Phenomenology and calculations for spin polarisation in high energy electron/positron storage rings

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Desmond P. Barber

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Deutsches Elektronen–Synchrotron (DESY), Germany

University of Liverpool, UK

Cockcroft Institute, Daresbury, UK

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Basic phenomenology

- Electrons (positrons) in storage rings can become spin POLARISED due to emission of synchrotron radiation: Sokolov–Ternov effect (1964).

- The polarisation is perpendicular to the machine plane in simple rings.

- The maximum value is then $P_{st} = 92.4\%.$ (For today!)

BUT!

- Sync. radn. also excites orbit motion. This leads to DEPOLARISATION!
- In any case, the **value** of the polarisation is the same at all azimuths — time scales.

The T-BMT equation

$$\frac{d\vec{S}}{ds} = \vec{\Omega}(\gamma, \vec{v}, \vec{B}, \vec{E}) \times \vec{S}$$

$\vec{S}$: unit length single-particle spin expectation value (for fermions).

s: distance around the ring.

Periodic solution $\hat{n}_0$ on closed orbit.

$\hat{n}_0$: **direction** of measured equilibrium radiative polarization.

Closed orbit **spin tune** ν_0: number of precessions per turn around $\hat{n}_0$ for a spin on the closed orbit.

In a flat simple ring, $\nu_0 = a\gamma$.

$a = (g-2)/2 \approx 0.001159...$ where g is the electron g factor.

At 27.54 GeV $\quad a\gamma \approx 62.5$

Having $a = (g-2)/2 \neq 0$ is essential for high-energy rotator design!

The **value** of the polarization is the same at all azimuths — time scales.

The Baier-Katkov-Strakhovenko equilibrium radiative polarisation

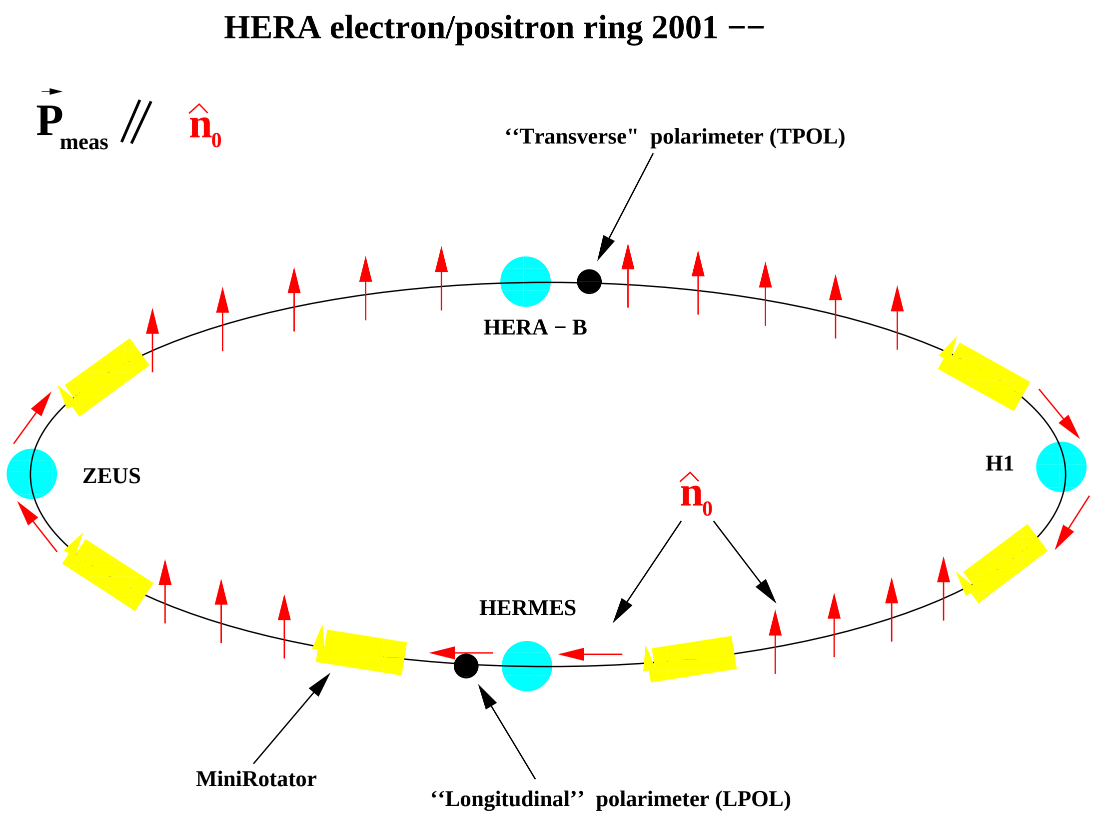
$$P_{ST} \to P_{\rm bks}$$

$$\vec{P}_{\rm bks} = -\frac{8}{5\sqrt{3}}\,\hat{n}_0\,\frac{\oint ds \frac{\hat{n}_0(s)\cdot\hat{b}(s)}{|\rho(s)|^3}}{\oint ds \frac{\left[1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2\right]}{|\rho(s)|^3}}$$

=====> Need $\hat{n}_0$ vertical in the arcs to drive the **Sokolov-Ternov** effect.

But need longitudinal polarisation at the IP's !!

=====> **spin rotators!**



Polarisation vertical in the arcs – to drive the **Sokolov-Ternov** effect

The first and only $e^{\pm}$ ring to supply longitudinal polarisation at high energy

— via the **Sokolov-Ternov** effect – also at 3 IP's simultaneously!

Well balanced parameters, see later

Spin motions: protons vs. electrons

- Protons: largely deterministic — unless IBS etc.

- Electrons/positrons:
  If a photon causes a spin flip, what are the other $\approx 10^{10}$ photons doing? ===>

  - Stochastic/damped orbital motion due to synchrotron radiation
  - with inhomogeneous fields
  - and spin–orbit coupling via T–BMT

  ===> spin diffusion i.e. **depolarisation!!!**

Equilibrium polarisation: Balance of poln. and depoln. ===>

$$P_\infty \approx P_{\rm bks} \, \frac{1}{1 \quad + \quad \left(\frac{\tau_{\rm dep}}{\tau_{\rm bks}}\right)^{-1}}$$

$$\tau_{tot}^{-1} = \tau_{bks}^{-1} + \tau_{dep}^{-1}$$

In any case:

$\tau_{\rm dep}^{-1} \;\propto\; \gamma^{2N} \; \tau_{\rm bks}^{-1}$ (actually a polynomial in γ^{2N})

===> Trouble at high energy!

Spin–orbit resonances

$$\nu_{\text{spin}} = k + k_I \nu_I + k_{II} \nu_{II} + k_{III} \nu_{III}$$

ν_{spin} : amplitude dependent spin tune ≈ closed orbit spin tune = precessions /turn on CO

- Orbit "drives spins" ===> Resonant enhancement of spin diffusion
  AT FIXED ENERGY EVEN AWAY FROM RESONANCES!
- Resonance order: $|k_I| + |k_{II}| + |k_{III}|$
- First order: $|k_I| + |k_{II}| + |k_{III}| = 1$ e.g. SLIM like formalisms.
- Strongest beyond first order:
  synchrotron sidebands of first order parent betatron or synchrotron resonances

$$\nu_{\text{spin}} = k + k_i \nu_i + k_{III} \nu_{III}, \quad i = I, II \text{ or } III$$

- The so-called "resonance strengths" for proton spin dynamics are NOT helpful for estimating depolarising rates!
- Other concepts from proton spin dynamics also don't travel well to electrons

High order resonances already with purely linear orbital motion!

Sidebands of parent first order betatron resonances: a useful **approximation**

$$\tau_{dep}^{-1} \propto \frac{A}{(\nu_0 \pm Q_y)^2} \quad \rightarrow \quad \tau_{dep}^{-1} \propto \sum_{m_s=-\infty}^{\infty} \frac{A\,B(\xi; m_s)}{(\nu_0 \pm Q_y \pm m_s\,Q_s)^2}$$

A is an energy dependent factor

$B(\xi; m_s)$'s: *enhancement factors*, contain modified Bessel functions $I_{|m_s|}(\xi)$ and $I_{|m_s|+1}(\xi)$ depending on the *modulation index*

$$\xi = (\frac{a\gamma\ \sigma_\delta}{Q_s})^2$$

in a simple flat ring.

===> very strong effects at high energy — dominant source of trouble

Analogous formula for sidebands of first order synchrotron resonances.

Rotators: need the T-BMT equation.

$$\frac{d\hat{n}_0}{ds} = \vec{\Omega} \times \hat{n}_0$$

- Dipole rotators: in transverse fields:

$$\delta\theta_{spin} = a\gamma \cdot \delta\theta_{orbit}$$

$a = (g-2)/2$ where g is the electron g factor.

At 27.54 GeV $a\gamma = 62.5$

===>Suitable for high energy

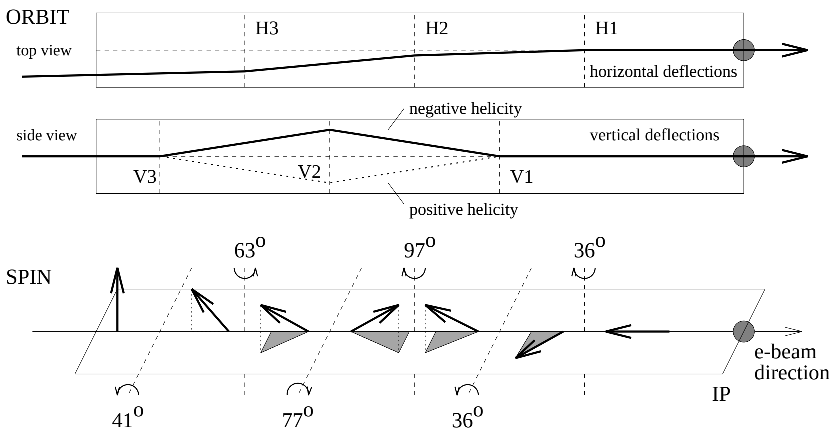
- Solenoid rotators:

$$\delta\theta_{spin} \propto \frac{B_{\mathrm{long}}}{E}$$

===>Only suitable for low energy.

. Also nontrivial spin–orbit coupling.

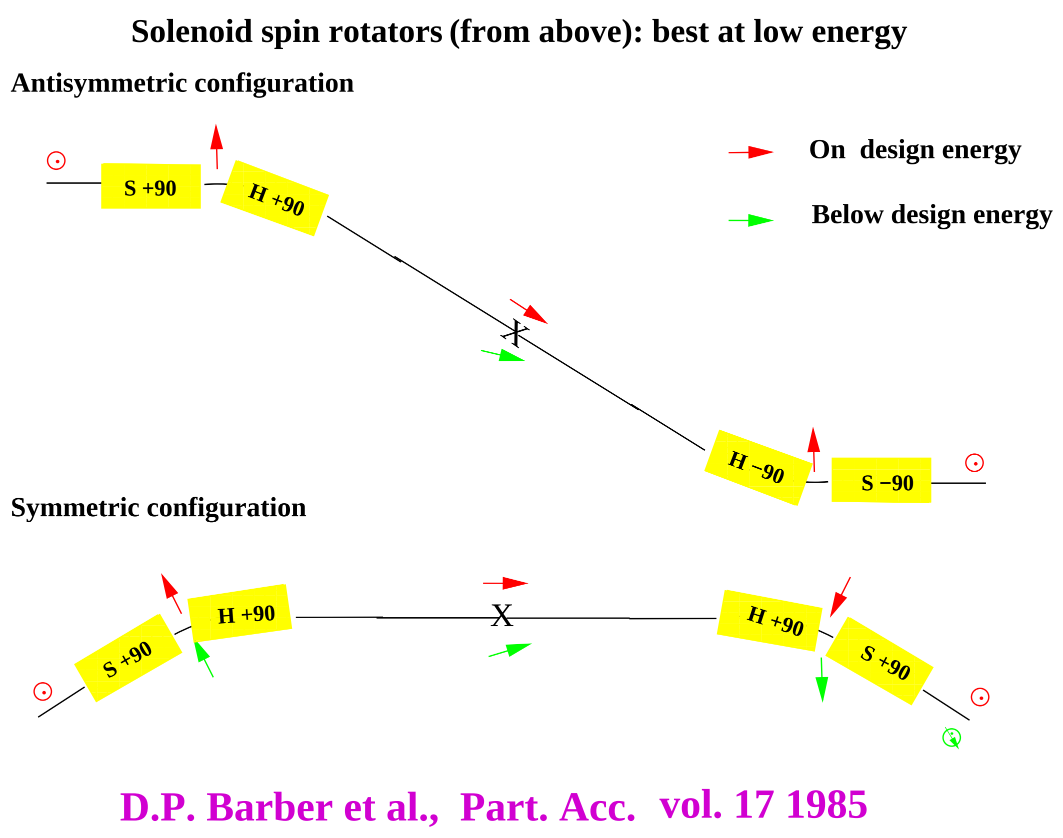
HERA MiniRotator: Buon + Steffen



56 m ("short") → no quads.

27 – 39 GeV, both helicities, variable geometry

Fixed ring geometry but variable rotator geometry and fields. NO QUADRUPOLES



The MEIC (Medium Energy Electron Ion Collider: JLab) and the ring-ring option of eRHIC (†) use(d) variations of these.

BUT! for **storage**

Spin rotators are usually potentially dangerous for polarisation!!

- Spin rotators orient the spins so that they are particularly susceptible to diffusion: regions of "spin-opacity"
- Spin rotators can themselves be "spin-opaque"
- In addition: depolarisation can be strongly enhanced by the tilt of $\hat{n}_0$ from the vertical in the arcs due to misalignments

The SLIM formalism for estimating depolarisation at first order (Chao 1981).

Skip the fancy theory (ISF, ADST..): heuristics instead for today!

$$\vec{S} = \sqrt{1-\alpha^2-\beta^2}\ \hat{n}_0(s) + \alpha\hat{m}_0(s) + \beta\hat{l}_0(s) \quad \approx \quad \hat{n}_0(s) + \alpha\hat{m}_0(s) + \beta\hat{l}_0(s)$$

$\hat{m}_0$ and $\hat{l}_0$ orthogonal to $\hat{n}_0$. All obey the T-BMT eqn.

α, β: 2 **small** spin tilt angles — have subtracted out the big rotations!

. ≡ an "INTERACTION PICTURE"

Linearised (T-BMT) equations of motion for α, β: ===> spin-orbit coupling matrix $\mathbf{G_{2\times 6}}$:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_2) \;=\; \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \\ \alpha \\ \beta \end{pmatrix}(s_1)$$

with $\delta = \delta E/E_0$

Unified transport formalism for trajectory and spin.

Spin–orbit covariance matrix

$$\left(\begin{array}{cccccc|cc}
\sigma_x^2 & \sigma_{xx'} & . & . & . & . & . & . \\
\sigma_{x'x} & \sigma_{x'}^2 & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . \\
. & . & . & . & . & \sigma_\delta^2 & . & . \\
\hline
. & . & . & . & . & . & \sigma_\alpha^2 & \sigma_{\alpha\beta} \\
\sigma_{\beta x}. & . & . & . & . & . & \sigma_{\beta\alpha} & \sigma_\beta^2
\end{array}\right)$$

$$\Delta P = 1 - \Delta < \sqrt{1-\alpha^2-\beta^2} > \approx -\frac{1}{2}\Delta(<\alpha^2+\beta^2>) = -\frac{1}{2}\Delta(\sigma_\alpha^2+\sigma_\beta^2) \Longrightarrow \frac{dP}{dt} \approx -\frac{1}{2}\frac{d}{dt}(\sigma_\alpha^2+\sigma_\beta^2)$$

Random walk in plane orthogonal to $\hat{n}_0$.

Calculate analytically (stochastic differential equations or Fokker-Planck methods) (Brownian motion). Everything needed is in 1-turn matrices.

Or brute force with a Monte-Carlo with multi-turn tracking – in fact the best in the end.

No damping mechanism for spin – but the S-T effect works to restore the polarisation along $\hat{n}_0$.

Just need the **slope** $\frac{dP}{dt}$ to get τ_{dep}^{-1}.

Linear spin matching

$$\hat{\mathrm{M}}_{8\times 8} \;=\; \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix}$$

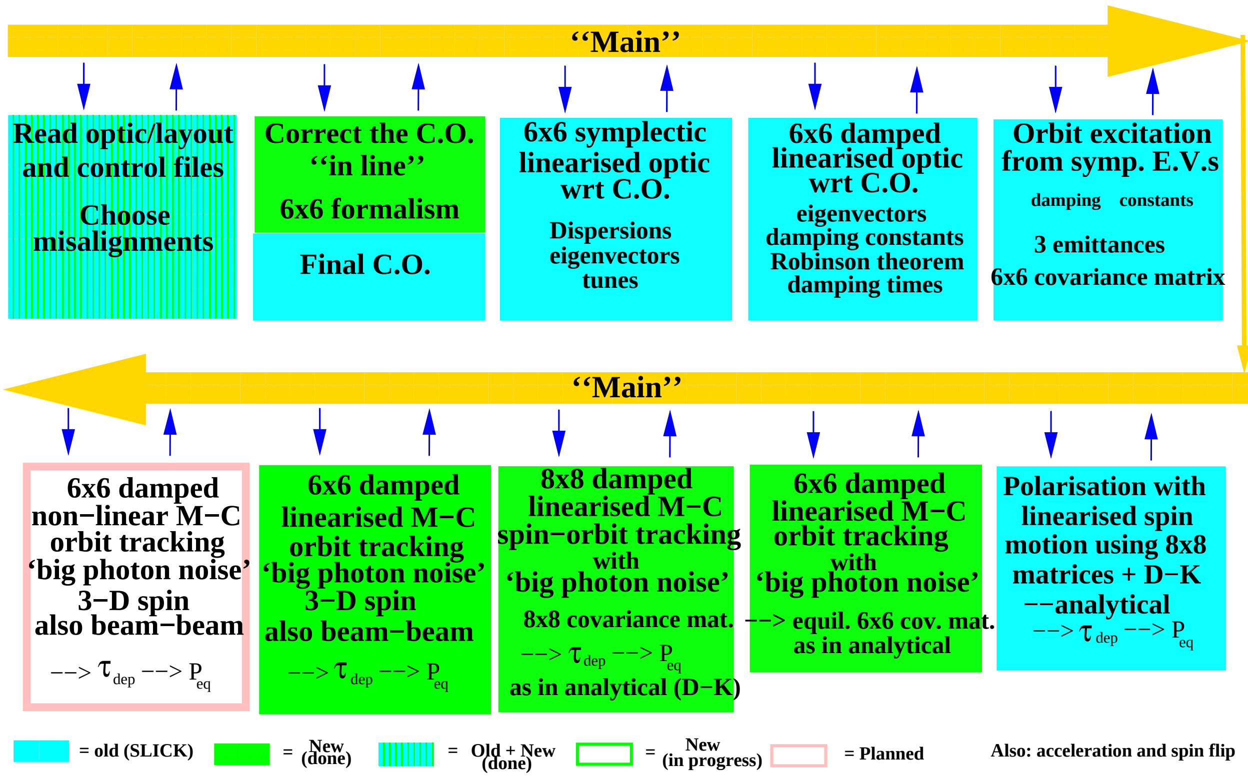
To minimize depolarization:
minimize appropriate bits of $\mathbf{G_{2\times 6}}$ for appropriate stretches of ring
===> lots of independent quadrupole circuits.

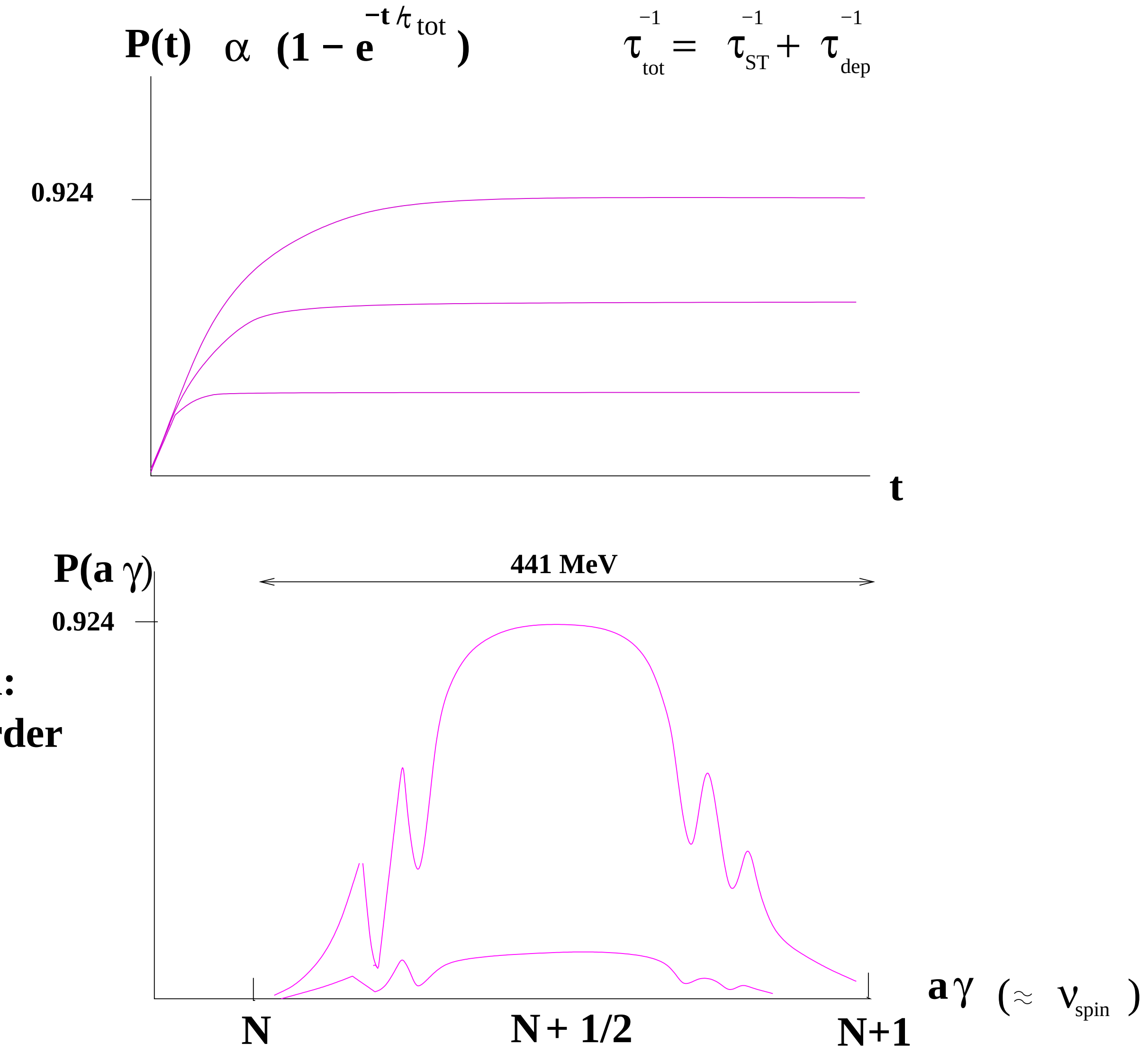
“Spin transparency”!!!
– but be wary of loose terminology: this is not the trivial kind on the design orbit !

The SLIM formalism and its high order forms based on the Derbenev-Kondratenko formalism are fine for first impressions

– but at high energy, there's no escaping model-independent, brute-force, multi-turn, Monte-Carlo, s-o tracking with full 3-D spin motion and with stochastic photon emission, to estimate the depolarisation rate, τ_{dep}^{-1}, and compare with the polarisation rate τ_{bks}^{-1}.

The structure of SLICKTRACK

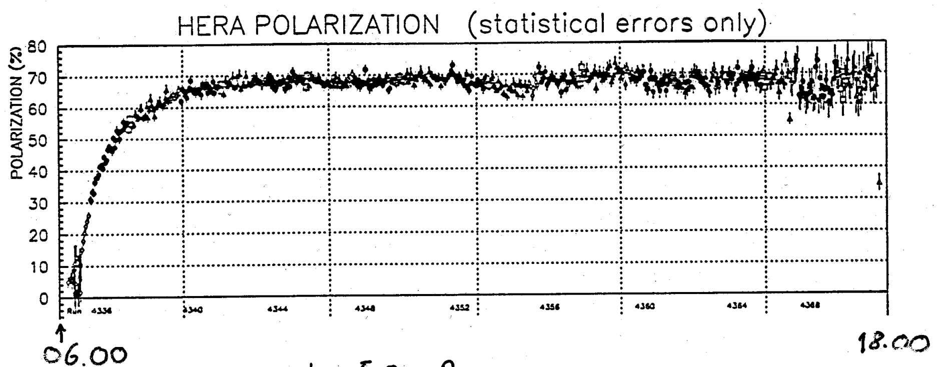




SLIM formalism:

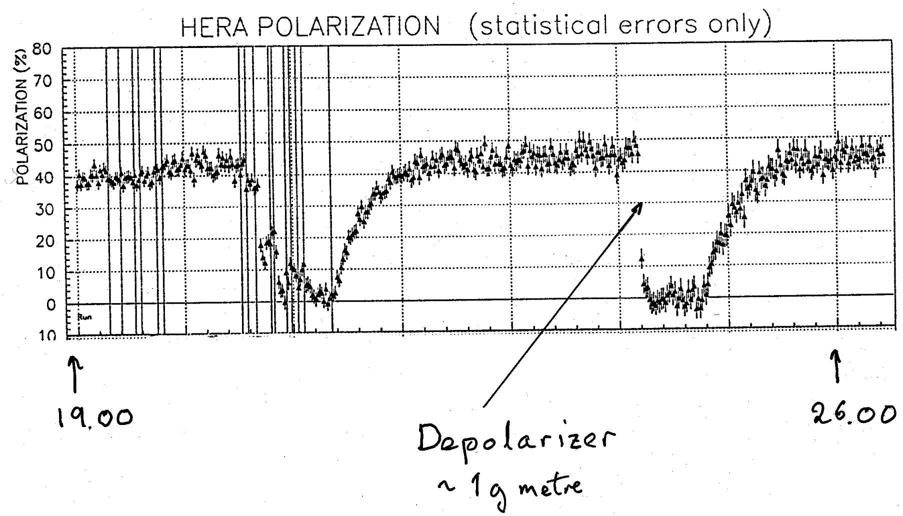
⟶ **only first order resonances**

July '95 : Hera Polarimeter Group.

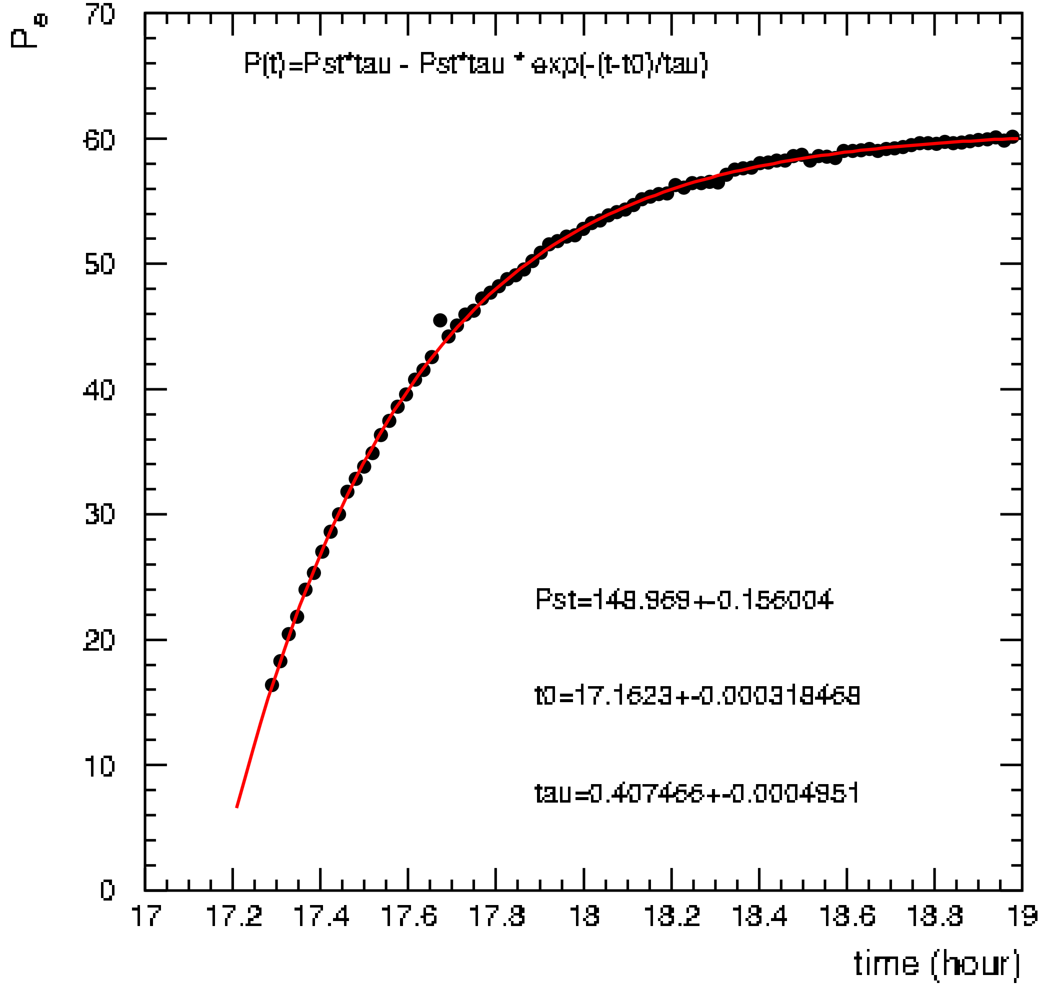


p: 50mA

e^+: 30mA → 10mA



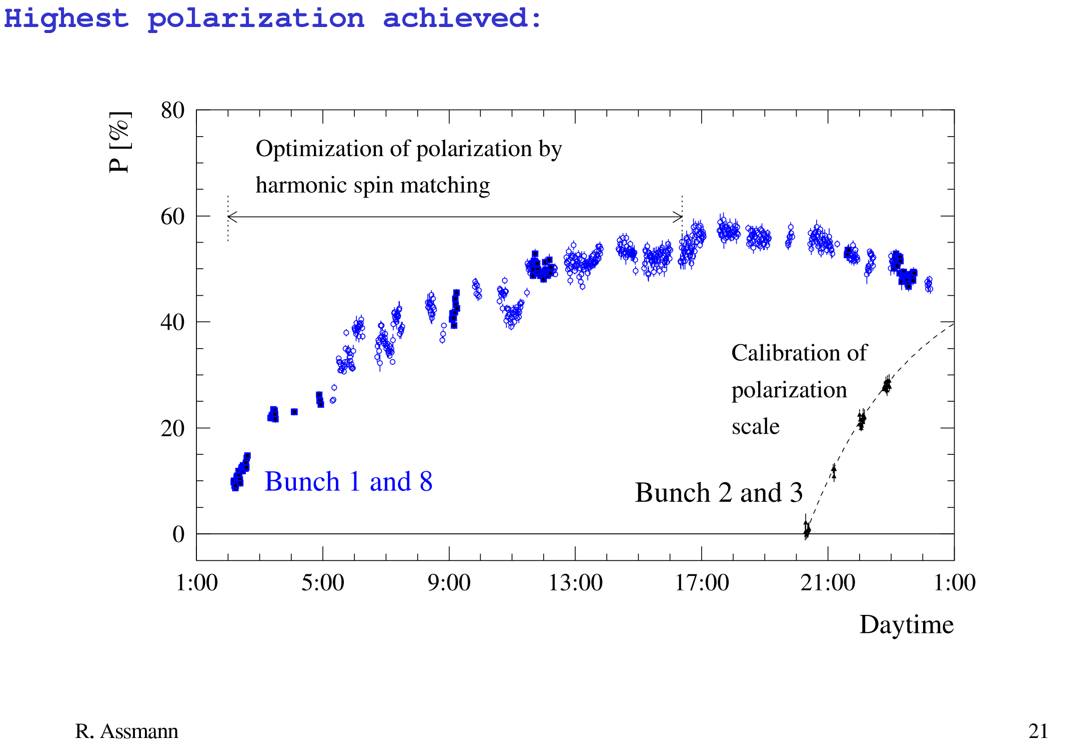
June 2007, the Fabry-Perot-Compton polarimeter of the POL2000 Project: Calibrating polarimeters



3 pairs of rotators (so max. **Sokolov-Ternov** polarisation = 83 %), solenoids on, no beam-beam

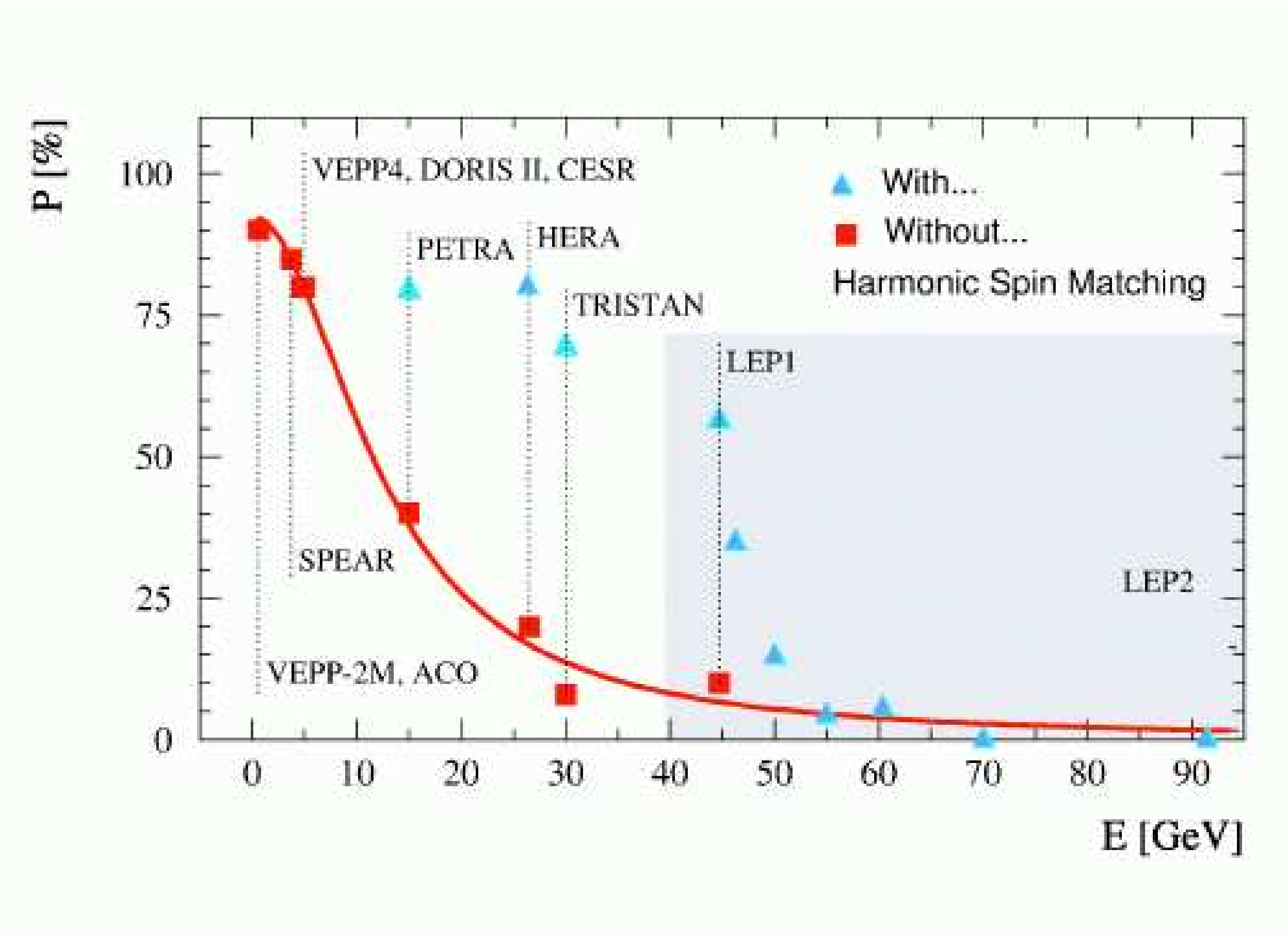
LEP

Polarisation from the Sokolov-Ternov effect at 46 GeV and above – the highest energy so far!



Vertical polarisation by the S-T effect, no rotators. "Deterministic" harmonic orbit correction.

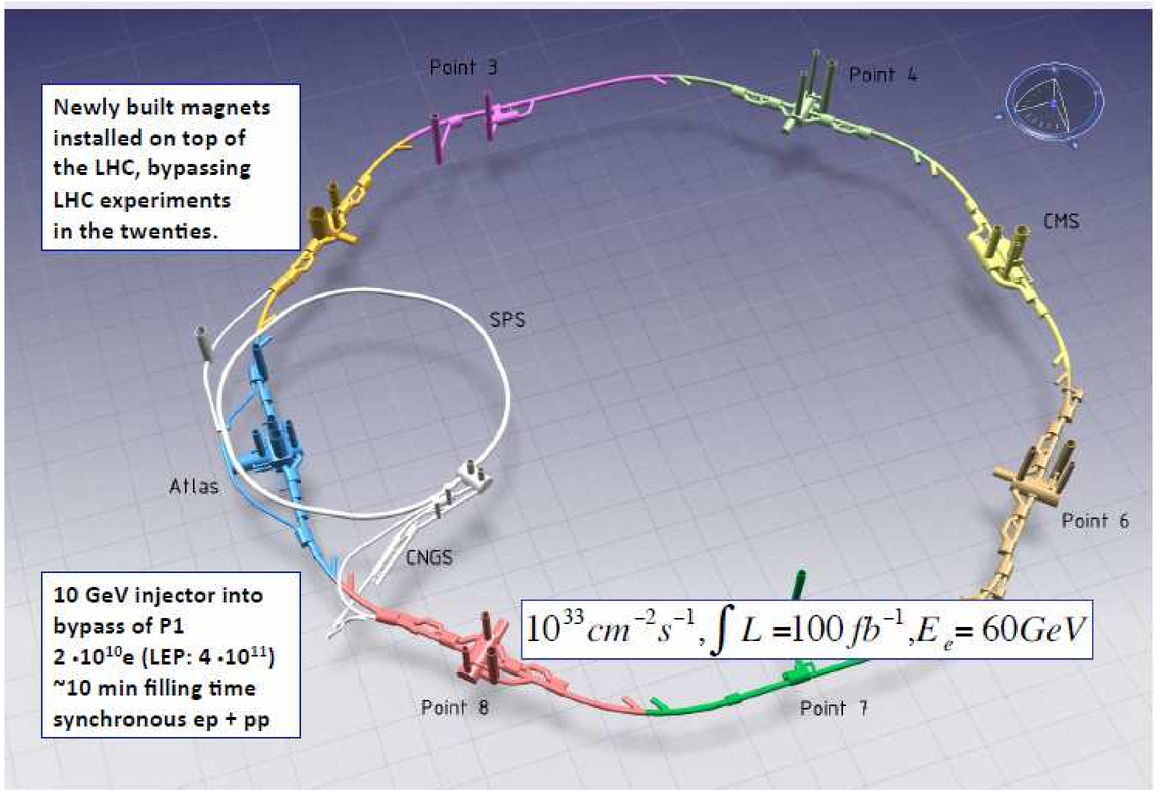
46 GeV, $\tau_{st} \approx 5$ hours



R. Assmann, SPIN2000, Osaka, Japan

The LHeC: ring - ring option.

Max Klein: at ICHEP Paris: Future Machines and Projects 24.7.2010



Now some work together with
H.-U. Wienands, M. Fitterer and H. Burkhardt

See for example:

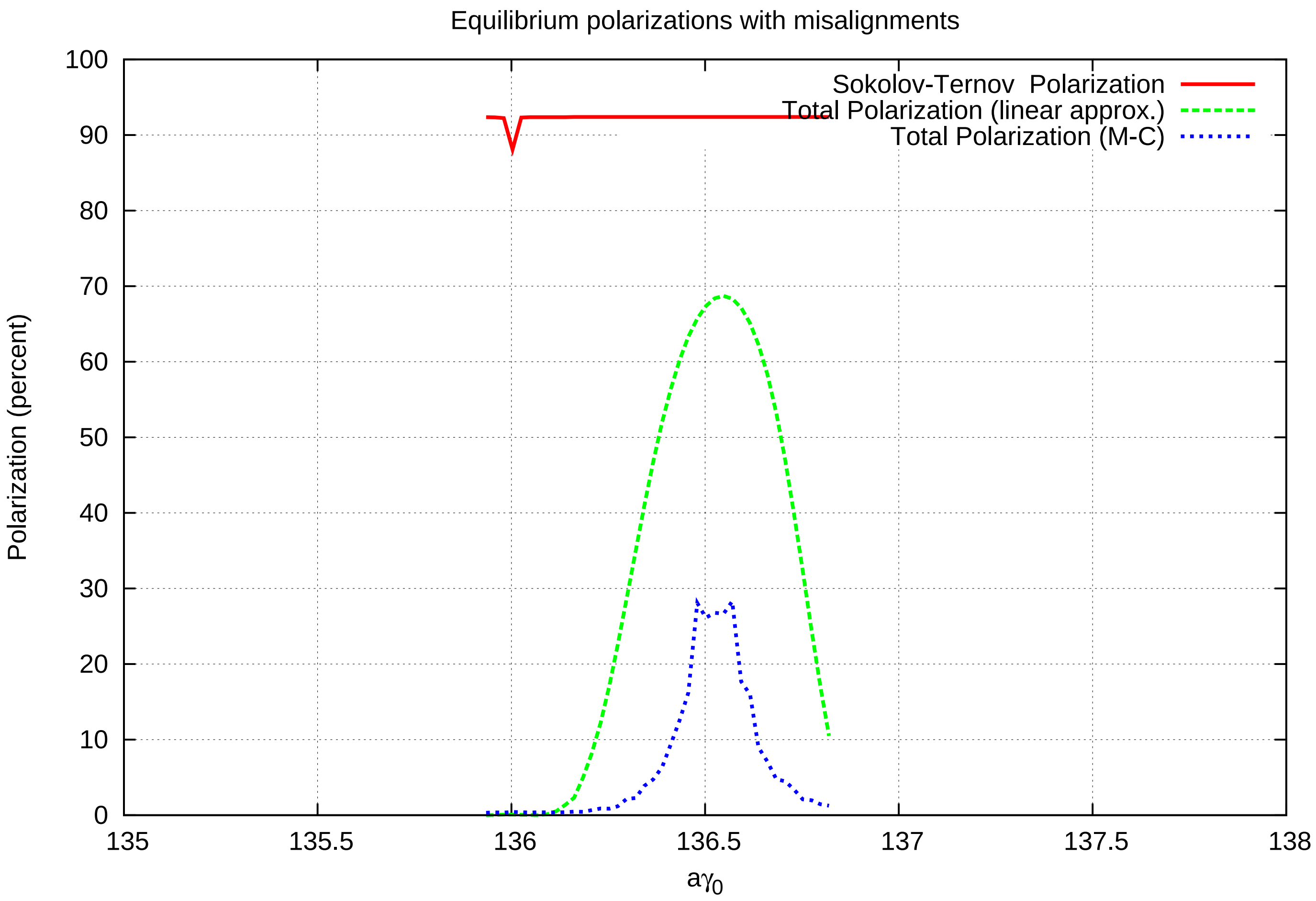
J. L. Abelleira Fernandez et al., Journal of Physics G: Nuclear and Particle Physics Volume 39 Number 7.

D.P. Barber, H.-U. Wienands, M. Fitterer and H. Burkhardt, Proceedings of SPIN2010, Juelich, Germany, October 2010.

Flat ring with vertical polarisation near 60 GeV

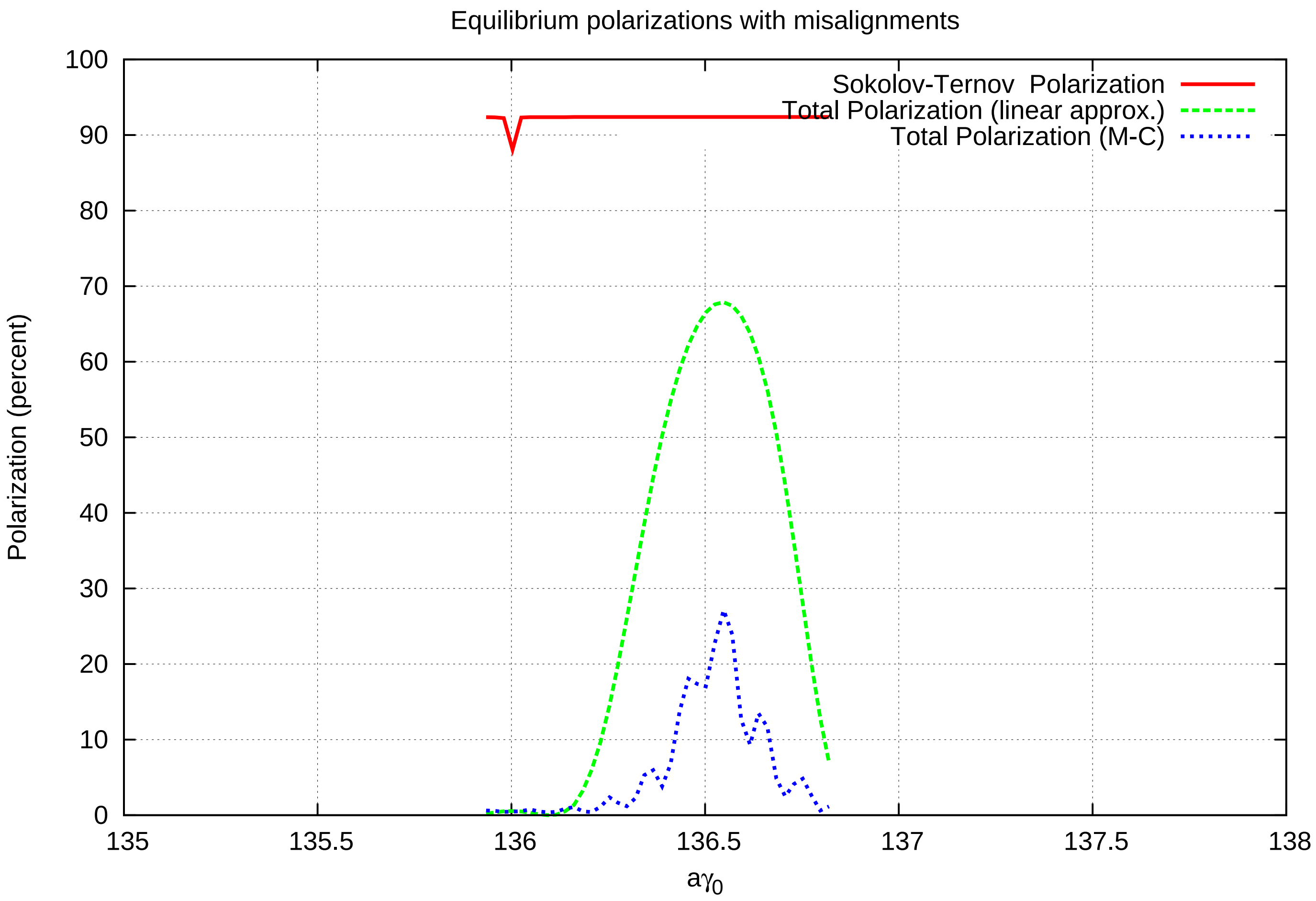
- $Q_x = 123.83$
- $Q_y = \ 85.62$
- $\sigma_{\rm vco} = 75$ microns
- R.m.s. tilt of $\hat{n}_0 \approx 4$ mrad near the peak polarisation.
  No harmonic closed-orbit spin matching so far.
- Radiative energy loss: 430 MeV per turn.
- Fractional spread in spin precession frequency: $a\gamma\frac{\sigma_\gamma}{\gamma} \approx 0.13$.

$Q_s = 0.06, \xi \approx 5$ **An example of just one misalignment**



Just linear orbital motion

$Q_s = 0.1, \xi \approx 1.9$. An example of just one misalignment



Just linear orbital motion

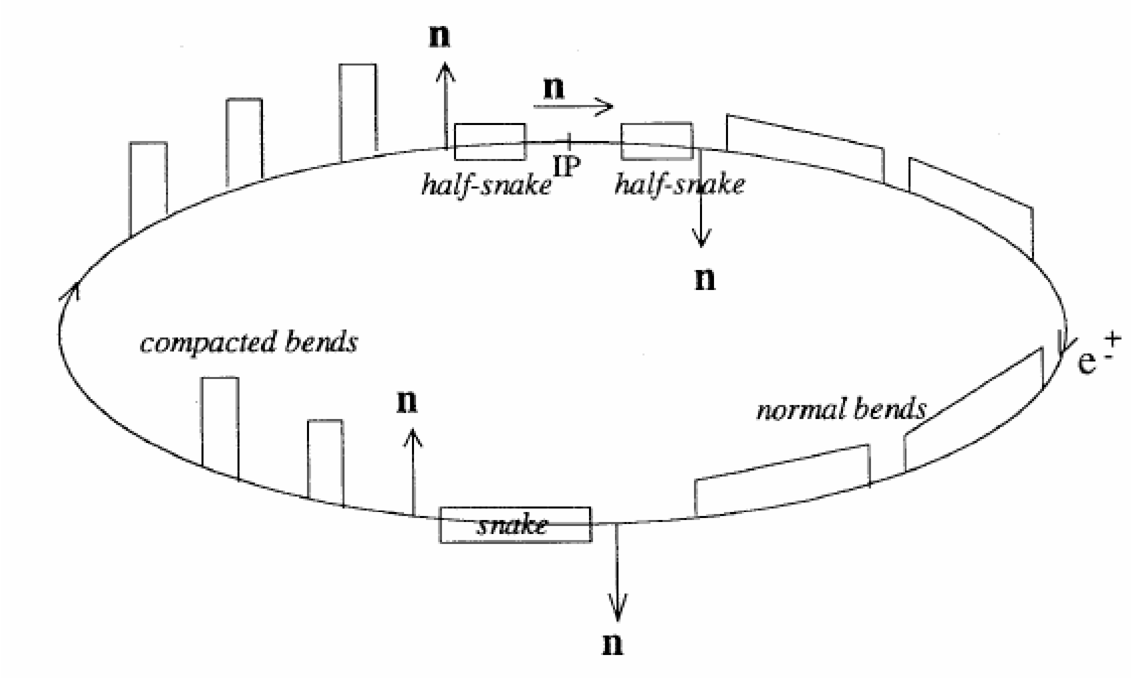
Summary on the flat ring with no rotators

- Initial calculations suggest that vertical polarisation would not be impossible with modern very good alignment.
- The dependence on Q_s is qualitatively as expected.
- The attainable equilibrium polarisation is highest at low energy as expected.

Longitudinal polarisation

- Need rotators ==> need serious spin matching.
- Rotators must be compatible with the contraints of the environment.
- Do Siberian Snakes help to suppress the effect of synchrotron sidebands by suppressing the oscillations of $a\gamma$?
- Naive use of snakes kills the Sokolov-Ternov polarisation!
- So need asymmetric distribution of radiation.
- Try the Derbenev-Grote scheme (1995).

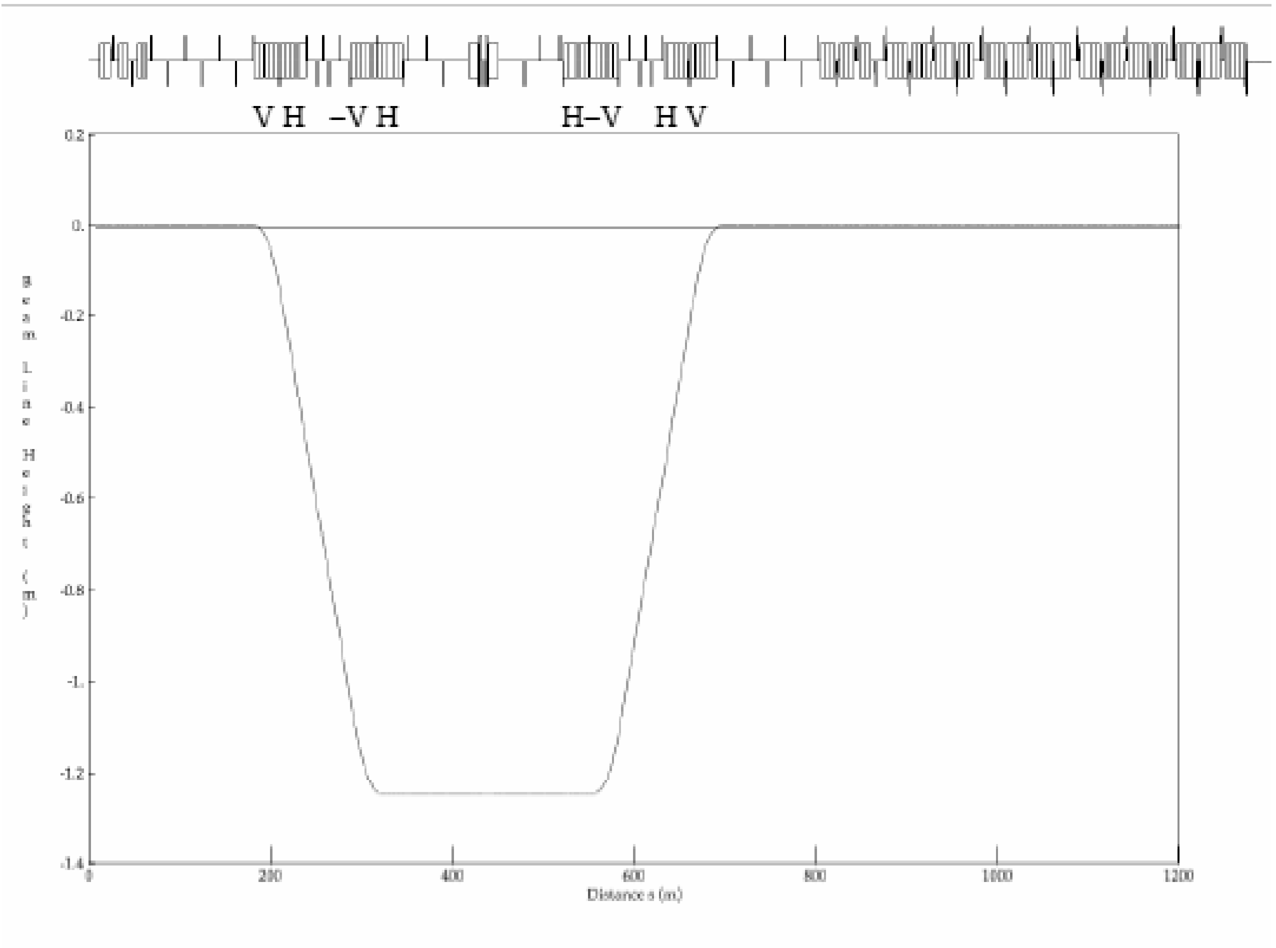
A suggestion by Ya. Derbenev and H. Grote



n means $\hat{n}_0$ here !!!

LHeC rotators

A vertical “dog leg”: by H.-U. Wienands



$\Longrightarrow$ very strong depolarisation – of course!

But we can switch spin-orbit coupling off/on to see what does what: the G matrix

So make the interaction region and rotators spin transparent in software.

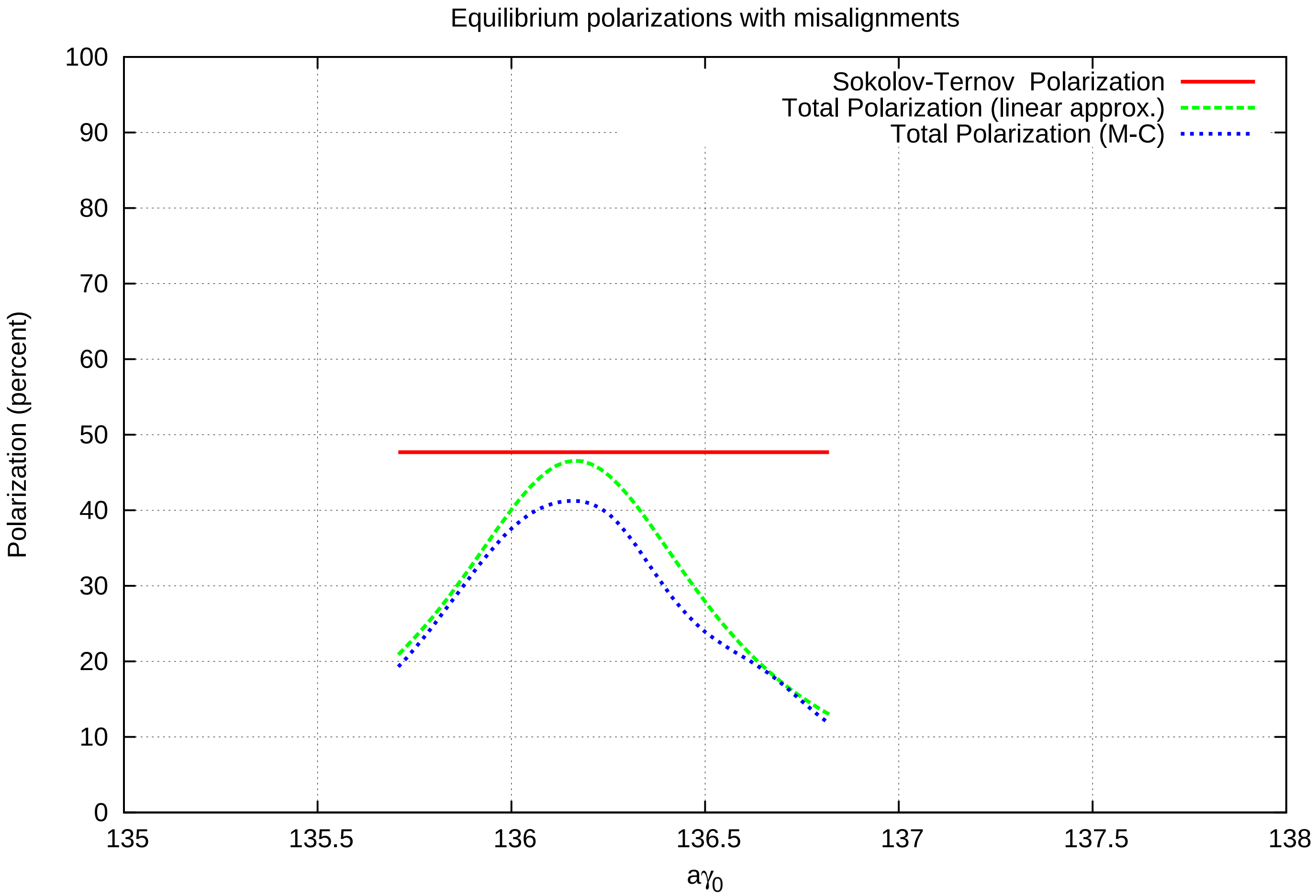
Diagnostics! – since 1982

The D–G set-up near 60 GeV

- $Q_x = 124.36$
- $Q_y = 88.80$
- $\sigma_{\rm vco} = 75$ microns
- R.m.s. tilt of $\hat{n}_0 \approx 8$ mrad near the peak polarisation. No harmonic closed-orbit spin matching so far.
- Radiative energy loss: 586 MeV per turn
- $a\gamma\frac{\sigma_\gamma}{\gamma} \approx 0.13$.
- An ideal thin lens snake which is transparent for orbital motion.
- ν_0 is almost independent of machine energy: around 0.41 (not 0.5 – because of the rotators).

Perfect alignment and with the IR G matrix off

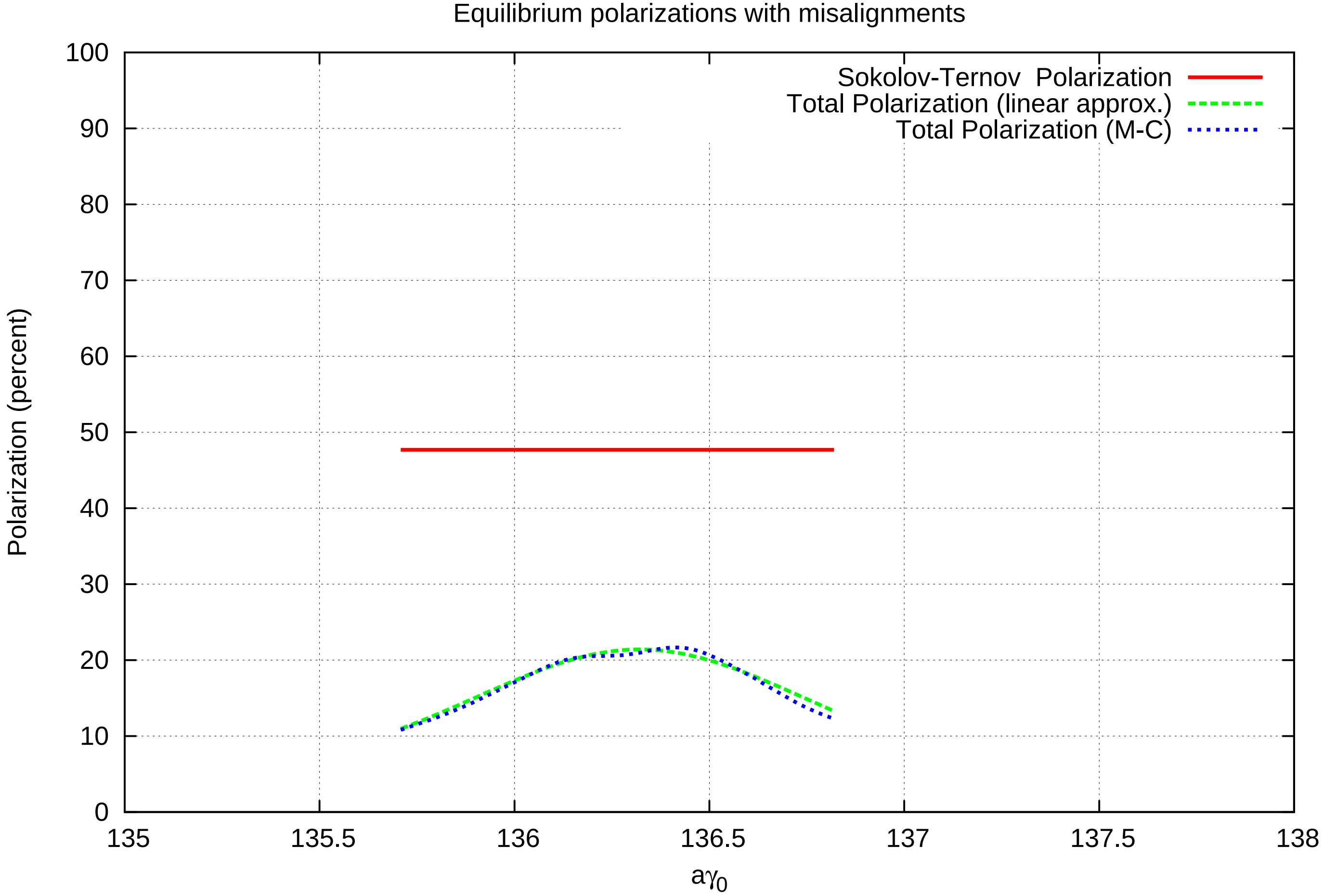
$Q_s = 0.1$



Just linear orbital motion

An example of just one misalignment and with the IR G matrix off

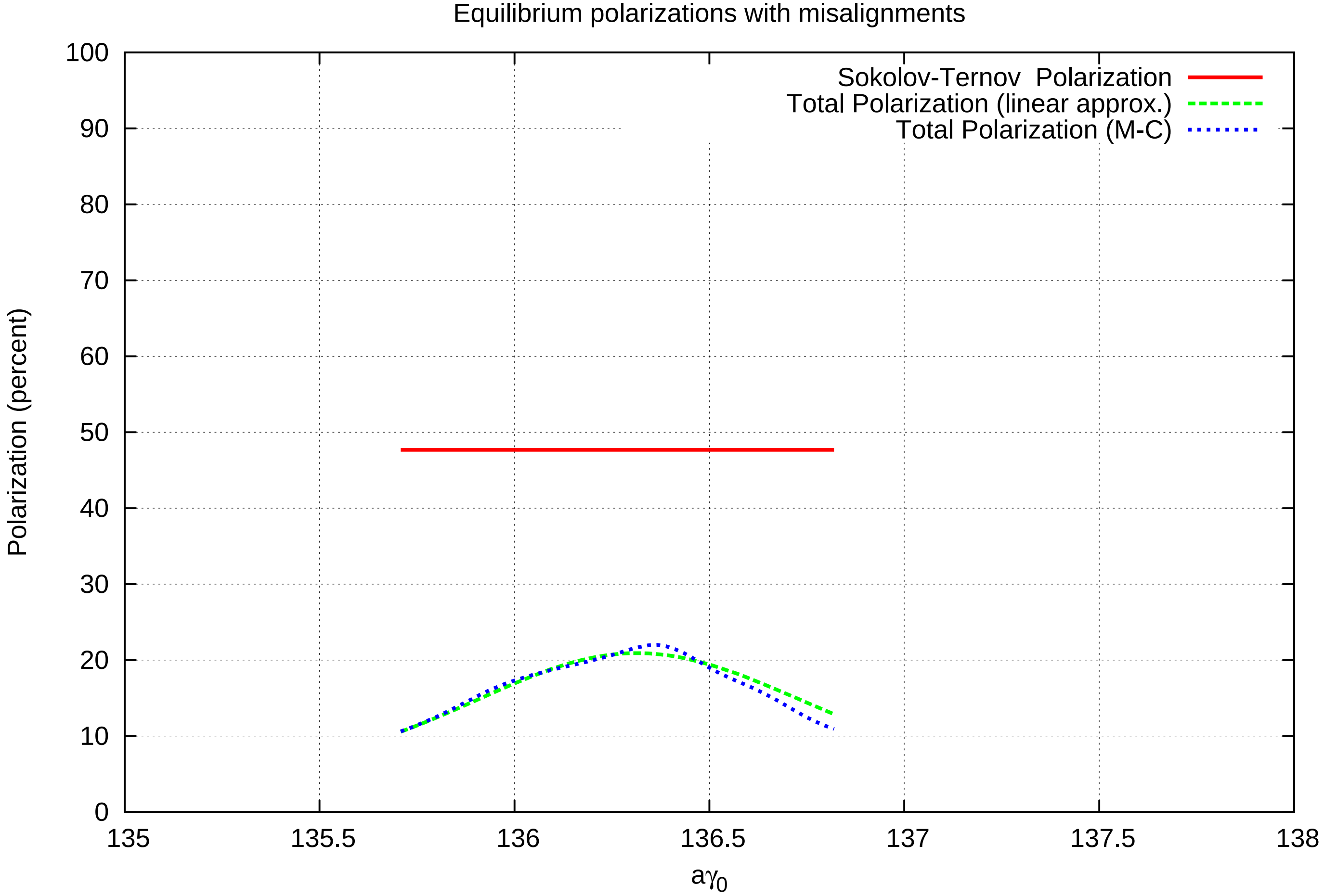
$Q_s = 0.06$



Just linear orbital motion

An example of just one misalignment and with the IR G matrix off

$Q_s = 0.10$



Just linear orbital motion

Summary on the model ring with rotators and a snake

- The maximum S-T polarisation is limited by the need for the asymmetric radiation distribution.
- In these calculations $\hat{n}_0$ is tilted from the vertical twice as much as for the flat ring $\Longrightarrow$ lower polarisation compared to the maximum S-T polarisation.
- With this rotator, $\hat{n}_0$ is tilted in the arcs away from design energy.
- Initial indications that the snake supresses the synchrotron sidebands. Much more investigation needed.
- —- the first time in the field that this topic had been invstigated.
- Essential to provide optical spin matching of the IR and arcs — obviously!.
- The dogleg rotator fits the need to bring the electron beam down to the proton beam.
- A practical snake design is needed.
- Optical spin matching is a big but necessary challenge.
- Harmonic closed orbit spin matching should be tested.
- In any case it would be essential to align the ring extremely well – but modern rings do have good alignment.

TLEP

$e^{-/+}$ self polarisation?

$80km, \quad 45GeV :\Longrightarrow \tau_{bks} \approx 150$ hours

To get $\tau_{tot} \approx 1$ hour we need $\tau_{dep}/\tau_{bks} \approx 1/150 \Longrightarrow P_{\infty} \approx \frac{92.4\%}{150} < 1\%$

D-G snakes:

$80km, \quad 90GeV :\Longrightarrow \tau_{bks} \approx 5$ hours

To get $\tau_{tot} \approx 30$ mins we need $\tau_{dep}/\tau_{bks} \approx 1/10 \Longrightarrow P_{\infty} \approx \frac{50\%}{10} = 5\%$

$\Longrightarrow$ another approach is needed!

Recommendation for self polarisation

It is not trivial to avoid depolarisation.

Build in appropriate facilities for electron polarisation right at the beginning!

Otherwise it is very unlikely that polarisation can be patched in later.

Preparations for electrons are also useful for polarised positrons

Finally:

In a regime where beam-beam forces are leading to beam instability,
the orbital motion can contain significant higher harmonics
$\Longrightarrow$ more spin-orbit resonances

Unrealistic dreams and hopes should not replace very careful calculations
based on details of a realistic design and optics !!