POLARISED PROTONS IN HERA*

D.P. Barber †, G.H. Hoffstaetter ‡ , Deutsches Elektronen–Synchrotron (DESY), Hamburg, Germany
M. Vogt §, Department of Mathematics, University of New Mexico, Albuquerque, NM 87131, USA

Abstract

Since 1994 the $e^{\pm} - p$ collider HERA has been providing longitudinally polarised electron (positron) beams at 27.5 GeV to the HERMES experiment. The beams become self–polarised by the Sokolov-Ternov effect and the polarisation is made longitudinal with spin–rotators. There is now strong interest in the study of collisions of $e^{\pm}$ with high energy polarised protons but there is no convincing self-polarising mechanism for protons at high energy. Therefore protons must be polarised almost at rest in a source and then accelerated to the working energy. At HERA, if no special measures are adopted, this means that the spins must cross several thousand spin-orbit resonances, each of which could depolarise the beam. While Siberian Snakes enable all first order resonances to be avoided, higher order resonances can still be destructive. We have searched for orbital tunes that reduce the number of dangerous higher-order resonances and we have searched for a suitable choice of Siberian Snakes. Long term spin-orbit tracking simulations demonstrate that a beam would lose at least about 15% of its polarisation during the acceleration cycle of HERA up to 803 GeV. Acceleration to 920 GeV, the current energy of HERA, would require a significant reduction of the vertical emittance.

1 INTRODUCTION

Following the successful attainment of longitudinal electron and positron spin polarisation at around 27.5 GeV in HERA [1], the $e^{\pm} - p$ collider at DESY, extensive numerical and theoretical studies have been made of the feasibility of obtaining high spin polarisation in the proton beam at energies above 800 GeV [2, 3, 4, 5, 6, 7, 8]. In the meantime, polarised proton beams have recently been attained at RHIC [9] at 100 GeV but as our calculations have shown, and as we explain below, at the much higher energies of HERA maintaining the polarisation would be much more difficult than at 100 GeV. At very high energy the chief features of spin motion are best understood in terms of advanced concepts such as those of the invariant spin field and the amplitude dependent spin tune. This, in turn, requires ways to calculate these quantities numerically and, if possible, analytically.

This paper reviews these concepts and very briefly summarises the studies for HERA carried out so far. More detailed information is available in [2, 3, 4, 5, 6, 7] and references therein.

2 INVARIANT SPIN FIELD AND AMPLITUDE DEPENDENT SPIN TUNE

Particle dynamics in storage rings is described in terms of three pairs of canonical coordinates $\vec{u} = (q_1, p_1, q_2, p_2, q_3, p_3)$ which could, for example, be $(x, p_x, y, p_y, \Delta t, \Delta E)$ where $x,\ p_x,\ y,\ p_y$ describe transverse motion with respect to the curved periodic orbit and $\Delta t, \Delta E$ are the time delay relative to a synchronous particle and the deviation from the "design" energy. The independent variable is the distance along the ring s ("the azimuth"). Spin motion for protons moving in electric and magnetic fields is described by the T–BMT equation [10, 11] $d\vec{S}/ds = \vec{\Omega} \times \vec{S}$ where $\vec{S}$ is the rest frame spin expectation value of the particle ("the spin") and $\vec{\Omega}$ depends on the electric and magnetic fields, the velocity and the energy so that it depends on $\vec{u}$ and s. If the beam is stable the phase space density $\rho(\vec{u}; s)$ of a particle bunch is the same from turn to turn, i.e. $\rho(\vec{u}; s + C) = \rho(\vec{u}; s)$ where C is the circumference. If the spin distribution is also stable, the value $P(\vec{u}; s)$ and the direction $\hat{n}(\vec{u}; s)$ of the polarisation at each point in phase space are also 1–turn periodic: $P(\vec{u}; s + C) = P(\vec{u}; s)$ and $\hat{n}(\vec{u}; s + C) = \hat{n}(\vec{u}; s)$. Thus ρ and P are 1-turn periodic *scalar fields* and $\hat{n}$ is a 1–turn periodic *vector field*. Moreover, although the *"invariant spin field"* $\hat{n}$ viewed as a whole is 1–turn periodic, $\hat{n}(\vec{u}; s)$ obeys the T–BMT equation along particle orbits so that $\hat{n}(\vec{M}(\vec{u}; s); s + C) = \hat{n}(\vec{M}(\vec{u}; s); s) = R_{3\times3}(\vec{u}; s)\hat{n}(\vec{u}; s)$ where $\vec{M}(\vec{u}; s)$ is the new phase space vector after one turn starting at $\vec{u}$ and $R_{3\times3}(\vec{u}; s)$ is the corresponding spin transfer matrix. On the closed orbit $\hat{n}(\vec{u}; s)$ becomes $\hat{n}(\vec{0}; s)$ which we denote by $\hat{n}_0(s)$. This is the 1–turn periodic solution of the T-BMT equation on the closed orbit.

The attainment of high polarisation at an azimuth s requires both $P(\vec{u}; s)$ and $P_{\rm lim} = |\langle\hat{n}(\vec{u}; s)\rangle|$ to be high where $\langle\rangle$ denotes the average over phase space. The T–BMT equation shows that for motion in transverse fields, a deflection of the orbit by an angle $\Delta\theta_{\rm orb}$ in (say) the radial field of a quadrupole is accompanied by a rotation of the spin by an angle $(G\gamma + 1)\Delta\theta_{\rm orb}$ where $G \approx 1.7928$ is the gyromagnetic anomaly of the proton. At 800 GeV $(G\gamma + 1) \approx 1530$. Then at very high energy in HERA vertical betatron motion in the quadrupoles can cause the spread of $\hat{n}$ over phase space to be many tens of degrees so that methods of calculating $\hat{n}$ and minimising the spread are essential. The calculation of $\hat{n}(\vec{u}; s)$ is far from trivial beyond a first order approximation [12, 4, 5, 6, 13] and since the first order approximation is inadequate at high energy *non–perturbative* means such as stroboscopic aver-

* Contributed to the 8th European Particle Accelerator Conf., EPAC-2002, Paris, France, June 2002.
† mpybar@mail.desy.de
‡ From Oct. 2002 at Cornell University, Georg.Hoffstaetter@desy.de
§ From July 2002 at DESY,vogtm@mail.desy.de

aging [13], the SODOM algorithm [14] or adiabatic anti-damping [4] must be used. All three methods are incorporated in the computer code SPRINT [5, 6].

Another key quantity, which is also calculated in SPRINT, is the spin tune. This is the number of spin precessions around $\hat{n}$ for one turn around the ring. This *"amplitude dependent spin tune"* $\nu(\vec{J})$ can only depend of the amplitudes $\vec{J}$ of the synchrobetatron motion and not on the phases of the particle motion. On orbital resonance $\nu(\vec{J})$ may not exist. Spins are particularly strongly perturbed when the spin motion is coherent with the orbital motion, i.e. when the spin precession rate is near resonance with the orbital tunes: $\nu(\vec{J}) = k_0 + k_1 Q_1 + k_2 Q_2 + k_3 Q_3$ where the Q's are the tunes of the orbital modes and the k's are integers. Resonances with $|k_1| + |k_2| + |k_3| = 1$ are called first order resonances. Note that resonances are primarily due to the non–commutative nature of spin rotations so that they occur even with perfectly linear orbital motion. Close to spin–orbit resonance $P_{\rm lim}$ can be especially small and methods are therefore needed to avoid resonance or to keep $P_{\rm lim}$ large in other ways. On the closed orbit $\nu(\vec{J})$ reduces to $\nu(\vec{0})$ which we write as ν_0. In a ring with misalignments $\hat{n}_0$ is strongly tilted away from its nominal direction at integer resonances $\nu_0 = k_0$.

More background material and complete details on these concepts can be found in [2, 4, 5, 6, 7, 13].

3 SIBERIAN SNAKES

The first step in setting up a ring for polarisation is to ensure that $\hat{n}_0$ is vertical in the arcs so that spins are less sensitive to vertical fields in quadrupoles experienced during motion on horizontal betatron and dispersion trajectories, thereby suppressing horizontal betatron and synchrotron resonances. But spin motion on vertical betatron orbits must still be dealt with to suppress first order vertical betatron resonances and higher order resonances. In a perfectly aligned ring with no solenoids or vertical bends, $\nu_0 = G\gamma$ and it increases by one unit for every 523 MeV. Thus during acceleration several thousand spin–orbit resonances must be crossed and polarisation can be lost at many of these. The solution is to install pairs of Siberian Snakes. These are magnet systems designed to rotate spins through an angle π around an axis (the snake axis) in the machine plane independently of the position in phase space and the beam energy. With a suitable choice of the number of snakes, their positions and their axes, ν_0 can be fixed at $1/2$ independently of energy in a perfectly aligned ring. Then with a proper choice of orbital tunes, first order spin–orbit resonances can be avoided and the reduction of $P_{\rm lim}$ and polarisation loss during acceleration can be reduced. The local effect of orbital motion on spin (through $\Omega(\vec{u}; s)$) increases with the orbital amplitudes. Then, in general, global quantities like the spread of $\vec{n}$ increase with the orbital amplitudes. That this is not always the case can be seen in [5, 6, 7].

In HERA, where the proton ring has interleaved vertical and horizontal bends on each side of three straight sections, extra "flattening snakes" [15, 8, 6] are needed solely to ameliorate the effects of the vertical bends and ensure that $\hat{n}_0$ is vertical in the arcs.

4 FILTERING AND SNAKE MATCHING

Contrary to popular belief it is not the case that the polarisation is always improved by increasing the number of pairs of snakes. In a ring such as HERA with only approximate 4–fold symmetry, snake layouts and orbital tunes must be chosen carefully. The first step at HERA is to employ a "filtering" algorithm [4] in which $\hat{n}$ is calculated in linear approximation for snake layouts with 4 or 8 snakes. In each case a large number of snake angle combinations which give $\nu_0 = 1/2$ and vertical $\hat{n}_0$ is chosen and the layout is selected which maximises $P_{\rm lim}$, averaged over a set of *fixed energies* in some energy range, and for purely vertical betatron motion with a realistic amplitude. By this means one can find layouts which are much superior to naive "obvious" choices [5, 6] and one easily discovers schemes with 8 snakes which perform worse than schemes with 4 snakes. Since an increase of $P_{\rm lim}$ implies a reduction of overall spin–orbit coupling it is hoped that with a sufficiently large $P_{\rm lim}$ depolarisation during acceleration is reduced too. A chosen layout is then studied in more detail with all three modes of orbital motion.

The primary disturbance to spins comes from vertical betatron motion and at first order the disturbance can be written in terms of 4 one turn integrals involving the phase of spin precession and the betatron motion [5]. While filtering is an automated but rather unsystematic way of searching for good snake schemes, one can try to systematically choose betatron and spin phase advances between sections of the ring to reduce the size of the 4 integrals. This procedure is called "snake matching" [5]. It can be shown that for a 4–fold symmetric ring with 4 main snakes, snake matching can be achieved for all energies even if the geometrical and optical symmetry has been somewhat broken [5]. With 8 main snakes, snake matching can be achieved even without changing the betatron phase advance between different sections of the ring. Calculations of $P_{\rm lim}$ have shown that snake matching with 8 snakes allows a 16–fold increase in the useable vertical emittance compared to that with an uneducated choice of snake angles.

5 SELECTION OF ORBITAL TUNES

While the use of snakes together with a proper choice of orbital tunes allows first order resonances to be avoided, the orbital tunes should be chosen so that dangerous higher order spin–orbit resonances are avoided too. This is where a correct understanding and definition of spin tune becomes important since the spin tune depends on the orbital amplitudes. So before orbital tunes are chosen a survey of the variation of spin tune over the phase space should be made [6, 7].

6 RESULTS OF SIMULATIONS

Figure 1 shows the results of a simulation of acceleration from 40 GeV to 820 GeV for a perfectly aligned HERA with an optimised choice of 4 main snakes (figure 2) obtained by filtering and for typical energy dependent HERA optics with optimal orbital tunes. Since, owing to lack of space, it would be difficult to install 8 snakes in HERA, results for that case are not reported here. The protons move at fixed amplitudes of 2σ in all three planes. The invariant emittances are 4π mm.mrad for transverse motion and $1.8\ 10^{-2}\pi$ m.rad longitudinally. The polarisation shows a step–like fall at a strong residual resonance structure at about 803 GeV. An estimate for the achievable polarisation based on averaging over all amplitudes shows that up to 803 GeV, between 62 and 85% of the injected polarisation could be maintained. More polarisation could be maintained by a significant decrease in the vertical emittance.

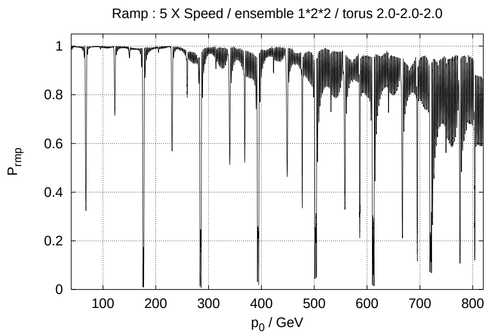

Figure 1: The polarisation during acceleration to 803 GeV for 2σ in all 3 phase space amplitudes for the snake layout of figure 2. From [6].

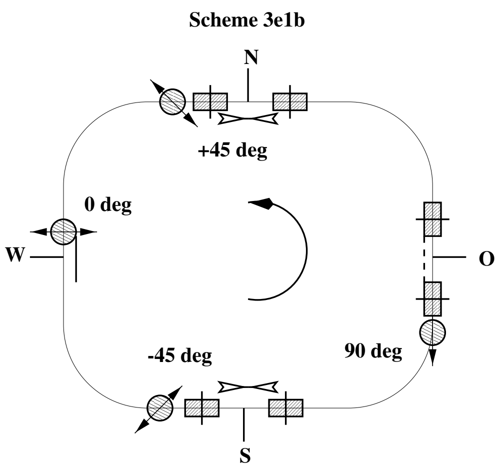

Figure 2: The 4 snake layout used for figure 1.

7 FURTHER WORK

The study described here deals with a perfectly aligned ring. Thus further studies are needed: (i) the effect of closed orbit distortions to determine the tolerable limits on such distortions and to develop orbit correction schemes. Simulation with distortions shows, so far, that maintaining polarisation during acceleration would be difficult beyond about 300 GeV [16]. (ii) further tracking simulations with "snake matched" schemes [5], (iii) the effect of beam–beam forces, other nonlinear effects and various sources of noise should be studied, (iv) studies of proton spin motion in DESY III and in PETRA, including a proposed electron cooler section [17], (v) feasibility study for high field (6 Tesla to save space) snakes and rotators, (iv) design study for practical rotator layouts [18].

Obtaining highly polarised proton beams at high energy in HERA would be a difficult undertaking but the success at RHIC gives grounds for optimism.

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