# Polarisation in electron rings

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**D.P. Barber**

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*Deutsches Elektronen–Synchrotron (DESY), Germany*

*University of Liverpool, UK*

*Cockcroft Institute, Daresbury, UK*

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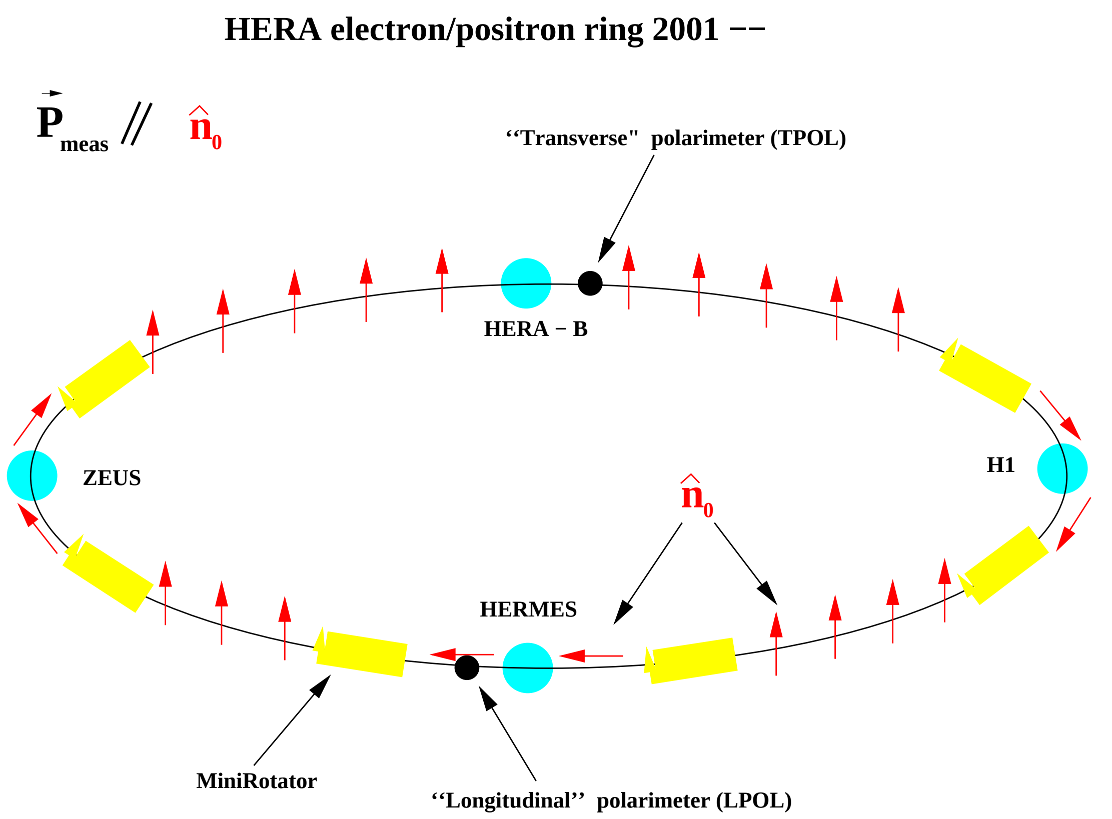
## Plan

- HERA – a reminder
- Some theory and phenomenology
- eRHIC ring-ring: calculations, including beam-beam effects
- ELIC: Comments and questions
- Things still to do

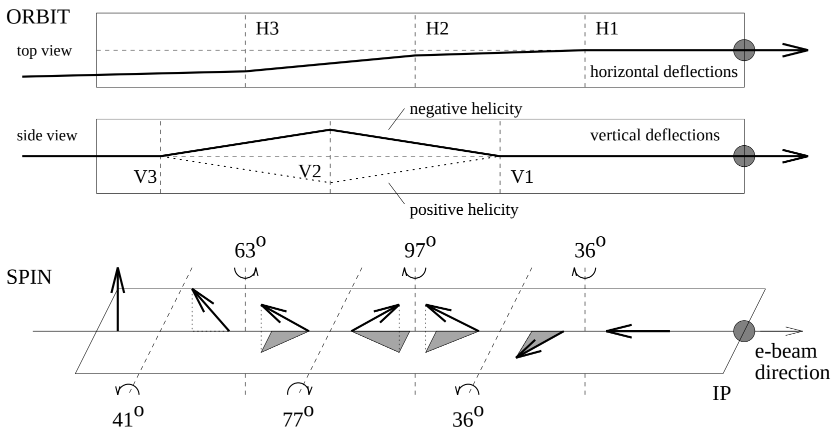
# HERA

The first and only $e^{\pm}$ ring to supply longitudinal polarisation at high energy
— via the Sokolov-Ternov effect – also at 3 IP's simultaneously!




Polarisation vertical in the arcs – to drive the Sokolov-Ternov effect

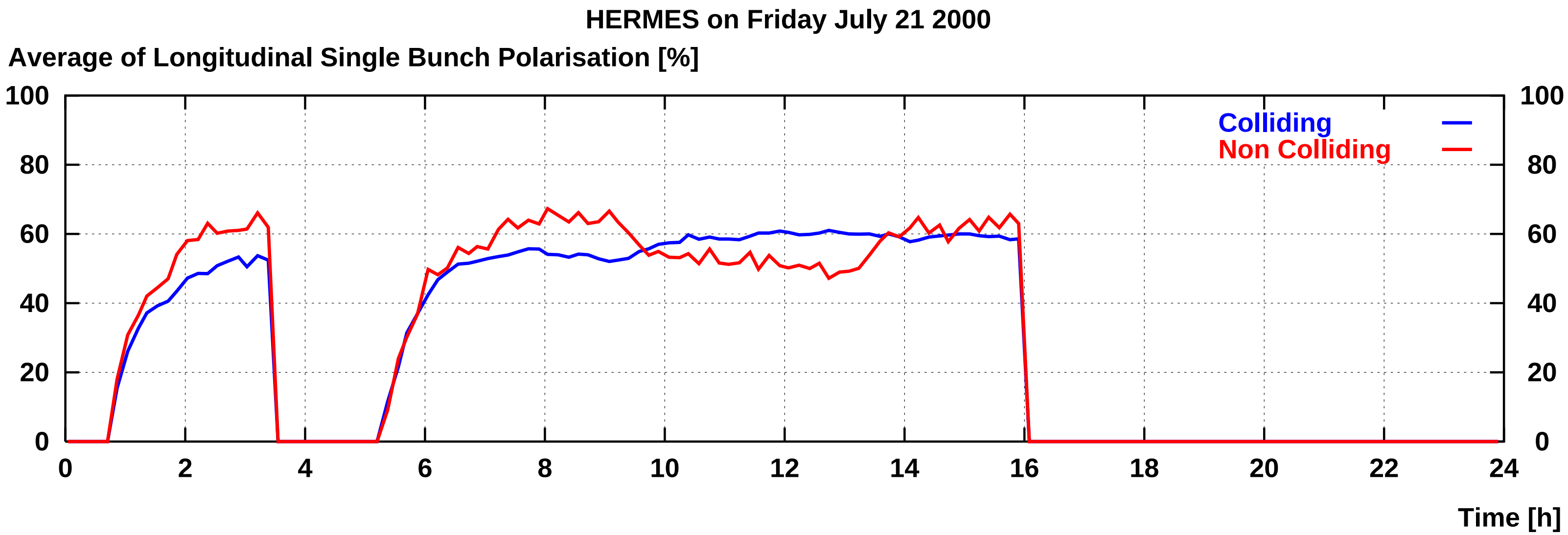
## HERA MiniRotator: Buon + Steffen



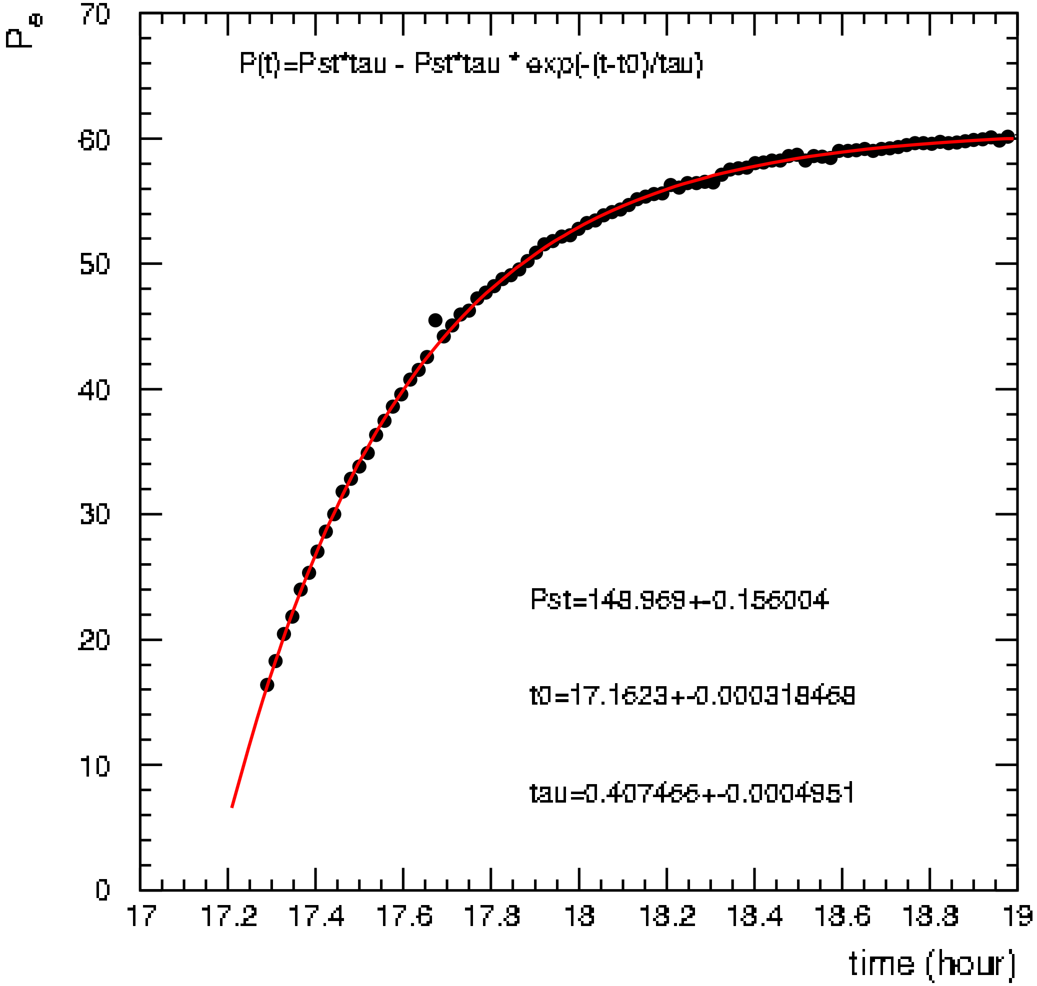

**56 m (“short”) → no quads.**

**27 – 39 GeV, both helicities, variable geometry**

NO INTERNAL QUADRUPOLES!


HERMES on Friday July 21 2000
Average of Longitudinal Single Bunch Polarisation [%]
100
80
60
40
20
0
0
2
4
6
8
10
12
14
16
18
20
22
24
Time [h]
Colliding
Non Colliding

June 2007, the Fabry-Perot-Compton polarimeter of the POL2000 Project: Calibrating polarimeters




3 pairs of rotators (so max. Sokolov-Ternov polarisation = 83 %), solenoids on, no beam-beam

# THEORY and PHENOMENOLOGY

## The T-BMT equation.

$$\frac{d\vec{S}}{ds} = \vec{\Omega}(\gamma, \vec{v}, \vec{B}, \vec{E}) \times \vec{S}$$

## Periodic solution $\hat{n}_0$ on closed orbit.

The real unit eigenvector of:

$$R_{3\times 3}(s + C, s)\hat{n}_0 = \hat{n}_0$$

$\hat{n}_0$ is 1–turn periodic: $\hat{n}_0(s + C) = \hat{n}_0(s)$

$\hat{n}_0$: direction of measured equilibrium radiative polarization.

Closed orbit spin tune $\nu_0$: number of precessions per turn around $\hat{n}_0$ for a spin on the closed orbit. Extract from the eigenvalues of $R_{3\times 3}(s + C, s)$

The **value** of the polarization is the same at all azimuths — time scales.

## Spin motions

- Protons: largely deterministic — unless IBS.
- Electrons/positrons:
  If a photon causes a spin flip, what are the other $\approx 10^{10}$ photons doing? ===>

  Stochastic/damped orbital motion due to synchrotron radiation
  + inhomogeneous fields
  + spin–orbit coupling via T–BMT
  ===> spin diffusion i.e. depolarisation!!!

  Self polarisation: Balance of poln. and depoln. ===>

  $$P_\infty \approx P_{BK} \frac{1}{1 + (\frac{\tau_{dep}}{\tau_{BK}})^{-1}} \qquad (P_{ST} \rightarrow P_{BK})$$

  In any case:

  $$\tau_{dep}^{-1} \propto \gamma^{2N}\, \tau_{st}^{-1}$$ (actually a polynomial in $\gamma^{2N}$)

  ===> Trouble at high energy!

## Spin–orbit resonances

$$\nu_{\rm spin} \;=\; k + k_I\nu_I + k_{II}\nu_{II} + k_{III}\nu_{III}$$

$\nu_{\rm spin}$ : amplitude dependent spin tune ≈ closed orbit spin tune = precessions /turn on CO

- Orbit "drives spins" ===> Resonant enhancement of spin diffusion.
- Resonance order: $|k_I| + |k_{II}| + |k_{III}|$
- First order: $|k_I| + |k_{II}| + |k_{III}| = 1$ e.g. SLIM like formalisms.
- Strongest beyond first order:
  synchrotron sidebands of first order parent betatron or synchrotron resonances

$$\nu_{\rm spin} \;=\; k + k_i\nu_i + k_{III}\nu_{III}, \quad i = I, II \text{ or } III$$

## More on spin–orbit resonances

(1) Linear orbit motion with linearized spin motion (SLIM/SLICK):
just first order spin–orbit resonances.

(2) Linear orbit motion with full 3–D spin motion:
all orders of spin–orbit resonances.

(3) Non–linear orbit motion with linearized spin motion:
orders of spin–orbit resonances just reflecting the orbital spectrum.

(4) Non–linear orbit motion with full 3–D spin motion:
all orders of spin–orbit resonances.

**Diagnostics : With (1) and (3) we use spin motion to Fourier analyse the orbital motion!**

## Linear spin matching

Skip the invariant spin field and the Derbenev-Kondratenko formula for today!

Heuristics instead!

$$\vec{S} \approx \hat{n}_0(s) + \alpha \hat{m}_0(s) + \beta \hat{l}_0(s)$$

$\alpha, \beta$: 2 small spin tilt angles — have subtracted out the big rotations!

$$\hat{\mathbf{M}}_{8\times 8} = \begin{pmatrix} \mathbf{M_{6\times 6}} & \mathbf{0_{6\times 2}} \\ \mathbf{G_{2\times 6}} & \mathbf{D_{2\times 2}} \end{pmatrix}$$

acting on $\vec{u} = (x,\ x',\ y,\ y',\ l,\ \delta)$ and $\alpha, \beta$

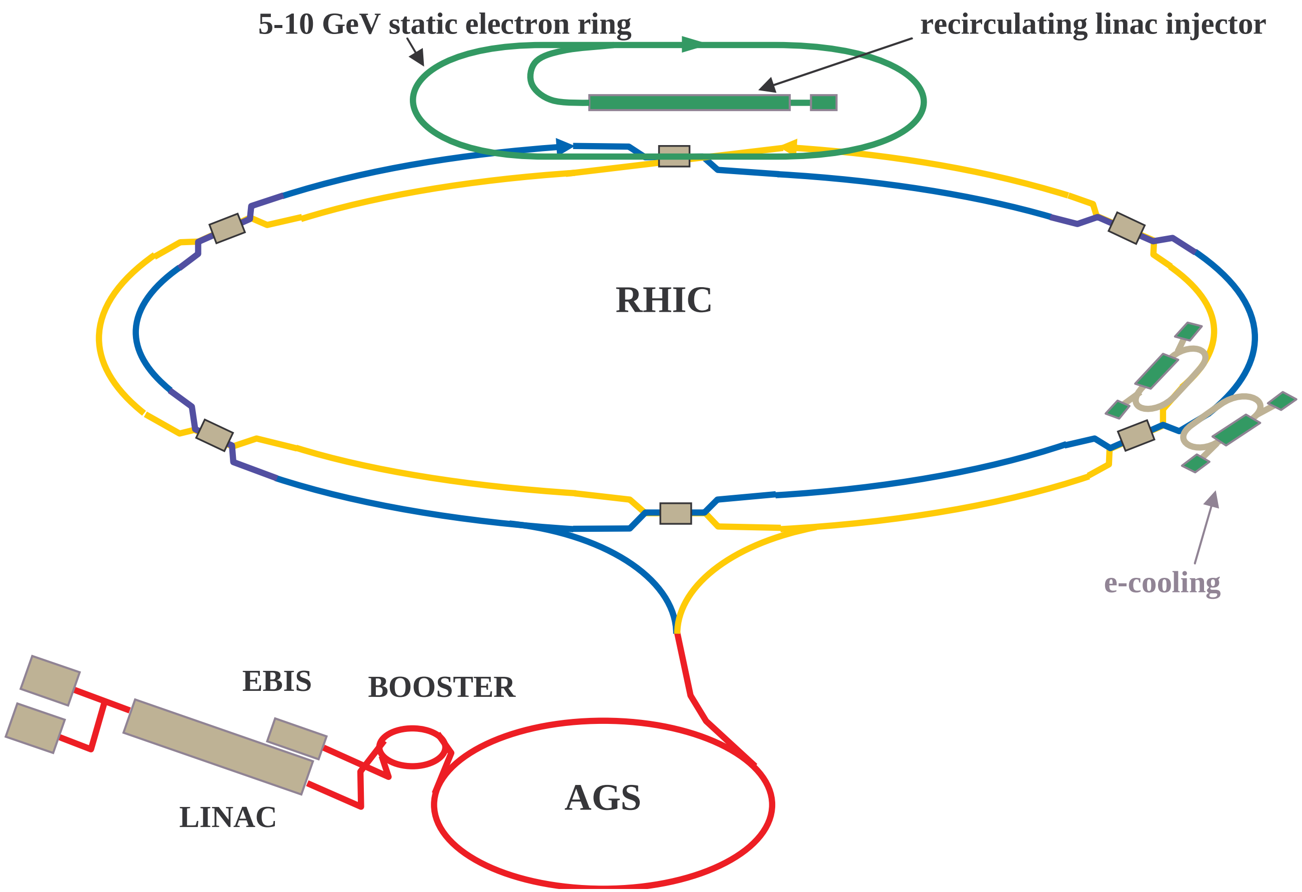
This is the SLIM formalism for estimating depolarisation analytically at first order (Chao 1981).

To minimize depolarization:
minimize appropriate bits of $\mathbf{G_{2\times 6}}$ for appropriate stretches of ring
===> lots of independent quadrupole circuits.

Spin transparency!!! – not the trivial kind to which this term is applied at certain labs.

# eRHIC: ring-ring option

# The Layout

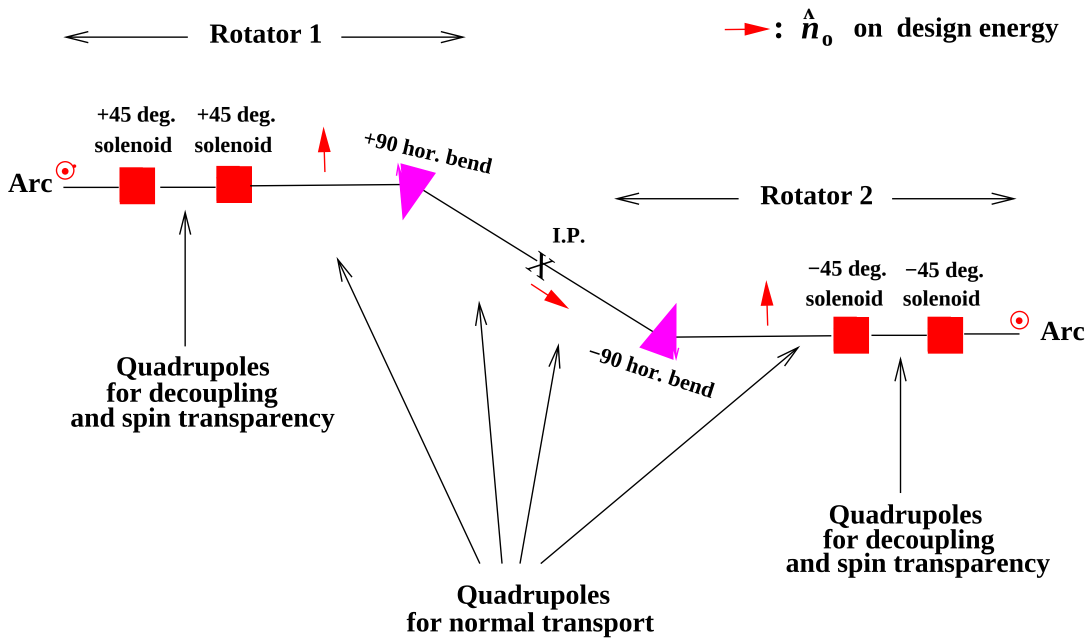

5-10 GeV static electron ring
recirculating linac injector
RHIC
e-cooling
EBIS
BOOSTER
LINAC
AGS

# Parameters

Table 1.1. Luminosities and main beam parameters for electron(positron)-proton collisions.

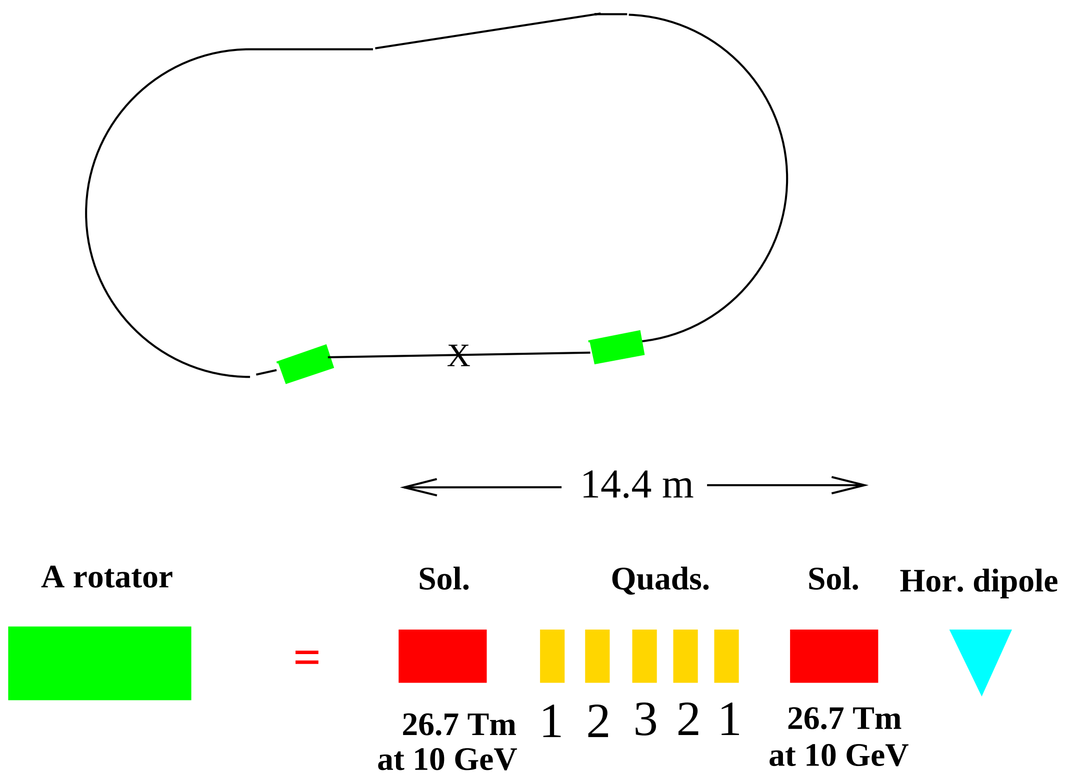
| | High energy setup | | Low energy setup | |
|---|---|---|---|---|
| | p | e | p | e |
| Energy, GeV | 250 | 10 | 50 | 5 |
| Bunch intensity, $10^{11}$ | 1 | 1 | 1 | 1 |
| Ion normalized emittance, $\pi$ mm $\cdot$ mrad , x/y | 15/15 | | 5/5 | |
| rms emittance, nm, x/y | 9.5/9.5 | 53/9.5 | 16.1/16.1 | 85/38 |
| $\beta$*, cm, x/y | 108/27 | 19/27 | 186/46 | 35/20 |
| Beam-beam parameters, x/y | 0.0065/0.003 | 0.03/0.08 | 0.019/0.0095 | 0.036/0.04 |
| $\kappa=\varepsilon_y/\varepsilon_x$ | 1 | 0.18 | 1 | 0.45 |
| Luminosity, 1.e32 $cm^{-2}s^{-1}$ | 4.4 | | 1.5 | |

## The solenoid spin rotators




$\hat{n}_0$ exactly longitudinal at just at one energy.

# The basic eRHIC geometry for spin−−exaggerated

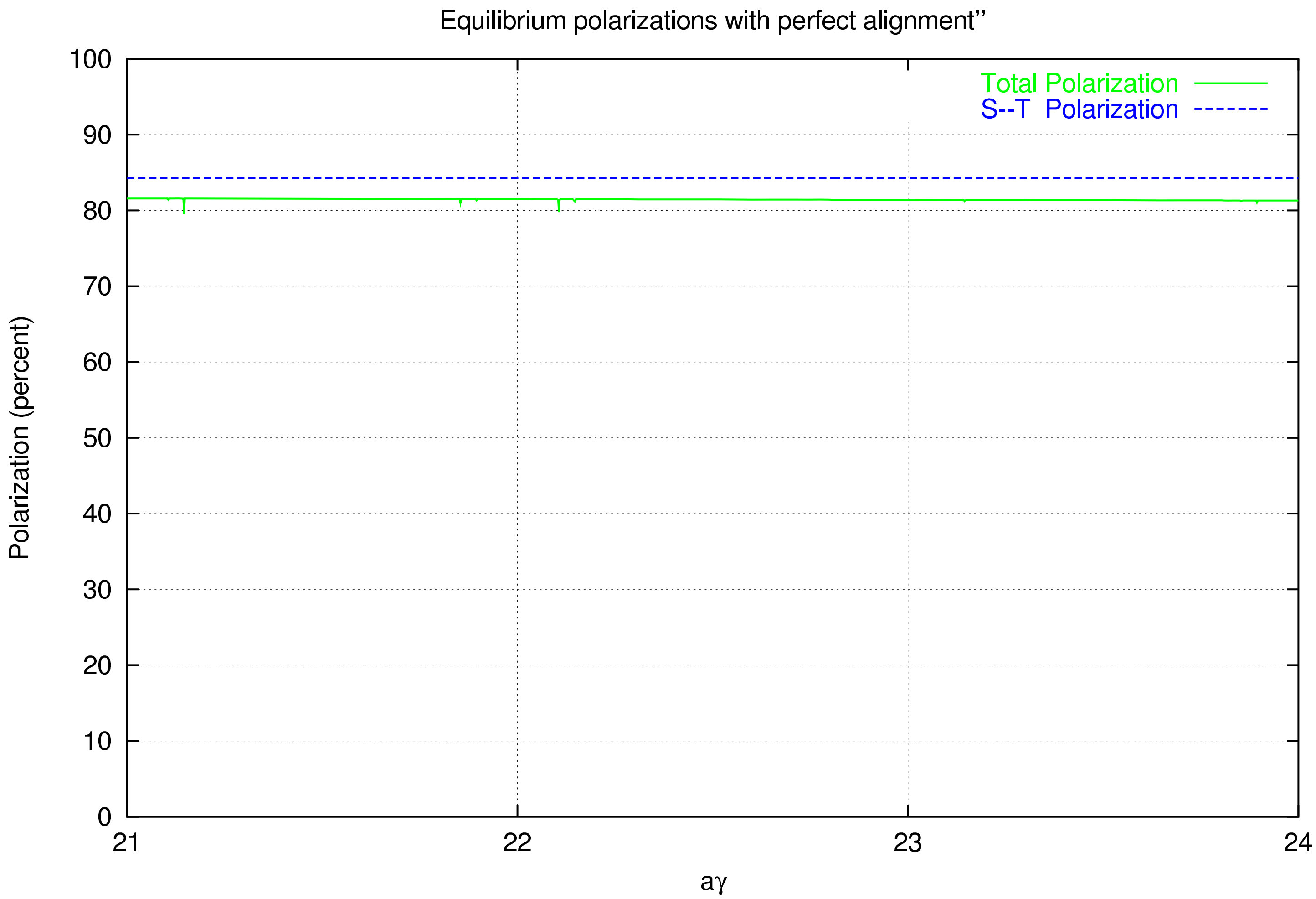


The $4 \times 4$ transfer matrix for the transverse motion through a pair of solenoids:

$$\begin{pmatrix} 0 & -2r & 0 & 0 \\ 1/2r & 0 & 0 & 0 \\ 0 & 0 & 0 & 2r \\ 0 & 0 & -1/2r & 0 \end{pmatrix}$$

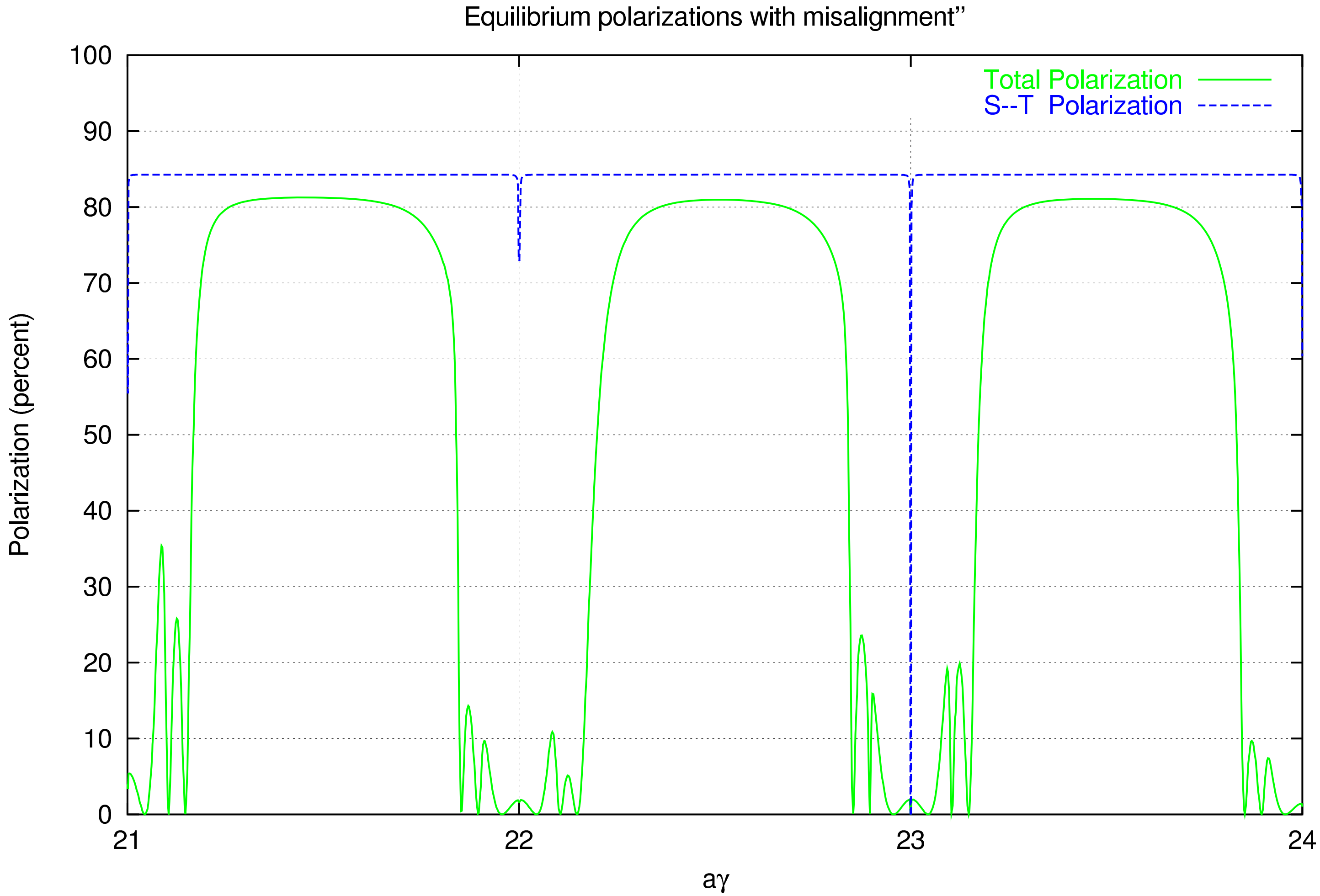
where $r$ is the radius of orbit curvature in the longitudinal field.

Use 5 back–to–back symmetric quadrupoles.

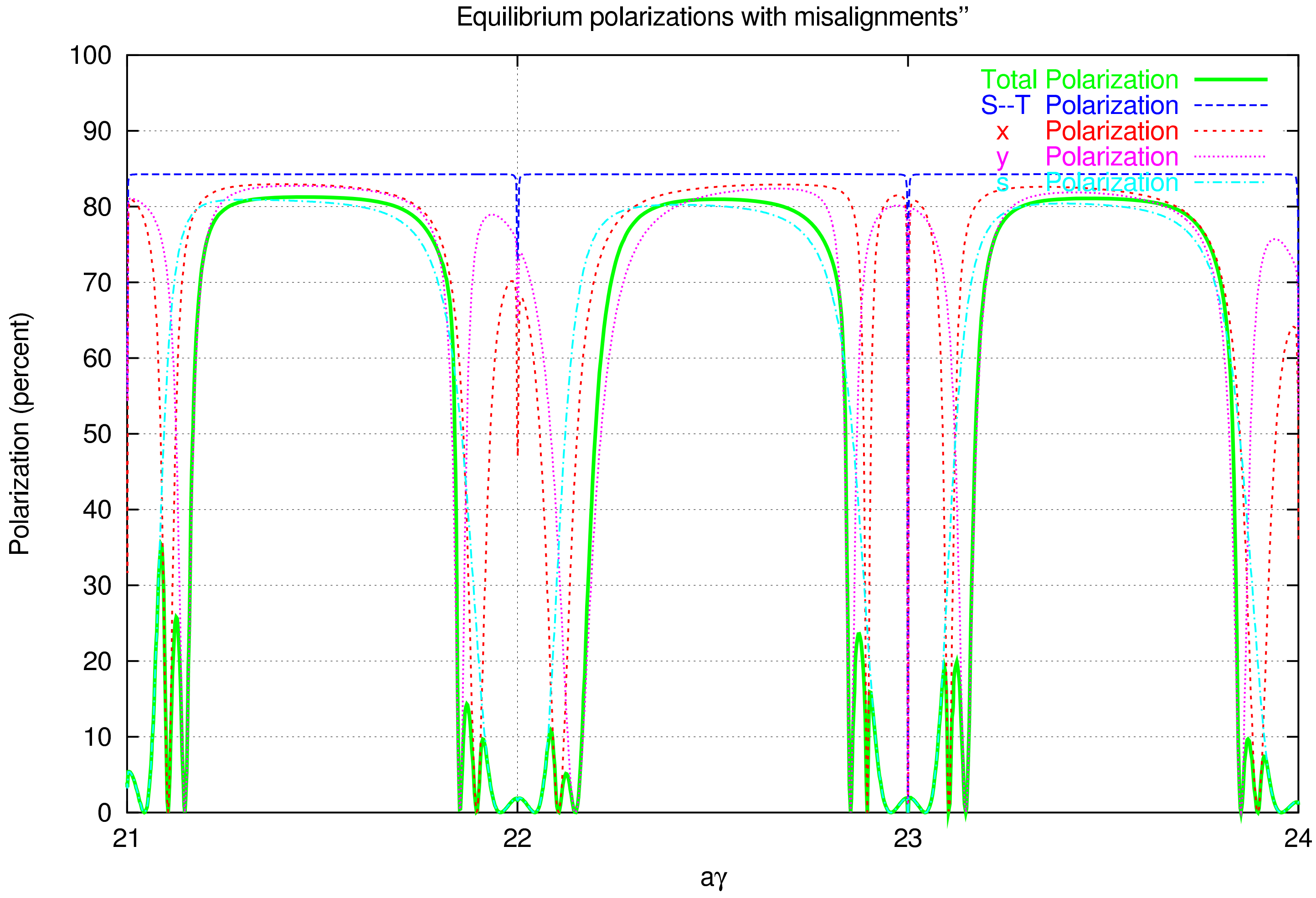

Equilibrium polarizations with perfect alignment”
Total Polarization
S--T Polarization
Polarization (percent)
aγ
100
90
80
70
60
50
40
30
20
10
0
21
22
23
24

$$\vec{P}_{\rm bks} \;=\; -\frac{8}{5\sqrt{3}}\,\hat{n}_0\,\frac{\oint ds\,\frac{\hat{n}_0(s)\cdot\hat{b}(s)}{|\rho(s)|^3}}{\oint ds\,\frac{\left[1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2\right]}{|\rho(s)|^3}}$$

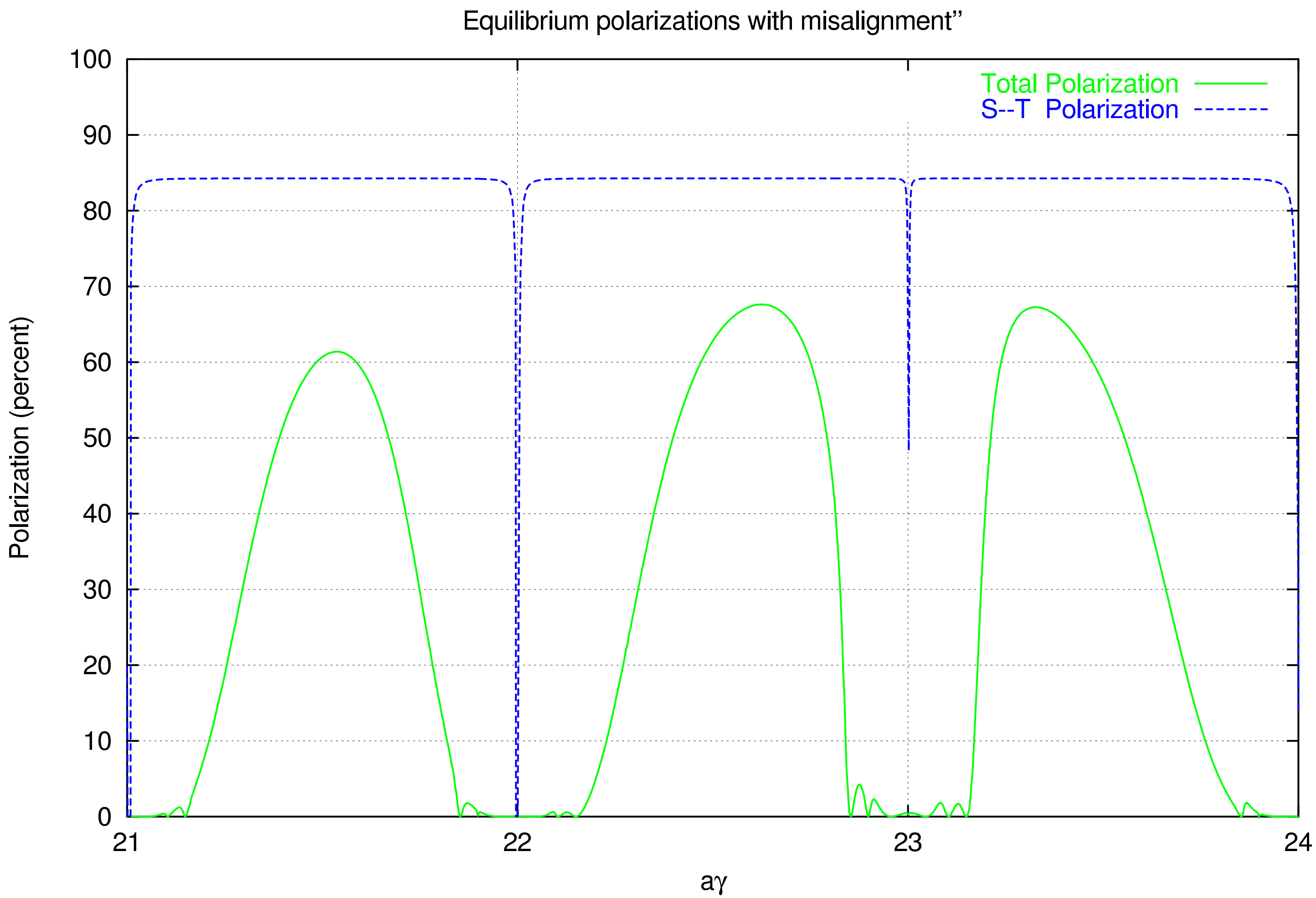
## All monitors on: just ONE example!

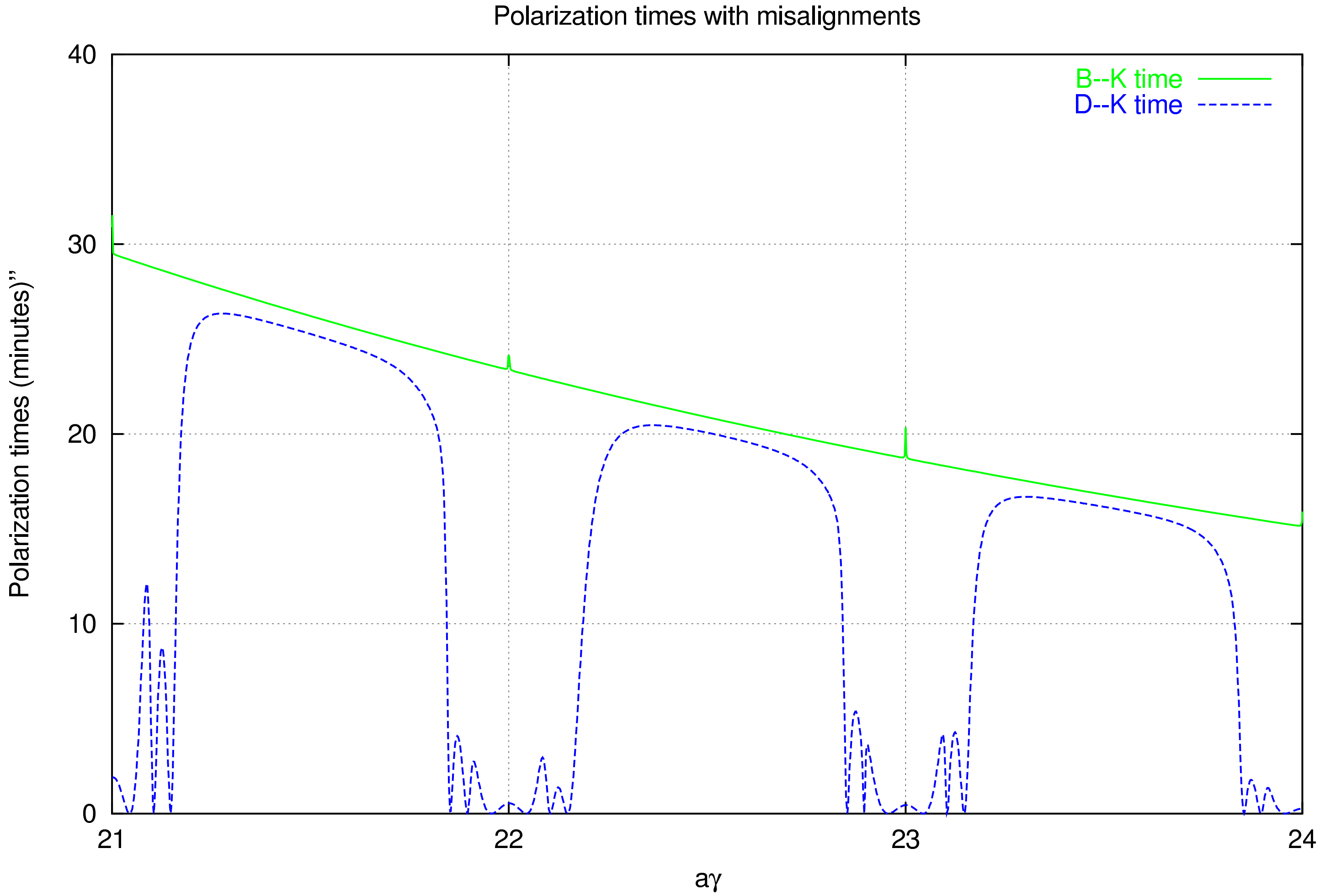


# All monitors on: DIAGNOSTICS

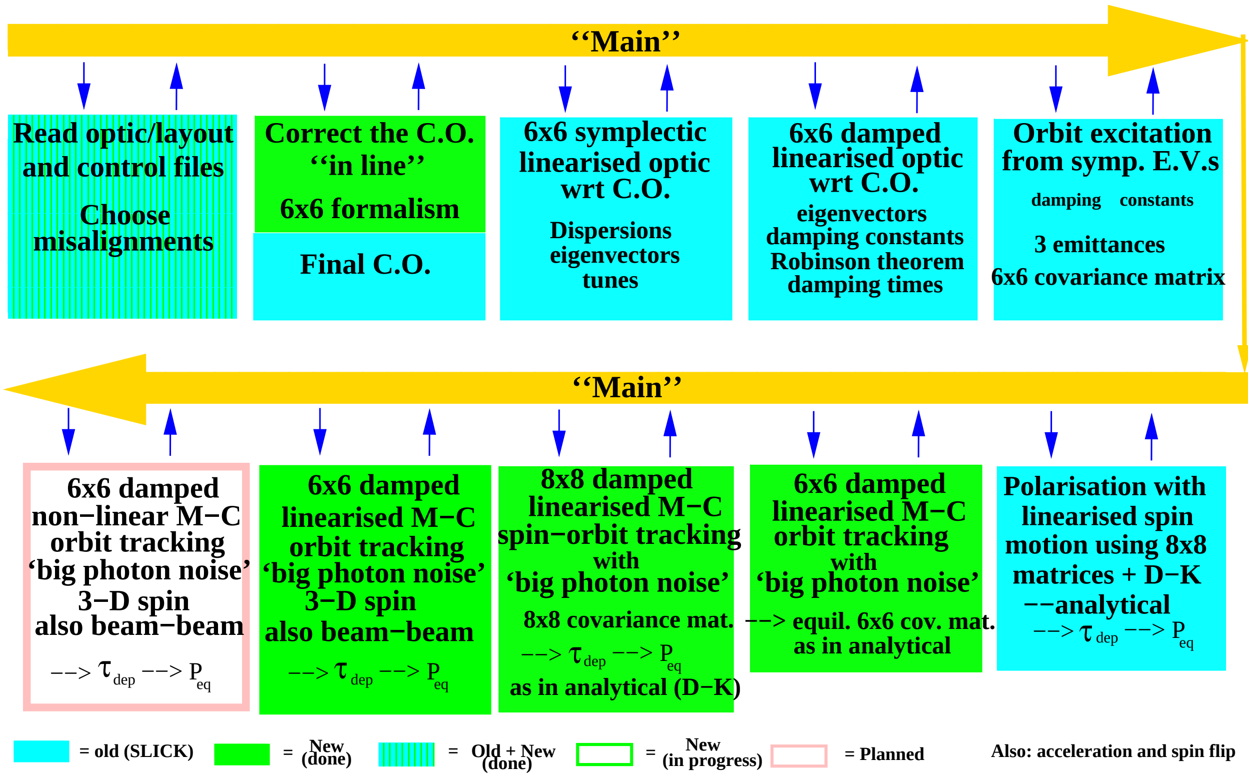


## 20 percent monitors on: just ONE example!




Polarization times with misalignments
B--K time
D--K time
Polarization times (minutes)"
aγ
40
30
20
10
0
21
22
23
24

## The structure of SLICKTRACK



## Spin coordinates

$$\hat{S} = \sqrt{1-\alpha^2-\beta^2}\,\hat{n}_0 + \alpha\hat{m} + \beta\hat{l}$$

## Estimating depolarisation by M-C simulation $\alpha^2+\beta^2 << 1$

$$\Delta P \approx -\frac{1}{2}\Delta(<\alpha^2+\beta^2>) = -\frac{1}{2}\Delta(\sigma_\alpha^2+\sigma_\beta^2) \quad \Longrightarrow \frac{dP}{dt} \approx -\frac{1}{2} = -\frac{1}{2}\frac{d}{dt}(\sigma_\alpha^2+\sigma_\beta^2)$$

## Spin–orbit covariance matrix

$$\begin{pmatrix}
\sigma_x^2 & \sigma_{xx'} & . & . & . & . & | & . & . \\
\sigma_{x'x} & \sigma_{x'}^2 & . & . & . & . & | & . & . \\
. & . & . & . & . & . & | & . & . \\
. & . & . & . & . & . & | & . & . \\
. & . & . & . & . & . & | & . & . \\
. & . & . & . & . & \sigma_\delta^2 & | & . & . \\
- & - & - & - & - & - & - & - & - \\
. & . & . & . & . & . & | & \sigma_\alpha^2 & \sigma_{\alpha\beta} \\
\sigma_{\beta x}. & . & . & . & . & . & | & \sigma_{\beta\alpha} & \sigma_\beta^2
\end{pmatrix}$$

## Spin–orbit maps for sections

For linearised spin motion (SLIM/SLICK):

$$\hat{\mathbf{M}} = \begin{pmatrix} \mathbf{M_{6\times6}} & \mathbf{0_{6\times2}} \\ \mathbf{G_{2\times6}} & \mathbf{D_{2\times2}} \end{pmatrix}$$

The $\mathbf{G_{2\times6}} \times (\mathbf{x}, \mathbf{x'}, \mathbf{y}, \mathbf{y'}, \mathbf{\Delta l}, \delta)^{\mathbf{T}}$ delivers changes to the 2 small angles $\alpha$ and $\beta$

For full 3–D spin motion:

$$\hat{\mathbf{M}} = \begin{pmatrix} \mathbf{M_{6\times6}} & \mathbf{0_{6\times3}} \\ \mathbf{G_{3\times6}} & \mathbf{D_{3\times3}} \end{pmatrix}$$

The $\mathbf{G_{3\times6}} \times (\mathbf{x}, \mathbf{x'}, \mathbf{y}, \mathbf{y'}, \mathbf{\Delta l}, \delta)^{\mathbf{T}}$ delivers rotations around $\hat{n}_0,\ \hat{m}_0,\ \hat{l}_0$

The beam–beam (non–linear) kicks are applied at single points

# Diagnostics! Diagnostics! Diagnostics!

**Switch spin-orbit coupling off/on to see what does what.**

Sidebands of parent first order betatron resonances: a useful **approximation**

$$\tau_{dep}^{-1} \propto \frac{A}{(\nu_0 \pm Q_y)^2} \quad \rightarrow \quad \tau_{dep}^{-1} \propto \sum_{m_s=-\infty}^{\infty} \frac{A\, B(\xi; m_s)}{(\nu_0 \pm Q_y \pm m_s\, Q_s)^2}$$

$A$ is an energy dependent factor

$B(\xi; m_s)$'s: *enhancement factors*, contain modified Bessel functions
$I_{|m_s|}(\xi)$ and $I_{|m_s|+1}(\xi)$ depending on the *modulation index*

$$\xi = (\frac{a\gamma\ \sigma_\delta}{Q_s})^2$$

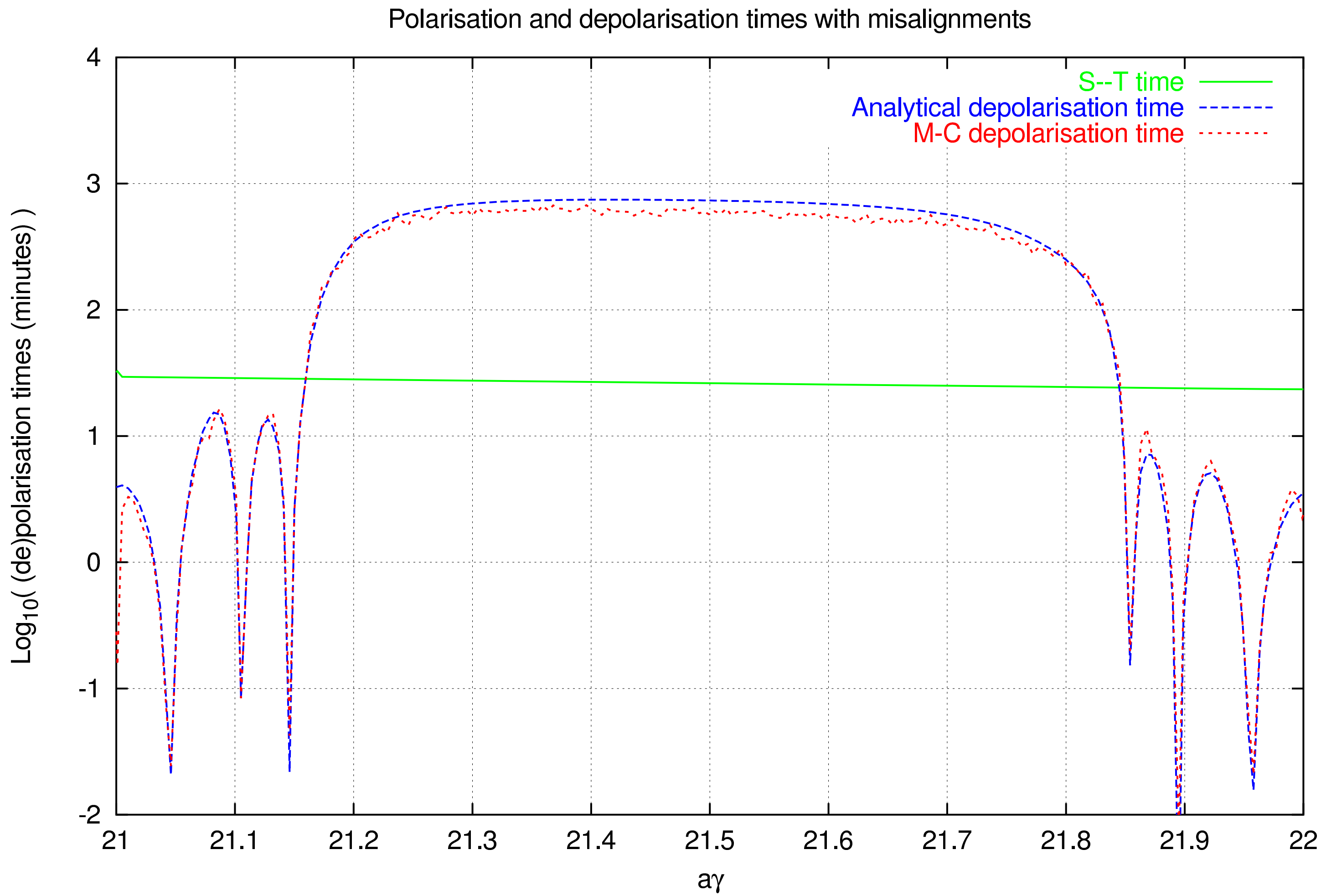
in a flat ring.

===> very strong effects at high energy — dominant source of trouble

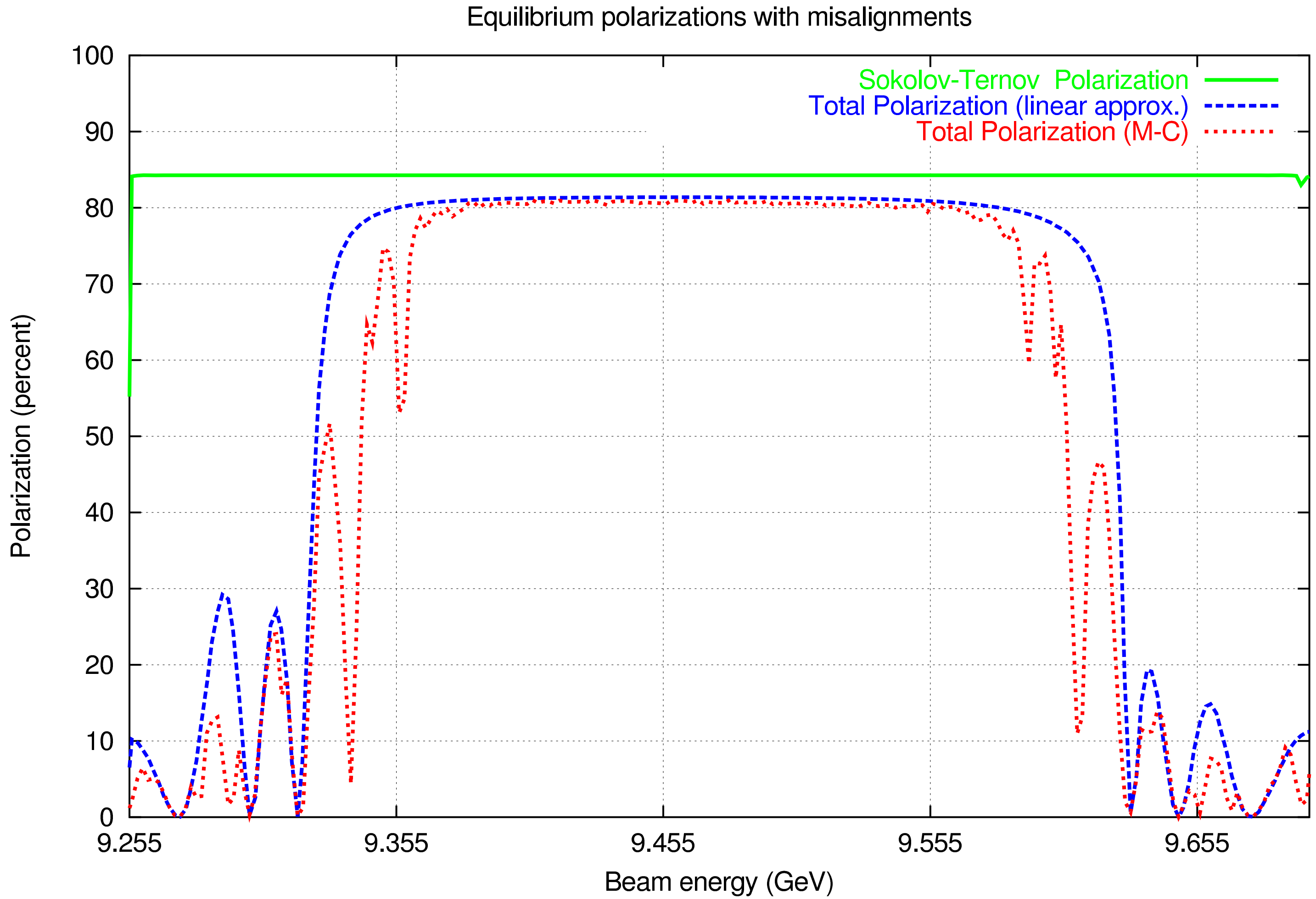
Recall the limitations at LEP!!

Analogous formula for sidebands of first order synchrotron resonances.

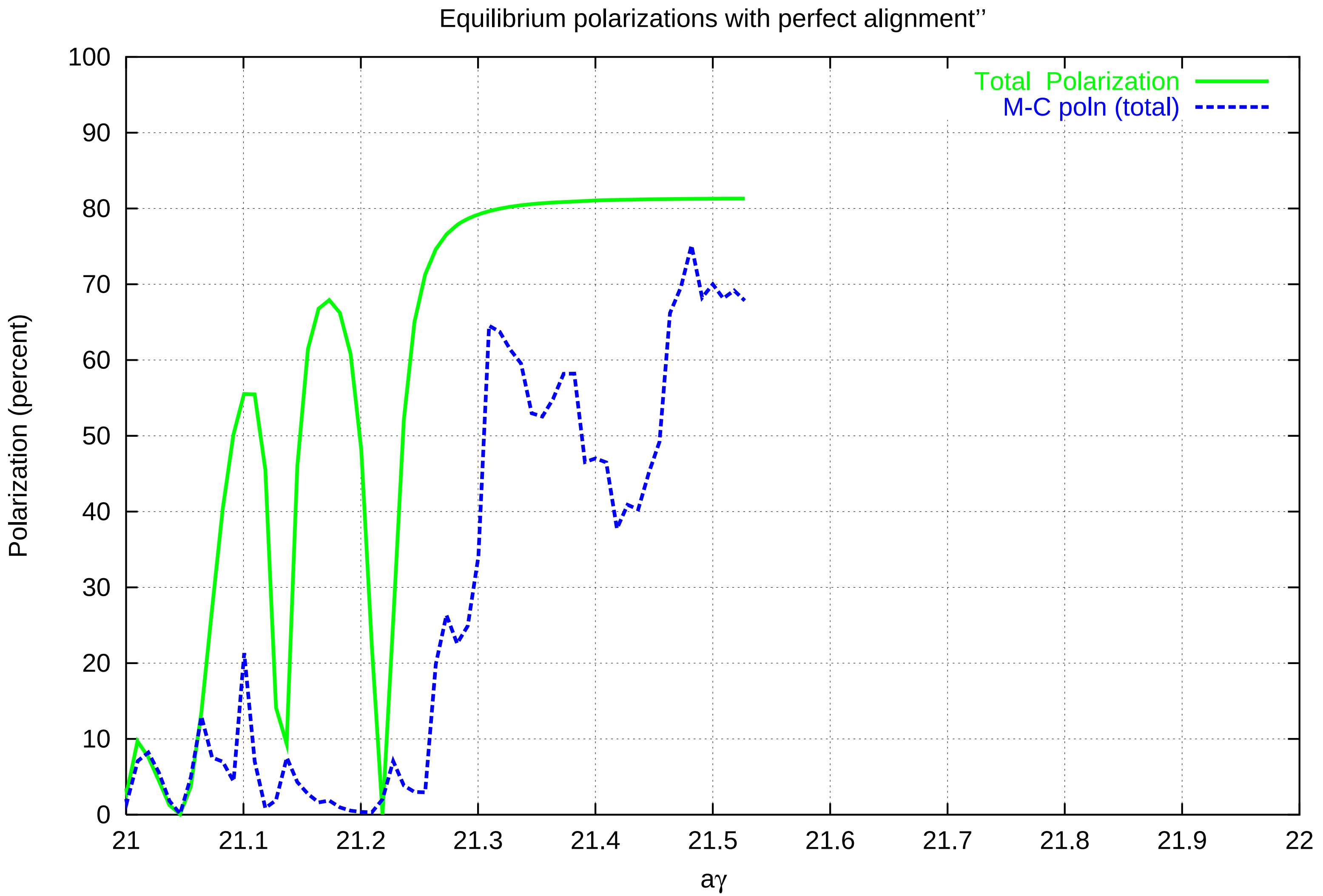
# Calibrating the (first order) M-C software structure against SLICK



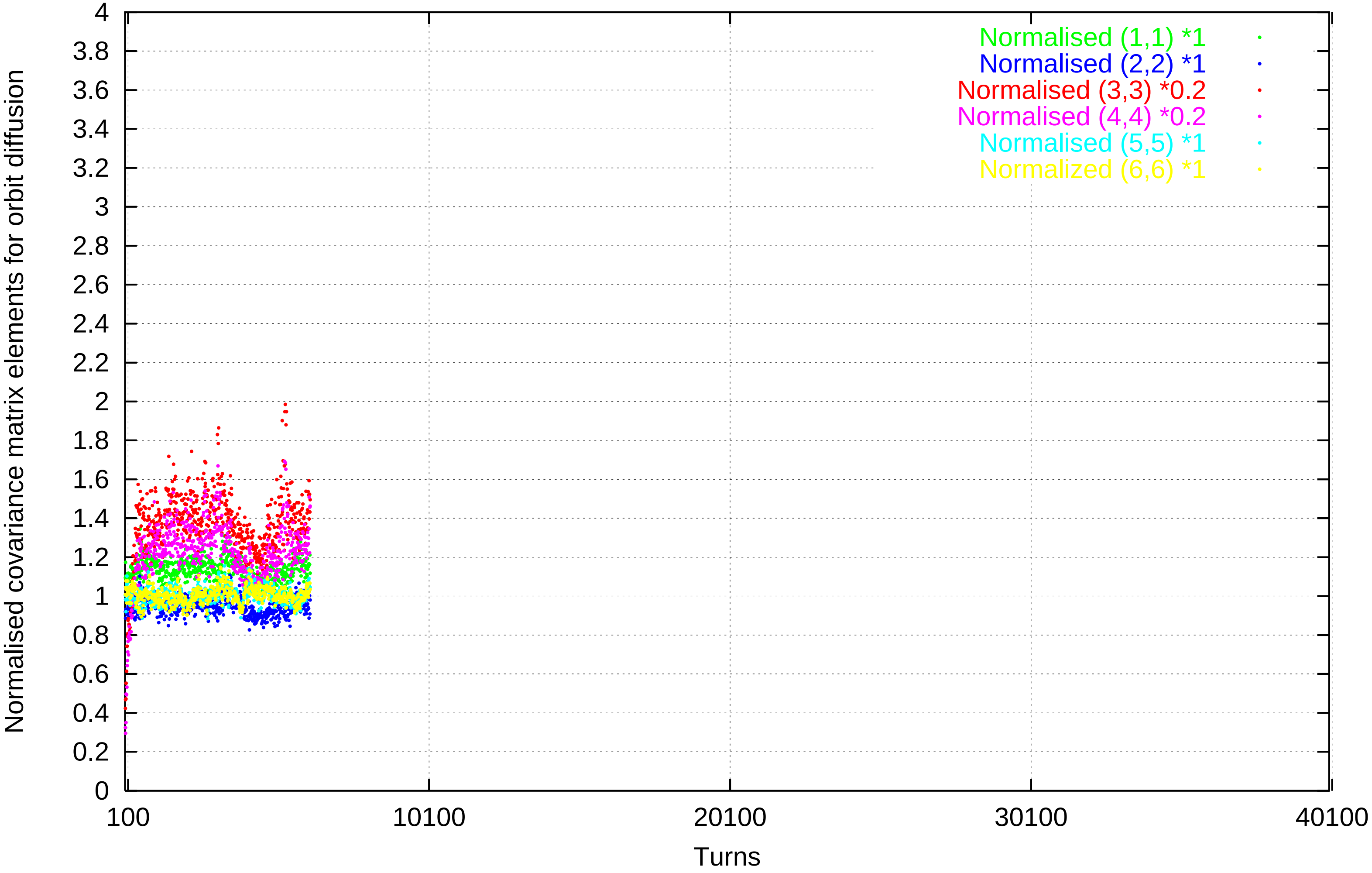
# Full 3–D spin motion



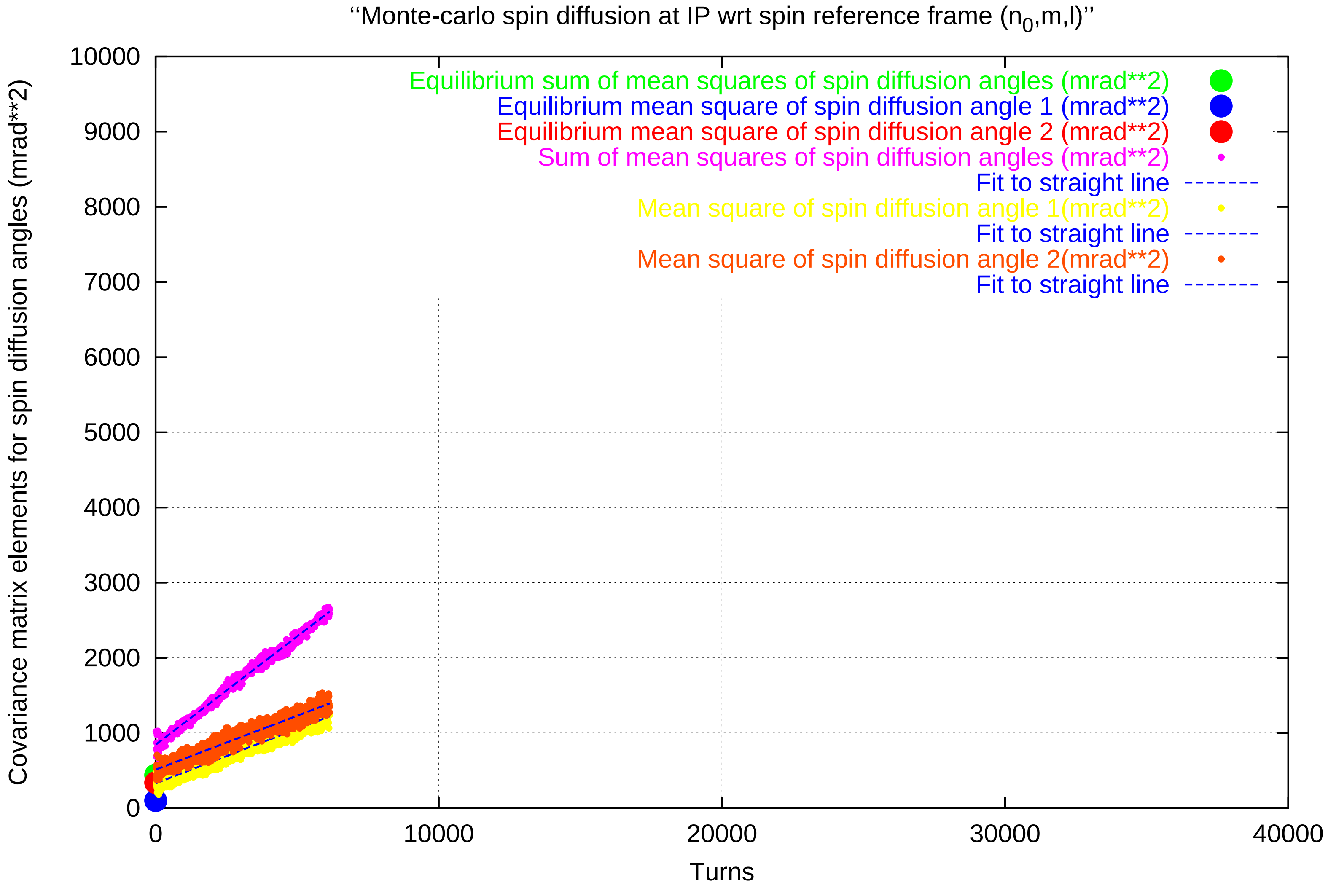
# Effect of beam-beam forces – preliminary



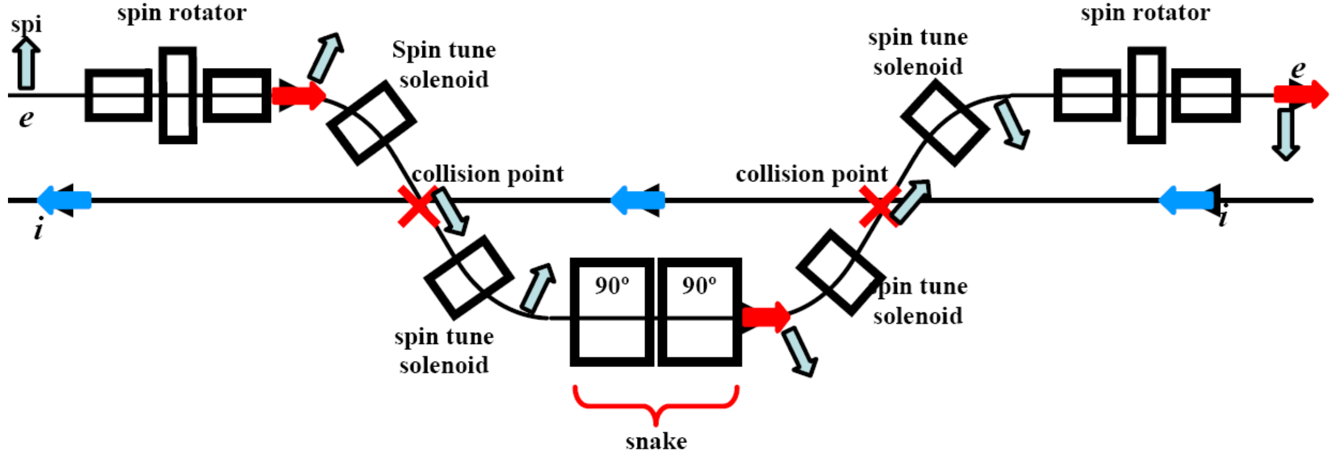
# Normalised diagonal elements of the beam covariance matrix

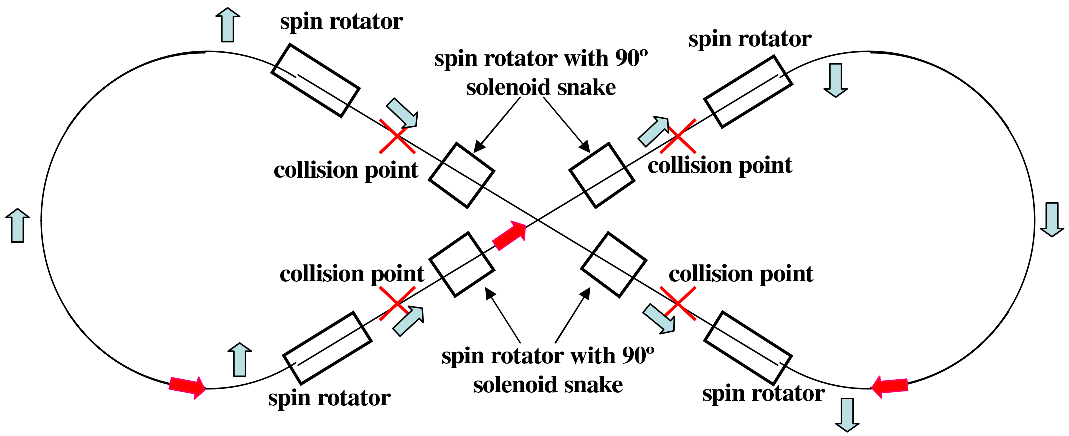


# Sum of diagonal elements of spin covariance matrix



**ELIC: latest version using stored beams**


spi
spin rotator
e
Spin tune
solenoid
collision point
i
spin tune
solenoid
90°
90°
snake
collision point
spin tune
solenoid
spin rotator
e
pin tune
solenoid
i


spin rotator
spin rotator with 90º
solenoid snake
spin rotator
collision point
collision point
collision point
collision point
spin rotator with 90º
solenoid snake
spin rotator
spin rotator

Features: $\nu_0 = 1/2$ with Siberian Snakes but the figure of 8 form ensures that the Sokolov-Ternov effect doesn't average to zero.

$$\vec{P}_{\rm bks} = -\frac{8}{5\sqrt{3}}\,\hat{n}_0\,\frac{\oint ds \frac{\hat{n}_0(s)\cdot\hat{b}(s)}{|\rho(s)|^3}}{\oint ds \frac{\left[1-\frac{2}{9}(\hat{n}_0(s)\cdot\hat{s})^2\right]}{|\rho(s)|^3}}$$

Using vertical bends to get rotation of $\hat{n}_0$ to longitudinal at the IP's.

## Comments and Questions

- The polarisation lifetime should be larger than the beam lifetime. Probably no problem at the lowest energy. Self polarisation only helps at high energy.
- If no proper spin transparency has been built in, one cannot assume *a priori* that the depolarisation rate will be low just because the design spin tune $\nu_0$ is $1/2$.

  See D.P. Barber and G. Ripken in *"Handbook of Accelerator Physics and Engineering"*, Eds: A.W. Chao and M. Tigner, World Scientific, 3rd edition (2006).

- Why not put the vertical bends into the hadron ring?

  If spin matching were needed, it would be energy dependent and that would be a real mess.

- Page 52: why not try to use the terminology for spin matching that was established in 1982 (PEAS workshop DESY), so that we can all communicate?
- Does the energy independent $\nu_0$ imply that synchrotron sidebands are weak?
- What are the implications of the crab cavities? Effects of detector fields and compensation.
- How much polarisation survives beam-beam effects (direct and indirect) ?). Very large $\xi$ .
- Wigglers to decrease the polarisation time: they will decrease $P_{\rm bk}$ too. More depolarisation?

**Things still to do:**

# Many!

- First create a (MAD) optic file for a closed ring with solenoids, skew quads, rf cavities etc.
- Then check if a decoupled optic can be created at each energy.
- Then check if it gives the expected tunes (3!) and emittances and damping times ......
- Then get a first order analytical estimate of the polarisation and depolarisation time at one energy around 9 - 10 GeV using SLICK.
- No need for crab cavities, beam-beam forces, full 3-D spin motion if the first estimates are bad.
- If things look bad, find out why: Diagnostics!!!!
- Then let's.....

# The HERA Upgrade

| | LUMINOSITY UPGRADE | | DESIGN | | 2000 (average) | |
|---|---|---|---|---|---|---|
| | e-Beam | p-Beam | e-Beam | p-Beam | e-Beam | p-Beam |
| $E$ [GeV] | 27.5 | 920 | 30 | 820 | 27.5 | 920 |
| $I$ [mA] | 58 | 140 | 58 | 160 | 45 | 95 |
| $N_{ppb}$ ($N_e$ or $N_p$) $\times 10^{10}$ | 4.0 | 10.3 | 3.6 | 10.1 | 3.1 | 7.0 |
| $N_{b,tot}$ | 189 | 180 | 210 | 210 | 189 | 180 |
| $N_{b,col}$ | 174 | 174 | 210 | 210 | 174 | 174 |
| $\epsilon_x$ [nm rad] | 20 | $\frac{5000}{\beta\gamma}$ | 48 | $\frac{6000}{\beta\gamma}$ | 41 | $\frac{5000}{\beta\gamma}$ |
| $\epsilon_z / \epsilon_x$ | 0.17 | 1 | 0.05 | 1 | 0.1 | 1 |
| $\beta_x^*$ [m] | 0.63 | 2.45 | 2.2 | 10.0 | 0.9 | 7.0 |
| $\beta_z^*$ [m] | 0.26 | 0.18 | 0.9 | 1.0 | 0.6 | 0.5 |
| $\sigma_x \times \sigma_z$ [$\mu m^2$] | $112 \times 30$ | $112 \times 30$ | $325 \times 46$ | $262 \times 83$ | $192 \times 50$ | $189 \times 50$ |
| $\sigma_s$ [mm] | 10.3 | 191 | 8.3 | 200 (85) | 11.2 | 191 |
| $\Delta\nu_x$ / IP | 0.034 | 0.0015 | 0.019 | $8 \cdot 10^{-4}$ | 0.012 | 0.0012 |
| $\Delta\nu_z$ / IP | 0.052 | $4 \cdot 10^{-4}$ | 0.024 | $6 \cdot 10^{-4}$ | 0.029 | $3 \cdot 10^{-4}$ |
| min. aperture [$\sigma_x$] | 20 | 12 | 23 | 16 | 14 | 10 |
| $\mathcal{L}_s$ [$cm^{-2}s^{-1}mA^{-2}$] | $1.8 \cdot 10^{30}$ | | $3.4 \cdot 10^{29}$ | | $7.4 \cdot 10^{29}$ | |
| $\mathcal{L}$ [$cm^{-2}s^{-1}$] | $7.5 \cdot 10^{31}$ | | $1.5 \cdot 10^{31}$ | | $1.5 \cdot 10^{31}$ | |

Table 1: