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Spin Decoherence in Electron Storage Rings *

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Abstract

A simple model of spin decoherence in electron storage rings is presented and its relevance to rf spin flipping at high energy is discussed.

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1 Prologue

The original version of this paper was based on a talk at the 11th International Symposium on High Energy Spin Physics, Bloomington, Indiana, USA, September 1994 and it can be found in the proceedings, edited by Kenneth J. Heller and Sandra L. Smith, and published as AIP Conf.Proc. 343 (1995). The proceedings (and the paper) are also available at the CERN Document Server.

Since then, a formula has been corrected and more details have been added but now, following important numerical demonstrations and the appearance of a relevant important paper, a section called Developments has been added by D.P. Barber to cover them. Necessary extra citations have been added too.

2 Introduction

Stored electron(positron) beams can become spin polarised by the emission of synchrotron polarisation—-the so-called Sokolov-Ternov effect [1, 2, 3]. In rings without vertical bends and solenoids, the polarisation is vertical, antiparallel(parallel) to the guide field. It has recently been demonstrated at HERA that spin rotators can be used to rotate the polarisation vector into the beam direction just before an interaction point and back again after the interaction point so that longitudinally polarised electrons or positrons are available for the high energy physics experiment [4].

Periodic reversal of the helicity is essential for the physics programme and it is clear that it would also be useful to have a means of flipping the polarisation direction for short periods in order to check for systematic errors. The helicity at the interaction point can be reversed by changing the geometry and fields of the rotators but that would mean a temporary loss of polarisation or even dumping the beam and refilling. However, a faster, more convenient method was already considered many years ago [5, 6] and would utilize a horizontal radio-frequency (rf) magnetic field.

The rf magnetic field (or a combination of fields forming a closed bump) [5, 7] would be installed at a position on the ring where the polarisation were vertical and it would run in resonance with the natural spin precession frequency i.e. at a frequency close to $f_{flip} = f_c \cdot \tilde{\nu}_0$ or $f_{flip} = f_c \cdot (1 - \tilde{\nu}_0)$ where f_c is the circulation frequency and $\tilde{\nu}_0$ is the fractional part of the closed-orbit spin tune, ν_0, which is the number of spin precessions per turn around the ring on the closed orbit [8, 9, 10]. In calculations, $\tilde{\nu}_0$ is extracted from the complex eigenvalues of the one-turn 3 x 3 spin-transport matrix on the closed orbit [8, 9, 10].

Flipping would involve sweeping slowly enough across resonance to ensure that the polarisation vector were tipped over adiabatically. This would require that the spins in a bunch remain tightly bundled. Such flipping techniques are routine at the Budker Institute for Nuclear Physics (BINP) [11, 12] at low energy. These techniques are very closely related to the method used to depolarise a beam and hence measure its energy by noting the required rf frequency [13, 14, 15, 16, 17].

It has been suggested that if flipping were repeated at the suitable intervals it would perhaps be possible to reach a periodic limit cycle for the polarisation [18, 19, 20].

However, if the projections of the spins on the horizontal plane were to become spread out uniformly over the range $\pm\pi$ (in an appropriate coordinate system) during the sweep process, i.e. if there were complete decoherence, the polarisation vector would not be flipped but instead the polarisation would vanish. As we will see, one such source of decoherence is the stochastic nature of synchrotron-radiation photon emission. In proton rings, decoherence of this nature cannot occur and full spin flip is not difficult to achieve [21].

Spin flip was sometimes observed at LEP during energy calibrations [15] using rf fields of just a few gauss-metres but the value of the polarisation was much reduced and the effect was not consistently reproducible. It is also unclear which are the best ranges of sweep rate and rf field strength [22].

But the fact that flip *can* be achieved suggests that the spin projections remain coherent at least for several seconds during the sweep. Thus in order to better understand the measurements it would be useful to estimate the decoherence rate. One such calculation suggests that the characteristic decoherence time is proportional to the fourth power of the synchrotron tune and could indeed be several minutes at LEP [23].

In this article we show, by a more complete treatment of the photon emission process and the subsequent development of the spin distribution function, that with the same linear "smooth ring" model for the synchrotron motion as in [23], the spin distribution actually reaches *equilibrium* in a few damping times and that there need not be full decoherence. We then consider other sources of decoherence and their consequences.

The calculation presented below is a very abbreviated version of a full treatment based on a well defined and trusted formalism. The full calculation together with other material has been published elsewhere as detailed in the section on Developments.

3 Equations of linearized orbital motion

The linearized equation of orbit motion with respect to the closed orbit in the presence of stochastic excitation and damping due to synchrotron radiation takes the form used in the SLIM program [8, 9, 10]:

$$\frac{d}{ds}\,\vec{y} \;=\; \underline{\hat{A}}\cdot\vec{y}+\delta\underline{\hat{A}}\cdot\vec{y}+\delta\vec{\hat{c}}\,, \tag{1}$$

where s is the distance around the ring and $\vec{\hat{y}}$ is the vector of orbit variables $(\hat{x},\hat{p}_x,\hat{z},\hat{p}_z,\hat{\sigma},\hat{p}_\sigma)$. Here, $\hat{\sigma}$ is the distance to the centre of the bunch and $\hat{p}_\sigma$ is the fractional energy deviation. $\underline{\hat{A}}$ represents the "Hamiltonian" motion due to the Lorentz forces and $\delta\underline{\hat{A}}$ describes damping. Both are s–dependent 6 x 6 matrices. The vector $\delta\vec{\hat{c}}=(0,0,0,0,0,\delta c)$ accounts for the stochastic excitation in the energy variable due to photon emission [24]:

$$\delta c \;=\; \sqrt{\omega}\cdot\xi(s)\,, \tag{2}$$

where, in terms of the curvatures K_x and K_z, $\omega=(|K_x|^3+|K_z|^3)\cdot C$ with

$$C \;=\; \frac{55}{24\sqrt{3}}r_e\lambda\!\!\!^{-}_e\gamma_0^5 \tag{3}$$

where γ_0 is the Lorentz factor on the design orbit for the chosen energy, where r_e is the classical electron radius, where $\lambda\!\!\!^{-}_e$ is the reduced Compton wavelength of the electron and where the stochastic averages of the kicks $\xi(s)$ are

$$<\xi(s)\cdot\xi(s')> \;=\; \delta(s-s')\;;\quad <\xi(s)>\;=\;0\;. \tag{4}$$

Thus, as is usual and sufficient [24, 25, 26], we take the synchrotron radiation to be a white noise process. For our current purpose it will be more convenient to work with dynamical variables which allow a clearer separation of the influence of energy oscillations from the purely "betatron" motion due to the quadrupoles. To achieve this we introduce the dispersion by means of a canonical transformation to obtain a new set of variables $\vec{y}\equiv(x,p_x,z,p_z,\sigma,p_\sigma)$ defined by:

$$x \;=\; \hat{x}-\;\hat{p}_\sigma\cdot D_x\;; \tag{5}$$
$$p_x \;=\; \hat{p}_x-\;\hat{p}_\sigma\cdot D'_x\;; \tag{6}$$
$$z \;=\; \hat{z}-\;\hat{p}_\sigma\cdot D_z\;; \tag{7}$$
$$p_z \;=\; \hat{p}_z-\;\hat{p}_\sigma\cdot D'_z\;, \tag{8}$$
$$\sigma \;=\; \hat{\sigma}-\hat{p}_x\cdot D_x+\hat{x}\cdot D'_x-\hat{p}_z\cdot D_z+\hat{z}\cdot D'_z\;; \tag{9}$$
$$p_\sigma \;=\; \hat{p}_\sigma \tag{10}$$

where the $D's$ are the components of the dispersion vector [27].

In terms of the variables $x,\ p_x,\ z,\ p_z,\ \sigma,\ p_\sigma$ the equation of motion now takes the form:

$$\frac{d}{ds}\,\vec{y} \;=\; \underline{A}\cdot\vec{y}+\delta\underline{A}\cdot\vec{y}+\delta\vec{c}\,. \tag{11}$$

If the dispersion is zero at the position of the rf cavities, there is no transverse–longitudinal coupling and the matrices $\underline{A}$ and $\delta\underline{A}$ have a simple block diagonal form. For example:

$$\underline{A}(s)=\left(\begin{array}{cc} \underline{A}^{(\beta)}_{(4\times4)}(s) & \underline{0}_{(4\times2)} \\ \underline{0}_{(2\times4)} & \underline{A}^{(\sigma)}_{(2\times2)}(s) \end{array}\right). \tag{12}$$

The matrix $\underline{A}^{(\beta)}_{(4\times4)}(s)$ describes betatron motion in the focussing fields. $\underline{A}^{(\sigma)}_{(2\times2)}(s)$ describes the synchrotron motion. When acting alone this gives :

$$\frac{d}{ds}\,\sigma \;=\; -[K_x\cdot D_x+K_z\cdot D_z]\cdot p_\sigma\ ; \tag{13}$$

$$\frac{d}{ds}\,p_\sigma \;=\; h\cdot\frac{2\pi}{L}\cdot\frac{eV(s)}{E_0}\cos\varphi\cdot\sigma\,, \tag{14}$$

where the symbols have their usual meanings.

In this calculation we also work in the "smooth ring" approximation and consider only synchrotron motion. Thus we will follow exactly the philosophy of [23]. So the matrix elements in $\underline{A}^{(\sigma)}_{(2\times2)}$ and $\delta\underline{A}^{(\sigma)}_{(2\times2)}$ are averaged over one turn (of length L) and we obtain :

$$\left(\begin{array}{c}\sigma' \\ p'_\sigma\end{array}\right) \;=\; \left(\begin{array}{cc} 0 & -\kappa \\ \Omega_s^2/\kappa & 0\end{array}\right)\cdot\left(\begin{array}{c}\sigma \\ p_\sigma\end{array}\right)+\delta\underline{A}^{(\sigma)}_{(2\times2)}\cdot\left(\begin{array}{c}\sigma \\ p_\sigma\end{array}\right)+\delta\vec{c}\,, \tag{15}$$

where $\delta\underline{A}^{(\sigma)}_{(2\times2)}$ and $\delta\vec{c}$ take the forms:

$$\delta\underline{A}^{(\sigma)}_{(2\times2)}\equiv\left(\begin{array}{cc} 0 & 0 \\ 0 & -2\cdot\alpha_s/L\end{array}\right)\,,\qquad \delta\vec{c}\equiv\sqrt{\tilde{\omega}}\cdot\left(\begin{array}{c} 0 \\ \xi(s)\end{array}\right)\,. \tag{16}$$

Here, α_s is the one turn synchrotron damping decrement and $\tilde{\omega}$ is the one-turn averaged ω. Also, $\Omega_s = 2\pi\cdot Q_s/L$ where Q_s is the synchrotron tune and κ is the compaction factor.

The equilibrium covariance matrix for σ and p_σ then takes the usual value [28], namely :

$$\underline{\sigma}_2(\infty)=\left(\begin{array}{cc}\sigma_\sigma^2 & 0 \\ 0 & \sigma^2_{p_\sigma}\end{array}\right)\,,\qquad \sigma^2_{p_\sigma}=\frac{\tilde{\omega}\cdot L}{4\cdot\alpha_s}\,,\qquad \sigma^2_\sigma=\frac{\kappa^2}{\Omega_s^2}\cdot\sigma^2_{p_\sigma}\,. \tag{17}$$

4 Inclusion of spin

After this recapitulation of the basis for the matrix formulation of the standard smoothed description of damped stochastic synchrotron motion we are in a position to introduce spin motion. Although spin is a quantum mechanical phenomenon, spin motion in high energy storage rings can be treated at the semiclassical level using the Thomas-BMT equation [8, 9, 10]

$$\frac{d}{ds}\,\vec{S} \;=\; \vec{\Omega}(\vec{y};s,\gamma_0)\times\vec{S}\,, \tag{18}$$

describing the precession of a single-particle spin-expectation value $\vec{S}$ ("the spin") in electric and magnetic fields. The precession vector $\vec{\Omega}(\vec{y};s,\gamma_0)$ is a function of the magnetic and electric fields and of the particle velocity and energy. As is usual for spin calculations in storage rings we now write $\vec{\Omega}$ as a sum of a piece $\vec{\Omega}_0(s,\gamma_0)$ accounting for the fields on the closed orbit and a piece $\vec{\Omega}_{osc}(\vec{y};s,\gamma_0)$ accounting for synchro-betatron motion with respect to the closed orbit.

We will assume that the ring has no vertical bends, solenoids or skew quadrupoles, and that it is perfectly aligned so that there is no vertical closed orbit deviation. For electrons the vertical emittance can then be taken to be zero and only motion in the horizontal plane need be considered but in accordance with our picture of a smoothed ring, we also take the closed orbit to be the design orbit. For this naive estimate the betatron motion and the radial rf magnetic field will be ignored. Spin motion will be calculated with respect to a pair of mutually orthogonal axes precessing at the rate $|\vec{\Omega}_0|$ in the horizontal plane around the vertical dipole field. The direction of a horizontal spin in this frame is denoted by a phase angle ψ so that we have $\psi' = \Omega_{osc}(\vec{y};s,\gamma_0)$. After averaging we then obtain $\psi' = 2\pi\nu_0/L\cdot p_\sigma$ where ν_0 is the design-orbit spin tune, namely $(g-2)/2\cdot\gamma_0$. Thus ψ only couples to and is only driven by p_σ. When the spin phase ψ is included, the stochastic differential equation for the system takes the form :

$$\begin{pmatrix}\sigma'\\ p_\sigma{}'\\ \psi'\end{pmatrix} = \underbrace{\begin{pmatrix}0&a&0\\ b&0&0\\ 0&d&0\end{pmatrix}\cdot\begin{pmatrix}\sigma\\ p_\sigma\\ \psi\end{pmatrix}}_{\text{Hamiltonian motion}} + \underbrace{\begin{pmatrix}0&0&0\\ 0&c&0\\ 0&0&0\end{pmatrix}\cdot\begin{pmatrix}\sigma\\ p_\sigma\\ \psi\end{pmatrix}}_{\text{Damping}} + \underbrace{\sqrt{\tilde{\omega}}\cdot\begin{pmatrix}0\\ \xi\\ 0\end{pmatrix}}_{\text{Excitation}}, \tag{19}$$

where the constants a, b, c and d are defined as :

$$a = -\kappa\;,\, b = \Omega_s^2/\kappa\;,\, c = -2\cdot\alpha_s/L\;,\, d = 2\pi\nu_0/L\,. \tag{20}$$

This can be rewritten in the form :

$$\vec{x}' \;=\; \underline{\mathcal{A}} \cdot \vec{x} + \delta\vec{c}_3 \;, \tag{21}$$

where

$$\vec{x} \equiv \begin{pmatrix} \sigma \\ p_\sigma \\ \psi \end{pmatrix} \;, \qquad \underline{\mathcal{A}} \equiv \begin{pmatrix} 0 & a & 0 \\ b & c & 0 \\ 0 & d & 0 \end{pmatrix} \;, \qquad \delta\vec{c}_3 \equiv \sqrt{\tilde{\omega}} \cdot \begin{pmatrix} 0 \\ \xi \\ 0 \end{pmatrix} \;. \tag{22}$$

This linear Langevin equation is interpreted according to the Stratonovich convention and leads to the following Fokker-Planck equation [29, 30] for the distribution function $W(\sigma, p_\sigma, \psi)$:

$$\frac{\partial W}{\partial s} \;=\; -\sum_{j=1}^{3} \frac{\partial}{\partial x_j}[\mathcal{D}_j \cdot W] + \sum_{i,j=1}^{3} \frac{\partial^2}{\partial x_i \partial x_j}[\mathcal{D}_{ij} \cdot W] \;, \tag{23}$$

where

$$\mathcal{D}_j \equiv \sum_{k=1}^{3} \mathcal{A}_{jk} \cdot x_k \;, \qquad \mathcal{D}_{ij} \equiv \frac{\tilde{\omega}}{2} \cdot \delta_{ij} \cdot \delta_{i2} \qquad (i,j = 1,2,3) \;. \tag{24}$$

So the Fokker-Planck equation has the final form :

$$\begin{aligned} \frac{\partial W}{\partial s} \;=\; & -c \cdot W - a \cdot p_\sigma \cdot \frac{\partial W}{\partial \sigma} - [b \cdot \sigma + c \cdot p_\sigma] \cdot \frac{\partial W}{\partial p_\sigma} - d \cdot p_\sigma \cdot \frac{\partial W}{\partial \psi} \\ & + \frac{\tilde{\omega}}{2} \cdot \frac{\partial^2 W}{\partial {p_\sigma}^2} \;. \end{aligned} \tag{25}$$

With such Fokker-Planck formulations for this and more complicated models we can carry out perfectly standard detailed studies of spin decoherence under all possible conditions just by looking for the possible solutions for $W(\sigma, p_\sigma, \psi)$ compatible with the initial conditions. In our model, by starting with a delta function distribution in σ, p_σ and ψ, corresponding to a pointlike beam and a tight bundle of spin projections, the distribution function (i.e. the transition probability in this case) evolves so that the covariance matrix for the σ, p_σ and ψ is given by [29, 30] :

$$\underline{\sigma}_3(s) \;=\; 2 \cdot \int_0^s ds' \; \underline{M}(s') \cdot \underline{\mathcal{D}} \cdot \underline{M}^T(s') \;, \tag{26}$$

where M is the real valued transfer matrix solving :

$$\underline{M}' = \underline{\mathcal{A}} \cdot \underline{M} \;, \qquad \underline{M}(s=0) = \underline{1} \;. \tag{27}$$

After some initial damped oscillatory behaviour, in a few synchrotron damping times the elements of $\underline{\sigma}_3$ reach the asymptotic values :

$$\underline{\sigma}_3(\infty) = \begin{pmatrix} \sigma_\sigma^2 & 0 & \frac{d}{a}\cdot\sigma_\sigma^2 \\ 0 & \sigma_{p_\sigma}^2 & 0 \\ \frac{d}{a}\cdot\sigma_\sigma^2 & 0 & \frac{d^2}{a^2}\cdot\sigma_\sigma^2 \end{pmatrix} . \tag{28}$$

This result follows *exactly* from the stochastic differential equation 19. Thus the σ and p_σ distributions acquire the equilibrium spreads given earlier. This is to be expected since in these approximations the spin has no influence on the orbital motion. However, and this is perhaps unexpected, equation 28 shows that the distribution of ψ *also* reaches equilibrium (on the same time scale) with

$$\sigma_\psi^\infty \equiv \sigma_\psi(\infty) = |\frac{d}{a}|\cdot\sigma_\sigma = \nu_0\sigma_{p_\sigma}/Q_s \tag{29}$$

for the asymptotic variance of ψ which we call the *decoherence index*. Thus apart from an initial decoherence lasting a few dampimg times there is no continual decoherence in this model with these starting conditions! But of course, if $\sigma_\psi(\infty)$ were very large the spins *would be effectively* decoherent.

In the HERA electron ring at 27.5 GeV, ν_0 is about 62.5, σ_{p_σ} is about 10^{-3} and Q_s is about 0.06. So the asymptotic σ_ψ is about 60 degrees. So far, we have allowed the azimuthal angle ψ to cover the range $\pm\infty$ whereas the physical range is $\pm\pi$. To account for this we calculate the corresponding asymptotic polarisation, $P(\infty)$, for the Gaussian distribution of ψ as

$$P(\infty) = \frac{1}{\sqrt{2\pi}\sigma_\psi^\infty}\int_{-\infty}^{+\infty} e^{-\psi^2/2(\sigma_\psi^\infty)^2}\cos\psi\, d\psi = e^{-(\sigma_\psi^\infty)^2/2} \tag{30}$$

which is about 58%.

However, several extra points should be noted. Firstly, ψ is correlated to σ, not as one might have expected, to the energy deviation p_σ. Secondly, the last column of $\underline{\mathcal{A}}$ is empty and the $\underline{\sigma}_3(s)$ is singular for all s. For a linear problem such as this, one expects that the asymptotic W is always a generalized Gaussian distribution in σ, p_σ and ψ [33]. But, the coefficients of the quadratic form in the exponent of this Gaussian clearly cannot be obtained by inverting $\underline{\sigma}_3(s)$. So another method suitable for problems of this type must be used [33]. Then one finds that the asymptotic W function is not unique but reaches an equilibrium form depending on the initial conditions. For example to discuss decoherence according to the picture in the Introduction, one begins with Gaussian distributions in σ and p_σ with their equilibrium asymptotic variances and with a delta function distribution $\delta(\psi)$ in ψ. Then the asymptotic ψ distribution has a variance of

$$\sqrt{2}(\nu_0\sigma_{p_\sigma}/Q_s) = \sqrt{2}\sigma_\psi^\infty \tag{31}$$

which is about 85 degrees corresponding to an asymptotic polarisation, $P(\infty)$, in this case of about 33%. This calculation and further aspects of the problem will be treated in another article [33]. The asymptotic variances of σ and p_σ are unique. We note, with interest, that $\sigma_\psi(\infty)$ is equal to the well-known, so-called, *spin-tune modulation index* appearing as a scale factor for the strengths of synchrotron-sideband resonances in equilibrium electron(positron) polarisation, as shown, for example, in [31, 32].[1]

According to our model, in machines running at one or two GeV, the asymptotic σ_ψ is just a few degrees. So within this simple linear model there is no complete decoherence in such machines. Conventional wisdom suggests instead that σ_ψ should increase as $\sqrt{s}$. This is not the case as we have just seen. However, in the simpler 2 x 2 pure diffusion problem for p_σ and ψ without synchrotron oscillations the $\sqrt{s}$ growth does emerge after a few damping times and for HERA quickly results in complete decoherence. A similar conclusion emerges at the beginning of [19] before the effects of an rf field are included. So the synchrotron motion is an essential ingredient in our calculation.

So far we have neglected the detailed structure of the ring and misalignments which tilt the equilibrium polarisation axis and generate vertical dispersion. Horizontal and vertical betatron motion have been neglected as have the effects of sextupoles and spin rotators as well as any nonlinear dependence of the spin precession rate on orbital variables. So our model is perhaps too simple to represent a realistic storage ring but it has enabled us to reconsider the calculation in [23]. If a Siberian Snake is included in the otherwise smoothed ring there *is* decoherence [33].

By considering decoherence in isolation, a key component, the rf field itself, was ignored. Recall, however, that partial spin flip *was* sometimes seen at LEP with small rf fields. This suggests that the predictions of the model are not too unrealistic. For a treatment involving an rf field see [19].

5 Developments

There have been at least three important developments following the original paper.

Firstly, the paper [33] foreseen earlier came to fruition. This covers the model presented here, and other models, in great mathematical detail and it is therefore an important source.

Then, with the need for a damping ring for the efficient injection of longitudinally polarised positrons into the positron linear accelerator of the proposed International Linear Collider (ILC) [34], Monte-Carlo spin-orbit track-

[1] Note that the term "spin chromaticity" in the title of [31] is a misnomer. The quantity $\gamma\partial\mathbf{n}/\partial\gamma$ is now usually, and correctly, called the "spin-orbit coupling function".

ing simulations for the so-called OCS6 design [35] with SLICKTRACK [10] for full linearized synchro-betatron motion and large initial transverse beam size, showed almost exactly the level of decoherence for horizantal spin components predicted by our model. This shows that even with the complex ring layout involving asymmetric wigglers, the reservations above about a possible lack of relevance for such a simple model are unfounded. It also shows that one cannot assume that an initial horizontal polarisation component will necessarily damp away before extraction and injection of the beam into the linear accelerator via a spin rotator. This provides a nice illustration of the importance of our simple, apparently naive model.

The third development concerns studies by Monte-Carlo spin-orbit tracking of the effect of rf fields on the polarisation in LEP and the proposed FCC-ee rings [36]. Thus we return to the motivation for this paper. In particular, simulations of the motion of horizontal spins with the Xsuite software [37] at 45.6 GeV show almost perfect agreement with the level of decoherence, and its dependence on Q_s, predicted by our model [38] even though the model appears to be too simple.

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