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THEORY AND OBSERVATION OF ELECTRON POLARIZATION IN HIGH ENERGY STORAGE RINGS [1]

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ABSTRACT

A short status report is given describing recent measurements of polarization in high energy electron storage rings and recent advances in ways of calculating equilibrium spin polarization in electron storage rings.

1. Introduction

The synchrotron radiation emitted by relativistic electrons in the guide field of an electron storage ring can cause the beam to become spin polarized antiparallel to the field[28]. In ideal circumstances a maximum polarization, P_{ST}, of $\frac{8}{5\sqrt{3}} = 92.4\%$ can be reached. The Sokolov-Ternov polarizing process is slow on the time scale of other dynamical phenomena occuring in storage rings, and the inverse time constant for the exponential build up is [10,11]

$$\tau_{ST}^{-1} = \frac{5\sqrt{3}\gamma^5 \hbar e^2}{8\rho^3 m^2 c^2} \tag{1}$$

where γ is the Lorentz factor, ρ is the radius of curvature in the magnets and the other symbols have their usual meanings. At HERA the time constant is 40 minutes at 27 GeV, at LEP at 46GeV the time constant is about 300 minutes and at TRISTAN at 29GeV it is about 2 minutes.

Although the polarization mechanism is weak, polarization has been observed in most low energy electron storage rings. It has been detected at

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ACO and high levels have been seen in all Novosibirsk machines (VEPP-2,VEPP3,VEPP4) and at CESR, SPEAR and DORIS[27]. Polarization was observed at PETRA at 16.5 GeV where levels exceeding 70% were obtained in 1982[9].

However, synchrotron radiation is not only responsible for polarization but can also cause depolarization so that the attainable polarization, P_{eq} is less than P_{ST}. As described later, the depolarization effects become relatively stronger as the energy increases and this has led to pessimistic forcasts for the level of polarization at high energy[3,4].

But in 1990 polarization was also observed at TRISTAN (at 29GeV) and at LEP (at 46GeV). More recently a polarization level of almost 60% was reached at HERA at 26.7GeV and in this article we will report on some of the results from these three machines.

In addition, advances in methods for calculating polarization will be described.

2. Some Theory

The original treatment[28] dealt with uniform magnetic fields. In real rings however, the fields experienced by the stored electrons are much more complicated. Thus a more complete treatment is needed and this begins with a study of spin precession in general fields.

The polarization vector, defined in the rest frame, of a relativistic electron in a storage ring precesses according to the Thomas-BMT equation[7,29]

$$\frac{d\vec{P}}{ds} = \frac{e\vec{P}}{mc\gamma} \times ((1+a)\underline{B}_{\parallel} + (1+a\gamma)\underline{B}_{\perp}) \quad , \quad a = \left(\frac{g-2}{2}\right) \qquad (2)$$

where $\underline{B}_{\parallel}$, $\underline{B}_{\perp}$ are the magnetic fields parallel and perpendicular to the trajectory , s is the longitudinal coordinate and g is the electron g factor.

An essential consequence of eqn.2 is that the number of spin rotations per turn around a perfectly flat ring, the spin tune, is $a\gamma$ (denoted by ν), and represents the natural spin precession frequency of the system. For example at 46.5GeV ν is 105.5. Away from integer values of ν, a stable unique periodic solution of eqn.2 on the periodic (closed) orbit exists and is denoted as $\hat{n}_0$. In a perfectly flat ring, $\hat{n}_0$ is vertical and so is the equilibrium polarization, $\vec{P}_{eq}$. In machines with vertical bends and/or where there are misalignments, there are horizontal fields on the closed orbit so that $\hat{n}_0$ is no longer everywhere vertical. But $\vec{P}_{eq}$ is still parallel to $\hat{n}_0$. Thus we know the direction of the polarization vector in the general case also.

So far we have ignored spin diffusion processes but electrons experience stochastic disturbances as they recoil during photon emission and in a simple classical picture where spins 'ride' on electrons one would expect the spins to undergo extra stochastic precessions according to eqn.2 in the magnetic fields in the quadrupoles and that the spins would then diffuse or depolarize[10,11].

However in a flat perfectly aligned ring, i.e. when $\hat{n}_0$ is everywhere vertical and the beam is a flat ribbon, the stochastic motions do not lead to depolarization. Nevertheless in real rings with misalignments the optical state of the machine becomes modified and $\hat{n}_0$ becomes tilted. Then depolarization does occur and becomes larger as the misalignments increase and, in particular, as the tilt increases. Thus synchrotron radiation can create polarization but it can also lead to its destruction! The final equilibrium polarization depends on the relative magnitudes of the two effects. Furthermore, spin diffusion is strong and hence the polarization small when the frequencies of the orbital motion (which drives the spin diffusion) obey the following resonance condition:

$$\nu = k + m_x Q_x + m_z Q_z + m_s Q_s \ , \tag{3}$$

where k, m_x, m_z, m_s are integers, and the $Q_{x,z,s}$ are orbital tunes. Thus detailed energy scans reveal energies where the polarization is zero. These are the positions of the so called depolarizing resonances. A good example of such a scan can be found in ref.17.

A detailed inspection of the spin diffusion mechanism reveals that the rate of depolarization is given by the polarization rate (eqn.1) multiplied by a polynomial in powers of ν and where each term contains resonance denominator factors corresponding to the depolarizing resonances. Thus as the energy increases spin diffusion effects begin to dominate and one can expect that it is more difficult to achieve high polarization at high energy. One class of resonance is expected to be particularly dangerous, namely the synchrotron side band resonances of parent first order resonances ($m_x = \pm 1$ or $m_z = \pm 1$ while $m_s =$ a small integer)[22].

3. Polarimetry

At HERA,LEP and TRISTAN electron polarization is measured using the method proposed by Baier and Khoze in 1969[6], namely by observing the angular distribution of high energy photons from the Compton scattering of circularly polarized laser light off the electron beam. If the electrons are vertically polarized there is an up-down asymmetry in the scattering distribution projected on the vertical plane and the shift of the centroid under reversal of the laser beam helicity is proportional to the vertical electron polarization. Details of the three polarimeters can be found in the talks of M. Düren, K. Nakajima and R. Schmidt in these Proceedings. An impression of the high performance achievable can be seen in a fit of a Monte-Carlo simulation[5] to data in fig.2 of ref. 13.

4. The measurements at HERA, LEP and TRISTAN

The observation of significant levels of polarization in these three machines is an important milestone in obtaining usable levels of polarization for high energy physics experiments at high energy electron storage rings.

The first step to getting polarization is to choose an energy near to a half integer value of ν i.e. a working point centrered between the first order resonances. In this context it is also useful to choose small values for the fractional parts of Q_x and Q_z. Solenoid fields should be locally compensated or switched off. The machine must be well aligned and the closed orbit must be corrected with correction coils so that the typical rms horizontal and vertical deviations from the design orbit are less than 1 mm. However quadrupole misalignments of as little as 200 microns together with the required correction fields are enough to ensure that the polarization is small as a result of the $\hat{n}_0$ tilt[15] (see Section 2). So a method of reducing the tilt is needed. Fortunately, it can be shown that the tilt of the $\hat{n}_0$ axis is dominated by certain harmonics in a Fourier decomposition of a spin-orbit function appearing in the expression relating the tilt to the fields on the closed orbit[2,15,20,25]. Thus one can control the tilt using special sets of vertical correction coils with excitation currents chosen so as to artificially excite each of the dangerous harmonics. Then the measured polarization is maximized by cancelling each of the dangerous harmonics empirically in turn. The approach used at HERA is based on the program FIDO[2,15,20] and since the periodic reference frame required by the algorithm is constructed using just the fractional part of the spin tune, it is necessary to cancel the real and imaginary parts of the -1^{st},0^{th},$+1^{st}$ and 2^{nd} harmonics. This is achieved using eight short closed vertical bumps which allow the eight harmonics to be targeted individually[15,20]. Fig.1 shows an example of preliminary data from HERA[5] for an orbit optimization at 26.7 GeV ($\nu = 60.5$) where the polarization was increased to over 30% by adjusting the amplitude of the imaginary $+1^{st}$ harmonic to be about -1.8mm. Additional orbit corrections led to a polarization of over 56%.

Closed orbit optimization has also been successfuly used at TRISTAN at 28.86GeV, $\nu = 66.5$ [24]. The rms vertical closed orbit deviation was first corrected down to 0.3 mm without any increase in the polarization. Then the correction scheme of ref.25 which was successful at PETRA[9] and which uses eight symmetrically distributed single vertical correction coils to correct harmonics close to the spin tune in the vertical slope of the closed orbit was applied. Fig.2 shows how adjustment of the 66^{th} harmonic increased the polarization from about 7% to $75 \pm 15\%$ [24]. A similar programme of orbit corrections will be tried at LEP once the τ_{ST} has been reduced from 300 minutes to 30 minutes using the twelve asymmetric 'polarization wigglers'[19].

It is also useful to be able to depolarize the beam. Indeed, after the polarization has been maximized as described, the 'harmonic bumps' can be used to depolarize the beam by setting them in 'the wrong direction'. Once the polarization has reached zero the optimum bump settings can be quickly reestablished and the natural rise of the polarization observed. The polarization of an initially unpolarized beam increases in time according to

$$P(t) = P_{eq}\left[1 - e^{\frac{-t}{\tau}}\right] \tag{4}$$

where τ is the characteristic build up time and P_{eq} is the asymptotic polarization. For flat machines P_{eq} and τ are given by

$$P_{eq} = P_{ST}\frac{\tau_{dep}}{(\tau_{ST} + \tau_{dep})} \tag{5}$$

$$\tau = \tau_{ST}\frac{\tau_{dep}}{(\tau_{ST} + \tau_{dep})} \tag{6}$$

where τ_{dep} is the depolarizing time. Thus the τ obtained by fitting eqn.4 to a polarization build up curve together with theoretical values of τ_{ST} and P_{ST} allows P_{eq} to be measured with a very low systematic error so that the calibration of the polarimeter can be checked. This is illustrated in fig.3 for LEP[19] at 46.5 GeV ($\nu = 105.5$) where $\tau = 35 \pm 10$ mins and $P_{eq} = 10.7 \pm 3.1\%$. At TRISTAN(fig.4) the measurement was made at 14.76 GeV ($\nu = 33.5$) where τ_{ST} is about 68 mins and P_{eq} was $69 \pm 24\%$ in agreement with the polarimeter scale[24]. Finally, fig.5 shows the results of a preliminary analysis of a measurement at HERA[13] at 26.7 GeV ($\nu = 60.5$) where the rise time leads to a P_{eq} of $46.5 \pm 5\%$ compared to the value $50.3 \pm 1.4\%(stat) \pm 5\%(syst)$ predicted by the polarimeter[5].

A polarized beam can also be depolarized in a controlled manner which provides a very accurate measurement of the beam energy. In this case the beam is subjected to an oscillating radial magnetic field provided, for example, by the feedback magnets which are used to stabilize the beam and which have fields of a few gauss-metres. Depolarization can occur when the kicks to the vertical spins add up over many turns i.e. when the depolarizer frequency f_d is related to the circulation frequency f_c by the resonance condition $f_d = f_c\hat{\nu}$ or $f_d = f_c(1 - \hat{\nu})$ where $\hat{\nu}$ is the fractional part of the spin tune. In calculating $\hat{\nu}$ one must be careful to take account of energy variations along the closed orbit caused by energy loss by synchrotron radiation and energy gain in the rf cavities. The influence of magnet misalignments and solenoid fields should also be included. Thus $\hat{\nu}$ should be calculated from the complex eigenvalues of the one turn spin transfer matrix on the closed orbit[10,11]. Since f_c is large, of the order of tens of kiloherz and f_d can be measured very precisely, $\hat{\nu}$ can be measured with high precision so that the full spin tune is known with very high precision and can be used to obtain the central beam energy at a high energy physics interaction point with an error much smaller than the rms energy spread of the beam. This knowledge then allows the masses of elementary particles to be measured with errors at the MeV level[1].

Usually, the resonance is found by setting a signal generator to sweep slowly over a narrow frequency range and moving the central frequency until the polarization decreases. At LEP[19] it has been found that one can not only depolarize but often also flip the polarization to negative values. This then implies that the horizontal components of the spins in the ensemble unexpectedly remain aligned for some seconds and do not quickly become decoherent as the result of spin diffusion[8]. Initial use of rf depolarization[19] enabled the mass of the Z_0 to be determined to a precision of about 3 MeV i.e. to about one part in 10^4. By measuring the systematic variation of the difference between the beam energy derived from the dipole currents and the depolarizer over a 24 hour period it has even been possible to measure the effect of tidal forces on the length of the LEP ring[19]. Polarimetry at LEP has now become an integral part of the HEP program.

At TRISTAN, rf depolarization has been used at 29.8 GeV to show that the beam energy derived from the known dipole currents differs from the value measured with the depolarizer by only about 3 MeV[24].

At the time of writing almost 60% polarization has been seen at HERA, up to about 20% at LEP and about 70% at TRISTAN[13,19,24].

5. Longitudinal polarization

The high energy physicists at HERA and LEP wish to have longitudinal electron polarization at the interaction points. Since the polarization vector must be vertical in the arcs in order to 'drive' the polarization process,the polarization must be rotated into the longitudinal direction just before the interaction points and back to vertical afterwards. This can be achieved with special arrangements of dipole magnets called spin rotators. More details about schemes planned for HERA and LEP can be found in refs.3 and 4.

At HERA one pair of rotators has been constructed and is ready to be installed on either side of the east interaction point early in 1994.

6. Calculations and simulations

Naturally, one also needs to be able to calculate the expected polarization so that optimization schemes can be tested and some idea of the achievable polarization can be obtained. In the last few years there have been further advances in this field which will be described below.

We begin with analytical methods. So far in this article we have been relying on a simple picture of quantum mechanical polarization competing with classical spin diffusion. But it is possible to describe the whole process in a unified manner using the semi-classical quantum mechanical treatments given by Derbenev and Kondratenko in 1973 and Mane in 1987[12,21]. An overview of the theory can be found in refs.3 and 4.

The first program which could read an optic file and calculate $\hat{n}$ and $\gamma\, \partial\hat{n}/\partial\gamma$[12,21] without linearization and hence in principle arbitrarily high orders of resonances was SMILE by Mane[21]. The calculations are analytical, based on a perturbation expansion for $\hat{n}$ [3,4] and automatically include interference between resonances.

Calculations with SMILE have been invaluable in illuminating the behaviour of the resonance structure. Mane has also produced a modified and approximate algorithm which concentrates on the most dangerous resonances,namely the synchrotron sidebands of the lowest order resonances[22,23]. This version of SMILE is faster and more tractable than the original version.

An alternative way of calculating $\gamma\ \partial\hat{n}/\partial\gamma$ has recently been presented by Yokoya and implemented in the program SODOM[30] where the periodicities of $\hat{n}$ are used to write $\hat{n}$ as a Fourier series using the orbital phases on an orbit. The spin transfer matrices from the Thomas-BMT equation for $\hat{n}$ are also written in Fourier form and the Fourier coefficients of $\hat{n}$ are extracted by numerical iteration. Thus here, there is no explicit reliance on a perturbation expansion. Initial tests indicate that SODOM and the new SMILE agree very closely. Further details can be found in refs. 16 and 30.

Another way of calculating $\hat{n}$ proposed by Yokoya uses a Lie algebraic technique to manipulate the classical spin-orbit Hamiltonian and has been implemented in the program SpinLie by Eidelman and Yakimenko. This program is still under development but reproduces first order results and is able to include non-linear orbit effects[14].

The algorithms described so far are analytical. However, a Monte-Carlo model of naive classical spin diffusion can also be used, as for example, in the program SITROS originally written by Kewisch[18] and now in extensive use at DESY for predicting the polarization at HERA[8]. In SITROS an equilibrium ensemble of electrons is tracked with the inclusion of orbital damping and excitation effects and τ_{dep} is estimated for spins originally aligned along $\hat{n}_0$. The asymptotic polarization is then obtained from eqn.5. SITROS results agree qualitatively with measurements at HERA[8,13].

So there are many options now available for predicting the polarization in a ring but since they only include single particle effects, since the equilibrium polarization depends sensitively on the misalignments and on the details of the particle dynamics and since these can at best only be approximately modelled, the calculations can usually only yield approximate information about the attainable polarization. But this is still useful for the optimization and design of the machine.

7. Conclusion

Despite fears to the contrary, it *is* possible to obtain high levels of vertical polarization in high energy storage rings. Several algorithms are now available for calculating polarization and SITROS results agrees qualitatively with measurements at HERA[8,13]. It now remains to be seen if longitudinal polarization can be achieved using spin rotators.

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Fig. 1 Polarization optimization with a harmonic bump at HERA

Fig. 2 Polarization optimization with a harmonic bump at TRISTAN

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Fig. 3 Polarization build up at LEP

Fig. 4 Polarization build up at TRISTAN

Fig. 5 Polarization build up at HERA

Fig. 5 | ⇒rf cavity problems