Polarisation in electron rings

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Plan

- HERA – a reminder
- Some theory and phenomenology
- eRHIC ring-ring: calculations, including beam-beam effects
- ELIC: Comments and questions
- Things still to do
HERA

The first and only $e^\pm$ ring to supply longitudinal polarisation at high energy — via the Sokolov-Ternov effect — also at 3 IP’s simultaneously!
HERA electron/positron ring 2001 ---

$\vec{P}_{\text{meas}} \parallel \hat{n}_0$

"Longitudinal" polarimeter (LPOL)

"Transverse" polarimeter (TPOL)

HERA – B

ZEUS

HI

HERMES

MiniRotator

Polarisation vertical in the arcs – to drive the Sokolov-Ternov effect
HERA MiniRotator: Buon + Steffen

56 m ("short") → no quads.

27 – 39 GeV, both helicities, variable geometry

NO INTERNAL QUADRUPOLES!
HERMES on Friday July 21 2000

Average of Longitudinal Single Bunch Polarisation [%]

- **Colliding**
- **Non Colliding**
June 2007, the Fabry-Perot-Compton polarimeter of the POL2000 Project: Calibrating polarimeters

3 pairs of rotators (so max. Sokolov-Ternov polarisation = 83 %), solenoids on, no beam-beam
THEORY and PHENOMENOLOGY
The T-BMT equation.

\[ \frac{d\vec{S}}{ds} = \vec{\Omega}(\gamma, \vec{v}, \vec{B}, \vec{E}) \times \vec{S} \]

Periodic solution \( \hat{n}_0 \) on closed orbit.

The real unit eigenvector of:

\[ R_{3 \times 3}(s + C, s)\hat{n}_0 = \hat{n}_0 \]

\( \hat{n}_0 \) is 1-turn periodic: \( \hat{n}_0(s + C) = \hat{n}_0(s) \)

\( \hat{n}_0 \): direction of measured equilibrium radiative polarization.

Closed orbit spin tune \( \nu_0 \): number of precessions per turn around \( \hat{n}_0 \) for a spin on the closed orbit. Extract from the eigenvalues of \( R_{3 \times 3}(s + C, s) \)

The value of the polarization is the same at all azimuths — time scales.
Spin motions

- Protons: largely deterministic — unless IBS.
- Electrons/positrons:
  If a photon causes a spin flip, what are the other $\approx 10^{10}$ photons doing? $\implies$>

Stochastic/damped orbital motion due to synchrotron radiation
+ inhomogeneous fields
+ spin–orbit coupling via T–BMT
$\implies$ spin diffusion i.e. depolarisation!!

Self polarisation: Balance of poln. and depoln. $\implies$

$$P_\infty \approx P_{BK} \frac{1}{1 + \left(\frac{\tau_{dep}}{\tau_{BK}}\right)^{-1}} \quad (P_{ST} \to P_{BK})$$

In any case:

$$\tau_{dep}^{-1} \propto \gamma^{2N} \tau_{st}^{-1} \quad (\text{actually a polynomial in } \gamma^{2N})$$

$\implies$ Trouble at high energy!
Spin–orbit resonances

\[ \nu_{\text{spin}} = k + k_I \nu_I + k_{II} \nu_{II} + k_{III} \nu_{III} \]

\( \nu_{\text{spin}} \): amplitude dependent spin tune \( \approx \) closed orbit spin tune = precessions /turn on CO

- Orbit “drives spins” \( \implies \) Resonant enhancement of spin diffusion.
- Resonance order: \( |k_I| + |k_{II}| + |k_{III}| \)
- First order: \( |k_I| + |k_{II}| + |k_{III}| = 1 \) \quad e.g. SLIM like formalisms.
- Strongest beyond first order:
  synchrotron sidebands of first order parent betatron or synchrotron resonances

\[ \nu_{\text{spin}} = k + k_i \nu_i + k_{III} \nu_{III}, \quad i = I, II \text{ or } III \]
More on spin–orbit resonances

(1) Linear orbit motion with linearized spin motion (SLIM/SLICK):
just first order spin–orbit resonances.

(2) Linear orbit motion with full 3–D spin motion:
all orders of spin–orbit resonances.

(3) Non–linear orbit motion with linearized spin motion:
orders of spin–orbit resonances just reflecting the
orbital spectrum.

(4) Non–linear orbit motion with full 3–D spin motion:
all orders of spin–orbit resonances.

Diagnostics: With (1) and (3) we use spin motion to Fourier analyse the orbital motion!
Linear spin matching

Skip the invariant spin field and the Derbenev-Kondratenko formula for today!

Heuristics instead!

\[ \vec{\mathcal{S}} \approx \hat{n}_0(s) + \alpha \hat{m}_0(s) + \beta \hat{l}_0(s) \]

\( \alpha, \beta \): 2 small spin tilt angles — have subtracted out the big rotations!

\[ \hat{\mathcal{M}}_{8 \times 8} = \begin{pmatrix} M_{6 \times 6} & 0_{6 \times 2} \\ \mathbf{0}_{2 \times 6} & D_{2 \times 2} \end{pmatrix} \]

acting on \( \vec{u} = (x, x', y, y', l, \delta) \) and \( \alpha, \beta \)

This is the SLIM formalism for estimating depolarisation analytically at first order (Chao 1981).

To minimize depolarization:

minimize appropriate bits of \( G_{2 \times 6} \) for appropriate stretches of ring

\[ \Rightarrow \text{ lots of independent quadrupole circuits.} \]
Spin transparency!!! – not the trivial kind to which this term is applied at certain labs.
eRHIC: ring-ring option
The Layout

- 5-10 GeV static electron ring
- recirculating linac injector
- RHIC
- e-cooling
- AGS
- LINAC
- EBIS
- BOOSTER
- recirculating linac injector
## Parameters

Table 1.1. Luminosities and main beam parameters for electron(positron)-proton collisions.

<table>
<thead>
<tr>
<th></th>
<th>High energy setup</th>
<th>Low energy setup</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( p )</td>
<td>( e )</td>
</tr>
<tr>
<td>Energy, GeV</td>
<td>250</td>
<td>10</td>
</tr>
<tr>
<td>Bunch intensity, (10^{11})</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Ion normalized emittance, (\pi , \text{mm} \cdot \text{mrad}, x/y)</td>
<td>15/15</td>
<td></td>
</tr>
<tr>
<td>rms emittance, nm, x/y</td>
<td>9.5/9.5</td>
<td>53/9.5</td>
</tr>
<tr>
<td>(\beta^*), cm, x/y</td>
<td>108/27</td>
<td>19/27</td>
</tr>
<tr>
<td>Beam-beam parameters, x/y</td>
<td>0.0065/0.003</td>
<td>0.03/0.08</td>
</tr>
<tr>
<td>(\kappa = \varepsilon_i / \varepsilon_a)</td>
<td>1</td>
<td>0.18</td>
</tr>
<tr>
<td>Luminosity, (1 \times 10^{32} \text{cm}^{-2} \text{s}^{-1})</td>
<td>4.4</td>
<td></td>
</tr>
</tbody>
</table>
The solenoid spin rotators

\[ \hat{n}_0 \text{ exactly longitudinal at just one energy.} \]
The basic eRHIC geometry for spin—exaggerated

A rotator


26.7 Tm at 10 GeV

1 2 3 2 1

26.7 Tm at 10 GeV

14.4 m
The $4 \times 4$ transfer matrix for the transverse motion through a pair of solenoids:

$$
\begin{pmatrix}
0 & -2r & 0 & 0 \\
1/2r & 0 & 0 & 0 \\
0 & 0 & 0 & 2r \\
0 & 0 & -1/2r & 0 \\
\end{pmatrix}
$$

where $r$ is the radius of orbit curvature in the longitudinal field.

Use 5 back–to–back symmetric quadrupoles.
Equilibrium polarizations with perfect alignment

- **Total Polarization**
- **S-T Polarization**

Polarization (percent)

<table>
<thead>
<tr>
<th>a</th>
<th>Polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>80</td>
</tr>
<tr>
<td>22</td>
<td>80</td>
</tr>
<tr>
<td>23</td>
<td>80</td>
</tr>
<tr>
<td>24</td>
<td>80</td>
</tr>
</tbody>
</table>
\[ \vec{P}_{\text{bks}} = - \frac{8}{5 \sqrt{3}} \hat{n}_0 \int ds \frac{\hat{n}_0(s) \cdot \hat{b}(s)}{|\rho(s)|^3} \frac{\int ds \left[ 1 - \frac{2}{3} (\hat{n}_0(s) \cdot \hat{s})^2 \right]}{|\rho(s)|^3} \]
All monitors on: just ONE example!

Equilibrium polarizations with misalignment

Total Polarization
S-T Polarization
All monitors on: DIAGNOSTICS

Equilibrium polarizations with misalignments

Total Polarization — green
S-T Polarization — blue
x Polarization — red
y Polarization — purple
20 percent monitors on: just ONE example!
Polarization times with misalignments

- B-K time
- D-K time
Spin coordinates

\[ \hat{S} = \sqrt{1 - \alpha^2 - \beta^2} \hat{n}_0 + \alpha \hat{m} + \beta \hat{l} \]

Estimating depolarisation by M-C simulation \( \alpha^2 + \beta^2 \ll 1 \)

\[ \Delta P \approx -\frac{1}{2} \Delta(<\alpha^2 + \beta^2>) = -\frac{1}{2} \Delta(\sigma^2_{\alpha} + \sigma^2_{\beta}) \quad \Rightarrow \quad \frac{dP}{dt} \approx -\frac{1}{2} = -\frac{1}{2} \frac{d}{dt}(\sigma^2_{\alpha} + \sigma^2_{\beta}) \]

Spin–orbit covariance matrix

\[
\begin{pmatrix}
\sigma^2_{x} & \sigma_{xx'} & . & . & . & . & . \\
\sigma_{xx'} & \sigma^2_{x'} & . & . & . & . & . \\
. & . & . & . & . & . & . \\
. & . & . & . & . & . & . \\
. & . & . & . & \sigma^2_{\delta} & . & . \\
- & - & - & - & - & - & - \\
. & . & . & . & . & \sigma^2_{\alpha} & \sigma_{\alpha\beta} \\
\sigma_{\beta x'} & . & . & . & . & \sigma_{\beta\alpha} & \sigma^2_{\beta} \\
\end{pmatrix}
\]
Spin–orbit maps for sections

For linearised spin motion (SLIM/SLICK):

\[
\dot{\mathbf{M}} = \begin{pmatrix}
\mathbf{M}_{6\times6} & 0_{6\times2} \\
\mathbf{G}_{2\times6} & \mathbf{D}_{2\times2}
\end{pmatrix}
\]

The \( \mathbf{G}_{2\times6} \times (x, x', y, y', \Delta l, \delta)^T \) delivers changes to the 2 small angles \( \alpha \) and \( \beta \)

For full 3–D spin motion:

\[
\ddot{\mathbf{M}} = \begin{pmatrix}
\mathbf{M}_{6\times6} & 0_{6\times3} \\
\mathbf{G}_{3\times6} & \mathbf{D}_{3\times3}
\end{pmatrix}
\]

The \( \mathbf{G}_{3\times6} \times (x, x', y, y', \Delta l, \delta)^T \) delivers rotations around \( \hat{n}_0, \hat{m}_0, \hat{l}_0 \)

The beam–beam (non–linear) kicks are applied at single points
Diagnostics!  Diagnostics!  Diagnostics!

Switch spin-orbit coupling off/on to see what does what.
Sidebands of parent first order betatron resonances: a useful approximation

\[ \tau_{dep}^{-1} \propto \frac{A}{(\nu_0 \pm Q_y)^2} \rightarrow \tau_{dep}^{-1} \propto \sum_{m_s=-\infty}^{\infty} \frac{AB(\xi; m_s)}{(\nu_0 \pm Q_y \pm m_s Q_s)^2} \]

\( A \) is an energy dependent factor

\( B(\xi; m_s) \)'s: enhancement factors, contain modified Bessel functions

\( I_{|m_s|}(\xi) \) and \( I_{|m_s|+1}(\xi) \) depending on the modulation index

\[ \xi = \left( \frac{a\gamma}{\sigma \delta} \right)^2 \]

in a flat ring.

\( \Rightarrow \) very strong effects at high energy — dominant source of trouble

Recall the limitations at LEP!!

Analogous formula for sidebands of first order synchrotron resonances.
Calibrating the (first order) M-C software structure against SLICK
Full 3-D spin motion

Equilibrium polarizations with misalignments

- Sokolov-Ternov Polarization
- Total Polarization (linear approx.)
- Total Polarization (M-C)

Polarization (percent) vs. Beam energy (GeV)
Effect of beam-beam forces – preliminary

Equilibrium polarizations with perfect alignment”

Polarization (percent)

\(\gamma\)
Normalised diagonal elements of the beam covariance matrix

- Normalised (1,1) *1
- Normalised (2,2) *1
- Normalised (3,3) *0.2
- Normalised (4,4) *0.2
- Normalised (5,5) *1
- Normalised (6,6) *1
Sum of diagonal elements of spin covariance matrix

"Monte-carlo spin diffusion at IP wrt spin reference frame (n0,m,l)"

- Equilibrium sum of mean squares of spin diffusion angles (mrad**2)
- Equilibrium mean square of spin diffusion angle 1 (mrad**2)
- Equilibrium mean square of spin diffusion angle 2 (mrad**2)
- Sum of mean squares of spin diffusion angles (mrad**2)
- Mean square of spin diffusion angle 1 (mrad**2)
- Mean square of spin diffusion angle 2 (mrad**2)
ELIC: latest version using stored beams
Features: $\nu_0 = 1/2$ with Siberian Snakes but the figure of 8 form ensures that the Sokolov-Ternov effect doesn't average to zero.

\[
R_{bks} = -\frac{8}{5\sqrt{3}} \hat{n}_0 \int ds \frac{\hat{n}_0(s) \cdot \mathbf{b}(s)}{|ho(s)|^3} \int ds \frac{1 - \frac{2}{5} (\hat{n}_0(s) \cdot \mathbf{s})^2}{|ho(s)|^3}
\]

Using vertical bends to get rotation of $\hat{n}_0$ to longitudinal at the IP’s.
Comments and Questions

- The polarisation lifetime should be larger than the beam lifetime. Probably no problem at the lowest energy. Self polarisation only helps at high energy.

- If no proper spin transparency has been built in, one cannot assume *a priori* that the depolarisation rate will be low just because the design spin tune $\nu_0$ is $1/2$.

- Why not put the vertical bends into the hadron ring?
  If spin matching were needed, it would be energy dependent and that would be a real mess.

- Page 52: why not try to use the terminology for spin matching that was established in 1982 (PEAS workshop DESY), so that we can all communicate?

- Does the energy independent $\nu_0$ imply that synchrotron sidebands are weak?

- What are the implications of the crab cavities? Effects of detector fields and compensation.

- How much polarisation survives beam-beam effects (direct and indirect) ?). Very large $\xi$.

- Wigglers to decrease the polarisation time: they will decrease $P_{bk}$ too. More depolarisation?
Things still to do:

Many!

- First create a (MAD) optic file for a closed ring with solenoids, skew quads, rf cavities etc.
- Then check if a decoupled optic can be created at each energy.
- Then check if it gives the expected tunes (3!) and emittances and damping times ......
- Then get a first order analytical estimate of the polarisation and depolarisation time at one energy around 9 - 10 GeV using SLICK.
- No need for crab cavities, beam-beam forces, full 3-D spin motion if the first estimates are bad.
- If things look bad, find out why: Diagnostics!!!!
- Then let’s.....
The HERA Upgrade

<table>
<thead>
<tr>
<th></th>
<th>LUMINOSITY UPGRADE</th>
<th>DESIGN</th>
<th>2000 (average)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>e-Beam</td>
<td>p-Beam</td>
<td>e-Beam</td>
</tr>
<tr>
<td>$E$ [GeV]</td>
<td>27.5</td>
<td>920</td>
<td>30</td>
</tr>
<tr>
<td>$I$ [mA]</td>
<td>58</td>
<td>140</td>
<td>58</td>
</tr>
<tr>
<td>$N_{ppb}$ ($N_e$ or $N_p$) $\times 10^{10}$</td>
<td>4.0</td>
<td>10.3</td>
<td>3.6</td>
</tr>
<tr>
<td>$N_{h,tot}$</td>
<td>189</td>
<td>180</td>
<td>210</td>
</tr>
<tr>
<td>$N_{h,cel}$</td>
<td>174</td>
<td>174</td>
<td>210</td>
</tr>
<tr>
<td>$\epsilon_x$ [nm rad]</td>
<td>20</td>
<td>$\frac{5000}{\beta\gamma}$</td>
<td>48</td>
</tr>
<tr>
<td>$\epsilon_z/\epsilon_x$</td>
<td>0.17</td>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>$\beta_x$ [m]</td>
<td>0.63</td>
<td>2.45</td>
<td>2.2</td>
</tr>
<tr>
<td>$\beta_z$ [m]</td>
<td>0.26</td>
<td>0.18</td>
<td>0.9</td>
</tr>
<tr>
<td>$\sigma_x \times \sigma_z$ [$\mu$m²]</td>
<td>$112 \times 30$</td>
<td>$112 \times 30$</td>
<td>$325 \times 46$</td>
</tr>
<tr>
<td>$\sigma_s$ [mm]</td>
<td>10.3</td>
<td>191</td>
<td>8.3</td>
</tr>
<tr>
<td>$\Delta \nu_x$ / IP</td>
<td>0.034</td>
<td>0.0015</td>
<td>0.019</td>
</tr>
<tr>
<td>$\Delta \nu_z$ / IP</td>
<td>0.052</td>
<td>$4 \cdot 10^{-4}$</td>
<td>0.024</td>
</tr>
<tr>
<td>min. aperture [\sigma_x]</td>
<td>20</td>
<td>12</td>
<td>23</td>
</tr>
<tr>
<td>$\mathcal{L}$ [cm$^{-2}$ s$^{-1}$ mA$^{-2}$]</td>
<td>$1.8 \cdot 10^{30}$</td>
<td>$3.4 \cdot 10^{29}$</td>
<td>$7.4 \cdot 10^{29}$</td>
</tr>
<tr>
<td>$\mathcal{L}$ [cm$^{-2}$ s$^{-1}$]</td>
<td>$7.5 \cdot 10^{31}$</td>
<td>$1.5 \cdot 10^{31}$</td>
<td>$1.5 \cdot 10^{31}$</td>
</tr>
</tbody>
</table>

Table 1: