The arrangement is mirror-symmetric across diameters through the IPs and the mid-points of the arcs; two of the lines-of-symmetry are drawn. The optimum distance S was determined to be 119 m; the length of one octant is 792 m.

D.P. Barber et al. / Nucl. Instr. and Meth. in Phys. Res. A 338 (1994) 166–184 171

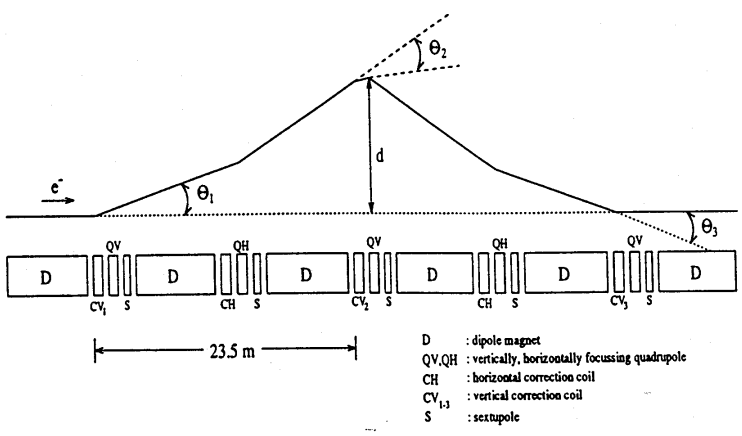


Fig. 1. The periodic magnet lattice in the HERA arcs. A single "FODO cell" contains two dipole magnets, two quadrupoles, two sextupoles, and two correction coils. The length of one cell is 23.5 m, and the phase advance of the betatron oscillations is 60° per cell. Also shown is a schematic drawing of a vertical closed orbit bump utilizing three consecutive vertical correction coils. The first coil produces a kick θ_1 and the subsequent coils produce kicks $\theta_2 \approx -\theta_1$ and $\theta_3 = \theta_1$; the kicks produced by the quadrupoles are also indicated. The maximum orbit deviation is denoted by d. The total length of a bump is 47.0 m.

Rotators: the T-BMT equation.

$$\frac{d\vec{S}}{ds} = \vec{\Omega} \times \vec{S}$$

- Dipole rotators: in transverse fields:

$$\delta\theta_{spin} = a\gamma \cdot \delta\theta_{orbit}$$

$a = (g-2)/2$ where g is the electron g factor.

At 27.5 GeV $\quad a\gamma = 62.5$

===>Suitable for high energy

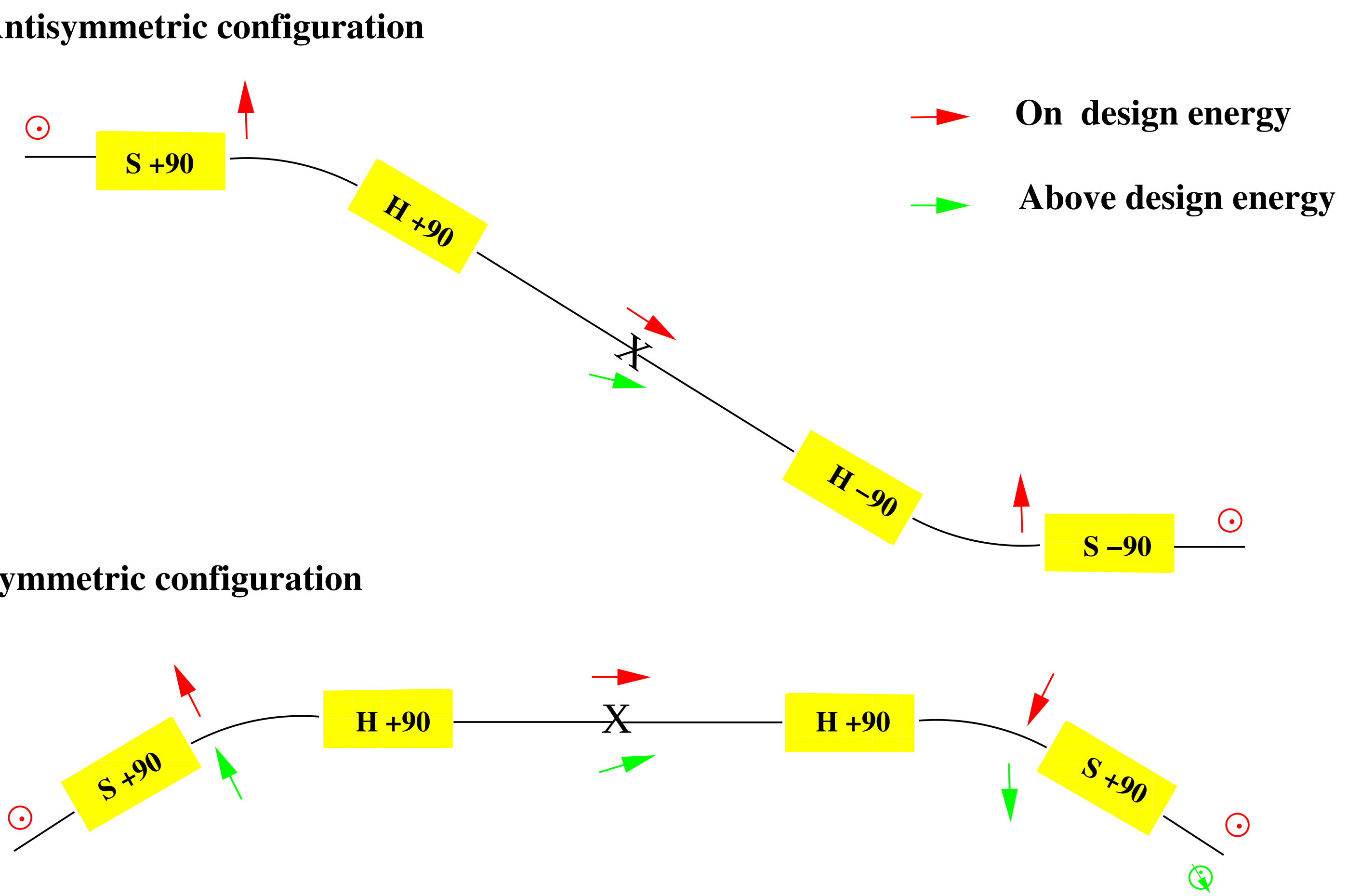
- Solenoid rotators:

$$\delta\theta_{spin} \propto \frac{B_{\mathrm{long}}}{E}$$

===>Only suitable for low energy.

. Also nontrivial spin–orbit coupling.

Solenoid spin rotators (from above): best at low energy



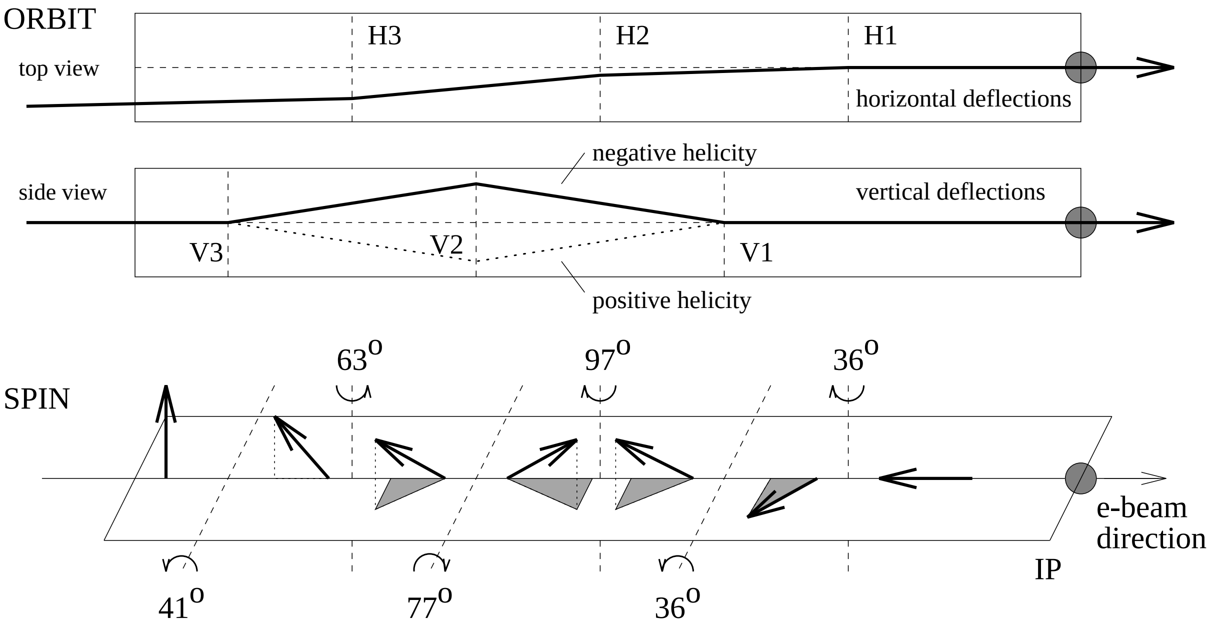
D.P. Barber et al.. Part. Acc. vol. 17 1985

Dipole spin rotators

- need $\vec{P}_{meas} \parallel \vec{B}$ in most of ring to drive Sokolov–Ternov.
- The natural polarization is vertical.
- But need $\vec{P}_{meas}$ longitudinal at IP.
- ===> Rotate $\vec{P}_{meas}(\hat{n}_0)$
  - Vertical $\rightarrow$ longitudinal just before an IP.
  - Longitudinal $\rightarrow$ vertical just after an IP.
- Recall: $\delta\theta_{spin} = a\gamma \cdot \delta\theta_{orbit}$
  Finite rotations do not commute
  ===>Use string of interleaved vertical and horizontal bends.

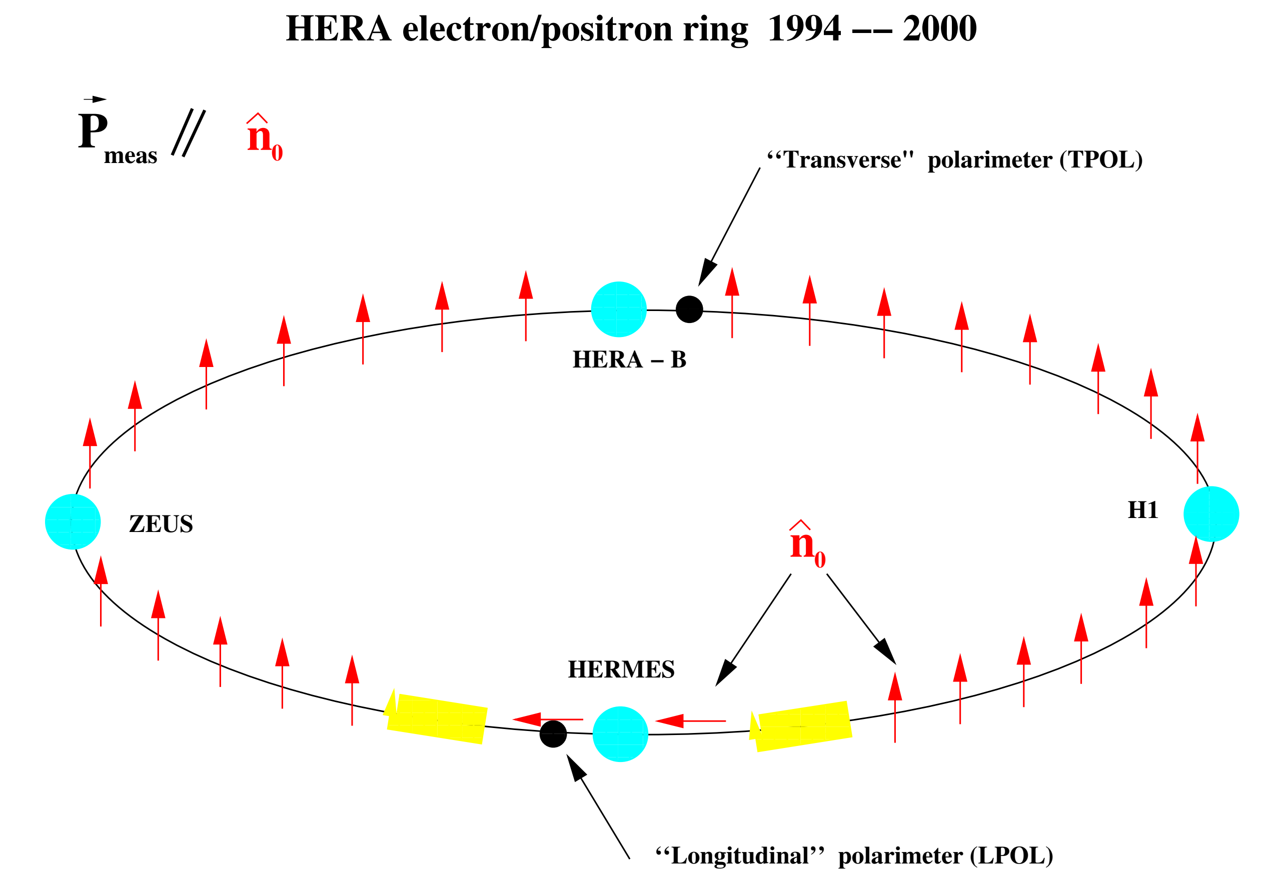
===> For HERA: MiniRotator by Buon and Steffen.

HERA MiniRotator: Buon + Steffen



56 m (“short”) → no quads.

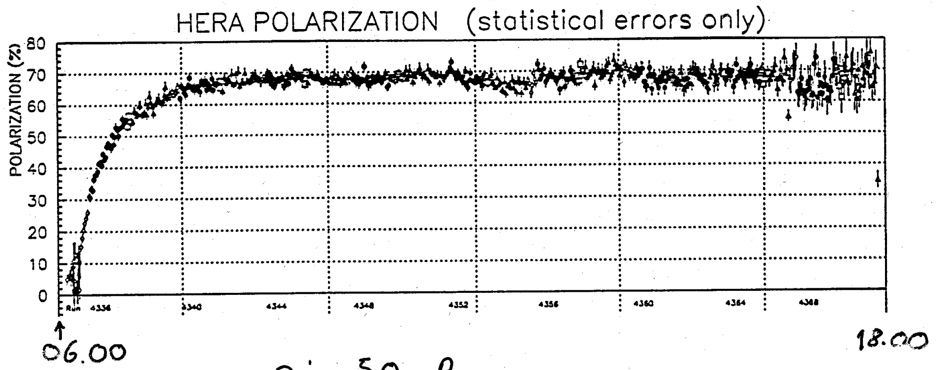
27 – 39 GeV, both helicities, variable geometry


HERA electron/positron ring 1994 –– 2000
$\vec{P}_{meas} \parallel \hat{n}_0$
“Transverse" polarimeter (TPOL)
HERA – B
ZEUS
H1
$\hat{n}_0$
HERMES
“Longitudinal” polarimeter (LPOL)

Chief features of the MiniRotator

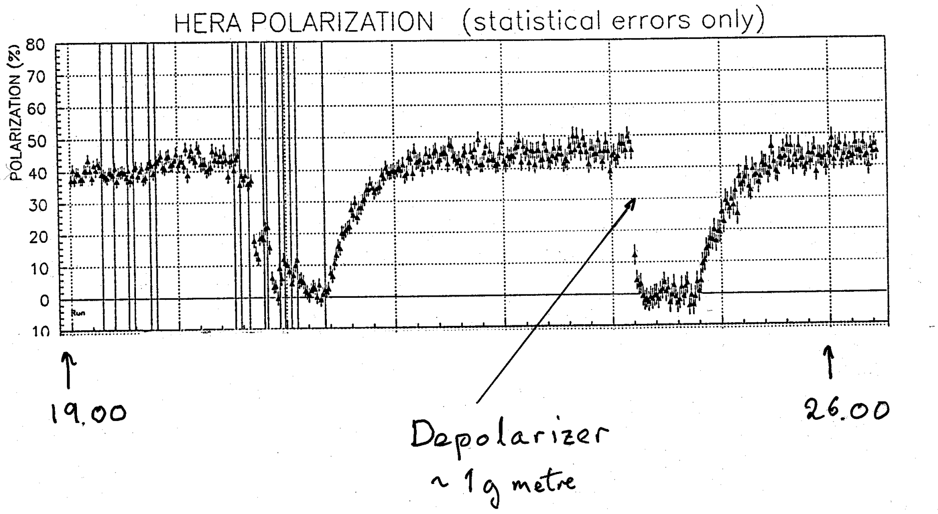
- Length limited to 56 metre: no quads needed, but $\delta P \propto L^{-2}$,
- Variable energy: 27–39 GeV, variable geometry.
- Symmetric w.r.t. IP in hor. plane. Antisymmetric in vert. plane.
- Maximum polarization: $P_{39} > P_{27}, \quad P_{BK}$ maximized.
- +ve and -ve helicity: reverse vertical bends.
- Ends fixed: middle section must me movable.
- Vertical orbit excursion small: 20 cm.
- Total vertical bend is zero.
- Total horizontal bend non–zero: include in arc to save space.
- Spin tune shift: $\delta\nu_{spin}$ small.

July '95 : Hera Polarimeter Group.



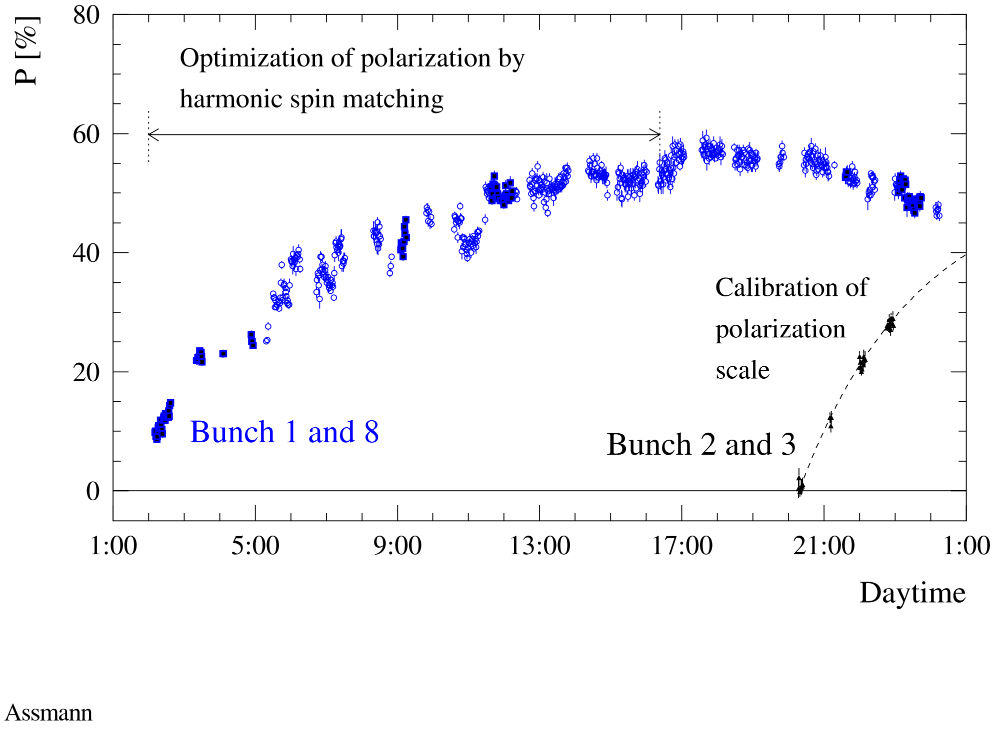
p: 50 mA

e^+: 30 mA → 10 mA



LEP: 46 GeV 1993. R. Assmann et al., SPIN2000, Osaka, Japan

Highest polarization achieved:



$\tau_{st} \approx 5$ **hours.**

The history of radiative spin polarization in storage rings.

Table 1:

Name	Year	Polarization	Energy
VEPP	1970	80% vert	0.65 GeV
ACO	1970	90% vert	0.53 GeV
VEPP–2M	1974	90% vert	0.65 GeV
VEPP–3	1976	80% vert	2 GeV
SPEAR	1975	90% vert	3.7 GeV
VEPP–4	1982	80% vert	5 GeV
CESR	1983	30% vert	5 GeV
DORIS	1983	80% vert	5 GeV
PETRA	1982	70% vert	16.5 GeV
TRISTAN	1990	70%? vert	29 GeV
LEP	1993	57% vert	47 GeV
HERA	1993	60% vert	26.7 GeV
HERA	1994	70% long	27.5 GeV
LEP	1999	7% vert	60 GeV

Software

- Linearised spin–orbit motion: SLIM, SLICK, SITF, ASPIRRIN, SOM.
- Linearised orbit motion, full 3–D spin by pert. expn.: SMILE.
- Linearised orbit motion, full 3–D spin by 1–turn maps: SODOM I.
- Linear + sext. orbit motion, full 3–D spin by ring section maps and Monte–Carlo radiative tracking; SITROS ==> SPRINT in future.

Tuning/optimising polarisation

(1) Set $a\gamma$ close to 1/2.

(2) Then choose betatron tunes intelligently, i.e. away from 1/2 but not too close to integers.

(3) Spin match the optic ("strong synchrobeta spin matching"), maintaining constraints on tunes, beam at polarimeters, beam at IP etc etc etc: use SPINOR.

(4) Flatten the orbit.

(5) Apply "harmonic closed orbit spin matching" with special vertical closed orbit bumps.

(6) Scan the energy to stay clear of synchrotron sideband resonances: $Q_s = 0.06 \rightarrow \approx 26 MeV$.

(7) Play with all tunes: c.f. beam–beam.

(8) Start again at (4).

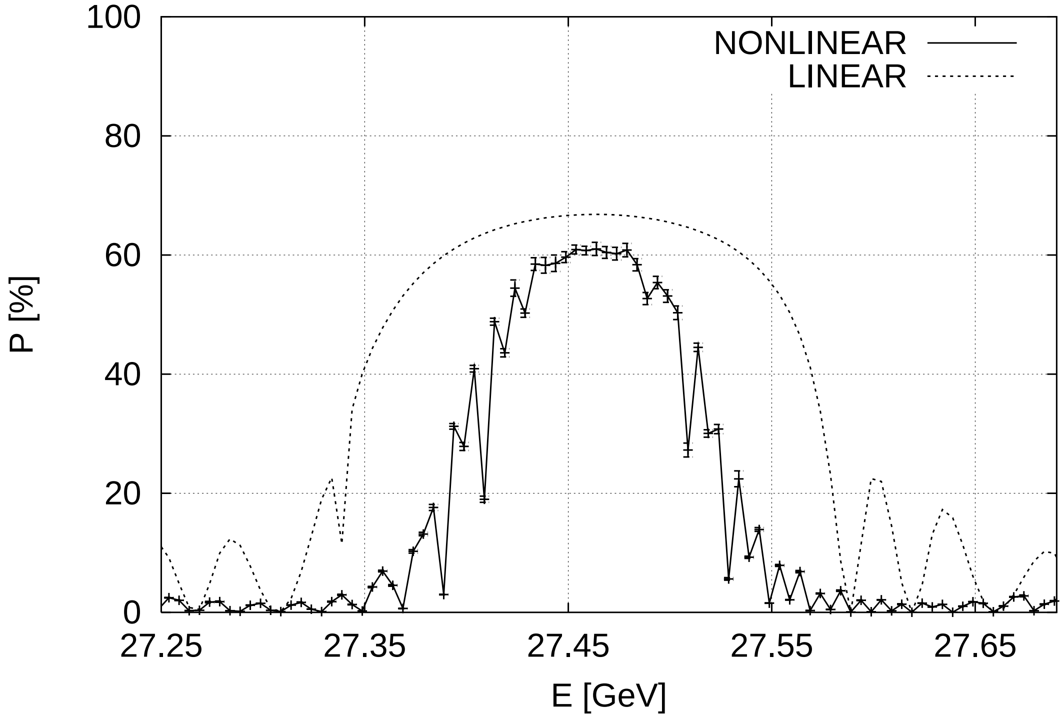
- Sidebands of parent first order betatron resonances: a useful approximation

$$\tau_{dep}^{-1} \propto \frac{A}{(\nu_0 \pm \nu_y)^2} \quad \rightarrow \quad \tau_{dep}^{-1} \propto \sum_{m_s=-\infty}^{\infty} \frac{A\,B(\xi; m_s)}{(\nu_0 \pm \nu_y \pm m_s\,\nu_s)^2}$$

  - A is an energy dependent factor
  - $B(\xi; m_s)$'s: *enhancement factors*, contain modified Bessel functions $I_{|m_s|}(\xi)$ and $I_{|m_s|+1}(\xi)$ depending on the *modulation index*

$$\xi = \left(\frac{a\gamma\ \sigma_\delta}{\nu_s}\right)^2$$

===> very strong effects at high energy — dominant source of trouble.



Polarization vs. energy for a HERA Upgrade lattice including the H1 and ZEUS solenoids: comparison of first order calculation (SLIM) and higher order calculations (SITROS).

M. Berglund, DESY–THESIS 2001–044 (2001).

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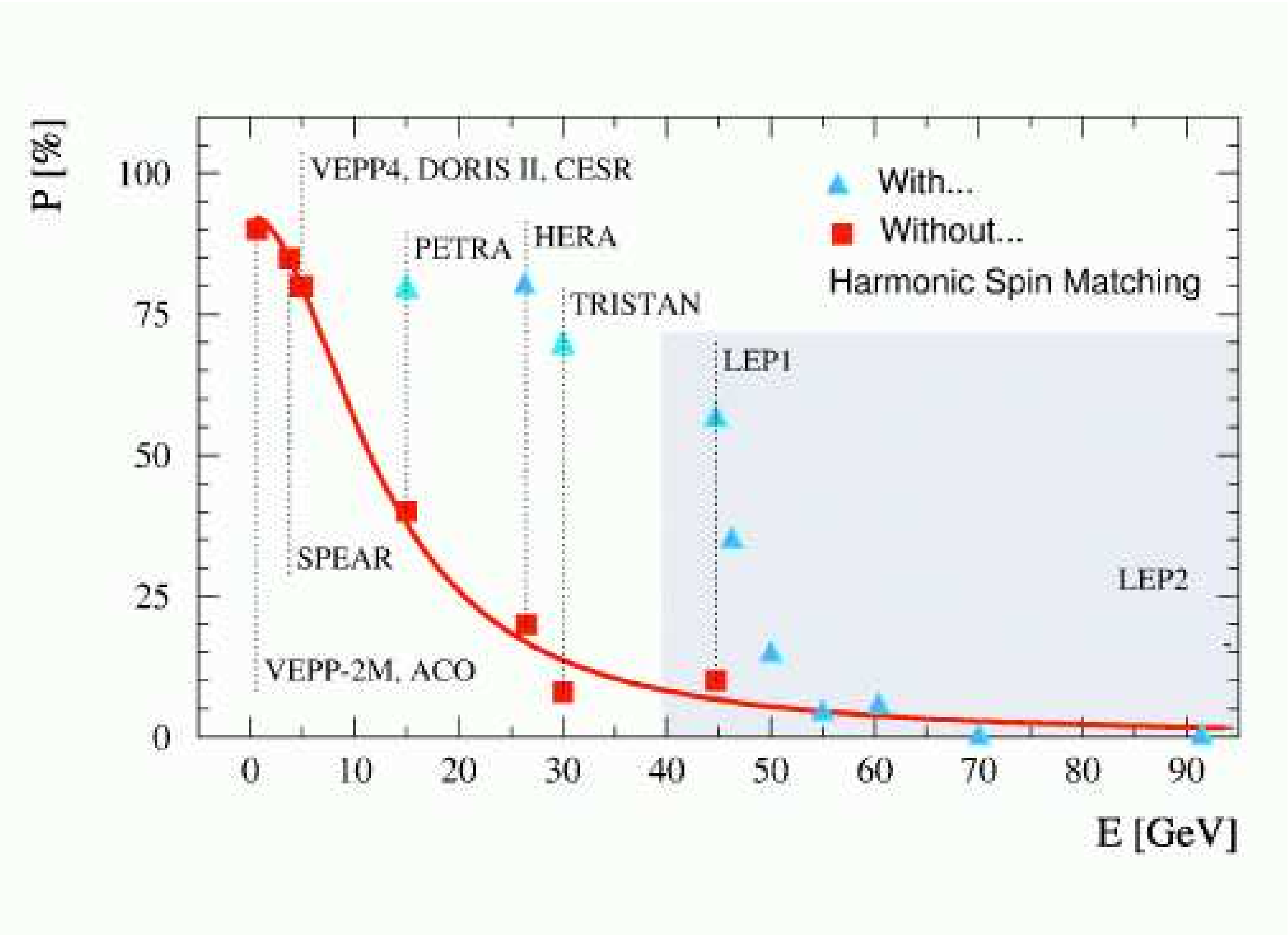

Recommendations for obtaining high self polarization or large lifetime for stored injected polarization

- Include polarization in the design (lattice, rotators, optic, spin matching) from the start — it should not be an "add on".
- Pay particular attention to:
  - alignment control and beam position monitoring
  - $\longrightarrow$ deterministic harmonic C.O. spin matching?
  - facilities for beam–based monitor calibration.
  - careful solenoid compensation $\longrightarrow$ locally with anti–solenoids if possible.
- Use spin transfer matrix formalism for spin matching in exotic machines and understand the physics of the spin–orbit coupling of each section of the ring. Ensure that there are enough independent quadrupole circuits.
- There is plenty of software available for detailed numerical calculations of linearized spin motion. The theory for linear orbit motion is well established.
- Very interesting depolarization effects due to beam–beam forces have been seen at HERA and LEP. For future high luminosity ring–ring colliders it will be very important to have a good understanding of these effects and to be able to carry out reliable simulations with tracking codes. This could become a high priority for running in the presence of intense proton beams.

/cont.

- Pay close attention to polarimetry: backgrounds!! $\longrightarrow$ build the machine around the polarimeter(s)! Fast precise polarimeters are essential for facilitating fast adjustment of the orbit or tunes. Build the machine around the polarimeter(s) so that bremsstrahlung and synchrotron radiation backgrounds are avoided
- Don't try to calibrate polarimeters during beam–beam collisions
  and be careful about the effects of kinetic polarization if the ring is not flat.