

### III. THE PERMISSIBLE EQUILIBRIUM POLARISATION DISTRIBUTION IN A STORED PROTON BEAM <sup>a</sup>

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We illustrate the use of the invariant spin field for describing permissible equilibrium spin distributions in high energy spin polarised proton beams.

#### 1 A problem and its solution

Following the successful attainment of longitudinal  $e^\pm$  polarisation in HERA (Article II) it is natural to consider whether it would be possible to complement the polarised  $e^\pm$  with 820 GeV polarised protons [1, 2].

As pointed out in Article I, a stored polarised proton beam can only be obtained by injecting and then accelerating a prepolarised beam provided by a suitable source. However, I comment on another concept in the Appendix.

A major obstacle to reaching high energy with the polarisation intact is that the spins must negotiate groups of spin-orbit resonances every 523 MeV (Article I) since the spin tune is approximately proportional to the energy.

However, this problem can be ameliorated by the inclusion of Siberian Snakes [3, 4]. These are magnet systems which rotate spins by 180 degrees around an axis in the horizontal plane independently of the energy of the particle. By the installation of suitable combinations of snakes, the spin tune  $\nu_{spin}$  can be fixed at one half and then by suitable choice of orbital tunes, resonances can be avoided at all energies, assuming that the dependence of spin tune on synchrobeta amplitude is weak (Article I).

Tracking simulations show that even with snakes, preservation of polarisation up to high energy is nontrivial. For example a 1 milliradian orbit deflection at 820 GeV causes a 90 degree spin rotation (Article I, Eq. (4)). Thus one should check first whether the spin distribution permitted by the requirement of equilibrium at a chosen high energy would be acceptable. There would be no point in trying to accelerate if the answer were negative. Moreover, to arrive at an answer we have the ideal tool at hand, namely the invariant spin field introduced in Article I. The measure for acceptability is the deviation of  $\hat{n}$  from  $\hat{n}_0$  averaged across phase space. If the average deviation were, say, 60 degrees, then even with  $|\vec{P}_{eq}(\vec{u}; s)| = 1$  at each point in phase space, a polarimeter would only record about 50% polarisation. Thus the optic and ring layout must be chosen so that the deviation is minimised. The invariant spin

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field can be calculated using the numerical technique ‘stroboscopic averaging’ of the computer code SPRINT [5]<sup>b</sup>.

Examples of the invariant spin field for a HERA proton optic with a suitable snake layout are shown in the figures. In this simulation the protons only execute integrable vertical betatron motion. Each figure shows the locus, on the surface of a sphere, of the tip of the  $\hat{n}$  vector ‘attached’ to its phase space ellipse at an interaction point on the ring where  $\hat{n}_0$  is vertical. The parameters are shown in the captions. An emittance of  $4\pi$  mm mrad corresponds to ‘1- $\sigma$ ’. The energy  $800$  GeV lies well below

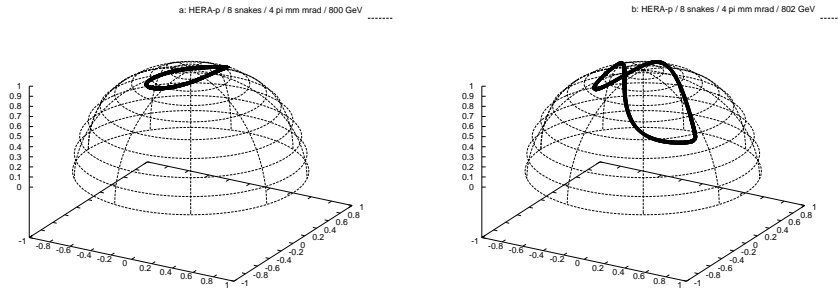


Figure 1: The  $\hat{n}$ -vector for the  $4\pi$  mm mrad ellipse at  $800$  GeV (left) and  $802$  GeV (right).

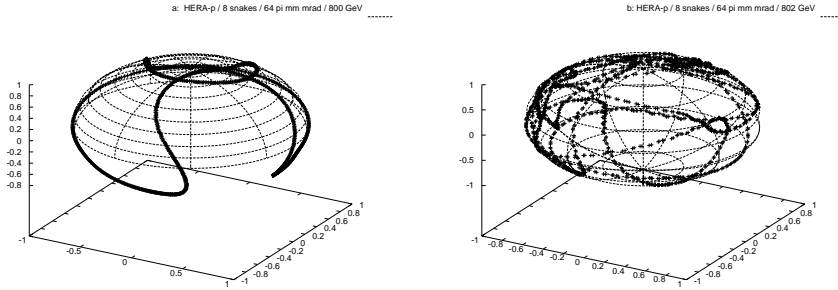


Figure 2: The  $\hat{n}$ -vector for the  $64\pi$  mm mrad ellipse at  $800$  GeV (left) and  $802$  GeV (right).

a resonance structure that survives even in the presence of snakes and  $802$  GeV is just below this structure. For particles at 1- $\sigma$  the spin field is well aligned at  $800$  GeV. At 4- $\sigma$  it has opened well beyond 90 degrees at some phases. At  $802$  GeV the 1- $\sigma$  locus deviates by more than 30 degrees from  $\hat{n}_0$  at some orbital phases and at 4- $\sigma$  the field is almost isotropic! In all four cases the locii are closed as required by the periodicity condition  $\hat{n}(\vec{u}; s) = \hat{n}(\vec{u}; s + C)$  (Article I).

A distribution of spins aligned along an invariant spin field is the ideal starting point for long term tracking studies of spin stability at fixed energy since deviations from equilibrium are then easy to discern.

<sup>b</sup>The new version of the SODOM algorithm [6] gives equivalent results. See Article I.

## Appendix

It has been suggested that by using Stern–Gerlach (SG) forces to drive coherent synchro-beta motion and thereby separate particle bunches into ‘spin-up’ and ‘spin-down’ parts, a proton beam could effectively be polarised [7]. The scheme using transverse SG forces requires running close to spin-orbit resonance but figure 2 illustrates that at high amplitude spin directions become isotropic so that the SG effect would average away. In any case the basic scheme might fail as a result of conservation laws [8, 9, 10]. The longitudinal SG effect [7] would be subject to mixing due to synchrotron oscillation unless some very special means were found to prevent it. Moreover, the longitudinal SG force is a total time derivative of a function of the fields and could integrate to zero [11, 12, 13, 14].

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