IV. UNRUH EFFECT, SPIN POLARISATION AND THE DERBENEV–KONDRATENKO FORMALISM [a]

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The relationship between the level of spin polarisation caused by Unruh radiation as calculated by Bell and Leinaas and that obtained from the Derbenev–Kondratenko formalism is explained.

1 Introduction

In 1986 in the course of investigating quantum fluctuations in accelerated reference frames and striving to assign spin temperatures, Bell and Leinaas (BL) [1] found that in a perfectly aligned, azimuthally uniform, weak focussing electron storage ring, the electron polarisation antiparallel to the dipole field is given by the formula

$$P_{eq} = \frac{8}{5\sqrt{3}} \frac{1 - \frac{f}{6}}{1 - \frac{f}{18} + \frac{13}{360} f^2}. \tag{1}$$

where $f = (g-2)Q_z^2/(Q_z^2 - \nu^2)$ and $\nu = a\gamma$ [b].

Over most of the energy range P_{eq} is $8/5\sqrt{3}$ i.e. 92.4%. But as one approaches the resonance point $Q_z = \nu$ from below, the polarisation dips to -17% and then rises through zero at the resonance energy to reach 99.2% before levelling off again at 92.4%.

Such behaviour is not exhibited in the DKM *formula* (Article I, Eq. (36)) which is based on a calculation of spin motion driven by synchrotron radiation emission in the laboratory frame. Indeed, in a perfectly aligned flat storage ring $\partial\hat{n}/\partial\delta$ is zero and the polarisation is 92.4% independently of energy. At the time, the BL result caused considerable surprise and bafflement in the accelerator community.

2 The solution

However, the BL effect can be accommodated within the DKM *formalism* and we were able to provide a detailed treatment [2]. The full story can be found in [2, 3] so that here, owing to space limitations, I will be exceedingly brief.

BL were primarily concerned with the effect of vertical orbit fluctuations driven by the background Unruh radiation [4]. In the laboratory frame these fluctuations stem from the fact that synchrotron radiation photons are emitted at a small angle of

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[b] The notation is the same as in Article I.

order $1/\gamma$ with respect to the horizontal plane and thus cause the particles to recoil vertically. This must also be taken into account when considering the change in the $\hat{n}$ axis under photon emission (Article I, Eq. (35)) and the DKM formula for the polarisation along $\hat{n}$ then becomes:

$$P_{dk} = -\frac{8}{5\sqrt{3}} \frac{\oint ds \left\langle \frac{1}{|\rho|^3} \left[\hat{b}\cdot\hat{n} - \hat{b}\cdot\vec{d} - \frac{1}{6}\hat{s}\cdot\vec{f} \right] \right\rangle_s}{\oint ds \left\langle \frac{1}{|\rho|^3} \left[1 - \frac{2}{9}\,(\hat{n}\cdot\hat{s})^2 + \frac{11}{18}\mid \vec{d}\mid^2 - \frac{1}{18}\frac{\dot{\hat{s}}}{|\dot{\hat{s}}|}\cdot(\hat{n}\times\vec{f}) + \frac{13}{360}\mid\vec{f}\mid^2 \right] \right\rangle_s} \tag{2}$$

where the vector $\vec{f} \equiv -(2/\gamma)\partial\hat{n}/\partial\beta_z$ and $\vec{d} = \partial\hat{n}/\partial\delta$. See [2, 3] for notation [c].

If $\partial\hat{n}/\partial\delta$ is zero as in the BL ring, the terms containing the very small quantity $\vec{f}$ come into play. Then we obtain:

$$P_{dk} = -\frac{8}{5\sqrt{3}} \frac{1 - \frac{F}{6}}{1 - \frac{F}{18} + \frac{13}{360}\,F^2} \tag{3}$$

where $F = \frac{2}{\gamma} + f$.

Thus we have recovered the BL result except for the extra piece $2/\gamma$. Near to the resonance this is negligible compared to the resonance term and so near to the resonance we may consider the two results to be in agreement. Thus the vertical kicks imparted to the orbit by the Unruh radiation of BL have been identified with vertical recoils caused by synchrotron radiation.

Further instructive interpretations of synchrotron radiation can be found in [5, 6]. In [6] synchrotron radiation emission is considered to result from 'inverse Compton scattering' of electrons from the virtual photons of the deflecting magnetic field and the spin dependent Compton cross–section is used to obtain the radiation distribution. It would be interesting to see if an extension of this calculation emphasising spin effects could simulate the Sokolov–Ternov effect.

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[c] In particular the vector $\vec{f}$ in Eq. (2) and the quantity f in Eq. (1) are distinct.