# Polarisation in electron rings

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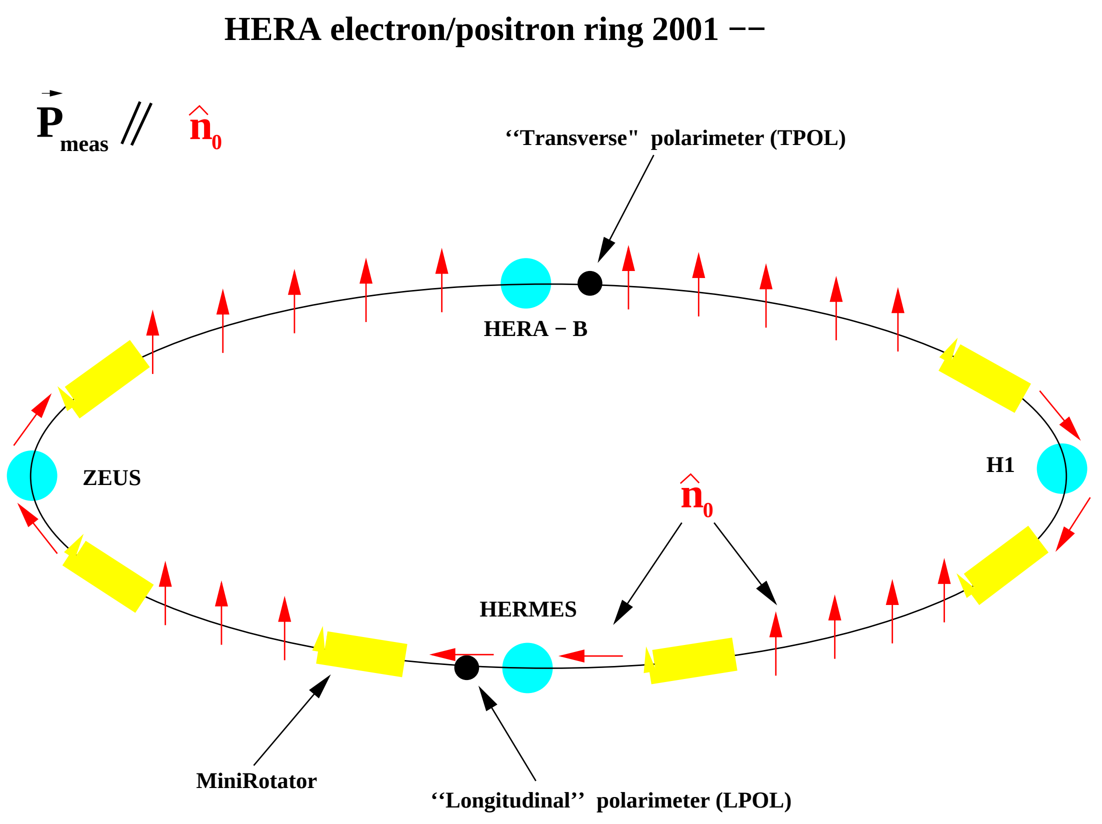
## Plan

- HERA – a reminder
- Some theory and phenomenology
- eRHIC ring-ring: calculations, including beam-beam effects
- ELIC: Comments and questions
- Things still to do

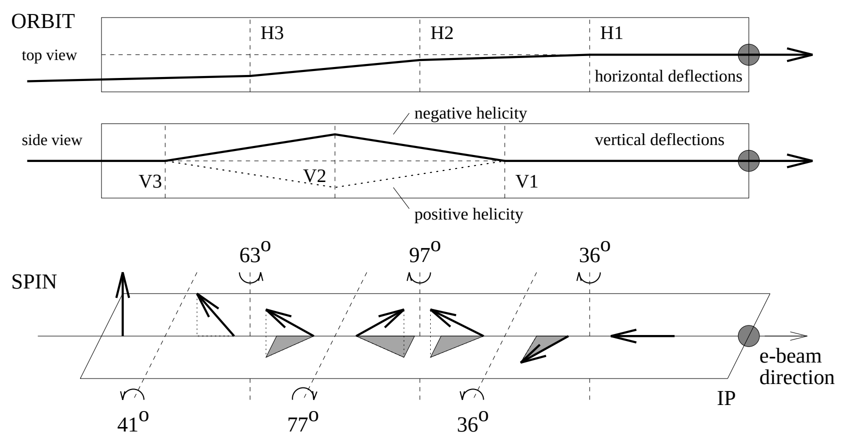
# HERA

The first and only $e^{\pm}$ ring to supply longitudinal polarisation at high energy — via the Sokolov-Ternov effect – also at 3 IP's simultaneously!




Polarisation vertical in the arcs – to drive the Sokolov-Ternov effect

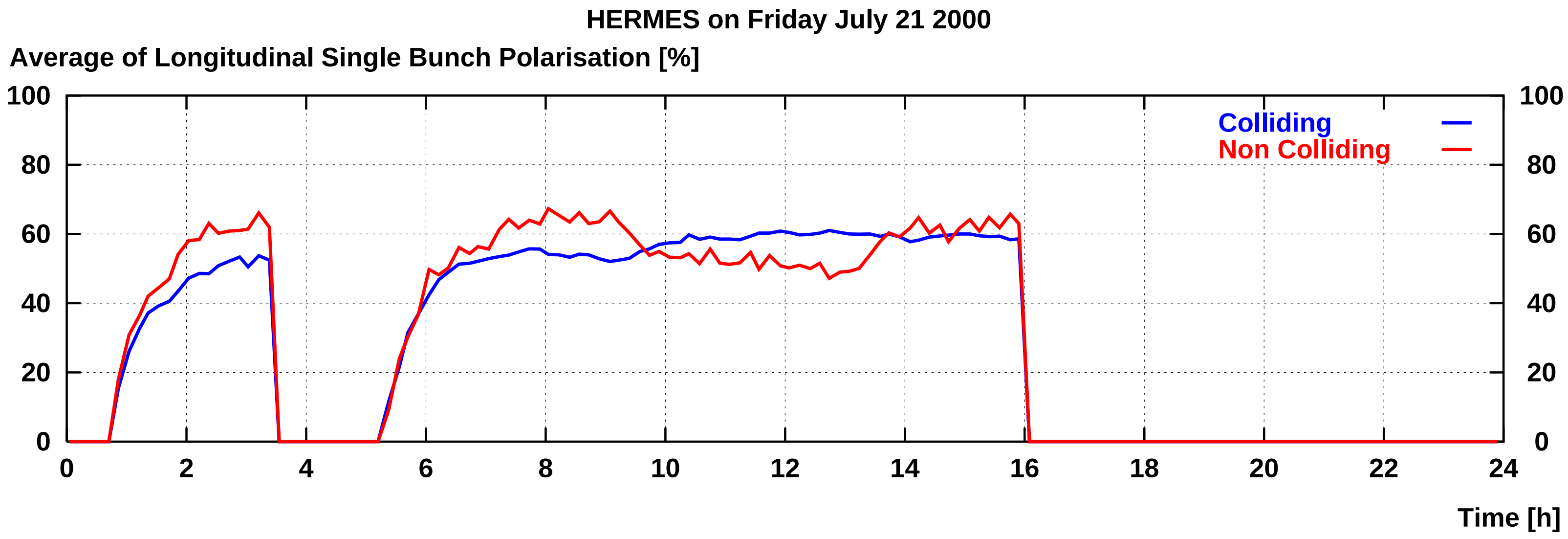
## HERA MiniRotator: Buon + Steffen



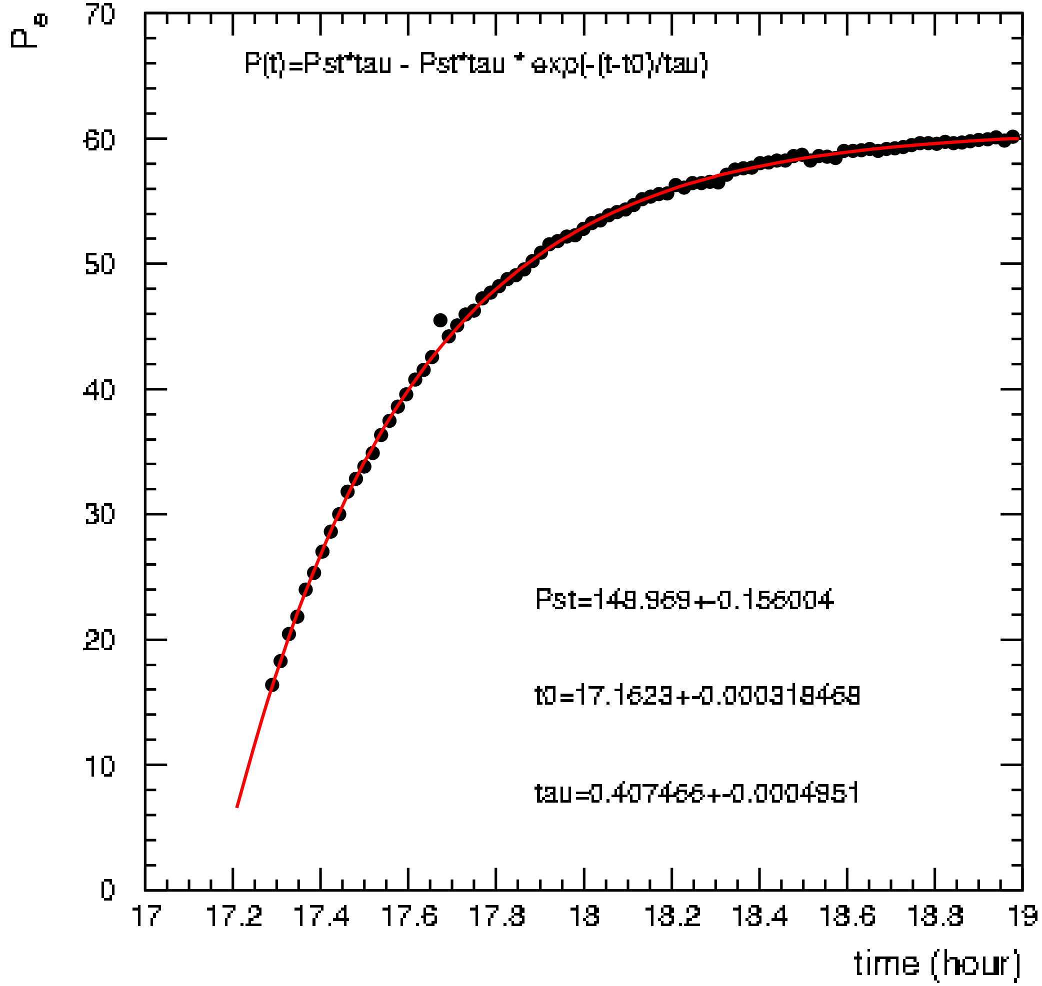

**56 m (“short”) → no quads.**

**27 – 39 GeV, both helicities, variable geometry**

NO INTERNAL QUADRUPOLES!


HERMES on Friday July 21 2000
Average of Longitudinal Single Bunch Polarisation [%]
Colliding
Non Colliding
Time [h]
0
20
40
60
80
100
0
2
4
6
8
10
12
14
16
18
20
22
24

June 2007, the Fabry-Perot-Compton polarimeter of the POL2000 Project: Calibrating polarimeters




3 pairs of rotators (so max. Sokolov-Ternov polarisation = 83 %), solenoids on, no beam-beam

# THEORY and PHENOMENOLOGY

## The T-BMT equation.

$$\frac{d\vec{S}}{ds} = \vec{\Omega}(\gamma, \vec{v}, \vec{B}, \vec{E}) \times \vec{S}$$

## Periodic solution $\hat{n}_0$ on closed orbit.

The real unit eigenvector of:

$$R_{3\times 3}(s+C, s)\hat{n}_0 = \hat{n}_0$$

$\hat{n}_0$ is 1–turn periodic: $\hat{n}_0(s+C) = \hat{n}_0(s)$

$\hat{n}_0$: direction of measured equilibrium radiative polarization.

Closed orbit spin tune $\nu_0$: number of precessions per turn around $\hat{n}_0$ for a spin on the closed orbit. Extract from the eigenvalues of $R_{3\times 3}(s+C, s)$

The **value** of the polarization is the same at all azimuths — time scales.

## Spin motions

- Protons: largely deterministic — unless IBS.
- Electrons/positrons:
  If a photon causes a spin flip, what are the other $\approx 10^{10}$ photons doing? ===>

  Stochastic/damped orbital motion due to synchrotron radiation
  + inhomogeneous fields
  + spin–orbit coupling via T–BMT
  ===> spin diffusion i.e. depolarisation!!!

  Self polarisation: Balance of poln. and depoln. ===>

$$P_{\infty} \approx P_{BK} \, \frac{1}{1 \; + \; (\frac{\tau_{dep}}{\tau_{BK}})^{-1}} \qquad (P_{ST} \to P_{BK})$$

In any case:

$$\tau_{dep}^{-1} \;\propto\; \gamma^{2N} \; \tau_{st}^{-1} \quad \text{(actually a polynomial in } \gamma^{2N}\text{)}$$

===> Trouble at high energy!

## Spin–orbit resonances

$$\nu_{\rm spin} \;=\; k + k_I \nu_I + k_{II} \nu_{II} + k_{III} \nu_{III}$$

$\nu_{\rm spin}$ : amplitude dependent spin tune ≈ closed orbit spin tune = precessions /turn on CO

- Orbit "drives spins" ===> Resonant enhancement of spin diffusion.
- Resonance order: $|k_I| + |k_{II}| + |k_{III}|$
- First order: $|k_I| + |k_{II}| + |k_{III}| = 1$ e.g. SLIM like formalisms.
- Strongest beyond first order:
  synchrotron sidebands of first order parent betatron or synchrotron resonances

$$\nu_{\rm spin} \;=\; k + k_i \nu_i + k_{III} \nu_{III}, \quad i = I, II \text{ or } III$$

## More on spin–orbit resonances

(1) Linear orbit motion with linearized spin motion (SLIM/SLICK):
just first order spin–orbit resonances.

(2) Linear orbit motion with full 3–D spin motion:
all orders of spin–orbit resonances.

(3) Non–linear orbit motion with linearized spin motion:
orders of spin–orbit resonances just reflecting the orbital spectrum.

(4) Non–linear orbit motion with full 3–D spin motion:
all orders of spin–orbit resonances.

**Diagnostics : With (1) and (3) we use spin motion to Fourier analyse the orbital motion!**

## Linear spin matching

Skip the invariant spin field and the Derbenev-Kondratenko formula for today!

Heuristics instead!

$$\vec{S} \approx \hat{n}_0(s) + \alpha \hat{m}_0(s) + \beta \hat{l}_0(s)$$

$\alpha, \beta$: 2 small spin tilt angles — have subtracted out the big rotations!

$$\hat{\mathbf{M}}_{8\times 8} = \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix}$$

acting on $\vec{u} = (x,\ x',\ y,\ y',\ l,\ \delta)$ and $\alpha, \beta$

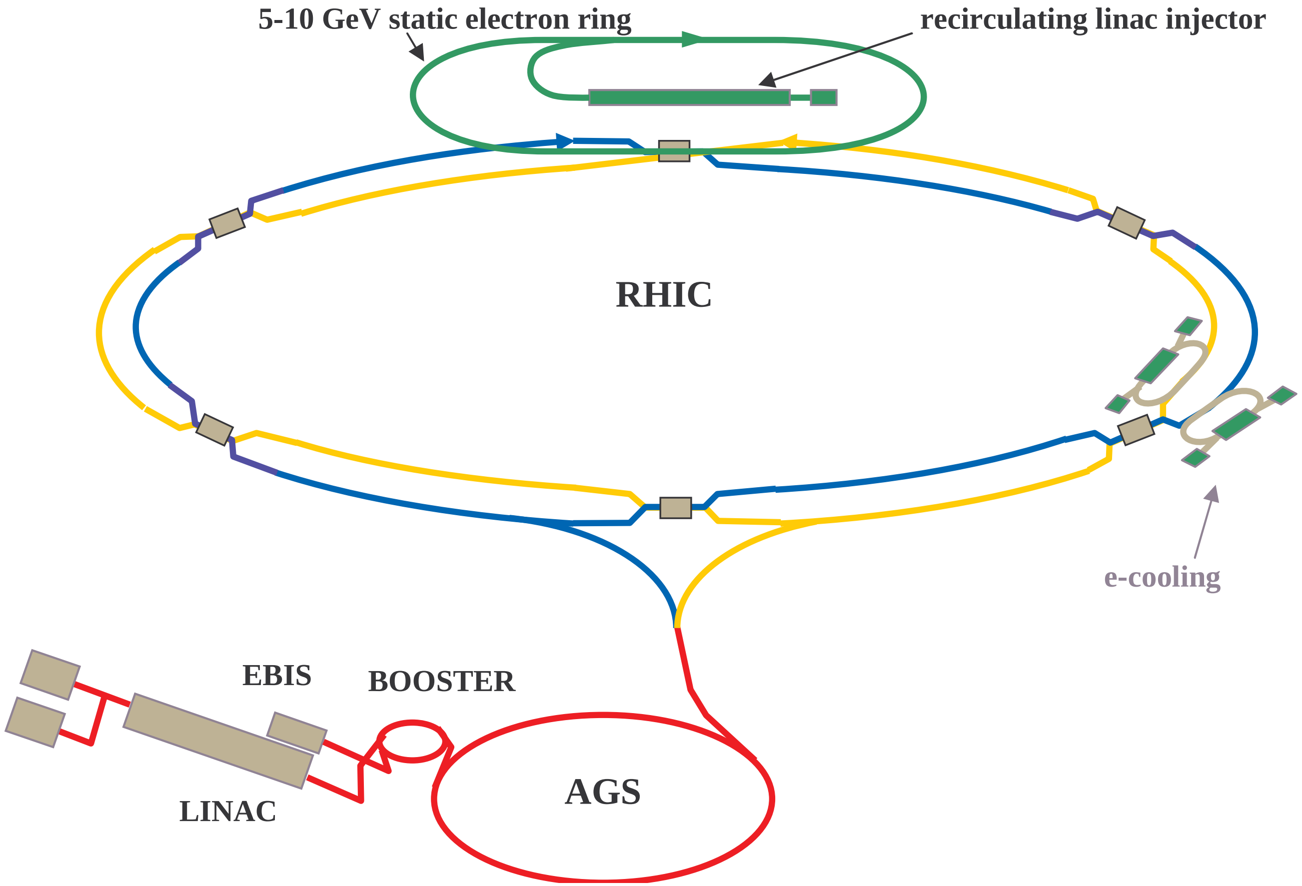
This is the SLIM formalism for estimating depolarisation analytically at first order (Chao 1981).

To minimize depolarization:
minimize appropriate bits of $\mathbf{G_{2\times 6}}$ for appropriate stretches of ring
===> lots of independent quadrupole circuits.

Spin transparency!!! – not the trivial kind to which this term is applied at certain labs.

# eRHIC: ring-ring option

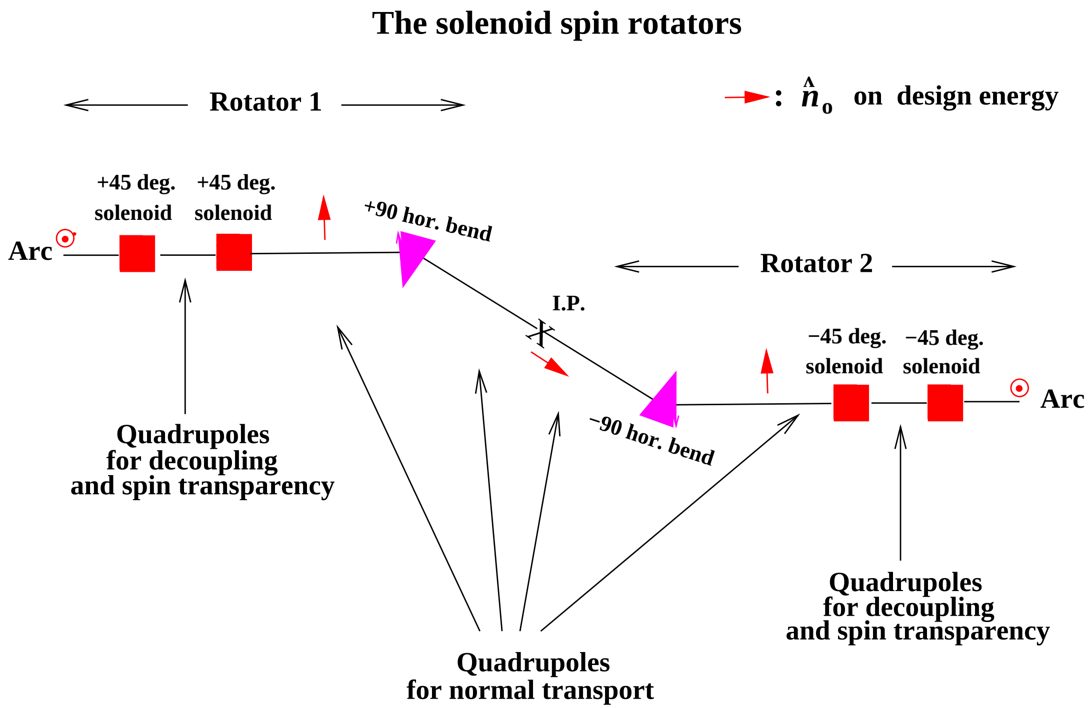
# The Layout


5-10 GeV static electron ring
recirculating linac injector
RHIC
e-cooling
EBIS
BOOSTER
LINAC
AGS

# Parameters

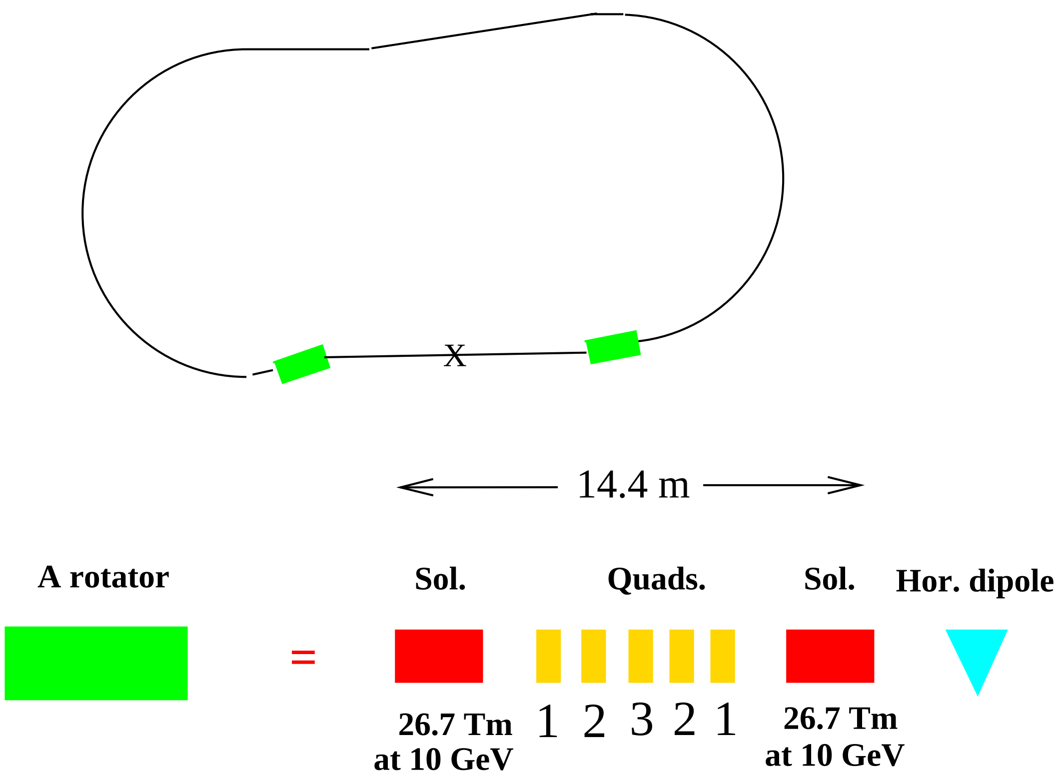
Table 1.1. Luminosities and main beam parameters for electron(positron)-proton collisions.

| | High energy setup | | Low energy setup | |
|---|---|---|---|---|
| | p | e | p | e |
| Energy, GeV | 250 | 10 | 50 | 5 |
| Bunch intensity, $10^{11}$ | 1 | 1 | 1 | 1 |
| Ion normalized emittance, $\pi$ mm$\cdot$mrad, x/y | 15/15 | | 5/5 | |
| rms emittance, nm, x/y | 9.5/9.5 | 53/9.5 | 16.1/16.1 | 85/38 |
| $\beta^*$, cm, x/y | 108/27 | 19/27 | 186/46 | 35/20 |
| Beam-beam parameters, x/y | 0.0065/0.003 | 0.03/0.08 | 0.019/0.0095 | 0.036/0.04 |
| $\kappa=\varepsilon_y/\varepsilon_x$ | 1 | 0.18 | 1 | 0.45 |
| Luminosity, 1.e32 $cm^{-2}s^{-1}$ | 4.4 | | 1.5 | |




$\hat{n}_0$ exactly longitudinal at just at one energy.

The basic eRHIC geometry for spin−−exaggerated

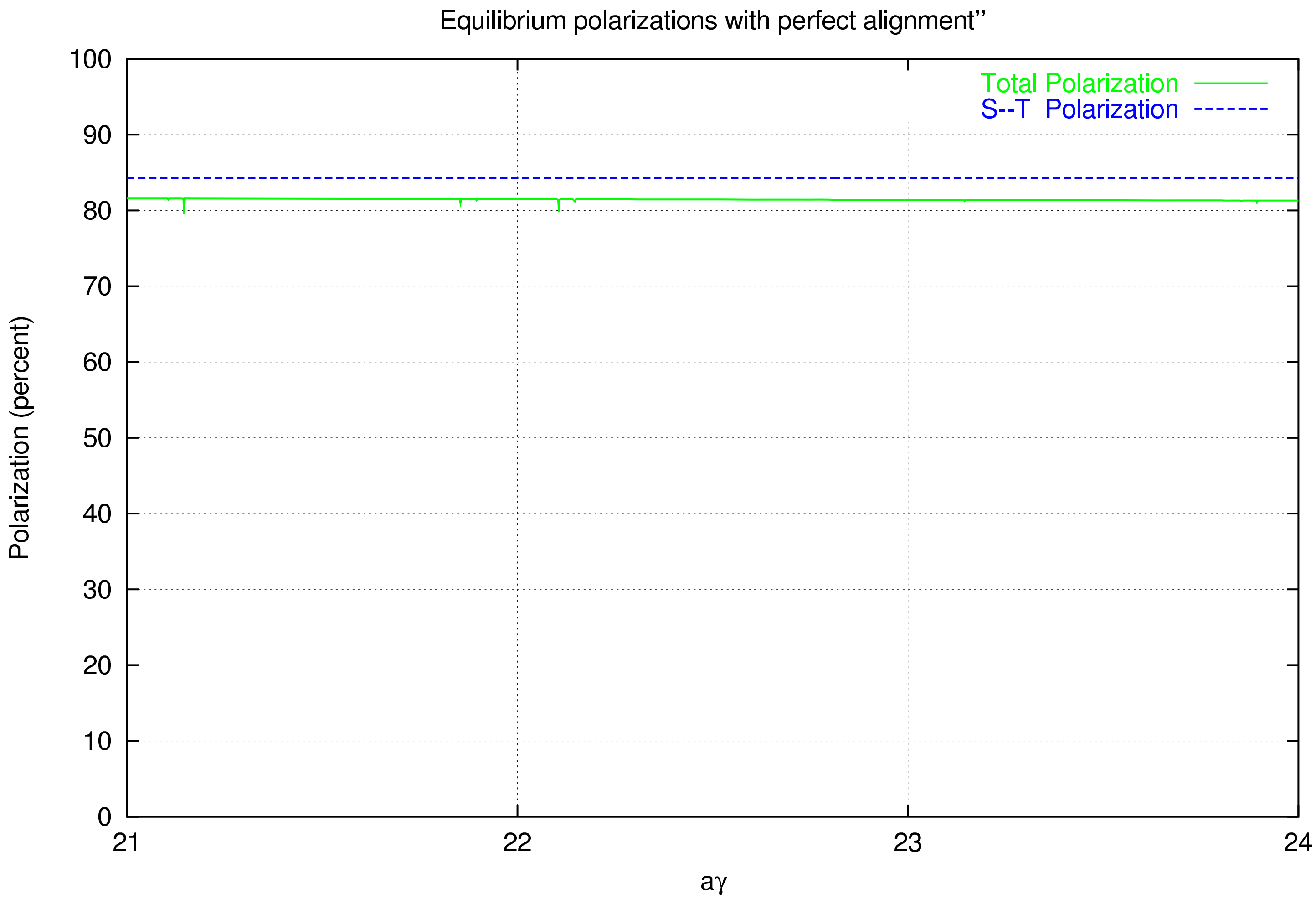


The $4 \times 4$ transfer matrix for the transverse motion through a pair of solenoids:

$$\begin{pmatrix} 0 & -2r & 0 & 0 \\ 1/2r & 0 & 0 & 0 \\ 0 & 0 & 0 & 2r \\ 0 & 0 & -1/2r & 0 \end{pmatrix}$$

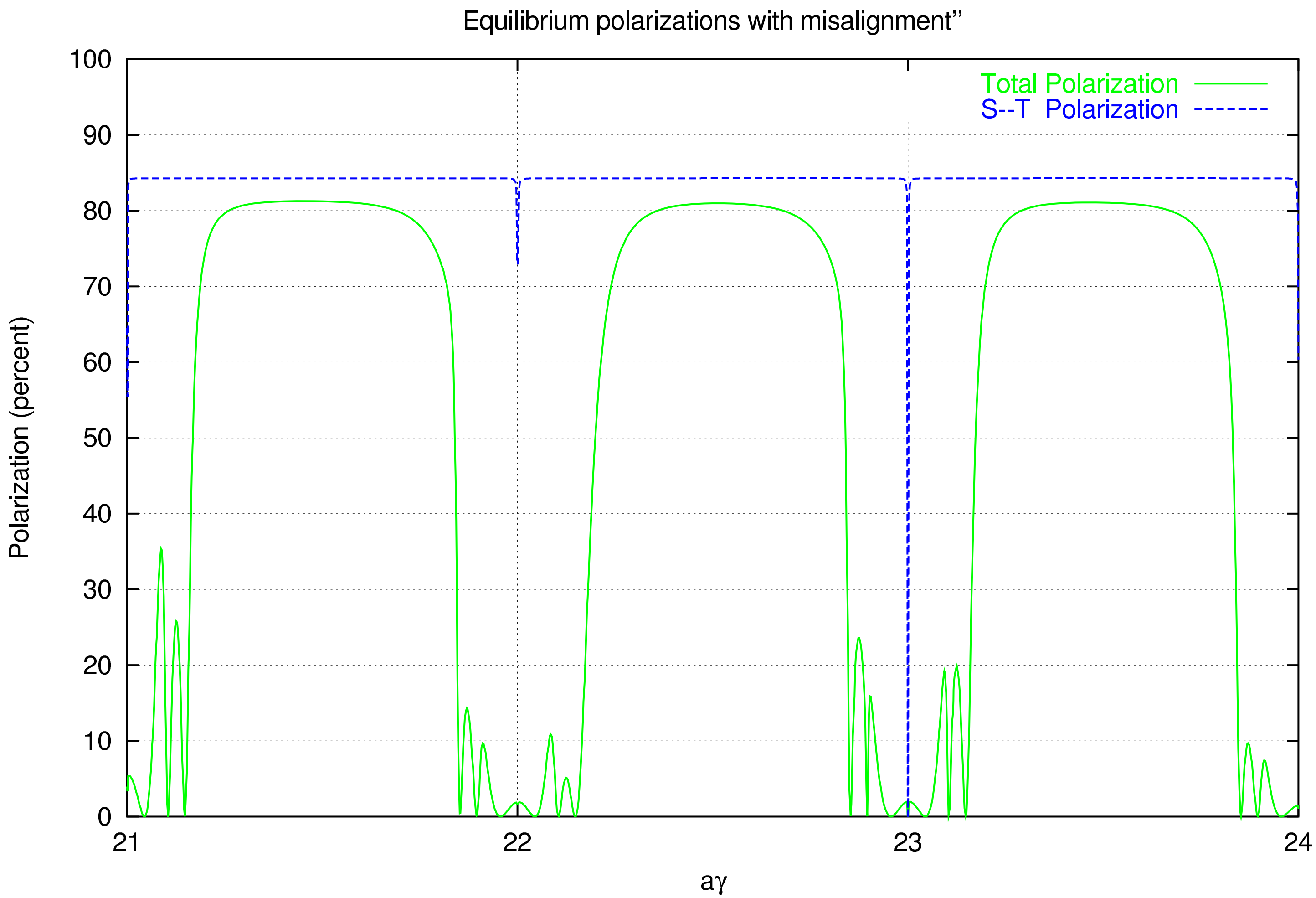
where $r$ is the radius of orbit curvature in the longitudinal field.

Use 5 back–to–back symmetric quadrupoles.

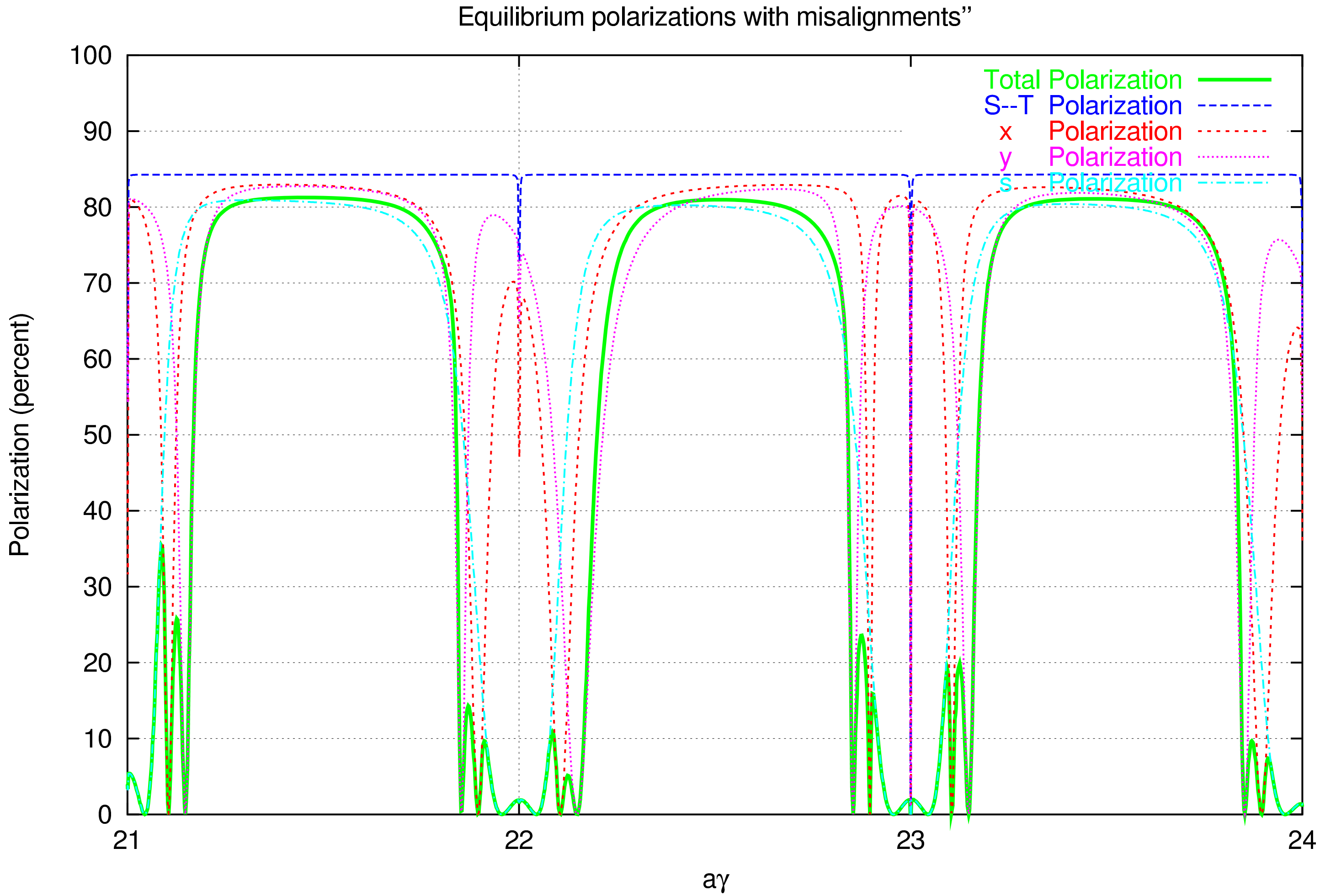

Equilibrium polarizations with perfect alignment"
Total Polarization
S--T Polarization
Polarization (percent)
0
10
20
30
40
50
60
70
80
90
100
21
22
23
24
aγ

$$\vec{P}_{\rm bks} \;=\; -\frac{8}{5\sqrt{3}}\,\hat{n}_0\,\frac{\oint ds\,\frac{\hat{n}_0(s)\cdot\hat{b}(s)}{|\rho(s)|^3}}{\oint ds\,\frac{\left[1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2\right]}{|\rho(s)|^3}}$$

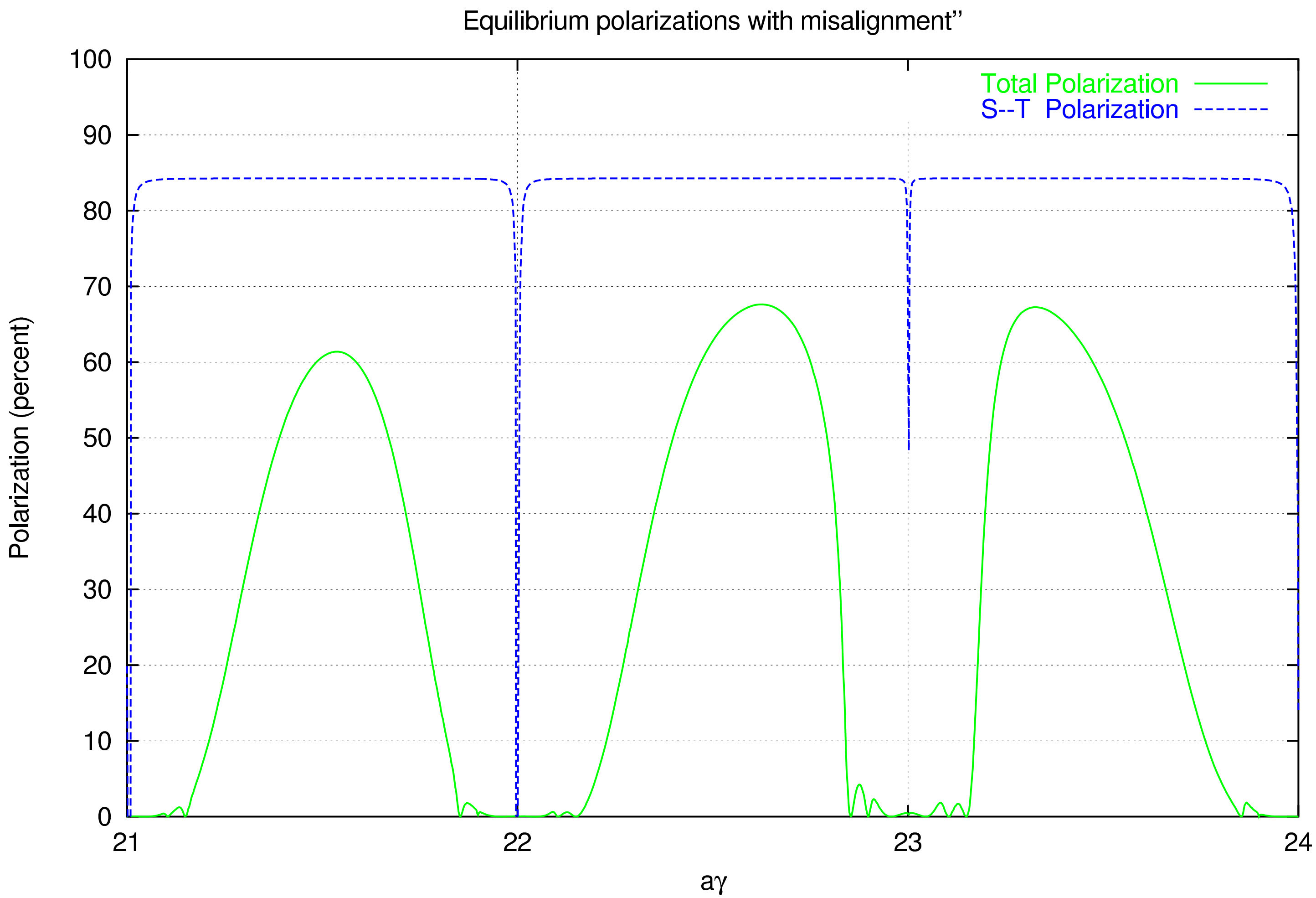
## All monitors on: just ONE example!

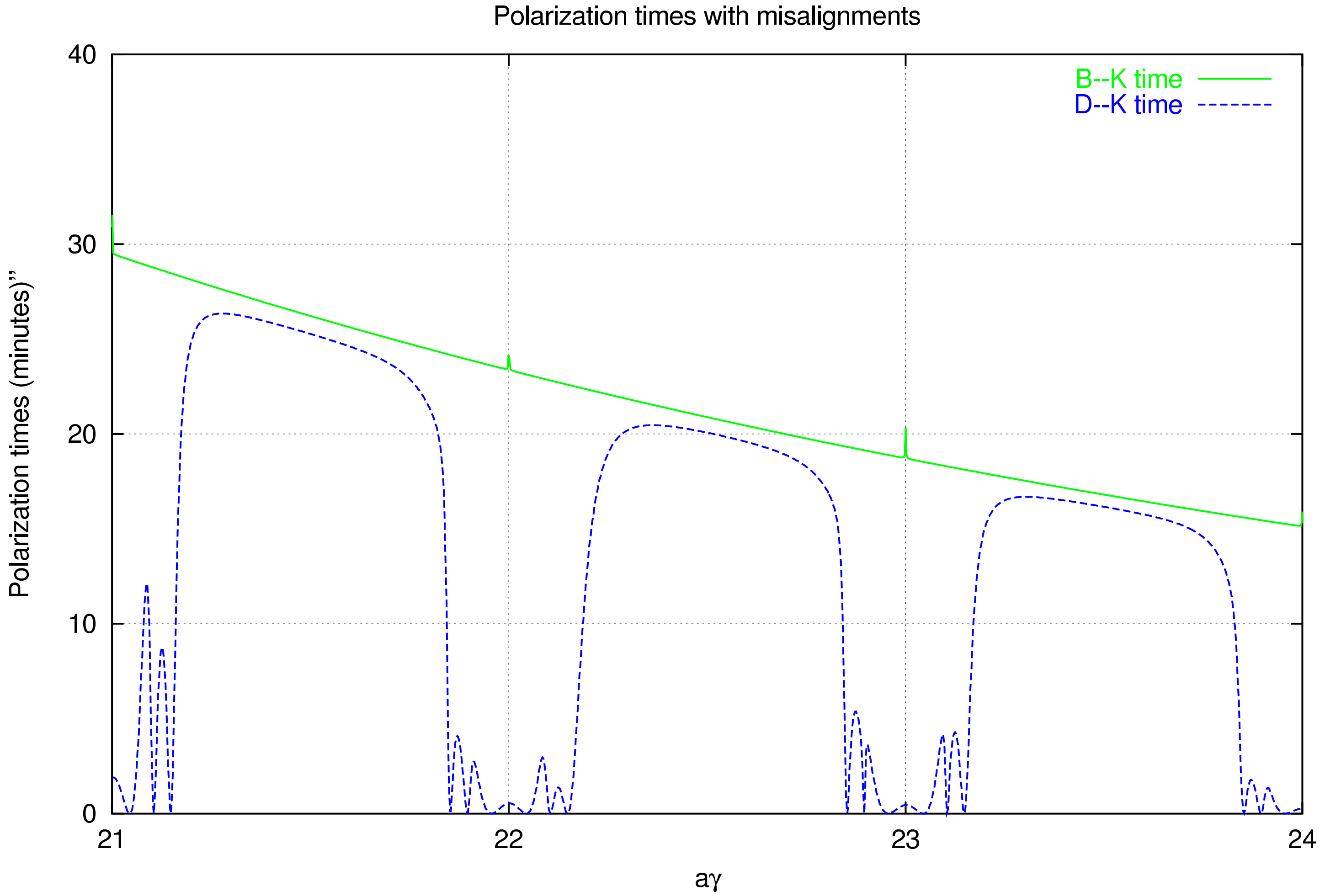


# All monitors on: DIAGNOSTICS

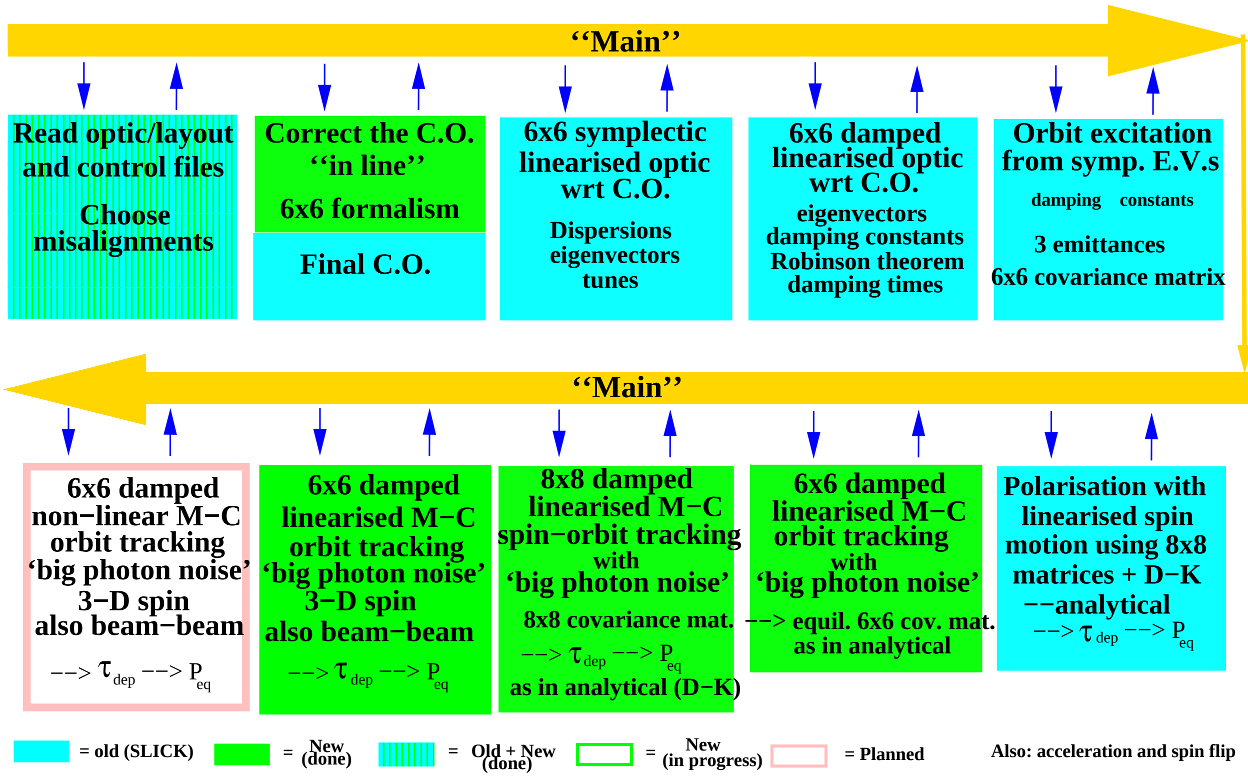


# 20 percent monitors on: just ONE example!




Polarization times with misalignments
B--K time
D--K time
Polarization times (minutes)"
aγ
40
30
20
10
0
21
22
23
24

The structure of SLICKTRACK



## Spin coordinates

$$\hat{S} = \sqrt{1-\alpha^2-\beta^2}\;\hat{n}_0 + \alpha\hat{m} + \beta\hat{l}$$

## Estimating depolarisation by M-C simulation $\alpha^2+\beta^2 << 1$

$$\Delta P \approx -\frac{1}{2}\Delta(<\alpha^2+\beta^2>) = -\frac{1}{2}\Delta(\sigma_\alpha^2+\sigma_\beta^2) \quad \Longrightarrow \frac{dP}{dt} \approx -\frac{1}{2} = -\frac{1}{2}\frac{d}{dt}(\sigma_\alpha^2+\sigma_\beta^2)$$

## Spin–orbit covariance matrix

$$\begin{pmatrix}
\sigma_x^2 & \sigma_{xx'} & . & . & . & . & | & . & . \\
\sigma_{x'x} & \sigma_{x'}^2 & . & . & . & . & | & . & . \\
. & . & . & . & . & . & | & . & . \\
. & . & . & . & . & . & | & . & . \\
. & . & . & . & . & . & | & . & . \\
. & . & . & . & . & \sigma_\delta^2 & | & . & . \\
- & - & - & - & - & - & - & - & - \\
. & . & . & . & . & . & | & \sigma_\alpha^2 & \sigma_{\alpha\beta} \\
\sigma_{\beta x}. & . & . & . & . & . & | & \sigma_{\beta\alpha} & \sigma_\beta^2
\end{pmatrix}$$

## Spin–orbit maps for sections

For linearised spin motion (SLIM/SLICK):

$$\hat{\mathbf{M}} = \begin{pmatrix} \mathbf{M_{6\times6}} & \mathbf{0_{6\times2}} \\ \mathbf{G_{2\times6}} & \mathbf{D_{2\times2}} \end{pmatrix}$$

The $\mathbf{G_{2\times6}} \times (\mathbf{x}, \mathbf{x}', \mathbf{y}, \mathbf{y}', \mathbf{\Delta l}, \delta)^{\mathbf{T}}$ delivers changes to the 2 small angles $\alpha$ and $\beta$

For full 3–D spin motion:

$$\hat{\mathbf{M}} = \begin{pmatrix} \mathbf{M_{6\times6}} & \mathbf{0_{6\times3}} \\ \mathbf{G_{3\times6}} & \mathbf{D_{3\times3}} \end{pmatrix}$$

The $\mathbf{G_{3\times6}} \times (\mathbf{x}, \mathbf{x}', \mathbf{y}, \mathbf{y}', \mathbf{\Delta l}, \delta)^{\mathbf{T}}$ delivers rotations around $\hat{n}_0,\ \hat{m}_0,\ \hat{l}_0$

The beam–beam (non–linear) kicks are applied at single points

## Diagnostics! Diagnostics! Diagnostics!

**Switch spin-orbit coupling off/on to see what does what.**

Sidebands of parent first order betatron resonances: a useful **approximation**

$$\tau_{dep}^{-1} \propto \frac{A}{(\nu_0 \pm Q_y)^2} \quad \rightarrow \quad \tau_{dep}^{-1} \propto \sum_{m_s=-\infty}^{\infty} \frac{A\, B(\xi; m_s)}{(\nu_0 \pm Q_y \pm m_s\, Q_s)^2}$$

$A$ is an energy dependent factor

$B(\xi; m_s)$'s: *enhancement factors*, contain modified Bessel functions $I_{|m_s|}(\xi)$ and $I_{|m_s|+1}(\xi)$ depending on the *modulation index*

$$\xi = (\frac{a\gamma\ \sigma_\delta}{Q_s})^2$$

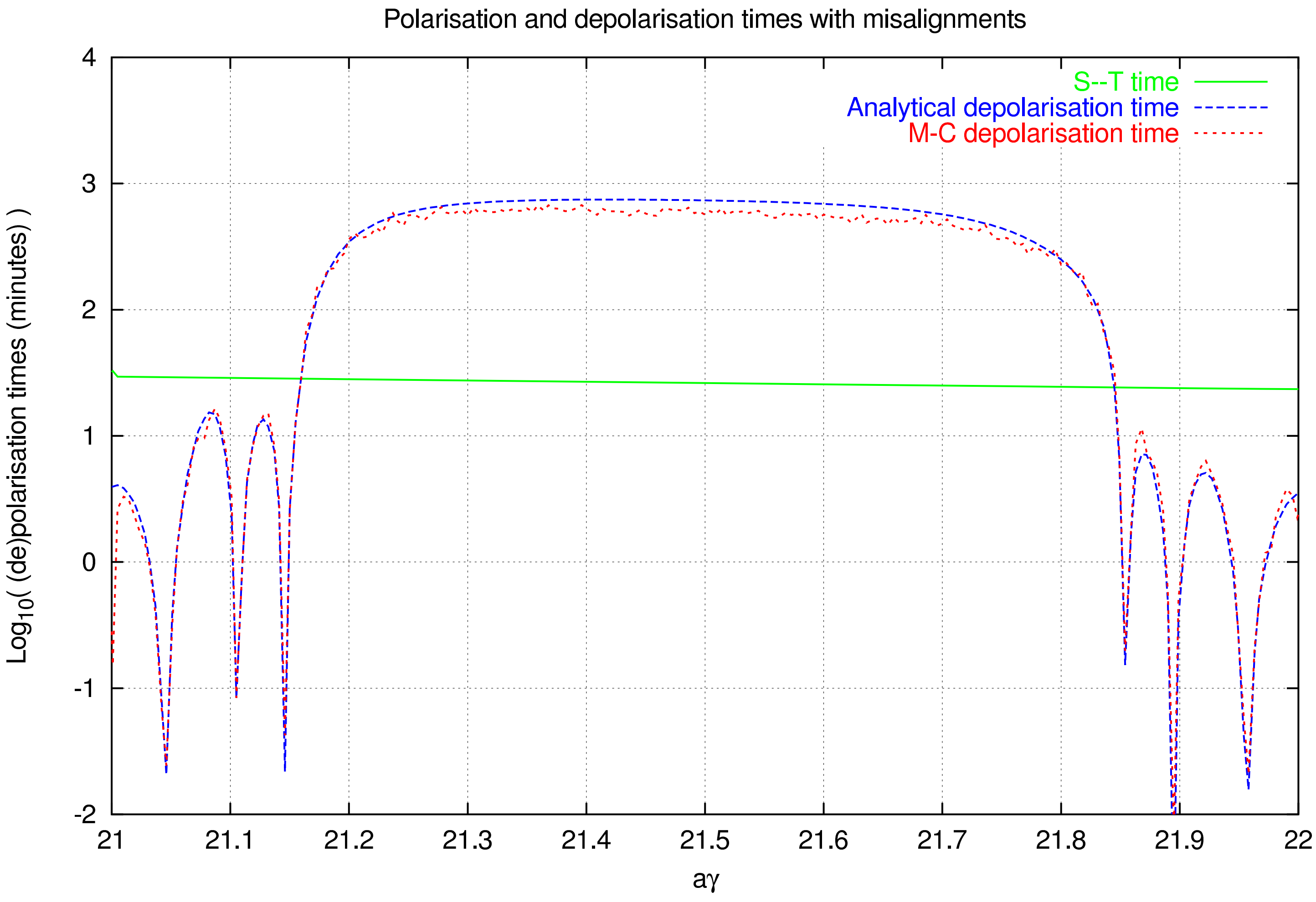
in a flat ring.

===> very strong effects at high energy — dominant source of trouble

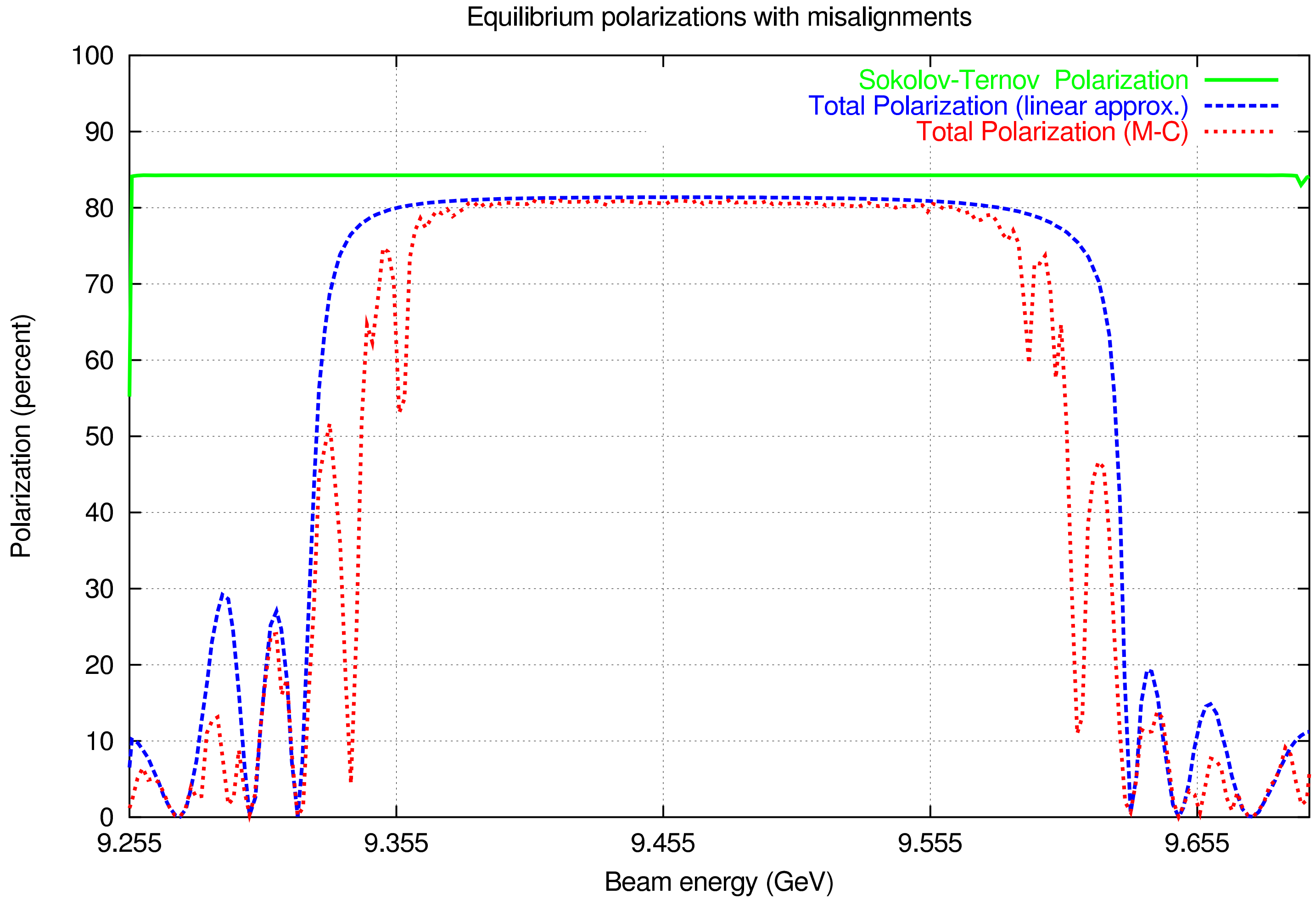
Recall the limitations at LEP!!

Analogous formula for sidebands of first order synchrotron resonances.

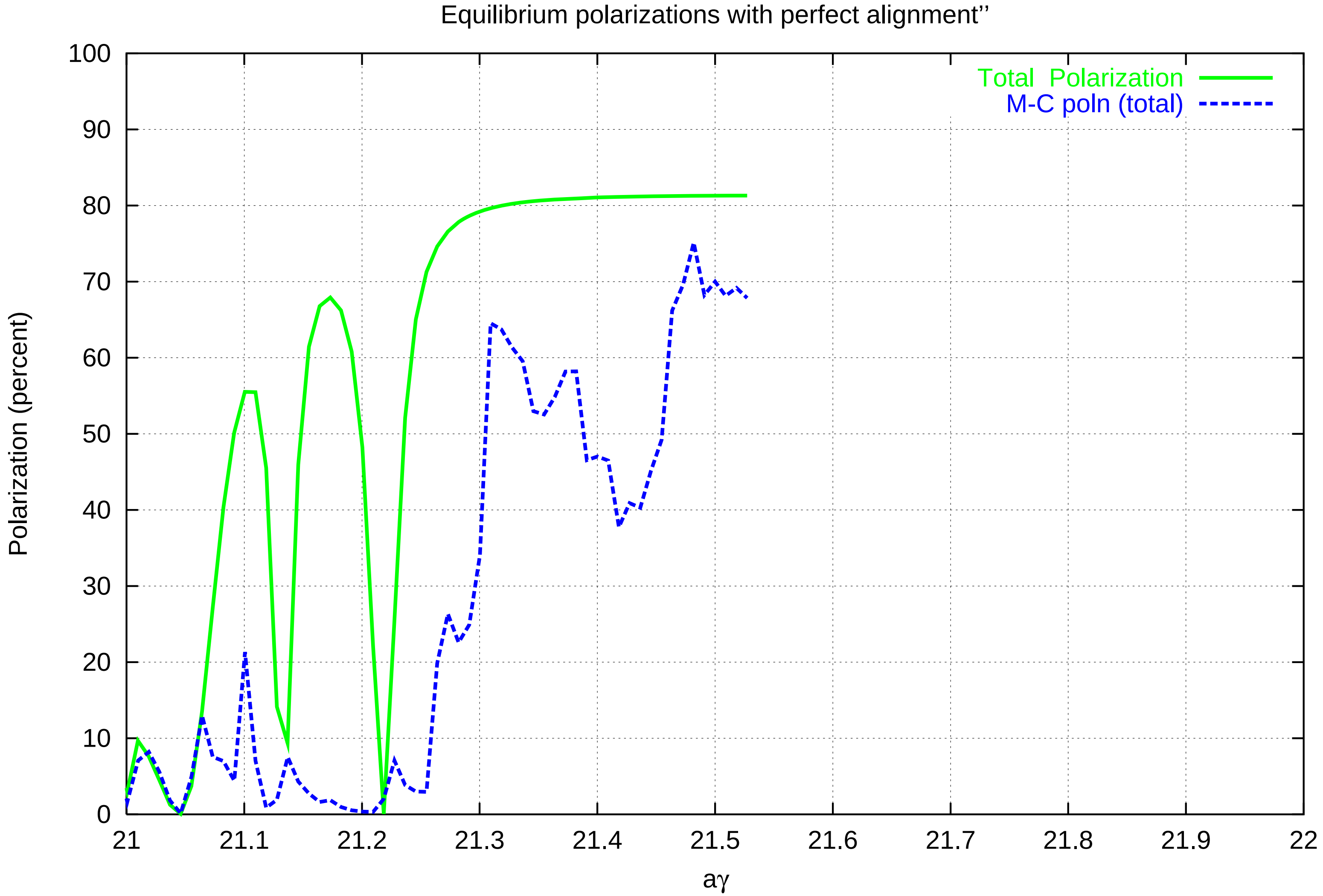
# Calibrating the (first order) M-C software structure against SLICK



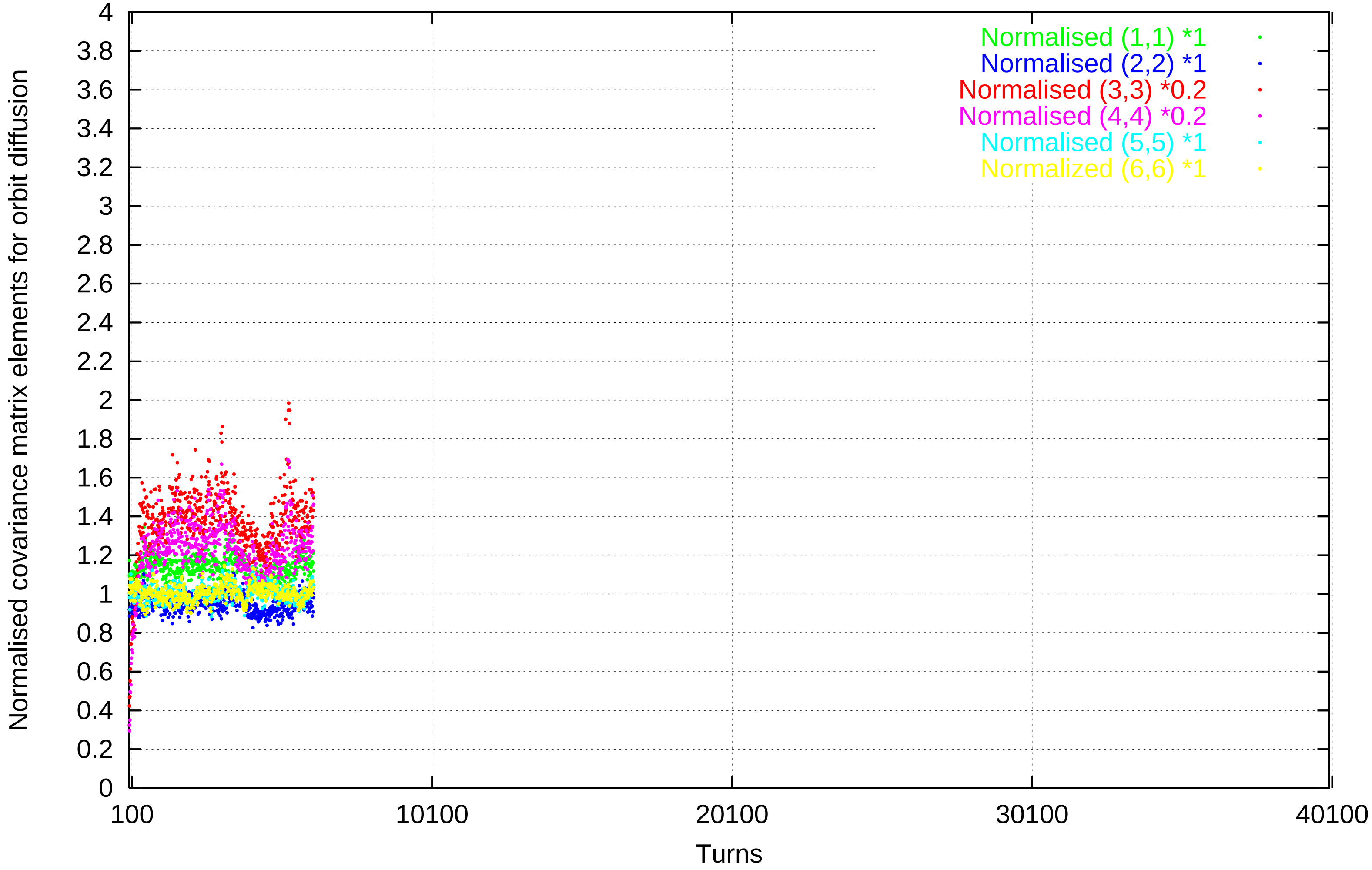
# Full 3–D spin motion



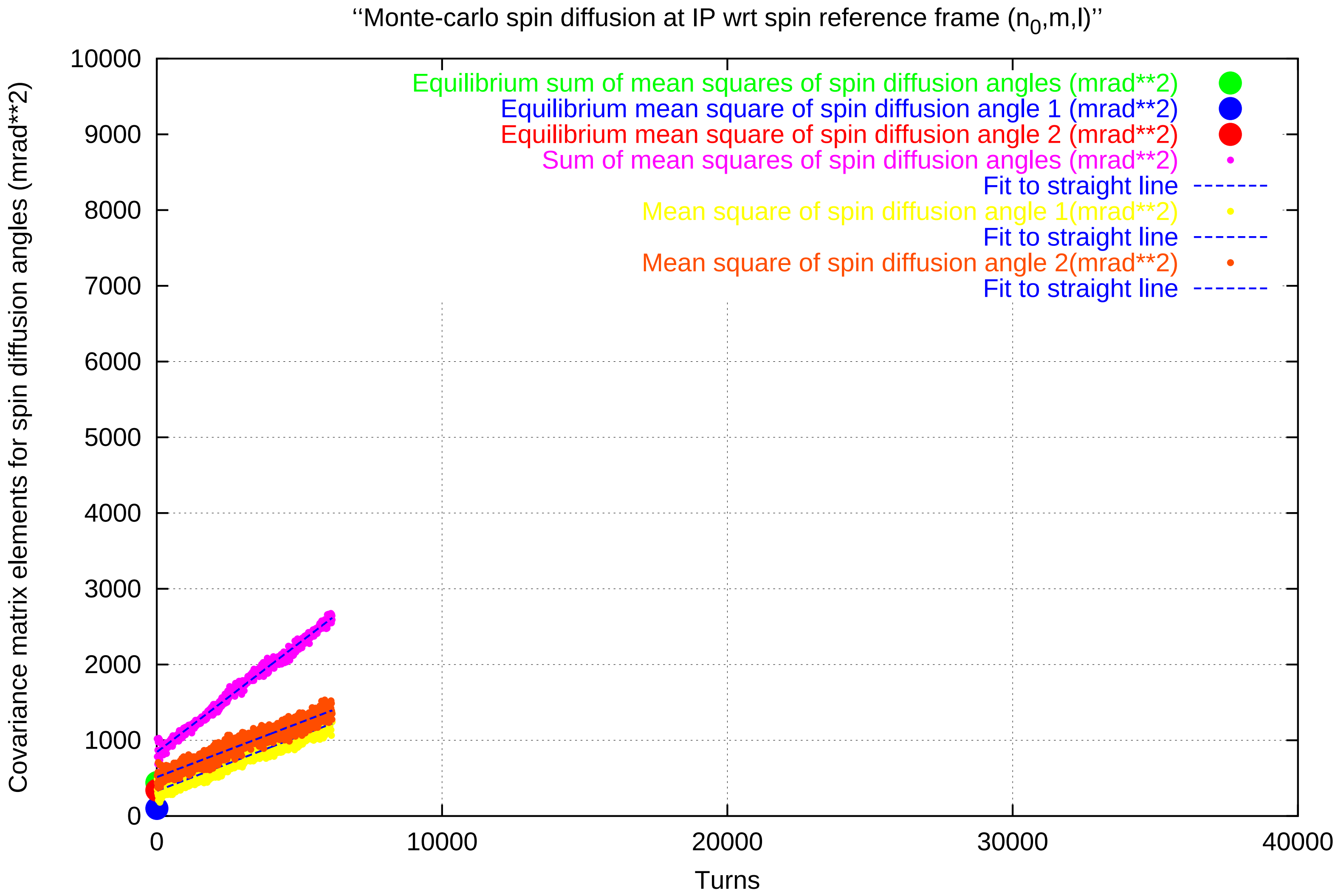
# Effect of beam-beam forces – preliminary



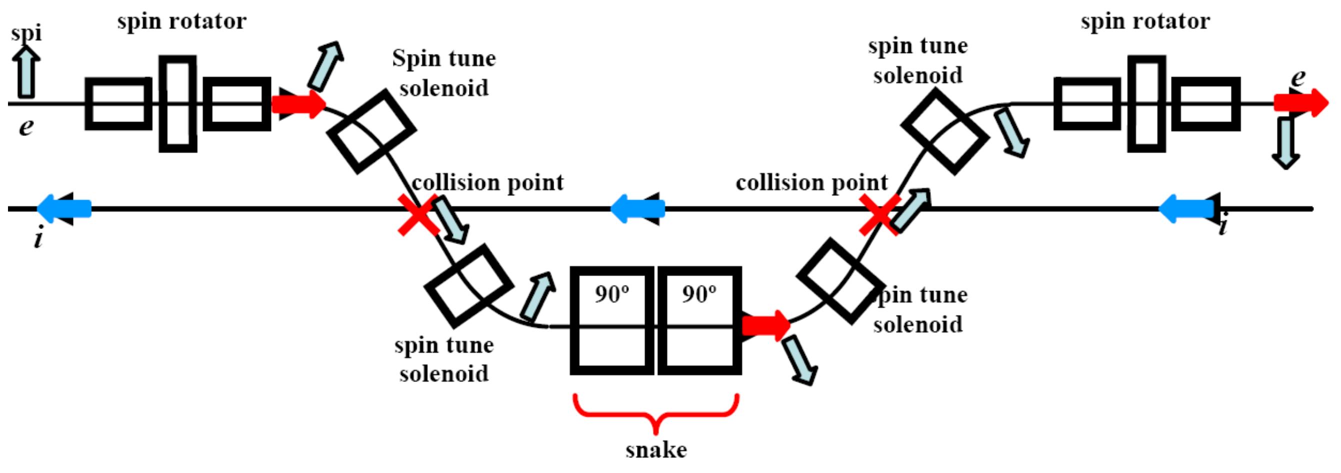
## Normalised diagonal elements of the beam covariance matrix

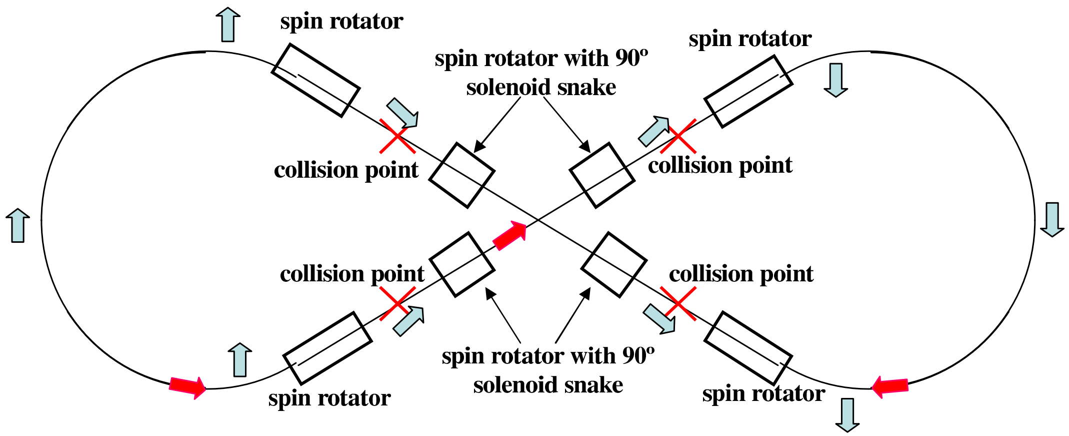


# Sum of diagonal elements of spin covariance matrix



**ELIC: latest version using stored beams**


spi
spin rotator
e
Spin tune
solenoid
spin tune
solenoid
spin rotator
e
collision point
collision point
i
i
90°
90°
pin tune
solenoid
spin tune
solenoid
snake


spin rotator
spin rotator with 90º
solenoid snake
spin rotator
collision point
collision point
collision point
collision point
spin rotator
spin rotator with 90º
solenoid snake
spin rotator

Features: $\nu_0 = 1/2$ with Siberian Snakes but the figure of 8 form ensures that the Sokolov-Ternov effect doesn't average to zero.

$$\vec{P}_{\rm bks} \;=\; -\frac{8}{5\sqrt{3}}\,\hat{n}_0\,\frac{\oint ds \frac{\hat{n}_0(s)\cdot\hat{b}(s)}{|\rho(s)|^3}}{\oint ds \frac{\left[1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2\right]}{|\rho(s)|^3}}$$

Using vertical bends to get rotation of $\hat{n}_0$ to longitudinal at the IP's.

# Comments and Questions

- The polarisation lifetime should be larger than the beam lifetime. Probably no problem at the lowest energy. Self polarisation only helps at high energy.
- If no proper spin transparency has been built in, one cannot assume *a priori* that the depolarisation rate will be low just because the design spin tune $\nu_0$ is $1/2$.

  See D.P. Barber and G. Ripken in *"Handbook of Accelerator Physics and Engineering"*, Eds: A.W. Chao and M. Tigner, World Scientific, 3rd edition (2006).
- Why not put the vertical bends into the hadron ring?

  If spin matching were needed, it would be energy dependent and that would be a real mess.
- Page 52: why not try to use the terminology for spin matching that was established in 1982 (PEAS workshop DESY), so that we can all communicate?
- Does the energy independent $\nu_0$ imply that synchrotron sidebands are weak?
- What are the implications of the crab cavities? Effects of detector fields and compensation.
- How much polarisation survives beam-beam effects (direct and indirect) ?). Very large $\xi$ .
- Wigglers to decrease the polarisation time: they will decrease $P_{\rm bk}$ too. More depolarisation?

**Things still to do:**

# Many!

- First create a (MAD) optic file for a closed ring with solenoids, skew quads, rf cavities etc.
- Then check if a decoupled optic can be created at each energy.
- Then check if it gives the expected tunes (3!) and emittances and damping times ......
- Then get a first order analytical estimate of the polarisation and depolarisation time at one energy around 9 - 10 GeV using SLICK.
- No need for crab cavities, beam-beam forces, full 3-D spin motion if the first estimates are bad.
- If things look bad, find out why: Diagnostics!!!!
- Then let's.....

# The HERA Upgrade

| | LUMINOSITY UPGRADE | | DESIGN | | 2000 (average) | |
|---|---|---|---|---|---|---|
| | e-Beam | p-Beam | e-Beam | p-Beam | e-Beam | p-Beam |
| $E$ [GeV] | 27.5 | 920 | 30 | 820 | 27.5 | 920 |
| $I$ [mA] | 58 | 140 | 58 | 160 | 45 | 95 |
| $N_{ppb}$ ($N_e$ or $N_p$) $\times 10^{10}$ | 4.0 | 10.3 | 3.6 | 10.1 | 3.1 | 7.0 |
| $N_{b,tot}$ | 189 | 180 | 210 | 210 | 189 | 180 |
| $N_{b,col}$ | 174 | 174 | 210 | 210 | 174 | 174 |
| $\epsilon_x$ [nm rad] | 20 | $\frac{5000}{\beta\gamma}$ | 48 | $\frac{6000}{\beta\gamma}$ | 41 | $\frac{5000}{\beta\gamma}$ |
| $\epsilon_z / \epsilon_x$ | 0.17 | 1 | 0.05 | 1 | 0.1 | 1 |
| $\beta_x^*$ [m] | 0.63 | 2.45 | 2.2 | 10.0 | 0.9 | 7.0 |
| $\beta_z^*$ [m] | 0.26 | 0.18 | 0.9 | 1.0 | 0.6 | 0.5 |
| $\sigma_x \times \sigma_z$ [$\mu$m$^2$] | 112 × 30 | 112 × 30 | 325 × 46 | 262 × 83 | 192 × 50 | 189 × 50 |
| $\sigma_s$ [mm] | 10.3 | 191 | 8.3 | 200 (85) | 11.2 | 191 |
| $\Delta\nu_x$ / IP | 0.034 | 0.0015 | 0.019 | $8 \cdot 10^{-4}$ | 0.012 | 0.0012 |
| $\Delta\nu_z$ / IP | 0.052 | $4 \cdot 10^{-4}$ | 0.024 | $6 \cdot 10^{-4}$ | 0.029 | $3 \cdot 10^{-4}$ |
| min. aperture [$\sigma_x$] | 20 | 12 | 23 | 16 | 14 | 10 |
| $\mathcal{L}_s$ [cm$^{-2}$s$^{-1}$mA$^{-2}$] | $1.8 \cdot 10^{30}$ | | $3.4 \cdot 10^{29}$ | | $7.4 \cdot 10^{29}$ | |
| $\mathcal{L}$ [cm$^{-2}$s$^{-1}$] | $7.5 \cdot 10^{31}$ | | $1.5 \cdot 10^{31}$ | | $1.5 \cdot 10^{31}$ | |

Table 1: